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Harvesting of a Transboundary Replenishable Fish Stock: A Noncooperative Game Solution

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Abstract *In this study we use a N-person differential game structure to represent a renewable resource industry in which the decision agents are few in number and noncooperative (as would be the case, for example, in international fishing wars). As an illustration we assume an environment similar to that presented by Levhari and Mirman (1980) to derive a set of tractable strategies. Although there is no guarantee that the stock size would always be positive with human harvesting in the Levhari and Mirman case, our model provides growth dynamics that rule out negative stocks. Explicit solutions of equilibrium game strategies and a steady-state level of stock are derived. Finally, we demonstrate that in situations when stock size enters the production function, combined maximization such as an international treaty is more "conservative" than individual maximization.*

Introduction

Fishing is one of the oldest commercial production activities. Since its natural resource base is generally of open access, rules and institutions have evolved to settle territorial disputes or in some way to legislate fairness of exploitation. Sometimes such rules have been responses to efficiency needs, sometimes to equity problems.

For example, in the early development of the Northern Atlantic fishery, it was decreed that it would be illegal to develop year-round fishing communities in Newfoundland since those residents would have an "unfair advantage" in harvesting the cod stock.[1] Similarly, fishing traditions evolved in the whaling and fishing industry to establish rules of fair play.

More recently, quota rules were established by joint agreement for fish stock management. For instance, the North Atlantic was divided into well-defined regions by ICNAF[2] with rules and quotas in each region.

Such institutional arrangements represented two established propositions of economic analysis. The first was that fish stock management was essential to maximize social welfare (or social rents), and the second was that joint maximization (or colluded settlement) was resource conserving.

The introduction of 200-mile territorial limits defining fishing property rights caused existing sharing arrangements to expire and resulted in territorial disputes over fishing jurisdiction. These disputes occur where territorial boundaries overlap such as in the Georges Bank between the United States and Canada, and in 1987 between Canada and

France when the French islands of St. Pierre and Miquelon, a few miles off the Newfoundland coast, claimed extended fishing rights.

In such territorial disputes, cooperative solutions are generally sought that are Pareto optimal equilibria (see Lewis and Cowens 1982; Kaitala et al. 1985; Munro 1979). As an example, an international commission recently settled the territorial dispute between the United States and Canada over the Georges Bank.

However, the possibility of noncooperation exists. At the present time, Canada and France have not been able to agree on a settlement of their territorial struggle in the Atlantic. Such a situation would be modeled as a noncooperative differential game, or "fish war" as in Levhari and Mirman (1980).

To be more specific about the environment of our model, there are assumed to be two (or more) agents who intend to harvest an international fish stock. Each has complete information concerning the size and dynamics of this stock and the set of harvesting strategies available to other agent(s). Each fishing country is aware that whatever stock it does not harvest may or may not be available for further growth and harvesting depending upon the harvesting decision(s) of the other fishing countries. Since each country must choose its harvest plan (or strategy) without knowledge of which strategy the other (competing) harvester(s) will choose, the scenario defines a "game." The outcome depends upon each country's harvesting action. Moreover, the accumulated harvest of all agents at any one time will determine the size of the remaining resource and hence the availability of stock for all future harvests. Hence the game is *dynamic.*

The only "strategy" available in this game is the choice of harvesting effort. Each country will choose its harvesting rate at time t_i (as part of its intertemporal optimizing strategy) in such a way as to maximize its objective (such as the satisfaction it achieves from selling the harvest), but subject to the effect its catch will have on the stock dynamics, while taking full account of the assumed known harvesting effects that the other countries will have on the stock dynamics. Moreover, its harvesting strategy should display dynamic optimality (or equivalently subgame perfection) in the following sense: at initial time t_o it chooses not only its harvesting rate for that period, but its complete strategy, which consists of harvesting rates for all subsequent periods. Of course, at some later time t_j, it could adopt a different harvesting rate from that previously chosen for period t_j, but *dynamic optimality* requires that there is never any reason to do so. Such strategy is often called "subgame perfect."

This dynamic game is assumed to be *Nash* in the usual sense that each participating agent assumes that the other agents will adopt a known and fixed intertemporal optimizing strategy (which will be similar to their own).

As a final comment, the game is also *noncooperative.* Several characteristics of an overlapping jurisdiction fish stock define the harvesting by two or more countries as a noncooperative game. First, noncooperation exists by definition, or the rules of the game. Cooperative agreement may be sought, but there are transaction costs and perhaps other noneconomic reasons for noncooperation. We will point out later that "cooperation" could imply that some agents are better off and none worse off (if side payments are allowed).

In summary, an individual fishing agent must formulate harvesting strategy with full knowledge of the interdependencies of all harvesting and all effects upon the stock now and its dynamic evolution. (The environment is not passive). In reality, the agent need make only his decision about the current harvest rate at any one time, but an optimal strategy will define not only his current harvesting rate but all future rates. [This last feature, called subgame perfection, means that if he were to start the game next year, he would choose the same harvesting strategy from then onward at the same rate as he has already chosen for those future years when formulating his current strategy.]

Several authors have developed models applying theories of dynamic games to such transboundary fishing disputes. Munro (1979) provides a Cournot-Nash analysis as a noncooperative game using discrete time and in a steady-state context. Lewis and Schmalensee (1982) provide analysis and properties of a cooperative solution that they show to be Pareto optimal.

Levhari and Mirman (1980) model a common-property noncooperative game (or "fish war") using a Cournot-Nash analysis, which they extend to include a Stackelberg leader-follower. The LM model is dynamic and in discrete time. It suffers from a technical problem in that the model allows the fish stock to become zero, which is clearly not in the best interests of the harvesting countries.

In this study we analyze a "fish war" using specifications of objective functions and constraints common for this kind of model (see Hamalainen et al. 1983; Lewis and Cowens 1982; Levhari and Mirman 1980 for similar specifications). Our model is quite similar to a continuous analogue of the LM model. In contrast to the LM model, however, the fish population is always positive with or without human intervention.

We use an N-person noncooperative game structure to represent a renewable resource industry and subsequently develop a set of tractable closed-loop feedback strategies that constitute a closed-loop feedback Nash equilibrium that is subgame perfect.[3] In a closed-loop feedback equilibrium, each participant takes into consideration feedback actions of all other participants. Since no commitment is made by participants, the strategy of each participant is, in general, a decision rule depending on the current state and time.

The Model

We develop this N-person game with subgame perfect solution using as illustration the model of Levhari and Mirman (1980) for which they provide Cournot-Nash solutions. The concepts we employ, however, are generally suitable to apply to many other similar problems.

We specify a biological rule of growth dynamics,

$$\dot{x}(t) = \alpha x(t) - x(t) b \ln x(t) \tag{1}$$

where $x(t)$ stands for the quantity of resource at time t.[4] One can easily observe that $\exp(\alpha/b)$ is a natural steady-state and for any positive initial stock size, future stock size will never be negative.

The number of participating agents (nations or fishermen) is $N \geq 2$ and agent i will choose his harvesting effort w_i such that

$$c_i(t) = w_i(t)\, x(t) \tag{2}$$

where $c_i(t)$ is the amount of fish harvested at time t by agent i and $w_i(t)$ is the amount of effort applied.[5] Hence, equation (2) can be viewed as a production function of the Schaefer type common in the fishery literature.[6]

The growth dynamics of the fish population with human harvesting become:

$$\dot{x} = \left(\alpha - \sum_{j=1}^{N} w_i)\right)x - xb \ln x \qquad (1')$$

Once again, one can observe that given a positive initial value of x, the stock size is always positive irrespective of the magnitudes of α, N and w_i's[7]

Following LM, we express the utility of agents in logarithmic form (as do Kaitala et al. 1985):

$$u_i(c_i) = u_i(w_i x) = a_i \ln w_i x \qquad \text{for } i = 1, 2, \ldots, N$$
$$0 \leq w_i \leq \bar{w}_i \qquad (3)$$

where a_i is a constant.

We assume this to be a finite horizon game to be played within time $t \epsilon [O,T]$ (where T can approach infinity) and each agent has its own discount rate, r_i, and we assume that each participating agent would get an equal share of the remaining stock. Since the objective function of agents is in logarithmic form, the terminal valuation of resource stock is in the form:

$$e^{-r_i T} S_i \ln(x(T))/N \qquad \text{for } i = 1, 2, \ldots, N$$

where S_i is a nonnegative constant.

Therefore, each agent faces the problem:

$$\max_{w_i} \quad \int_0^T a_i \ln(w_i x) e^{-r_i s} ds + S_i[\ln(x(T)) - \ln N] e^{-r_i T}$$

subject to

$$\dot{x} = \left(\alpha - \sum_{j=1}^{N} w_j\right)x - bx \ln x$$
$$x(0) = x_0 \qquad (4)$$

A set of control strategies $w^* = [w_1^* (t,x), w_2^* (t,x), \ldots, w_n^* (t,x)]$ provides a closed-loop feedback Nash equilibrium if there exist functions satisfying the following conditions:

$$V^i(t,x) = [\int_t^T a_i \ln(w_i^* (s, x^*(s)) x^* e^{-r_i s} ds + e^{-r_i T} S_i \ln (x^* (T))]$$

$$\geq [\int_t^T a_i \ln(w_i (s, x^i(s)) x_i^{-r_i s} ds + e^{-r_i T} S_i \ln(x^i (T))]$$

$$V^i(T,x) = e^{-r_i T} [S_i \ln(x(T)) - S_i \ln N] \quad \text{for } i = 1, 2, \ldots, N$$

where on the interval [0,T]

$$\dot{x}^*(s) = \left(\alpha - \sum_{j=1}^{N} w_j^*(s,x^*)\right)x^* - bx^* \ln x^*$$

$$\dot{x}^i(s) = \left(\alpha - \sum_{\substack{j \neq 1 \\ j=1}}^{N} w_j^*(s,x^i) - w_i(s,x^i)\right) x^i - bx^i \ln x^i \tag{5}$$

Note that strategies $w_1^*(t,x), \ldots, w_N^*(t,x) = w^*$ are rules that depend on time and the current state x(t) and hold for any subgame interval [t,T], $0 < t < T$.

Using Bellman's technique of dynamic programming, one can show that w^* must satisfy the following system of Hamilton-Jacobi equations:

$$-V_t^i = \max_{w_i} \left\{ a_i \ln (w_i(t)x)e^{-r_it} + V_x^i \left[\alpha - \sum_{\substack{j \neq 1 \\ j=1}}^{N} w_j^*(t,x) - w_i(t) - b \ln x \right] x \right\}$$

$$V_x^i(T) = S_i e^{-{}_iT} \frac{1}{x(T)} \quad \text{for } i = 1, 2, \ldots, N \tag{5'}$$

In general, the set of tractable closed-loop feedback strategies w^* is difficult to obtain. To date there are only a few classes of differential games with explicitly solvable feedback solutions.[8] Game structure (4) does not appear to belong to any known class of differential games that provides a tractable solution.[9] Our approach in this case is to provide a transformation of variable, which, in fact, allows us to find an open-loop solution. This open-loop solution is shown to qualify as a closed-loop solution.

To accomplish this solution, we propose the following transformation:

$$\text{Let } y = \ln x \tag{6}$$

$$\dot{y} = \left(\alpha - \sum_{j=1}^{N} w_j\right) - by \tag{7}$$

The objective functional of agents expressed in terms of the transformed variable y now becomes

$$\max_{w_i} \int_0^T (a_i \ln w_i + a_i y)e^{-r_is}\, ds + e^{-r_iT} S_i[y - \ln N] \tag{8}$$

First we deduce a Nash equilibrium in open-loop controls. Let $\lambda = \{\lambda_1(t), \lambda_2(t), \ldots, \lambda_N(t)\}$ be the set of costate variables for the set of N agents. The Hamiltonian of agent i can be expressed as:

$$H^i = (a_i \ln w_i + a_i y)e^{-r_i t} + \lambda_i(t)\left(\alpha - \sum_{j=1}^{N} w_j - by\right) \tag{9}$$

A set of control strategies $w^*(t) = \{w_1^*(t), w_2^*(t), \ldots, w_N^*(t)\}$ constituting an open-loop Nash equilibrium must satisfy the following conditions:

$$\text{i) } \dot{y}^* = \alpha - \sum_{j=1}^{N} w_j^* - by$$

$$H^i(y^*, w^*, \lambda_i, t) \geq H^i(y^*, \hat{w}_i, \lambda_i, t) \quad \text{for } i = 1, 2, \ldots, N$$

$$\text{where } \hat{w}_i = \{w_1^*(t), w_2^*(t), \ldots, w_{i-1}^*(t), w_i(t), w_{i+1}^*(t), \ldots, w_N^*(t)\}.$$

An interior solution implies:

$$\text{ii) } a_i \frac{1}{w_i^*} e^{-r_i t} = \lambda_i \quad \text{for } i = 1, 2, \ldots, N$$

$$\text{iii) } \dot{\lambda}_i = -a_i e^{-r_i t} + b\lambda_i \quad \text{for } i = 1, 2, \ldots, N$$

$$\text{iv) } \lambda_i(T) = S_i e^{-r_i T} \quad \text{for } i = 1, 2, \ldots. N \tag{10}$$

The adjoint equations (10iii) are linear differential equations that are independent of the state variable and w_i^* is chosen independent of the state variable. Hence, this game is state-separable. The open-loop solutions of state-separable games are indeed closed-loop feedback solutions.[10] Hence, the set of control strategies w_i^* that satisfies (10) is a set of closed-loop feedback strategies.

Solving the set of linear differential equations in (10iii) with terminal conditions (10iv), we obtain a closed-form solution for the co-state variables of each agent.

$$\lambda_i(t) = \left[\frac{S_i - \dfrac{a_i}{b + r_i}}{e^{(b+r_i)T}}\right] e^{bt} + \frac{a_i}{b + r_i} e^{-r_i t} \quad \text{for } i = 1, 2, \ldots, N \tag{11}$$

Substituting (11) into (10ii), with $w_i = a_i/\lambda_t\, \epsilon^{-r_i t}$ we obtain the game strategy $w_i^*(t)$ of agent i time $t \epsilon [0,T]$. Observe that this strategy, expressed in terms of fishing effort of agent i at time t, can be substituted into the catch function (2) to give the catch of agent i at time t. This would imply an optimal harvesting strategy (at time t) of

$$c_i(t) = w_i(t)x(t) = \left[\frac{a_i}{\left[\dfrac{S_i - \dfrac{a_i}{b + r_i}}{e^{(b+r_i)T}}\right] e^{(b+r_i)t} + \dfrac{a_i}{b + r_i}}\right] x(t)$$

Since S_i represents agent i's share of the stock remaining at the terminal date, a_i represents the intensity of his preferences and r_i his discount parameter, we observe that he will decrease his effort over time faster if either his share S_i is large relative to his utility intensity parameter a_i, or his rate of time preference r_i is large.

In the extreme case where $S_i = 0$ he has no incentive to slow down his harvest rate. Alternatively, as $a_i \to 0$ or $r_i \to \infty$ he will prefer to delay harvesting in order to achieve a larger "bequest" at the terminal time. Differentiating $w_i^*(t)$ with respect to time yields:

$$\dot{w}_i^*(t) = \frac{-(b+r_i)a_i e^{-r_i t}\left[S_i - \dfrac{a_i}{b + r_i}\right]e^{-(b+r_i)T}e^{bt}}{\lambda_i^2} \tag{12}$$

Hence, harvesting effort is decreasing over time for each agent if and only if $S_i > a_i/b + r_i$. Moreover, one can show that for any nonnegative value of S_i, $\lambda_i(t)$ is nonnegative in the time interval [0,T].

Next, we extend the game to a situation where T approaches infinity. In a game of infinite horizon S_i, which represents a net evaluation is assumed to be zero for i = 1, 2, . . . , N. Using current-valued Hamiltonians and letting $\lambda = (\lambda_1(t), \lambda_2(t), \ldots \lambda_N(t))$ be corresponding costate variables, we have,

$$\hat{H}^i = e^{r_i t}H^i = a_i \ln w_i + a_i y + \gamma_i\left[\alpha - \sum_{j=1}^{N} w_j - by\right]$$

where

$$\gamma_i = e^{r_i t}\lambda_i \qquad \text{for } i = 1, 2, \ldots, N \tag{13}$$

Conditions (10ii) and (10iii) are replaced respectively by

$$\frac{a_i}{w_i^*} = \gamma_i$$

and

$$\dot{\gamma}_i = r_i\gamma_i - a_i + b\gamma_i \qquad \text{for } i = 1, 2, \ldots, N \tag{14}$$

In steady state γ_i has to be equal to $a_i/(b + r_i)$. One can observe that with T approaching infinity and S_i equalling zero it follows that

$$\lim_{t\to\infty} \lambda_i(t) = \frac{a_i}{b + r_i} e^{-r_i t} \qquad \text{for } i = 1, 2, \ldots, N \tag{15}$$

and by definition $\lambda_i(t) = e^{r_i t}\lambda_i(t) = a_i(b + r_i)$.

From the necessary conditions we obtain a harvesting rate for agent i

$$w_i^{*s}(t) = b + r_i \qquad \text{for } i = 1, 2, \ldots, N \tag{16}$$

Hence, agents with higher discount rates will harvest at higher rates. In the next section, it will be possible to show that the above harvesting rate is too high from a social point of view.

Substituting w_i^{*s} for i = 1, 2, . . . , N into the growth dynamics equation, we obtain

$$\dot{y}^{*s} = \alpha - \sum_{j=1}^{N} (b + r_j) - by \tag{17}$$

which implies a steady state level of stock of

$$y^{*s} = \frac{\alpha - \sum_{j=1}(b + r_j)}{b} \tag{18}$$

or in terms of the original stock variable

$$x^{*s} = \exp\left[\frac{\alpha - \sum_{j=1}(b + r_j)}{b}\right] \tag{19}$$

The steady-state harvest of agent i can now be expressed in terms of the original specification as

$$c_i^s = w_i^{*s}x^{*s} = (b + r_i)\exp\left[\frac{\alpha - \sum_j(b + r_j)}{b}\right]$$

The steady-state stock, which is always positive in size, results as the solution of the original dynamic non-cooperative game (with infinite horizon).

It is easily demonstrated that (19) represents the appropriate steady-state by direct substitution into (1′)

$$\begin{aligned}\dot{x} &= \left(\alpha - \sum_j w_j\right)x - xb\ln x \\ &= \left(\alpha - \sum_j w_j\right)w_j)x^* - xb\left[\frac{\alpha - \Sigma(b + r_j)}{b}\right]\end{aligned}$$

where from (16) $w_j = b + r_j$ in a steady state. If folllows that $\dot{x} = 0$.

To further demonstrate the optimality of $w_i^* = b + r_i$ for agent i, suppose instead he chooses $\tilde{w}_i = (b + r_i(1 + \delta_i)$ for some δ_i where other agents have $w_j^* = b + r_j$. If the appropriate catch rates given by (2) (5') for steady-state values, the *difference* between the values of the objective function for choice w_i^* with steady-state stock value x^* and $\tilde{w}_i$ with steady-state stock value $\tilde{x}^*$ can be shown to be $D \equiv \delta_i(1 + r_i/b) - \ln(1 + \delta_i)$. This difference is minimized when $\delta_i = 0$ hence $w_i = b + r_i$ is optimal. The steady-state value x^* will be compared next to the optimal stock under common jurisdiction.

Combined Maximization

In this section as in Levhari and Mirman (1980) and Lewis and Cowens (1982) we compare the results of the previous section to those that are based on combined efforts of agents to maximize the discounted sum of the objectives of all agents. In order to avoid the unresolved issue of utility comparability, we assume that $a_i \ln(w_i x)$ represents the instantaneous net revenue for agent i. The Hamiltonian to be maximized is:

$$H = \sum_{j=1}^{N} (a_j \ln w_j + a_j y)e^{-r_j t} + \Omega\left(\alpha - w_j - by\right) \tag{20}$$

Necessary conditions for an optimum control $w^* = \{w_1^*(t), w_2^*(t), \ldots, w_N^*(t)\}$ are:

$$\dot{y}^* + \alpha - \sum_{j=1}^{N} w_j - by^*$$

$$H(y^*, w^*, \Omega, t) \geq H(y^*, \hat{w}_i, \Omega, t)$$

which implies

$$a_i \frac{1}{w_i^*} e^{-r_j t} = \Omega \qquad \text{for } i = 1, 2, \ldots, N$$

$$\dot{\Omega} = -\sum_{j=1}^{N} \alpha_j e^{-r_j t} + \Omega b$$

$$\Omega(T) = \sum_{j=1}^{N} S_j e^{-r_j T} \tag{21}$$

The costate variable can be solved explicitly from (21) as

$$\Omega(t) = \left[Ae^{bt} + \sum_{j=1}^{N} \frac{a_j}{b + r_j} e^{-r_j t}\right]$$

where

$$A = \sum_{j=1}^{N}\left(S_j - \frac{a_j}{b + r_j}\right) e^{-(b+r_j)T} \tag{22}$$

Comparing (22) with (11), we observe that $\Omega(t) > \lambda_i(t)$ for $i = 1, 2, \ldots, N$ in the interval [0,T]. Hence, w_i^* will be lower in combined maximization. Similarly, as T approaches infinity, the steady-state value of $\lambda(t)$ with combined effort is

$$\sum_{j=1}^{N} \frac{a_j}{(b + r)}$$

which is larger than any single $\lambda_i(t)$ in the case of individual maximization. The steady-state harvesting efforts for combined maximization are:

$$w_i^{*s}(t) = \frac{a_i}{\sum_{j=1}^{N} a_j} (b + r_i) \qquad \text{for } i = 1, 2, \ldots, N \tag{23}$$

The steady-state stock under combined maximization is:

$$y^{*s} = \frac{\alpha - b - \sum_{k=1}^{N} \frac{a_k r_k}{\sum_{j=1}^{N} a_j}}{b} \tag{24}$$

Comparing (23) and (24) with (16) and (17), one observes that in the steady-state, harvesting effort is smaller while resource stock is higher with combined maximization. For the sake of comparison, we assume that agents are identical (i.e., $a_i = a_j$ and $r_i = r_j$ for $i \neq j$) so that (23) and (24) become:

$$w_i^{*s} = \frac{1}{N} (b + r) \qquad \text{for } i = 1, 2, \ldots, N \tag{23'}$$

and

$$y^{*s} = \frac{\alpha - (b + r)}{b} \tag{24}$$

In terms of the original specification, the volume of catch for each agent in steady-state under combined maximization is

$$w_i^{*s}\exp(y^{*s}) = \frac{1}{N} (b + r) \exp\left(\frac{\alpha - (b + r)}{b}\right) \tag{25}$$

whereas the corresponding volume of catch in steady-state under individual maximization is

$$(b + r) \exp\left(\frac{\alpha - N(b + r)}{b}\right) \tag{26}$$

One can verify that the expression in (25) is greater than the expression in (26). Hence in the steady state, combined management leads to a higher catch for each agent. Since production is influenced by the size of the fish stock, "proper" valuation of the stock by combined decision making leads to a higher sustainable yield. However, although the combined management solution is in *aggregate* more "profitable," it is not always achieved and fish wars persist. This is easily explained if we consider other aspects of importance in deciding on a sharing arrangement such as transaction costs. Presumably, an agent would rather compete as above than accept an "unfair" share. (In terms of standard language of game theory, we have shown the "usual" outcome that cooperation would imply a Pareto improvement over a noncooperative solution if side payments were allowed).

Finally, we depart from the assumption made above that the number of agents is given, perhaps by legal rights, licenses, etc. Assume that there is a group of identical potential participants and there is freedom of entry and exit. Each participant is required to pay a sum, f, perhaps as an entrance fee or set-up costs to enter the game.[12] Thus industry entry will require that

$$\int_0^T a_i(\ln w_i + y)e^{-r_i s}ds + S_i e^{-r_i T}y - f \geq 0 \qquad \text{for } i = 1, 2, \ldots, N \qquad (28)$$

Earlier, we showed that w_i is independent of y. Therefore, with more agents the value of y at each time instance in the interval [0,T] will be lower and hence the volume of catch will be lower. Thus since the left side of (28) is a decreasing function of N, there exists a unique number of firms that satisfies (28).

If f, a licence fee were to be appropriately chosen, such as by the value $[\Omega(t) - \lambda_i(t)]$ where a seasonal licence is allowed to vary over time, or by the above expression evaluated in the limit as $T \to \infty$, then the market could determine industry size N consistent with optimal stock size.

Conclusions

In this study, we presented a differential game model of renewable resource harvesting. Specific functional forms are assumed. These forms are consistent with LM and other authors and allow us to provide a closed form solution to illustrate the nature of the solution. We share the suggestion of Levhari and Mirman (1980) about the inadequacy of existing technology to generate explicit results for more general models. Our model differs from the LM model by introducing continuous time and eliminates a technical error they make by allowing agents to possibly extinguish the resource that is clearly nonoptimal. In this environment, we develop a closed-looped feedback Nash solution. Its primary advantage is widely recognized to be the fact that it allows agents to take into consideration complete feedback of other agents. Such games do not necessarily have solutions, but in this case a solution exists and we have derived in explicit form the equilibrium strategies and steady-state level of resource stock. We also demonstrated in our model that combined maximization generates potentially greater net revenue for each agent as compared to individual game strategic maximization. This result is consistent with the results obtained by Lewis and Cowens (1982) by Kaitala et al. (1985).

In terms of policy, it is apparent that having to engage in a "fish war" is inefficient or wasteful in that the maximum potential benefit of the resource stock is not exploited. Hence, even if transactions are costly, cooperation is usually sought. One reason, however, that cooperative solutions are slow in formulation is that only those cooperative solutions that don't disadvantage either party are feasible. The prospects of "side payments" are usually unrealistic. Hence, bargaining for a best deal is difficult, and other well-known aspects of game theory can occur. For instance, the "fish war" between France and Canada over harvesting in the Grand Banks is unresolved and various threats and counterthreats have resulted, such as a temporary impounding of a French fishing boat by Canadian authorities. One interesting "side payment" that seems to be "unofficially on the bargaining table" is whether the Canadian armed forces adopt a French-built submarine.

We described how the strategy of an agent engaged in a dynamic noncooperative game may appear. We have not analyzed the political environment that created the game but simply described how it would be played.

It is interesting to observe, however, that there have been two highly publicized "fish wars" in Canada in the past decade. The Canada-U.S. contest over the Georges Bank resulted in a joint agreement that was perceived by some Canadians as unfavorable for them, and the Canada-France dispute over the joint Newfoundland-St. Pierre/Miquelon fishery, which is still unresolved but under negotiation. In the interim, we believe that the modeling and dynamic solution we have provided reasonably describes current behavior.

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Notes

1. As an early example of "rent seeking," small settlements were soon "illegally" established in Newfoundland generally by Basque fishermen.
2. International Commission for the North Atlantic Fishery.
3. "A Nash equilibrium is subgame perfect . . . if the continuation of the decision rules constitutes a Nash equilibrium when viewed from any intermediate (date, state) pair, i.e., if they form a Nash equilibrium in every subgame of the original game. Reinganum and Stokey (1981), p. 3.
4. The specification (1) is the continuous analog of the discrete time growth function $x_{t+1} = bx_t^\alpha$ of Levhari and Mirman (1980); see J. B. Smith (1980) for derivation details.
5. The use of $w_i(t)$ instead of $c_i(t)$ (which is used in Levhari and Mirman 1980 model) as the control variable enables us to avoid the problem of reaching negative stock sizes. Kaitala et al. (1985) compare harvest rates c_i and harvesting effort w_i as control variables in their model, p. 605.
6. See, e.g., Clark and Munro (1976) for details of the Schaefer model.
7. In Levhari and Mirman's (1980) paper, an uncoordinated policy may lead to $c_1 + c_2 > x$ and consequently the game is undefined. Our specification is chosen to avoid this problem.
8. See Clemhout and Wan (1974), Leitmann and Schmittendorf (1978), Reinganum (1982), Dockner et al. (1985), Jorgensen (1985), Yeung (1987, 1988).
9. In Clemhout and Wan (1985), a similar structure with stochastic shocks is developed. However, the closed-loop feedback strategies obtained by them using Kushner's method applies only in the case of an infinite horizon case. In particular, the equilibrium strategies in steady-state when T approaches infinity developed in this paper converge to their strategies when stochastic shocks are removed. See Plourde and Yeung (1987).
10. See Leitmann and Schmittendorf (1973) and subsequently, for a proof, Dockner et al. (1985).
11. Comparison of (23′) and (16) suggest the obvious (trivial) limiting case of sole ownership since equality will occur if and only if $n = 1$. Otherwise, for $N > 1$ the agent who is involved in a noncooperative game will always harvest at a higher rate.
12. 'f' is measured in the same unit as the objective function.

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