

Optimal Capacity Decisions in a Developing Fishery

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Abstract The problem of estimating optimal fishing capacity for a developing fishery is discussed, using the methods of Bayesian decision analysis. The results obtained indicate that quite good decisions can often be made on the basis of limited prior information as to fish stock productivity, particularly if a conservative approach allowing for subsequent increases in capacity is employed.

Introduction

Although many of the world's marine fishery resources are already fully developed, there do remain some areas and stocks—located particularly in the coastal zones of less developed countries and in the southern hemisphere—where further development can be expected. In this paper we consider the case of a completely unfished (or at most, only lightly fished) stock which has been chosen for development on the basis of preliminary stock surveys and economic forecasts. The question of interest to us is, given the limited evidence as to stock abundance and productivity, how can decisions be made regarding the appropriate level of investment in fishing capacity?

A commonly used rule of thumb is the " $\frac{1}{2}MB_0$ rule," as follows (Gulland 1971; Shepherd 1981): let B_0 represent the biomass of the unexploited stock, as estimated from a stock survey, and let M denote the estimated natural annual mortality rate of the stock (assumed to consist of a single species). Then $Y = \frac{1}{2}MB_0$ is taken as an estimate of maximum sustainable annual yield (MSY), and capacity decisions are made on the basis of this value.

A simple rule of this sort may be defended on the grounds that any more complex and sophisticated decision method would be inappropriate for the type of situation in which the rule is normally employed. But fish stocks, especially those of certain schooling species, are notoriously subject to overfishing, particularly when the level of fishing capacity built up turns out to exceed the long-term sustainable yield from the resource. Controlling the catch at levels well below capacity is difficult, and in many circumstances, impossible—at least until stocks become so depressed that large catches can no longer be taken at all. In view of the seriousness and frequency of such occurrences, it would seem advisable to subject the current approach to a more rigorous analysis.

In this direction, Beddington and Cooke (1983) have addressed the question of whether in fact $\frac{1}{2}MB_0$ is generally a reasonable approximation to MSY. Using the standard deterministic cohort model (Beverton and Holt, 1957), Beddington and Cooke have

compared $\frac{1}{2}MB_0$ and MSY over a wide range of model parameters. The general conclusion of this work is that, assuming that a single λMB_0 rule of thumb is to be used, the value $\lambda = \frac{1}{2}$ is probably biased on the high side.

Beddington and Cooke (1983) also use Monte Carlo simulation methods to extend their analysis to the case of stochastic variations in recruitment. Such variations have two implications: first, the spawning stock may fluctuate to an unacceptably low level not predicted by a deterministic model, and second, survey estimates of biological parameters, such as B_0 , may be in error. The simulations lead to the conclusion that, for high levels of recruitment variation, it may be necessary to adjust allowable catch levels downward in order to ensure protection for the spawning stock.

In this paper we study the problem of optimal fishing capacity in a developing fishery. We shall take the viewpoint of Bayesian decision theory, a methodology that explicitly takes account of uncertainty in decision making. Beginning with a "prior" probability distribution for annual recruitment, obtained from survey data, we show how this limited information can be employed in the formulation of capacity decisions.

Bayesian methods have been applied to fishery management problems in a number of recent papers, notably Mendelsohn (1980), Ludwig and Walters (1981), and Charles (1983c). The capacity question has been considered, but in a different setting, by Charles (1983c).

In the next section we set down our basic cohort model with stochastic recruitment. In order to keep the analysis tractable, we ignore all sources of uncertainty except that associated with the long-run average of recruitment. However, our methods are clearly adaptable to deal with other sources of uncertainty, although treating several sources simultaneously can lead to computationally infeasible models. After setting up the general decision framework, we consider a greatly simplified (noncohort) version of our model, as a pedagogical exercise. In this version, the stock-recruitment function is suppressed, and most of the computations can be carried out analytically. The following section reintroduces stock recruitment and requires time-consuming

calculations. Finally, we return to the cohort model and report on numerical results.

The General Model

Let $\mathbf{N} = [N_t(1), \dots, N_t(A_m)]$ denote the population vector in year t . Explicitly, $N_t(i)$ represents the number of fish of age i in the population at the beginning of year t . The maximum possible age is denoted by A_m .

In modeling the relationship between fishing capacity and the annual catch of fish, we assume first that existing capacity is always fully utilized, in the sense that no effective controls exist on either fishing effort or total catch. This assumption appears realistic for the case of developing countries with limited management and enforcement superstructure.

We also assume a direct relationship between fishing capacity and fishing effort E . The relationship between effort and fishing mortality F then depends upon the concentration profile (Clark 1982) of the given fish stock. We shall here consider two alternatives: (1) a linear relationship between E and F (Schaefer type, or Clark's Type II) and (2) a constant-concentration relationship (Clark's Type IV) in which total annual catch is proportional to E , and hence to capacity. The latter model is thought to be appropriate for certain pelagic schooling species (Saville 1980).

For the Type II model we have

$$N_{t+1}(k+1) = \begin{cases} e^{-M}N_t(k) & \text{for } 1 \leq k < A_f \\ e^{-M-F}N_t(k) & \text{for } A_f \leq k < A_m \end{cases} \quad (1)$$

where A_f denotes the age of first capture, and F is total annual fishing mortality. For simplicity we assume here that F is the same for all fishable age classes. The total annual catch by weight from the k th age class is given by

$$C_t(k) = \frac{F}{M+F} [1 - e^{-(M+F)}] N_t(k)w(k) \quad (2)$$

where $w(k)$ represents the average weight of a fish of age k . Intra-seasonal changes in weight are ignored in this formulation; inter-seasonal changes are modeled in von Bertalanffy form:

$$w(k) = w_{\infty}\{1 - \exp[-K(k - k_0)]\}^3 \quad (3)$$

where the symbols w_{∞} , K , and k_0 have the usual interpretation (Beverton and Holt 1957). (We treat age k as an integer variable, in order to simplify the calculation. By appropriate selection of k_0 in equation 3, k can be taken to represent midyear age, so that the error introduced by this simplification is minimal.)

Total annual catch, in the Type II model, is given by

$$\begin{aligned} C_t^{\text{TOT}} &= \sum_{k=A_f}^{A_m} C_t(k) \\ &= \frac{F}{M + F} [1 - e^{-(M+F)}] B_f(N_t) \end{aligned} \quad (4)$$

where $B_f(N_t)$ denotes the fishable biomass:

$$B_f(N_t) = \sum_{k=A_f}^{A_m} N_t(k)w(k) \quad (5)$$

In the Schaefer form of the model, annual fishing mortality F will be assumed to be directly proportional to fishing capacity, that is, the number of (standardized) fishing units. (As noted above, we assume that capacity is always fully utilized.)

In the case of the Type IV (constant concentration) model, equations 1 are modified to

$$N_{t+1}(k + 1) = e^{-M}[N_t(k) - C_t^{\#}(k)] \quad k \geq 1 \quad (6)$$

where $C_t^{\#}(k)$ denotes the catch in numbers from $N_t(k)$. Assuming that the total annual catch C_t^{TOT} is distributed over fishable age classes in proportion to their numbers, we have

$$C_t^\#(k) = \frac{C_t^{\text{TOT}} N_t(k)}{B_f(\mathbf{N}_t)} \quad A_f \leq k \leq A_m \quad (7)$$

and $C_t^\#(k) = 0$ for $k < A_f$.

Next, the recruitment component of our model is

$$N_{t+1}(1) = R_t \phi[B_b(\mathbf{N}_t)] \quad (8)$$

where R_t is a random variable with mean \bar{R} and variance σ_R^2 , and where the function ϕ is a stock-recruitment function which acts multiplicatively on recruitment; $B_b(\mathbf{N}_t)$ denotes the breeding stock:

$$B_b(\mathbf{N}_t) = \sum_{k=A_b}^{A_m} N_t'(k) w(k) \quad (9)$$

where A_b = age of first breeding and $N_t'(k)$ denotes escapement.

In our numerical simulations we will assume that recruitment R_t has a lognormal distribution, so that $X_t = \log R_t$ is normal $N(\mu, \sigma^2)$. We have

$$\bar{R} = e^{\mu + \sigma^2/2}, \quad \sigma_R^2 = \bar{R}^2(e^{\sigma^2} - 1) \quad (10)$$

or inversely,

$$\sigma^2 = \log(1 + \sigma_R^2/\bar{R}^2), \quad \mu = \log \bar{R} - \sigma^2/2 \quad (10a)$$

Note that the variance σ^2 of log recruitment is a function of the coefficient of variation (σ_R/\bar{R}) of recruitment. The lognormal distribution accords well with data from many fish stocks (Hennemuth et al. 1980).

The difficulties of estimating stock-recruitment relationships from data are well known (Parrish 1973; Ludwig and Walters 1981; Shepherd 1982). In this study (with one exception described later) we adopt the following ad hoc formulation of this relationship:

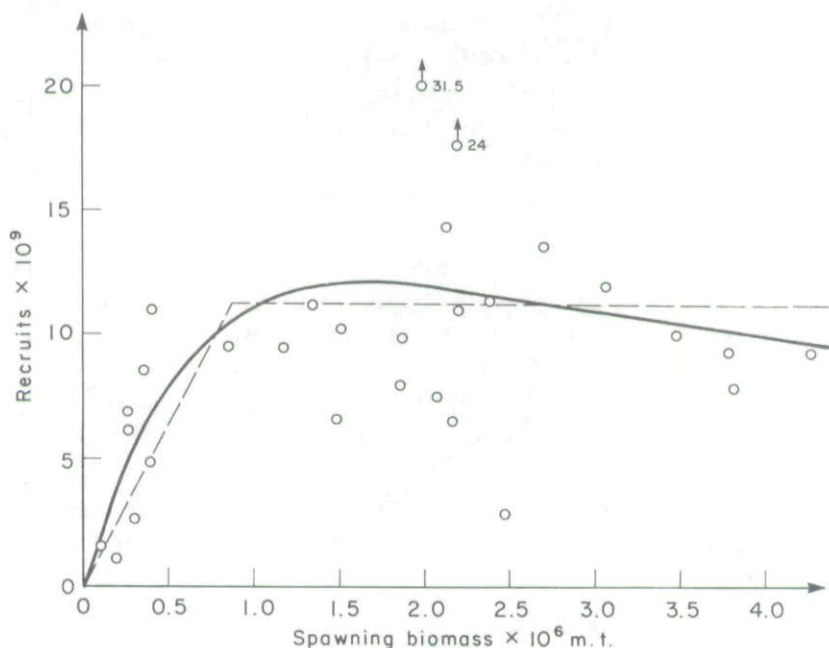


FIGURE 1. Spawning biomass vs. recruits for North Sea herring, with curve derived from Shepherd's (1982) model, and linear-threshold model.

$$\phi(B) = \begin{cases} 1 & \text{for } B \geq B_{\theta} \\ B/B_{\theta} & \text{for } B < B_{\theta} \end{cases} \quad (11)$$

where B_{θ} denotes a threshold biomass level. Typically, choosing $B_{\theta} \approx 0.2\bar{B}$ (where \bar{B} denotes long-run average unfished breeding biomass) provides as good a fit to data as any rigorously estimated parametric form (Figure 1). At any rate, since the estimation of stock-recruitment relations requires data that would not generally be available in an undeveloped fishery, some such ad hoc formulation is a necessity here.

The values of the mean \bar{R} and variance σ_R^2 of recruitment, which are estimated from a finite time series of recruitment data, necessarily involve uncertainty. In the present study we adopt the Bayesian approach, in which such uncertainty is captured using prior probability distributions. Our analysis will be greatly

simplified if we treat the mean μ of log recruitment as the only uncertain parameter. In principle, Bayesian methods could also be used to handle uncertainty in other parameters, but as is well known, a dynamic Bayesian analysis involving several uncertain parameters simultaneously becomes computationally difficult.

An appropriate value for the variance σ^2 of log recruitment can either be taken from comparative studies (Beddington and Cooke 1983), or else the sample variance $\hat{\sigma}_0^2$ can be used. Beddington and Cooke's study indicates that the coefficient of variation remains fairly constant over different populations of fish belonging to related groups.

Because of the nonlinearity of the stock-recruitment relation (equation 11) there is in general no simple formula relating long-term unfished biomass \bar{B} to the average recruitment \bar{R} of equation 10. However, if the probability that the natural biomass B_t falls below the threshold B_θ is negligible, we obtain the (approximate) formula

$$\bar{B} = \bar{R} \sum_{k=1}^{A_m} w(k) e^{-(k-1)M}$$

With σ^2 fixed, the appropriate prior distribution π_0 for the mean μ is given by (de Groot 1970)

$$\pi_0 \sim N(\hat{\mu}_0, \sigma^2/N_0) \quad (12)$$

where

$$\hat{\mu}_0 = \sum_1^{N_0} (\log R_{-i})/N_0$$

and N_0 = number of data points. Here we assume that the historical recruitment data R_{-i} are themselves error-free (see Ludwig and Walters 1981 for methods of handling errors in data), and also that the breeding stock levels $B(N_t)$ that gave rise to the recruitment data all exceeded the threshold B_θ . This latter assumption seems reasonable for a newly developed fishery;

otherwise, any recruitment data resulting from subthreshold breeding stock can simply be rejected in performing the estimation of μ .

Next, an optimization objective must be specified. We assume in this study that existing capacity is always fully utilized. Thus our model does not allow for annual catch quotas, fishery closures, and other such forms of management. (Gear restrictions affecting the age of first capture are allowed in the model, by altering A_f , but we do not discuss this possibility.) The assumption of unrestricted utilization of available capacity seems appropriate for fisheries in underdeveloped countries with limited management infrastructure. (See Charles 1983c for studies of optimal capacity when annual catches are controllable.)

Our optimization objective will be the expected net return, that is, the expected discounted present value of future catches, net the capital cost of capacity. The price of fish (per kilogram) p , is assumed constant. We also assume a constant cost γ per unit of fishing capacity; since capacity is always fully utilized, this single cost term can be assumed to cover both variable and amortized fixed costs.

In the Schaefer form of the model, fishing mortality F is taken to be proportional to fishing capacity. The optimization objective can therefore be expressed as

$$J(F) = E \left\{ p \sum_{t=1}^H \alpha^t C_t^{\text{TOT}} \right\} - \gamma F \quad (13)$$

where H is the time horizon, and α denotes the discount factor ($0 < \alpha \leq 1$). The expectation $E\{ \dots \}$ in equation 13 is actually a double expectation, first with respect to the stochastic recruitment process R_t , conditioned on μ , and second with respect to the prior distribution π_0 . The model thus encompasses both fluctuations in recruitment and uncertainty as to the long-run average of recruitment.

In equation 13 we are assuming that the capacity decision (i.e., F) is a *single* decision made at the outset of the planning period H . This assumption will be relaxed later.

For the alternative "schooling" model, it is assumed that total annual catch C_t^{TOT} is proportional to capacity. The optimization objective now becomes

$$J^S(C^{\text{TOT}}) = E \left\{ p \sum_{t=1}^N \alpha^t C^{\text{TOT}} \right\} - \gamma^S C^{\text{TOT}} \quad (14)$$

where N is either the given time horizon H or the year in which the fishery collapses because of depletion of the stock—a possibility that has to be accounted for in the schooling model. We will define "collapse" to mean that the breeding biomass $B_b(N_t)$ falls below some specified threshold level B' (in the extreme case, $B' = 0$).

Determination of the level of optimal capacity F (or C^{TOT}) for the above models is straightforward; numerical examples using Monte Carlo simulation are reported below. Before presenting these results, however, we wish to discuss a second form of optimization objective.

The initial stock survey consists of a sample drawn from an underlying probability distribution for annual recruitment. If the sample is small, the resulting estimate of average recruitment will involve significant uncertainty. It may therefore prove advisable not to treat the initial capacity decision as final. If subsequent observations indicate that average recruitment is larger than first estimated, capacity can later be increased accordingly. However, if the initial capacity turns out to be too large, the costs of excess capacity cannot be recovered. Moreover, since capacity is always fully utilized, the fish stock may become depleted if capacity turns out to be excessive, leading to further economic losses. A conservative approach may then be appropriate for the initial capacity decision, especially when long-run prospects are uncertain.

Specifically, we shall suppose that a second decision regarding capacity is to be made after N_1 years, at which time the estimate of μ is "updated" according to the N_1 new observations of recruitment. Additional capacity can then be purchased, but ex-

isting capacity cannot be reduced—that is, capacity is “non-malleable” (Clark et al. 1979; Charles 1983a, 1983b).

Let $J_1(F_0, \hat{\mu}')$ denote the maximum expected net return for the post-updating period, given an existing capacity F_0 , and an updated estimate $\hat{\mu}'$ for the mean log recruitment. Thus

$$J_1(F_0, \hat{\mu}') = \max_{F_1 \geq 0} \left\{ E \left[p \sum_{t=1}^{H_1} \alpha^t C_t^{\text{TOT}}(F_0 + F_1) \right] - \gamma F_1 \right\} \quad (15)$$

where $C_t^{\text{TOT}}(F_0 + F_1)$ is given by equation 4 with F replaced by $F_0 + F_1$ (for the Schaefer version). Here H_1 denotes the post-updating time horizon. The expectation in equation 15 is with respect to future recruitment, and also with respect to the updated (“posterior”) distribution $\pi' \sim N[\hat{\mu}', \sigma^2/(N_0 + N_1)]$.

The overall optimization problem is then characterized by

$$\max_{F_0 \geq 0} E \left[p \sum_{t=1}^{N_1} \alpha^t C_t^{\text{TOT}}(F_0) - \gamma F_0 + \alpha^{N_1} J_1(F_0, \hat{\mu}') \right] \quad (16)$$

The expectation again has a multiple scope, with an outer expectation over μ and an inner expectation over the recruitment sequence; in particular, $\hat{\mu}'$ is a random variable determined by the recruitments R_1, \dots, R_{N_1} . With R_1, \dots, R_{N_1} given, and $X_i = \log R_i$, we have

$$\begin{aligned} \hat{\mu}' &= \frac{1}{N_0 + N_1} (X_1^0 + \dots + X_{N_0}^0 + X_1 + \dots + X_{N_1}) \\ &= a\hat{\mu}_0 + b\bar{X} \end{aligned} \quad (17)$$

where $X_1^0, \dots, X_{N_0}^0$ are the original (predecision) log recruitments,

$$\bar{X} = \frac{1}{N_1} \sum_{i=1}^{N_1} X_i$$

and where

$$a = \frac{N_0}{N_0 + N_1} \quad \text{and} \quad b = \frac{N_1}{N_0 + N_1}$$

Thus, given μ , the updated estimate $\hat{\mu}'$ is normally distributed $N(a\hat{\mu}_0 + b\mu, \sigma_1^2)$ where $\sigma_1^2 = N_1\sigma^2/(N_0 + N_1)^2$. (Note that all recruitments are tacitly assumed to exceed the critical level B_θ .)

In general it does not seem possible to obtain analytic solutions to the optimization models proposed above. Later we will describe a Monte Carlo simulation technique that can be used to obtain approximate solutions. In the next section we show that an analytic solution can be obtained if a number of further assumptions are adopted.

Before proceeding to the simplified model, however, we wish to point out that the optimization problem in equation 16 refers to a *given* decision maker, working on the basis of the survey data available at the time. This decision maker is forced to assume that the sample mean $\hat{\mu}_0$ also represents the mean of the prior distribution for the "true" (unknown) mean of log recruitment. Since in general $\mu \neq \hat{\mu}_0$, the initial capacity decision F_0^* will almost always appear to have been "wrong," when viewed on the basis of subsequent information. As noted above, a "wrong" capacity decision on the high side is likely to have more serious consequences than one on the low side.

A Simplified Model

Let us now assume, for the purpose of illustration, that there are no overlapping age classes, and also that annual recruitment is independent of past stock size. Specifically, annual recruitment is a sequence of independent random variables, having identical lognormal distributions with an uncertain mean but a "known" variance. A model of this kind appears appropriate for tropical penaeid shrimps (Clark and Kirkwood 1979) and should also be appropriate for the case of migratory species passing through the fishing zone of a given coastal state. In both

these examples, recruitment can be expected to be highly variable and unpredictable.

In addition, we ignore natural mortality during the fishing season, that is, $M = 0$. This assumption permits a completely analytic solution to our problem; if $M \neq 0$, the solution involves a (single) transcendental equation which must be solved numerically.

Let recruitment R_t now be specified in terms of biomass, rather than numbers of fish. The lognormal distribution for R_t implies that $\log R_t \sim N(\mu, \sigma^2)$. However, from the point of view of the decision maker, μ is uncertain, with $\mu \sim N(\hat{\mu}', \sigma'^2)$ being the subjective probability distribution. Hence the prior estimate of log recruitment becomes $\log R_t \sim N(\hat{\mu}', \sigma^2 + \sigma'^2)$ where σ'^2 is the variance of the estimate $\hat{\mu}'$.

The post-updating return function is given by

$$\begin{aligned} J_1(F_0, \hat{\mu}') &= \max_{F_1 \geq 0} \left[E \left\{ p \sum_{t=1}^{H_1} \alpha^t R_t [1 - e^{-(F_0 + F_1)}] \right\} - \gamma F_1 \right] \\ &= \max_{F_1 \geq 0} \left\{ p \left[\sum_1^{H_1} \alpha^t \right] e^{\hat{\mu}' + \sigma^2/2 + \sigma'^2/2} [1 - e^{-(F_0 + F_1)}] - \gamma F_1 \right\} \end{aligned} \quad (19)$$

where the harvest is $H_t = R_t [1 - e^{-(F_0 + F_1)}]$, and we have used the fact that

$$\begin{aligned} E(R_t) &= E_{\pi'} [E(R_t | \mu)] \\ &= E_{\pi'} (e^{\mu + \sigma^2/2}) = e^{\hat{\mu}' + \sigma^2/2 + \sigma'^2/2} \end{aligned}$$

Here $\hat{\mu}'$ is the mean, and $\sigma'^2 = \sigma^2/(N_0 + N_1)$ is the variance of the posterior distribution π' .

Introduce the notation

$$A(N) = p \sum_{t=1}^N \alpha^t = p\alpha \frac{1 - \alpha^N}{1 - \alpha} \quad (20)$$

and

$$\beta = \log [A(H_1)/\gamma] \quad (21)$$

It follows from equation 19 that the optimal additional capacity F_1 (for given initial capacity F_0) is

$$F_1^* = \begin{cases} \hat{\mu}' + \sigma^2/2 + \sigma'^2/2 + \beta - F_0 & \text{if this is positive} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

In other words, new investment is warranted only if the current estimate of log recruitment $\hat{\mu}'$ is large compared with current capacity F_0 . The optimal new capacity $F_1 = F^*$ can now be inserted in equation 19:

$$J_1(F_0, \hat{\mu}') = A(H)e^{\hat{\mu}' + \sigma^2/2 + \sigma'^2/2} [1 - e^{-(F_0 + F^*)}] - \gamma F^*$$

The overall optimization problem becomes

$$J_0 = \max_{F_0 \geq 0} \{A(N_1)e^{\hat{\mu}_0 + \sigma^2/2 + \sigma_0^2/2} (1 - e^{-F_0}) - \gamma F_0 + \alpha^{N_1} E_{\pi_0} E_{\hat{\mu}} [J_1(F_0, \hat{\mu}')]\} \quad (23)$$

where $\sigma_0^2 = \sigma^2/N_0$. Since both expectations here involve normal distributions, the double expectation can be reduced to a single integral:

$$E_{\pi_0} E_{\hat{\mu}} [J_1(F_0, \hat{\mu}')] = \int_{-\infty}^{\infty} J_1(F_0, a\hat{\mu}_0 + b\zeta) n(\zeta; \hat{\mu}_0, \sigma_2) d\zeta \quad (24)$$

where $a = N_0/(N_0 + N_1)$, $b = 1 - a$, $\sigma_2^2 = \sigma^2(1/N_0 + 1/N_1)$, and

$$n(\zeta; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp [-(\zeta - \mu)^2/2\sigma^2] \quad (25)$$

is the normal density.

Upon substituting the expressions for J_1 into equation 24 and rearranging, we can express the result in terms of the cumulative normal distribution function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$$

We obtain

$$\begin{aligned} E_{\pi_0} E_{\hat{\mu}}(J_1) &= A(H) \exp [\hat{\mu}_0 + \frac{1}{2}(\sigma^2 + \sigma'^2 + \sigma_2^2)] \\ &\quad - \frac{b\gamma\sigma_2}{\sqrt{2\pi}} \exp [-(w - \hat{\mu}_0)/2\sigma_2^2] \\ &\quad - \frac{1}{2} A(H) \exp [\hat{\mu}_0 - F_0 + \frac{1}{2}(\sigma^2 + \sigma'^2 + b^2\sigma_2^2)] \\ &\quad \times (1 + 2\Phi\{[w - (\hat{\mu}_0 + b\sigma_2^2)]/\sigma_2\}) \\ &\quad + \frac{\gamma}{2} [F_0 - \hat{\mu}_0 - 1 - \frac{1}{2}(\sigma^2 + \sigma'^2) - \beta] \\ &\quad \times \{1 - 2\Phi[(w - \hat{\mu}_0)/\sigma_2]\} \end{aligned} \quad (26)$$

where

$$w = \frac{1}{b} [F_0 - a\hat{\mu}_0 - \frac{1}{2}(\sigma^2 + \sigma'^2) - \beta] \quad (27)$$

In spite of its lengthy appearance, equation 26 takes only a few milliseconds of CPU time to calculate numerically when coded in FORTRAN. Thus the final maximization over F_0 in equation 23 can be performed efficiently and accurately by means of a simple search routine.

As an illustration we consider the following data, pertaining to a simplified version of the model described by Clark and Kirkwood (1979) for the banana prawn (*Penaeus merguensis*) fishery of Northern Australia:

$$\begin{aligned} R_1 &= 1.5 \times 10^7 \text{ kg}, \quad R_2 = 3.2 \times 10^6 \text{ kg}, \quad R_3 = 5.5 \times 10^6 \text{ kg} \\ \hat{\mu}_0 &= 15.67, \quad \sigma = 0.58, \quad N_0 = 3 \\ p &= \$1/\text{kg}, \quad \gamma = \$2.2 \times 10^7 \end{aligned}$$

For these values, and with $N_1 = 3$, $H = 20$, and $\alpha = 0.91$, we obtain

$$F_0^* = 1.05$$

This can be compared with (1) the value F_{no}^* if no subsequent updating is used, and (2) the value F_{det}^* derived from a deterministic model with $R = \bar{R}$, the apparent average recruitment. Thus F_{det}^* is obtained as the solution of

$$\max_{F \geq 0} [A(H)\bar{R}(1 - e^{-F}) - \gamma F] \quad (28)$$

whereas F_{no}^* (which allows for prior uncertainty) is the solution of

$$\begin{aligned} \max_{F \geq 0} E_{\pi_0} [A(H)e^{\mu + \sigma^2/2}(1 - e^{-F}) - \gamma F] \\ = \max_{F \geq 0} [A(H)e^{\hat{\mu}_0 + \sigma^2/2 + \sigma^2/2N_0}(1 - e^{-F}) - \gamma F] \end{aligned} \quad (29)$$

We obtain (since $\bar{R} = e^{\hat{\mu}_0 + \sigma^2/2}$)

$$F_{\text{det}}^* = \beta + \hat{\mu}_0 + \frac{\sigma^2}{2} \quad (30)$$

$$F_{\text{no}}^* = F_{\text{det}}^* + \frac{\sigma^2}{2N_0} \quad (31)$$

For our parameter values,

$$F_{\text{det}}^* = 1.08, \quad F_{\text{no}}^* = 1.13$$

We see that $F_0^* < F_{\text{det}}^* < F_{\text{no}}^*$, but that the differences are relatively small; the differences in the return functions $J(F)$ are even smaller, by about an order of magnitude. This suggests that, for these parameter values (and for this model), it is not worthwhile to take uncertainty into account in the capacity decision. It is clear from equations 30 and 31 why this is so: the "noise" terms $\sigma^2/2$ and $\sigma^2/2N_0$ are small ($\sigma^2/2 = 0.17$) relative to the values of F^* themselves. The prawn fishery in question is a highly profitable fishery, which experiences unusually high an-

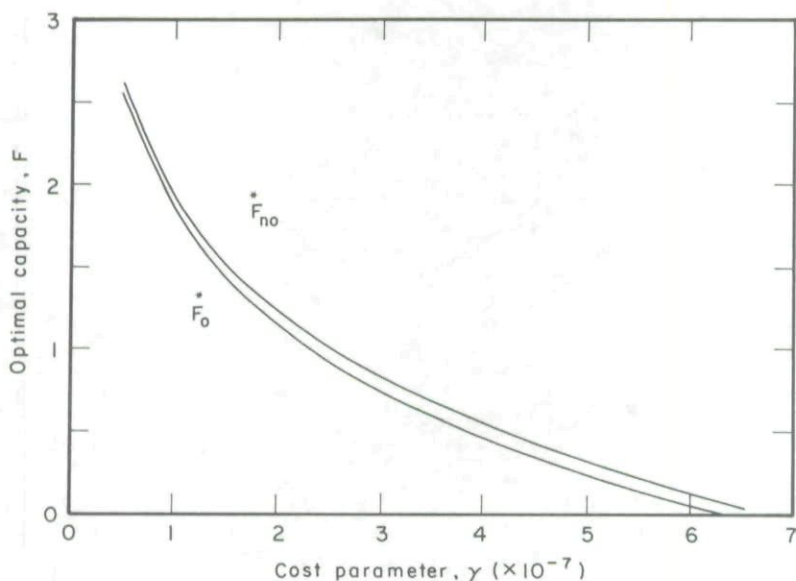


FIGURE 2. Dependence of optimal capacity on cost parameter: base case parameter values $\hat{\mu}_0 = 15.67$, $\sigma = 0.58$, $N_0 = N_1 = 3$.

nual exploitation rates, often exceeding 70% ($F = 1.20$). Under such circumstances, uncertainty about long-run average recruitment can have relatively little influence. (This might not be the case if high fishing mortality leads to reductions in recruitment, but the simplified model of this section ignores that possibility.)

The dependence of F_0^* and F_{no}^* on the cost parameter γ is illustrated in Figure 2. As cost increases, the optimal capacity levels decline. The *relative* importance of updating increases, although the *absolute* difference $F_{no}^* - F_0^*$ remains constant as γ is varied. (The latter is a computed result for which we have obtained no analytic derivation.)

The absolute difference between the computed capacity levels F_0^* and F_{no}^* depends critically on the value of N_0 and is obviously quite sensitive to N_0 when N_0 is small. Thus, for $N_0 = 3$, although the given recruitment levels show considerable variability, the (Bayesian) uncertainty in μ is already fairly small, and additional data cannot be expected to change the estimate much.

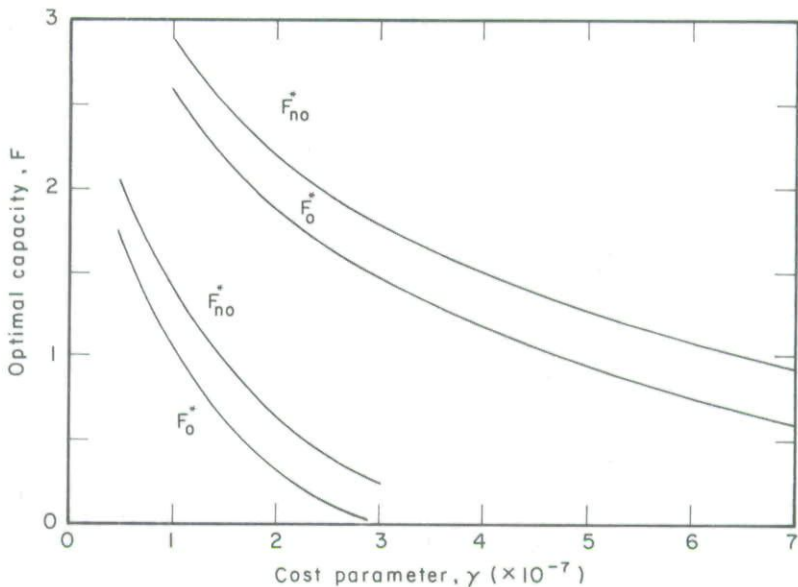


FIGURE 3. Dependence of optimal capacity on cost parameter: parameter values $N_0 = N_1 = 1$, $\sigma = 0.58$, $\hat{\mu}_0 = 16.52$ (upper pair of curves), $\hat{\mu}_0 = 14.98$ (lower pair of curves).

As a comparison, consider the possibility of making a capacity decision on the basis of a *single* data point. The results for $N_0 = 1$ are shown in Figure 3. Here the upper pair of curves correspond to the largest of the observed recruitments, $R = 1.5 \times 10^7$ kg, and the lower to the smallest, $R = 3.2 \times 10^6$ kg. As expected, there is now a much wider divergence between the updating and no-updating cases. Optimal capacity is of course overestimated when $R_1 > \bar{R}$, and conversely, but allowing for subsequent updating reduces the amount of overcapacity.

What about other sources of uncertainty? For example, recruitment data itself is invariably subject to error. Let x_i denote the true value of $\log R_i$, and let

$$y_i = x_i + \epsilon_i$$

denote the observed value. If the ϵ_i are mutually independent, and also independent of the x_i , and normally distributed

$N(0, \sigma_\epsilon^2)$, the appropriate prior distribution for x becomes normal $N(\bar{y}, \sigma^2/2 + \sigma_\epsilon^2/2)$. It is easily verified that σ^2 should be replaced by $\sigma^2 + \sigma_\epsilon^2$ throughout the optimal capacity analysis.

Figures 4a and 4b illustrate the dependence of the optimal capacity levels F_0^* and F_{no}^* upon the parameter σ . (In these calculations, variations in σ have been matched by opposite variations in $\hat{\mu}_0$ so as to keep the expected recruitment $\bar{R} = \exp(\hat{\mu}_0 + \sigma^2/2)$ constant. Thus the deterministic optimal capacity (equation 28) remains constant, but both F_{no}^* and F_0^* depend on σ by way of the prior density.) The nonupdating optimal capacity F_{no}^* always increases with σ , by equation 29 (see Charles 1983b), as shown in Figure 4a. Under updating, however, the optimal capacity F_0^* first decreases, but then ultimately increases, with increasing σ (see Figure 4b). At low levels, increased uncertainty calls for a more "conservative," wait-and-see investment decision, but at higher levels the need to have capacity to take advantage of possible high levels of recruitment outweighs the desirability of waiting for additional information.

Effects of Stock Recruitment

In this section we introduce a stock-recruitment relation into the simple model analyzed above. None of the simplifications of the previous section carry over to this case, so that a full Monte Carlo simulation approach is required.

To facilitate comparison with the previous results we adopt the Cushing form for the stock-recruitment function, namely

$$R = F(S)e^{\sigma z} = e^{\mu} S^\theta e^{\sigma z}$$

where z is a normal random variable with mean 0 and variance 1, making R lognormal. We have $E(R) = e^{\mu + \sigma^2/2} S^\theta$, and $\theta = 0$ reproduces our simple case above. We assume that μ is uncertain but that θ is known in terms of the "recruit per spawner" ratio at low stock levels. In effect this is equivalent to assuming knowledge of the *extent* to which stock-recruitment affects the pop-

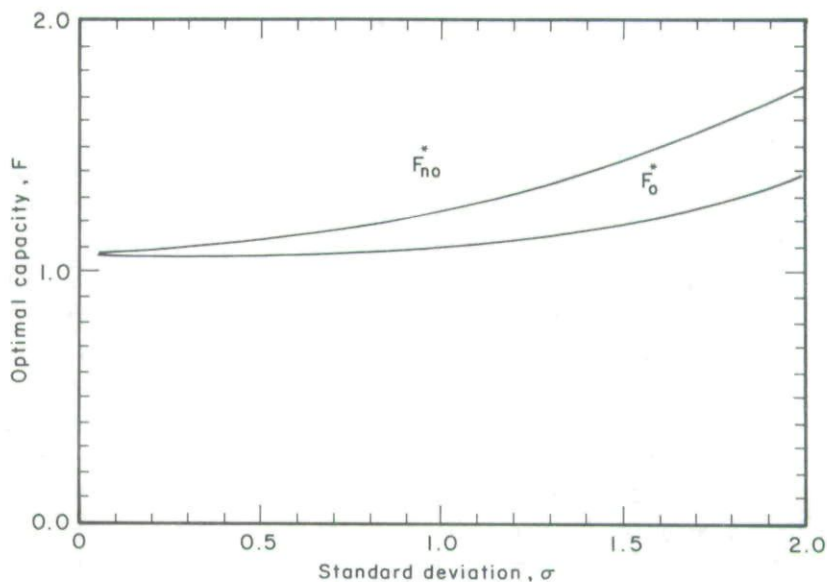


FIGURE 4a. Dependence of optimal capacity on standard deviation: base case parameters $\hat{\mu}_0 = 15.67$, $N_0 = N_1 = 3$, $\gamma = 2.2 \times 10^7$.

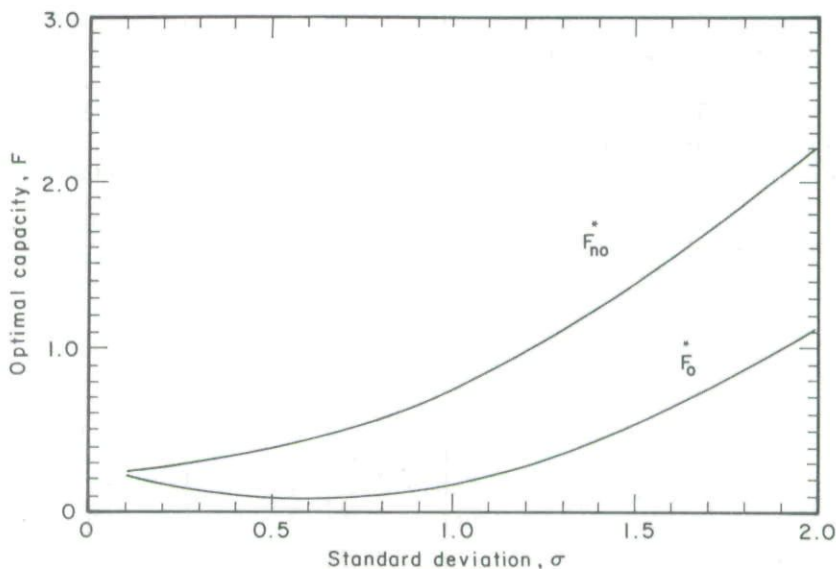


FIGURE 4b. Dependence of optimal capacity on standard deviation: parameter values $\hat{\mu}_0 = 15.67$, $N_0 = N_1 = 1$, $\gamma = 5 \times 10^7$.

ulation dynamics. The mean value of μ is then determined using the equation

$$S^* = \exp [(\mu + \sigma^2/2)/(1 - \theta)]$$

for the equilibrium unexploited escapement level, given S^* from survey data, together with θ and σ . In other words

$$\hat{\mu} = (1 - \theta) \log S^* - \sigma^2/2 \quad \text{and} \quad \mu \sim N(\hat{\mu}, \sigma_\mu^2)$$

where σ_μ^2 denotes the variance in the estimate of μ .

It is convenient to transform our fish stock variable by writing $x = \log R$ so that the stock-recruitment function becomes:

$$x_{t+1} = \mu + \theta \log S_t + \sigma z_{t+1} \quad (32)$$

Given full utilization of fleet capacity F , we have $S = Re^{-F} = e^{x-F}$, and $\log S_t = x_t - F$. Hence

$$x_{t+1} = \mu + \theta(x_t - F) + \sigma z_{t+1} \quad (33)$$

The overall optimization problem becomes

$$J_0 = \max_{F_0 \geq 0} E \left[p(1 - e^{-F_0}) \sum_{t=1}^{N_1} \alpha^t e^{\mu + \theta(x_{t-1} - F_0) + \sigma z_t} - \gamma F_0 + \alpha^{N_1} J_1(x_{N_1}, F_0, \hat{\mu}') \right] \quad (34)$$

where

$$J_1 = \max_{F_1 \geq 0} E \left\{ p[1 - e^{-(F_0 + F_1)}] \times \sum_{t=N_1+1}^H \alpha^{t-N_1} e^{\mu + \theta(x_{t-1} - F_0 - F_1) + \sigma z_t} - \gamma F_1 \right\} \quad (35)$$

Post-updating returns, for a given investment F_1 , are averaged over the posterior distribution $\mu \sim N(\hat{\mu}', \sigma_\mu'^2)$ and over the possible future random recruitments. Here

$$\sigma_{\mu}^{\prime 2} = \frac{\sigma^2}{N_0 + N_1}$$

and

$$\hat{\mu}' = \left(\frac{N_0}{N_0 + N_1} \right) \hat{\mu}_0 + \left(\frac{N_1}{N_0 + N_1} \right) \hat{\mu}_{N_1}$$

where

$$\hat{\mu}_{N_1} = \left(\frac{1 - \theta}{N_1} \right) \sum_{t=1}^{N_1} x_t + \left(\frac{\theta}{N_1} \right) (x_{N_1} - x_0) + \theta F_0 \quad (36)$$

The initial choice of F_0 maximizes the expected discounted present value of rents, averaged over the prior μ -distribution $\mu \sim N(\hat{\mu}_0, \sigma_{\mu}^2)$ and the stochastic outcomes of the first N_1 years. (For each prior μ -value and each stochastic outcome, the optimal F_1 investment level is determined and used in J_1 .)

The no-updating version is similar, except that J_1 is not calculated and we set $N_1 = H$, reflecting the fact that there is no post-updating period.

In the numerical simulations that we performed, expectations with respect to environmental outcomes z were calculated by averaging over 30 possible scenarios. Expectations with respect to μ were determined by discretizing the normal distribution $N(\hat{\mu}, \sigma_{\mu}^2)$ into 1-standard-deviation steps between $-6\sigma_{\mu}$ and $+6\sigma_{\mu}$, and approximating the expectation integral by a sum. Optimal values of F_0 and F_1 were found by a simple search procedure.

Using the parameter values of the previous section, we tested this simulation model for $\theta = 0$. The results obtained previously were reproduced by the new model at least to within the F_0 step size of 0.05 used in the analysis. Specifically we obtained $F_0^* = 1.05$ and $F_{\text{no}}^* = 1.15$.

In order to study the effects of stock recruitment, we used the above model with a recruit-spawner ratio of 6.0 at low stock levels ($\theta = 0.222$). This choice is entirely ad hoc. The optimal F_0 values with and without updating now become

$$F_0^* = 0.60 \quad F_{\text{no}}^* = 0.70$$

respectively. These capacity levels are considerably lower than those obtained previously, reflecting the influence of the stock-recruitment assumption in reducing average recruitment for the exploited stock. Uncertainty pertaining to the stock-recruitment relationship may be an important consideration in certain fisheries (Ludwig and Walters 1981), but we do not address the question explicitly here. (In the case of most tropical penaeid prawn stocks, no discernible stock-recruitment relationship appears to exist.)

Although the F_0 -values have been reduced, the absolute difference between F_0^* and F_{no}^* has not changed, suggesting that the simpler model was sufficient to depict the quantitative effect of updating on capacity decisions.

Of course, this is not a conclusive result, since only one possible stock-recruitment curve has been used, and parameter uncertainty has been restricted to just one of the parameters. While the full Monte Carlo simulation model would generally be preferred, severe computational requirements hamper its usefulness. To obtain tolerable accuracy, the innermost calculations in equation 34 must be performed approximately 150 million times. Depending on how fast the search procedure locates F_0^* , a full solution required between 20 and 45 *minutes* of CPU time on the University of British Columbia IBM Amdahl computer! While savings could likely be achieved by adopting a dynamic programming approach, the enormous difference in costs between the analysis with and without stock recruitment must certainly be taken into account in deciding upon an appropriate population dynamics model for use in capacity decisions.

A Cohort Model

Next we consider the full cohort model, using the threshold stock-recruitment relation of equation 11. The possibility of updating and subsequently increasing capacity, however, will not be considered here.

The optimization objective, equation 13, can be written more explicitly as

$$J(F) = \int_{-\infty}^{\infty} n(\mu; \hat{\mu}_0, \sigma^2/N_0) E \left[p \sum_{t=1}^H \alpha^t C_t(F) \mid \mu \right] d\mu - \gamma F \quad (37)$$

where the expectation in the integrand is taken with respect to the recruitment process R_t , conditioned on μ . This expectation can be approximately calculated by Monte Carlo simulation of the fishery based on equations 1 through 9, employing a log-normal random number generator. The integration in equation 37 is then performed numerically.

The numerical results reported below were obtained using 20 simulations for each value of μ , and a 10-mesh-point numerical integration [with the normal distribution $n(\zeta; \mu, \sigma)$ truncated at $\mu \pm 5\sigma$ and correspondingly renormalized]. Increasing the number of simulations had a negligible effect on the outcome, except for large values of σ (the graphs became slightly smoother). A typical calculation (for a given value of F) required 0.65 seconds of CPU time, when coded in FORTRAN IV and run on an Amdahl 470 machine.

Figure 5 shows computed results, for the Schaefer catch model, using the following parameter values:

$$\begin{aligned} \bar{R} &= 10^6, & w_{\infty} &= 1.0, & a_0 &= -1.0, & K &= 0.5, & M &= 0.2, \\ A_b &= A_f = 3, & A_m &= 20, & B_0 &= 0.2\bar{B}, \\ p &= 1, & \alpha &= 0.99, & H &= 20, & N_0 &= 5 \end{aligned}$$

The curves in Figure 5 represent the gross discounted return $E(p \sum \alpha^t C_t)$ rather than the net return of equation 37. The curves have a characteristic profile, increasing with F up to a maximum, and then declining to a positive asymptotic level as $F \rightarrow +\infty$. This is similar to the shape of the usual yield-per-recruit curves constructed from the Beverton-Holt cohort model (e.g., Beverton and Holt 1957, p. 321), but in the present model the decline in total (discounted) yield is associated both with overfishing of young fish and with fishery collapse resulting from recruitment failure. For the parameter values used in obtaining Figure 5, in fact the decline in yield is almost entirely associated with re-

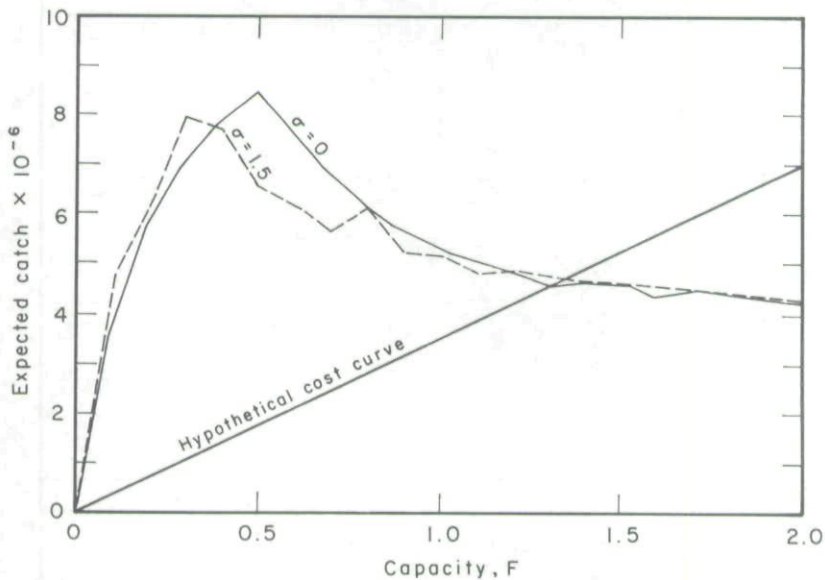


FIGURE 5. Expected catch vs. fishing capacity F , for $\sigma = 0$ and $\sigma = 1.5$. Parameter values: see text.

cruitment decline; the threshold breeding biomass ($0.20B_0$) is reached at $F = 0.5$.

The curves shown in Figure 5 indicate the effect of increasing the level of fluctuation and uncertainty. The sharp peak in the deterministic curve ($\sigma = 0$) is a direct consequence of the corner in our assumed stock-recruitment curve (equation 11). When uncertainty is introduced ($\sigma > 0$), the peak is smoothed out and shifted to the left. (The simulation techniques used in the calculations imply that the curve for $\sigma = 1.5$ is only an approximation to the actual curve. This also explains the kinkiness of the curve shown in the figure.)

The economically optimal capacity level F^* , which depends on the cost-price ratio (see equation 13), can also easily be determined approximately from the graphs. As before, F^* decreases as σ increases, reflecting the anticipated effect of uncertainty in calling for a smaller, more "cautious" level of fishing capacity. In the case of Figure 5, with $\sigma = 1.5$, a reduction of

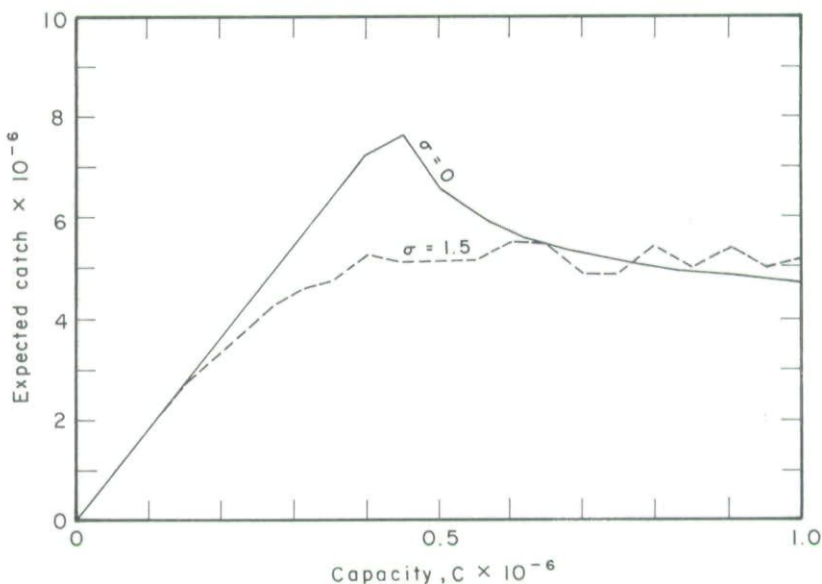


FIGURE 6. Expected catch vs. capacity C (Type IV model), for $\sigma = 0$ and $\sigma = 1.5$.

30–50% in the optimal capacity level from the value computed from a deterministic model appears appropriate for a wide range of cost-price ratios. In certain situations (relatively high cost-price ratio), the expected improvement in economic performance from such a reduction would be substantial. (If the possibility of updating were to be included, F^* would presumably be further reduced, although the results of the previous section suggest that the improvement might be minor in most cases.)

Figure 6 shows the outcome of the same model (and the same parameter combination as Figure 5, except $A_f = 1$), but for the case of a Type IV, constant-concentration fishery. Here the control variable is C^{TOT} = annual catch capacity. The deterministic curve ($\sigma = 0$) consists of a straight-line segment, joined at a sharp peak with an asymptotically declining curve. The effect of increasing uncertainty is similar to the previous case, although there is a more pronounced effect on the total expected catch than previously. This presumably results from the possibility of

complete collapse of the Type IV fishery. Again, uncertainty calls for a more conservative investment decision than would be obtained from a deterministic model, except possibly when the cost-price ratio is very low.

Conclusions

The importance of uncertainty in fishery systems is widely recognized. In this paper we have addressed the problem of estimating optimal fishing capacity under conditions of limited information as to the long-run productivity of the resource.

The effects of an error in the initial capacity decision are likely to be asymmetric. Too small a capacity will result in suboptimal initial catches, although capacity can usually be increased later if so indicated. Too large a capacity will result in financial losses of overinvestment, and also in potential long-term losses resulting from depletion of the stock. Reducing excess capacity often proves difficult, if not impossible, in practice. Because of this asymmetry, a conservative initial capacity decision appears intuitively appropriate.

In order to address this problem analytically we have used the methods of decision theory and adaptive control theory. Several alternative models have been discussed, based on alternative hypotheses concerning age structure, the catch-effort relationship, the stock-recruitment relationship, and the way in which uncertainty and information are handled.

If uncertainty is treated in a passive sense (i.e., if the possibility of future updating of information is ignored), then the effect of including uncertainty in the analysis may be an *increase* in the estimate of initial capacity, contrary to the intuitive response. Under an adaptive approach, however, the intuitive result becomes valid, at least if the degree of uncertainty is not too high.

The numerical computations that were performed also led to the tentative conclusion that in most cases a simple deterministic model would not perform too badly relative to more sophisticated decision-theoretic models. This finding, if it holds up to further analysis, could be of considerable help to fishery managers, who may tend to hesitate to recommend initial capacity

restrictions, on the grounds that uncertainty is too great to provide any accurate estimate of optimal capacity.

While it may often be true that the available estimates of resource productivity are subject to wide errors, our work suggests that a capacity decision based on "best estimates" may be close to the best possible decision under the circumstances. If this is not acceptable, then an analysis along the lines indicated in this paper becomes necessary. Experience has certainly demonstrated that the failure to make *any* attempt to control fishing capacity can have very serious long-term impacts on both the biology and the economics of the fishery.

In conclusion, we wish to emphasize the strong institutional tendencies towards overcapacity in fisheries. Decisions to limit capacity may fail to be made on the grounds that data on resource productivity is too fragmentary. Our analysis suggests that quite good decisions can be made with limited data, especially if a conservative wait-and-see attitude is adopted.

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