Marine Resource Economics, Volume 14, pp. 317–332
0738-1360/99 $3.00 + .00

Some Preliminary Evidence on Sampling of Alternatives with the Random Parameters Logit

KENNETH E. MCCONNELL
WEI-CHUN TSENG
University of Maryland

Abstract *Random utility models rely on the properties of the logistic distribution for ease of estimation, but this distribution implies the independence of irrelevant alternatives (IIA). The random parameters logit model offers a means of avoiding the IIA assumption as well as greater heterogeneity among agents, recreational anglers or beachgoers in the current application. A problem often encountered in the estimation of random utility models with many alternatives is the necessity of sampling alternatives or otherwise reducing the number of choices. Research has shown that in the random utility model, such changes in choice set still lead to consistent parameter estimates. However, with the random parameters logit, there is greater need to sample but no theoretical evidence that sampling is justified. In this paper we show the impact of sampling in a random parameters logit model. We find that sampling does not appear to change the parameter estimates substantially. We investigate two data sets: a study of beach use in the Chesapeake Bay and a study of marine recreational angling behavior for the Northeast of the U.S.*

Key words Random parameters logit, alternative sampling, parameter estimates, heterogeneity, choice set.

Introduction

This paper provides a preliminary investigation of the sampling of alternatives in the context of random parameter logit (RPL) models. This is a relevant investigation because the RPL has a number of attractive features compared to the standard random utility model (RUM) and so it is desirable to understand how variations in choice sets affect estimation of parameters and welfare. There is a close connection between the choice set and the stochastic structure of the RPL. Further, sampling of alternatives, often useful in the estimation of RUM's with many alternatives, becomes a necessity for estimation of RPL because estimation is so much more costly than with the RUM.

There are three different ways in which researchers can restrict a full choice set. One occurs when the full choice set is too large to allow practical estimations, and the choice set is sampled. This was the case for the Wisconsin lakes study by George Parsons and Mary Jo Kealy. The second case involves the exclusion of some alternatives, because they are not practical or feasible. For example, distances of more than 300 miles one way for single day recreation trips are not typically feasible. Nor are

---

Kenneth E. McConnell is professor in the Department of Agricultural and Resource Economics, University of Maryland, 2200 Symons Hall, College Park, MD 20742, e-mail: tedm@arec.umd.edu. Wei-Chun Tseng is a PhD student, Department of Agricultural and Resource Economics, University of Maryland, 2200 Symons Hall, College Park, MD 20742, e-mail: tweichun@arec.umd.edu.
We thank two anonymous referees, workshop participants and the editors for helpful comments.

trips with trailered boats to sites without boat ramps. A third approach is based on individual preferences or perceptions. One variant of this approach calls for restrictions to the alternatives that are known or are familiar to an individual (Parsons, Massey, and Tomasi; Hicks and Strand). A second makes the choice set endogenous to the individual (Haab and Hicks).

The first two reasons for restricted choice sets are not controversial, but the selection of individual-specific choice sets based on individual perceptions or preferences raises the issue of consistent modeling philosophy. Decisions about the structure of the choice set should be consistent with other modeling decision. A site choice model in recreation cannot be a precise description of behavior. Rather, such a model is a set of extreme simplifying assumptions that capture first order approximations of the basic forces at work. Actual decisions are infinitely more complex than models. A consistent modeling philosophy would suggest that the researcher make free use of simplifying assumption to construct a model. These assumptions can be abandoned for more complex assumptions when they are practical to compute and give greater generality or an intuitively more plausible model. A well-constructed model ought to approximate the various components of behavior with roughly the same degree of detail. The fact that individual-specific choice sets yield statistically different parameters and welfare estimates from a uniform, universal choice set is not reason enough to drop the universal set.

To understand why modeling consistency is important consider other model construction decisions that might be tested but typically are not. We might, for example, want to take the perceived distance or perceived costs to alternate sites, rather than systematically calculated costs. It is easy to imagine that this would give quite different results. Yet another deviation from current practice would be to let the cost coefficient vary by alternative. This would raise havoc with welfare estimation, yet might easily hold up statistically. Or we could experiment with various measures of central tendency of catch of fish, instead of the mean catch rate. The point of these examples is to illustrate the necessity of simplifying assumptions, made on the basis of intuition and judgment and the importance of balancing these assumptions across the model.

In this paper we investigate the implications of the RPL empirically. Our goal is to compare the RPL with conditional logit with estimation that includes the full set of alternatives. We do this with a small data set of beach users on the Chesapeake Bay. Then we investigate alternative sampling, using the very large Northeast Marine Recreational Fishing Statistics Survey, as well as the Chesapeake Bay data set.

The Random Parameters Model

The idea that the parameters of a discrete choice model could be made random is not new, but increases in computing power have made the logit version newly feasible. Train (1998) introduced the random parameter logit recently in recreation economics. (See also a longer exposition in Train 1999). We illustrate the RPL model briefly. Let the utility for individual i from alternative j be

$$U_{ij} = \beta x_{ij} + \varepsilon_{ij} \tag{1}$$

where $j = 1, \ldots, J$ is the number of alternatives. Assume that x_{ij} is an M-dimensional column vector of attributes and β an M-dimensional row vector of parameters. The conditional logit comes from the assumption that ε_{ij} is type I extreme value. Assume that ε_{ij} has mean zero and variance σ_ε^2. The RPL adds randomness in β. It is stochastic with a known distribution, independent of ε_{ij}. Suppose $f(\beta)$ is the density for β, with support B. Then the probability of individual i choosing alternative j is given by

$$\text{Prob}_i(j) = \int_B \frac{e^{tx_{ij}}}{\sum_{k=1}^{J} e^{tx_{ik}}} f(t)dt \tag{2}$$

This, the expected value of the conditional logit probability, is correct but not informative without more knowledge of the distribution of the β.

For the moment, suppose that β is distributed as N(b, Σ). Thus we can write $\beta = b + \theta$, where $\theta \sim N(0, \Sigma)$. Even with normality the researcher still has choices about the randomness of parameters. All can be made random or only one. In practice Σ is often diagonal although Train (1998) has recently estimated a model with covariance among random parameters. When Σ is not significantly different from zero, the RPL is equivalent to the conditional logit. Thus a test that the elements of Σ are zero is a test of the RPL versus the logit, so that the RPL is a generalization of the conditional logit, which is clear from the expression for $\text{Prob}_i(j)$. Intuitively it is the variance of the random parameters that provides for heterogeneity of individual preferences. High variance implies more heterogeneity.

The richness of the RPL can be seen by writing the utility function explicitly:

$$U_{ij} = bx_{ij} + \theta_i x_{ij} + \varepsilon_{ij} \tag{3}$$

Note that the vector θ_i has an individual specific index, meaning that a separate draw is made for each individual, but not each alternative. The random vector θ_i would have M elements and the term $\theta_i x_{ij}$ would naturally vary by j. Unobservable randomness for each individual is $\theta_i x_{ij} + \varepsilon_{ij}$ and because of the x_{ij} each individual has a unique distribution. Further, the random draw gives each individual a distinct marginal utility of the attribute. For attribute m the marginal utility for individual i is given by

$$\frac{\partial U_{ij}}{\partial x_{ijm}} = b_m + \theta_{im} \tag{4}$$

When the variance of the random parameter is high, a greater range of θ_{im} can be expected, and hence more diverse behavior. Further the RPL model does not impose the independence of irrelevant alternatives (IIA). The ratio of the probability of j to the probability of j^* for individual i is

$$\frac{\text{Prob}_i(j)}{\text{Prob}_i(j^*)} = \frac{\int_B \frac{e^{tx_{ij}}}{\sum_{j=1}^{J} e^{\beta x_{ij}}} f(t)dt}{\int_B \frac{e^{tx_{ij^*}}}{\sum_{j=1}^{J} e^{tx_{ij^*}}} f(t)dt} \tag{5}$$

which clearly does not factor, so that the ratio of probabilities between any two alternatives depends on the number of alternatives as well as the characteristics of other alternatives. The absence of IIA gives the RPL greater generality in representing preferences.

The covariance structure of the error terms is also relevant in the construction of choice sets. The covariance between two utilities is given by

$$
\begin{aligned}
& E(U_{ij} - EU_{ij})(U_{i^*j^*} - EU_{i^*j^*}) \\
&= x'_{ij}\Sigma x_{ij} + \sigma_\varepsilon^2 \quad \text{for } i = i^*, j = j^* \\
&\quad x'_{ij*}\Sigma x_{ij*} \quad \text{for } i = i^*, j \neq j^* \\
&\quad 0 \quad \text{for } i \neq i^*
\end{aligned} \tag{6}
$$

where prime denoted transpose. This structure differs in two ways from the conditional logit. For a given alternative, the variance differs across individuals. And for a given individual and different alternatives, there is a nonzero covariance. Thus each individual random disturbance has a different distribution, depending on the attribute of the site. The covariance matrix of utility is block diagonal, with $x'_{ij}\Sigma x_{ij} + \sigma_\varepsilon^2$ on the diagonal, $x'_{ij*}\Sigma x_{ij*}$ off-diagonal within the block, and the block size given by the dimensions of the choice set. This type of covariance matrix has the same structure as a panel of cross-section/time-series data with heteroskedasticity and unbalanced panels. When each individual has the universal choice set, the panel is balanced, and when the parameters are not random, heteroskedasticity disappears.

RPL models are estimated using the maximum simulated likelihood estimators (MSLE) because for most distributions, a closed-form solution for the integration associated with the likelihood function of RPL is not available. With a sufficient number of repetitions, the estimators are asymptotically equivalent to the maximum likelihood estimators (Train 1998).

The construction of choice sets for estimating the RPL differs in several ways from the standard model. First the urgency of sampling alternatives is much greater in the RPL. Because of numerical integration, these models are more difficult to estimate, taking more time and computer space. By way of example, the universal choice set of the Northeast Marine Recreational Fishing Statistical Survey (MRFSS), as constructed by Hicks *et al.*, with 63 sites and 13 mode-species combinations, cannot be estimated for the conditional logit without sampling alternatives and so it is not feasible to estimate the RPL. Because there is no theoretical support for sampling of alternatives when IIA does not hold, empirical evidence on the effects of sampling can broaden our understanding.

We have three objectives in this paper. First, we compare a conditional logit with two separate RPL models simply to assess the relative size of welfare estimates. In the long run, if RPL models give approximately the same results as the conditional logit, then their advantages dim. This comparison is accomplished with the Chesapeake Bay data set on beach use (Hicks and Strand). Second, in the spirit of this volume, we compare the RPL with a conditional logit when alternatives are sampled. This experiment uses the very large Northeast MRFSS data set, where sampling is a necessity. Third, we compare the RPL and logit, when estimated with sampled alternatives, with the same model from the full choice set. This exercise uses the Chesapeake Bay data set. The last two objectives should give some insight into the behavior of the RPL with sampled alternatives.

Comparing Results: Random Parameter Logit versus Conditional Logit

In this section we compare a conditional logit with the RPL, for the sake of continuing the investigation of the RPL. We use the relatively compact data set of beach use on the Chesapeake Bay. In the few models that have been estimated for recreational resources (Train 1998 and Herriges and Phaneuf) there is no evidence of systematic differences of the parameter estimates of RPL and the logit counterparts. However, welfare measures from different policy scenarios of a RPL model either all are larger (in absolute values) or all are smaller than their logit counterparts. Thus it is interesting to examine results for this data set. The models are estimated on a data set of beach visits on the western shore of the Chesapeake Bay (see Bockstael, Hanemann, and Strand and for details on the survey).[1]

For the choices among beaches, we specify the utility function as it depends on travel cost, travel time, water pollution and beach facilities.[2] The indirect utility function:

$$U_{ij} = \beta_1 TC_{ij} + \beta_2 TT_{ij} + \beta_3 WP_j + \beta_4 QI_j + \varepsilon_{ij} \tag{7}$$

where $j = 1,\ldots,10$ choices among beach sites on the western shore of Chesapeake Bay.

TC: round trip travel cost.
TT: travel time for individuals who are at corner solutions in the labor market.
WP: water pollution; is the presence of fecal coliform in the water, as measured by the most probable bacteria number count averaged over the measurement obtained for the 1984 season.
QI: quality index; a variable that captures other aspects of a site's quality such as the presence of bath facilities, boat docks, and pools.

This is a small data set with 388 trips and 10 sites (there is one trip per individual). This size of the data set makes it feasible to compare several variants of the RPL and conditional logit.[3] Two RPL models are specified. The first has only one random parameter, the parameter associated with WP; the second one has three random parameters, TT, WP, and QI.

We illustrated the RPL with an example that assumed the parameter to be normally distributed. The random parameter can also be log-normally distributed. The log-normal distribution permits the sign of the covariate effect to be specified. For example, suppose that we want the covariate to have a nonnegative effect. We specify the parameter

$$\gamma = e^{\beta} \tag{8}$$

where β is distributed $N(b, \sigma^2)$. Then the covariate contribution to utility is

$$x\gamma = xe^{\beta} \tag{9}$$

---

[1] We thank Ivar Strand and Rob Hicks for making the data available in their current form.
[2] This is the same specification used by Hicks and Strand in their study of the effects of choice set definitions.
[3] We also tried a nesting structure in which two beaches to the south were in one nest and the remaining northern beaches in the other nest. This structure worked fine for the nested logit model. However, the RPL version, which is created by assigning a dummy variable for one nest, and then allowing the coefficient on the dummy to be random, did not prove estimable. Further work along these lines is under way.

so that utility will always increase with increases in x, regardless of the sign of β. By changing the sign of γ we can specify a nonpositive effect for the covariate.[4] To compare these random parameters with their nonrandom counterparts, we calculate the mean of γ by

$$\bar{\gamma} = e^{b} \cdot e^{\frac{1}{2}*\sigma^2} \tag{10}$$

where the right-hand side is the mean of a (b, σ^2) lognormally distributed variable.

The results are reported in table 1. In RPL1, the only random parameter is on the variable WP. It is estimated as the negative of the log-normal, so that a negative covariate effect is assured. The estimated b_3 equals –2.47 and the estimated parameter $\gamma_3 = -\exp(-2.47 + 0.884^2/2)$. The test for the hypothesis that the RPL is the same as the conditional logit can be rejected because the standard deviation, σ_3, has a t-statistic of 4.80 under the null hypothesis that σ_3 equals zero.[5] In RPL2, there are random parameters for the three variables TT, WP and QI. The first two are assumed to have negative covariate effects, and the third, QI, is assumed to be positive. They are all estimated as log-normal. The nonrandom parameters vary little between the RPL and the logit, while certain random parameters, such as the parameter on WP in model RPL1 and on TT and WP in model RPL2, are quite different from the nonrandom parameters in the logit model. For example the mean value of the parameter for the WP covariate in RPL1 is about twice its value in the conditional logit model.
In RPL1, we can reject the hypothesis of no randomness for the parameter on WP because of the large t-statistic on its standard deviation. In RPL2, all of the parameters specified as random have significant standard deviations except the mean of the coefficient on QI variable. The standard deviation for the coefficient on QI is very close to zero and the mean of QI, 0.43, is very close to its logit estimate. This illustrates the relationship between the RPL and the conditional logit: when the standard deviation of a random parameter is zero, the random parameter degenerates to the standard parameter estimate.

The goal of estimating random utility models is the calculation of welfare valuations. The compensating variation (or willingness to pay, since the utility functions are all linear in income) for individual i for a conditional logit model is

$$C_i(\beta) = \left\{ \ln\left[\sum_j e^{\beta x_{ij}} \Big/ \sum_j e^{\beta x_{ij}^*} \right] \right\} \Big/ \beta^C \tag{11}$$

where x^* is a matrix of attributes of new state and β^C is the marginal utility of income, which is equivalent to the negative of marginal utility of travel cost (that is, β_1 in table 1).

For the RPL, where parameters are random, we need the expectation over the parameters for the welfare measure (Train 1998). The mean compensation variation for individual i:

$$CV_i = \int_B C_i(t) f(t) dt \tag{12}$$

[4] As we shall see below, the prior determination of the sign of the covariate is not without problems. The distribution of the parameter γ is skewed to the right.
[5] This hypothesis can also be tested using a likelihood ratio test.

Table 1
RPL Compared to the Conditional Logit: Chesapeake Bay Beach Trips

Variable	Logit	RPL 1	RPL 2	Parameters
TC	–0.0435	–0.0466	–0.0464	β_1
	(–6.24)	(–6.57)	(–6.60)	
TT	–0.0535	–0.0605	***–0.144***	$\bar{\gamma}_2$ or β_2
	(–2.24)	(–2.48)	***–3.28***	b_2
			(–4.25)	
			1.64	σ_2
			(2.43)	
WP	–0.0600	***–0.125***	***–0.135***	$\bar{\gamma}_3$ or β_3
	(–4.23)	***–2.47***	***–2.39***	b_3
		(–10.5)	***(–10.3)***	
		0.884	***0.887***	σ_3
		(4.80)	***(5.11)***	
QI	0.440	0.430	***0.430***	$\bar{\gamma}_4$ or β_4
	(8.01)	(7.98)	***–0.843***	b_4
			(–6.62)	
			0.0027	σ_4
			(0.017)	
Log-likelihood	–773.28	–767.07	–765.05	

Note: Values in parentheses are the ratio of parameter estimate to standard error. Random parameter estimates are in bold and italic.

where $f(.)$ is the probability density function of β. Simulation is used to calculate the value of the compensating variation because of the lack of a closed form solution to the integral. Thus welfare calculation is also computer-demanding and time-consuming.

We provide estimates of the sample mean of CV for four scenarios relating to the Chesapeake beaches:

Scenario 1: Eliminate Point Lookout
Scenario 2: Eliminate Sandy Point
Scenario 3: Double WP at Breezy Point
Scenario 4: Double WP at all sites

The first two scenarios do not specifically exploit the random parameters. Although simulation is necessary to calculate the welfare measure, the random coefficient is not directly related to the scenario itself. In the last two scenarios, the parameter on WP is random and so the welfare is directly connected. The welfare estimates are presented in table 2. For scenarios 1 and 2 the welfare estimates for logit and RPL's are virtually identical because the means of the parameters are quite close. In these two scenarios, the two methods give the same answer because nothing about the randomness prevents the model from predicting the choice proportions correctly, and that is what the welfare estimates depend on.[6] But when we examine the other cases, we see that the RPL is considerably bigger in absolute value, meaning the losses are larger in the RPL case. This follows from the differences in coefficients. The mean values of the covariate effects are larger (in absolute value) in the RPL. In table 1,

[6] The loss from eliminating a site j can be written as $-\log(1 - \pi_j)/\beta^C$ where π_j is the probability of choosing site j.

Table 2
Welfare Estimates for Conditional Logit and RPL

	Dollars	Ratios of RPL to logit	
Model	Logit	RPL1	RPL2
Scenario 1	–$1.87	*1.06*	*1.04*
Scenario 2	–$3.55	*0.96*	*1.00*
Scenario 3	–$0.63	*1.71*	*1.77*
Scenario 4	–$4.91	*1.62*	*1.79*

Note: Sample means of individual CV_i.

the mean coefficient on WP for the RPL is at least twice as big as the coefficient from the conditional logit. We have no convincing explanation of the larger parameters of the RPL. The log-normal density function, which shifts all the weight of the observations on WP to the negative range, remains a suspect.

The Random Parameters Logit Model with Alternative Sampling

In some cases with large numbers of alternatives, one has to sample to estimate the logit. Consequently, it is also necessary to sample alternatives to estimate the RPL. However, while there is theoretical support for consistency of parameter estimates in sampling of alternatives in the conditional logit (see McFadden 1978; Ben-Akiva, and Lerman 1985) no such theoretical support has been provided for the RPL. In the absence of theoretical support, we investigate the issue empirically. We consider two cases. The first is sampling of alternatives in the Northeast MRFSS data set. RPL estimation using all alternatives is not feasible for this data set and thus RPL with sampling is used. Logit with sampling is also estimated for the purpose of comparison. The second involves the Chesapeake beach data set, in which the number of alternatives is sufficiently small to permit the comparison of sampled alternatives with the universal choice set.

The Northeast MRFSS Data Set

This survey, the Marine Recreational Fishing Statistical Survey (MRFSS), was conducted by the National Marine Fisheries Service (NMFS) in 1994.[7] It is a combination of onsite and phone surveys covering the states from Maine to New Jersey. Hicks, Steinback, Gautam, and Thunberg explain the survey and analyze various nested logit models for this data set.

The assembled data set contains 819 alternatives, 2,976 observations, and with a reasonable specification, 10 variables. With this data set, one cannot estimate a conditional logit using a Pentium II PC. Thus it is impossible to run RPL using this data set and the same model specification. The solution in either

[7] We thank Rob Hicks for allowing us access not just to the data, but to the dataset prepared for random utility estimation. Anyone who has estimated a random utility model from a large dataset like the MRFSS knows that the most difficult is simply preparing the dataset for estimation.

case is to sample without replacement among alternatives. The theoretical maximum number of feasible alternatives per trip is 819 (63 sites and 13 mode/species combinations).[8] The site choice set is initially restricted by distance to reduce the maximum number to a feasible set.[9] The actual number of feasible alternatives ranges from 65 to more than 500.

The purpose of this estimation is to determine empirically how the RPL performs with sampling. However, since the model cannot be estimated with the full or universal choice set, we will compare it with a conditional logit model. We know that the parameters from a conditional logit are consistent. Further, we know that logit parameter estimates tend not to change much as the sampled alternatives increase (see Parsons and Kealy). Hence we can gain some insight into the behavior of the RPL under the sampling of alternatives by comparing it with a logit estimated with sampled alternatives.

In the model estimated, we choose the specification similar to Hicks *et al*. but we estimate a conditional logit, rather than a nested model. The original model was an nested logit model with overlapping nesting structure. We use all variables and, for each trip, sample seven alternatives among actual feasible alternatives and add the chosen alternative to make the choice set. With this data set, we estimate a logit model and two RPL models.[10]

Assume that the individual i's indirect utility for alternative j has the following form

[8] The three modes and thirteen mode/species combinations are the following:

Mode	Species Group
Party/Charter	Big Game
	Small Game
	Bottomfish
	Flatfish
	Not Targeting
Private/Rental	Big Game
	Small Game
	Flatfish
	Not Targeting
Shore	Small Game
	Bottomfish
	Flatfish
	Not Targeting

Source: Hicks *et al.* (1999), p. 13, table 3.1.

[9] If the closest site is within 30 miles from the angler's home then all sites within 150 miles are assumed to be in their choice set; otherwise, all sites within 400 miles are assumed to be in their choice set. This method of defining choice sets is based on the idea that 400 miles is the outer limit of one way travel for a day trip. But when people live close to the other sites, they have a much denser choice set, even when the one way distance is only 150 miles.

[10] For this paper, we estimated a variety of models including different random parameters using these two data sets. We use Pentium II PC with 400 MHz CPU and Pentium Pro PC's. The program we use is GAUSS and Maxlik procedure in GAUSS. The run time ranges from less than one minute for logit using the MD beach dataset to hours for some RPL models either using the MD beach data set or the Northeast dataset. When using the Northeast dataset, we sometimes encountered insufficient memory problems even using a Pentium II PC with 400 MHz CPU. Thus we have to use a portion of the dataset and do sampling among alternatives. Even so, it is still extremely time consuming to do the estimation. The assembling and sampling processes also take time. When calculating the compensating variation, we have to read the data set in a couple of steps because we have to use the full data set. These processes are extremely time consuming. Thus we did not estimate the Northeast model a couple of times, nor did we test the sensitivity of RPL toward the number of drawn alternatives as for the MD beach dataset.

$$\begin{aligned} U_{ij} = {} & \beta_1 TC_{ij} + \beta_2 TT_{ij} + \beta_3 LnM_{ij} + \beta_4 BGCR_j \\ & + \beta_5 SGCR_j + \beta_6 BTCR_j + \beta_7 FFCR_j + \beta_8 NSCR_j \\ & + \beta_9 PRDummy_{ij} + \beta_{10} CPRDummy_{ij} + \varepsilon_{ij} \end{aligned} \tag{13}$$

where $j = 1,\ldots,819$ choices among alternatives in the Northeast MRFSS data set. The variables are

TC = Travel cost
TT = Travel time
LnM = Log of the number of NMFS interview sites in aggregate sites
BGCR = Big game historical catch rate
SGCR = Small game historical catch rate
BTCR = Bottom fish historical catch rate
FFCR = Flat fish historical catch rate
NSCR = Nonseeking historical catch rate
PRDummy = equals 1 if Private/Rental mode and individual owns a boat, otherwise zero
CPRDummy = Cold Private/Rental boat ownership dummy, equals 1 if PRDummy = 1, and wave = 6, otherwise zero

In each of the catch rate variables, the catch rate is operative only if the angler is seeking the species. Thus the catch rate variables, BGCR, SGCR, BTCR, FFCR, and NSCR, have nonzero values only for trips in which the individual angler sought the species. For example, SGCR has nonzero values only for trips targeted small game fish. This means that when an angler seeks small game, the only catch rate that matters is the small game catch rate.

Hicks *et al*. use two variables to explain how individual's choose among the mode/species combinations. One who owns a boat is more likely to choose the private rental modes. This is captured by the variable, PRDummy. However, if the fishing activity occurs in wave 6 (November-December), the cold weather is likely to dampen the effect of owing a boat and choosing the private rental mode. This is captured by the variable, CPRDummy.

We estimate three models: logit, RPL1 with a random parameter for BGCR, and RPL2 with random parameters for TC, TT, and SGCR. Each model generates parameter estimates with anticipated signs; most parameters are significant at 99% level. Most nonrandom parameters are similar among the three models. However, the coefficient on BGCR of RPL1 is about four times its logit counterpart while other random parameters are similar to the logit estimates.

The policy scenario is to increase the small game catch rate by 50%. The compensating variations of logit and RPL1 are the same. In the calculation of CV for trips targeting small game, the values of the BGCR variable, the only random parameter in RPL1, are zero and the other parameters for the logit and RPL1 are close. Thus, the compensating variations calculated from these two models are very close. However, the compensating variation of RPL2 is 24% larger than the logit and RPL1.

The mean of the travel cost coefficient of RPL2 is larger (in absolute value) than for RPL1 and the conditional logit. Intuition tells us that the welfare effect should be smaller because this parameter enters the denominator of the formula of compensating variation and a larger denominator should generate a smaller compensating variation. The problem with making the travel cost coefficient random is that

Table 3
RPL with Sampling when the Number of Alternatives is Large

	Parameter Estimates			
Variables	Logit	RPL 1	RPL 2	Parameters
TC	–0.0546	–0.0554	***–0.0674***	$\bar{\gamma}_1$ or β_1
			–0.581S	b_1
			0.612S	σ_1
TT	–0.943	–0.971	***–0.974***	$\bar{\gamma}_2$ or β_2
			–0.173N	b_2
			0.542	σ_2
LnM	1.21	1.24	1 R	β_3
BGCR	3.7	***11.5***	3.8	$\bar{\gamma}_4$ or β_4
		–5.28S		b_4
		1.28S		σ_4
SGCR	2.51	2.51	***2.77***	$\bar{\gamma}_5$ or β_5
			–1.32S	b_5
			–0.273S	σ_5
BTCR	1.84	1.86	1.93	β_6
BFFCR	2.94	2.96	3.05	β_7
NSCR	2.50	2.51	2.56	β_8
PRDummy	1.24	1.26	1.27	β_9
CPRDummy	–0.26N	–0.27N	–0.26N	β_{10}
Log-likelihood	–882.3	–871.9	–877.9	
Welfare	$1.91	$1.91	$2.37	

Notes: Each choice set composed of eight alternatives: seven sampled sites and the chosen site.
N: Not significant at 95% level. The rest are significant at 99% level except the standard deviation of SGCR in RPL2, which is significant at 95% level.
R: Restricted. The coefficient on ln M is constrained to equal 1.
Random parameter estimates are in italics and blocked.
S: These estimates are estimated with scaling the covariates at various levels. The scale factors associated with the random parameters are: TC: 0.1; BGCR: 1000; SGCR: 10. For example, variable TC is multiplied by 0.1. We report the *b*'s and σ's at their estimated values. However, the means, $(\bar{\gamma}_1 \bar{\gamma}_4 \bar{\gamma}_5)$, are rescaled so that these means can be directly compared to estimates which are estimated without scaling**.** We report the results this way because the estimation of RPL1 without scaling does not converge and one cannot rescale *b*'s and σ's of a log-normal distribution.

it enters the denominator, and computing welfare measures essentially means computing the expected value of the reciprocal of the random variable, which in this case is log-normal. Consequently draws close to zero, though they occur with low probability, give very high values of CV, because they imply very small marginal utility of income. This characteristic overrides the larger mean value for the travel cost coefficient, so RPL2 generates a larger compensating variation than the other two models. The results could be different with other distributions, such as a normal. However, depending on the parameter estimates, a normal might lead to some draws with a negative marginal utility of income. All this suggests some peril in making the marginal utility of income a random parameter.

Judged strictly from the parameter estimates, the sampled logit and RPL's are close but not identical. The issue is not so much the difference in parameter estimates from the RPL's but in the various assumptions that are embodied in the random parameters and the implications of these assumptions for welfare calculations.

We can see that parameter estimates are not greatly different for a sampled logit and two sampled RPL's. However, we cannot make any firm conclusions about the effects of sampling because the data set is too large to permit a comparison with the full choice set, or even a choice set made of much larger sampling of alternatives. In an effort to understand how the RPL estimated from sampled alternatives compares one estimated with full set of alternatives, we return to the more manageable Chesapeake beach visit data set.

Comparing Sampled RPL and Conditional Logit with Models Using a Full Choice Set

Four models are compared in this section: full set logit, full set RPL, logit with sampling, RPL with sampling. The full set includes ten sites. For each choice occasion, we randomly sample, without replacement, three out of nine alternatives. These sampled alternatives together with the chosen alternatives become the new data set in which each angler has four alternatives. Because the sample of alternatives can cause variation in the estimates, we control by sampling repeatedly. Repeating the process fifteen times, we get fifteen sampled data sets. Then we estimate fifteen logit models and fifteen RPL models (with the same specification as in table 1) and take the means of estimated parameters and compensating variations. This is done in order to reduce the sample variations. The sampled logits are denoted L_{15} and the sampled RPL's are R_{15}. We also estimate the same model specification with the full data set of 10 alternatives per individual. These results are denoted L_F (full set logit) and R_F (full set RPL). Following the base case where we analyze four choice alternatives with fifteen repetitions, we show that these are not seriously limiting by choosing six and eight alternatives and by analyzing 100 repetitions of the four alternative choice sets.

In this instance, we specify the coefficient on the water quality variable (WP) as a log-normally distributed random parameter because the effect of this parameter is most likely to be individual-specific, and thus this parameter is most suitable to be a random parameter. Finally the ratios of parameters and compensating variations are taken from different models to see how they vary.

Table 4 provides estimates of parameters and standard errors for the full choice set and ratios and root mean squared errors of these estimates for the sampled choice sets. Recall that for each draw, the individual's choice set is composed of a sample of three alternatives, plus the chosen alternative. Two diagnostics for the estimated coefficients are provided: ratios and relative mean squared error (RMSE). The ratios are simply the ratio of the mean parameter estimate from the sampled data set to the parameter estimate from the full data set. The ratio can be quite close to one, which would be the ideal value for sampling, but individual draws could be off considerably if they cancel in either direction. To provide for this possibility we also calculate a kind of mean squared error, which is the root of the sum of squared deviations from the full set parameter, divided by the full set parameter. Column 3 gives the ratios and RMSE's for sampled relative to full choice set results for the logit, while the 4th column gives the same ratios and RMSE's for the RPL. For both models, there is a tendency to underestimate the travel cost and travel time coefficients, though RPL_{15} is very close to R_F for the important coefficient on travel cost. Also we see for the first two coefficients the influence of sampling on efficiency. The second quantities in each column are the ratios of t-statistics. They are lower for the sampled case, with no discernible difference between the logit and the RPL. Despite the more complicated error structure for the RPL, there seems to be no more loss in efficiency for the RPL versus the logit, at least in this limited case.

Table 4
Comparison of Parameters of Four Types of Models

	Estimated Coefficients		Ratios and RMSE's of Estimated Coefficients						
	(1)	(2)	(3)		(4)		(5)	(6)	Parameters for RPL and Logit
	L_F	R_F	L_{15}/L_F	RMSE[a]	R_{15}/R_F	RMSE	L_{15}/L_F	R_F/L_F	
TC	–0.0435	–0.0466	*0.88*	*0.45*	*0.97*	*0.11*	*0.82*	*1.07*	β_1
	(–6.24)[b]	(–6.57)	(*0.79*)[d]	—	(*0.80*)	—	(*0.75*)	(*1.06*)	
TT	–0.0535	–0.0605	*0.76*	*0.94*	*0.73*	*1.05*	*0.67*	*1.13*	β_2
	(–2.24)	(–2.48)	(*0.70*)[d]	—	(*0.66*)	—	(*0.63*)	(*1.11*)	
WP	–0.0600	**–0.125**	*1.16*	0.*62*	*1.53*	*2.23*	*0.55*	*2.08*	$\bar{\gamma}_3$ or β_3
	(–4.23)	**–2.47**	*1.16*	—	*0.90*	—	—	—	b_3
	—	**(–10.5)**	(*1.03*)[d]		(*0.91*)	—	—	—	
	—	**0.884**	—	—	*1.23*				σ_3
	—	**(4.8)**	—	—	(*1.10*)[d]				
QI	0.440	0.430	*1.22*	*0.85*	*1.36*	*1.38*	*1.25*	*0.98*	b_4
	(8.01)	(7.98)	(*1.03*)[d]	—	(*1.06*)	—	(*1.03*)	(*1.00*)	

Notes: [a] The full choice set is composed of all ten sites; the sampled choice set is composed of four sites: a sample of three sites and the chosen site.
[b] RMSE = $[\sum_{i=1}^{15}(\beta_i^c - \beta^F)^2 / 15]^{1/2} / \beta^F$ where β_i^c is the parameter estimate from the *i*th draw of case *c* where $c = L_{15}$ or R_{15} and β^F is the corresponding full choice set estimate.
[c] Values inside parentheses are t values.
[d] These rows to the right of column 2 are ratios of t-statistics for the sampled model to the t-statistics for the full model.
L_F: Full set logit.
R_F: Full set RPL.
L_{15}: Mean of 15 logit with sampling.
R_{15}: Mean of 15 RPL with sampling.

In terms of parameter estimates, the ratios R_{15}/R_F are bigger than L_{15}/L_F. The mean parameter estimate for WP for R_{15} is 1.53 times the R_F mean estimate. At least in this case, there is some evidence that the RPL does not get as close to the full choice set with samples as the logit does. When we look at the dispersion of parameter estimates, as measured by the RMSE, we see that the RPL has smaller RMSE for the TC parameter, about the same for TT, but much larger for the random parameter.

We analyze the same four policy scenarios that were explored in the third section. These are shown in table 5. Column 4 shows the ratio of the sampled RPL to the full choice set for the RPL as well as the RMSE. Even for scenarios 3 and 4, which involve increases in water pollution and thus depend directly on the random parameter, the maximum ratio is 1.14. This is much smaller than the maximum ratio of parameters, 1.53 in table 4. Thus in terms of sampling, the RPL seems to provide similar welfare estimates to the full choice set. Further, there is less dispersion of welfare estimates from the RPL than from the logit, despite the fact that the parameter estimates were more dispersed. All of the RMSE's for the RPL are lower than RMSE's for the logit. The difference as measured by the ratio of welfare estimates between the RPL and logit is seen to be similar, regardless of the sampling. Note that in columns 6 and 7 we have the logit to RPL welfare results. Whether with sampling or with the full choice set, the conditional logit yields welfare losses that are about 55% to 59% of their RPL counterparts. In this case, there is some evidence that the RPL gives systematically higher welfare measures. This is most likely due to the log-normal distribution of the random parameters.

Table 5
Comparison of Welfare of Four Types of Models

	Losses in Dollars		Ratios and RMSE's of Welfare Estimates						
	(1)	(2)	(3)		(4)		(5)	(6)	(7)
	L_F	R_F	L_{15}/L_F	RMSE[a]	R_{15}/R_F	RMSE	L_{15}/R_F	L_{15}/R_{15}	L_F/R_F
Scenario 1	–$1.87	–$2.25	*1.26*	*1.02*	*1.14*	*0.52*	*1.05*	*1.04*	*0.96*
Scenario 2	–$3.55	–$3.33	*1.13*	*0.50*	*1.04*	*0.17*	*1.20*	*1.15*	*1.00*
Scenario 3	–$0.64	–$1.48	*1.47*	*1.83*	*1.14*	*0.55*	*0.63*	*0.55*	*0.56*
Scenario 4	–$4.91	–$9.43	*1.25*	*0.97*	*1.09*	*0.36*	*0.65*	*0.59*	*0.56*

Notes: [a] RMSE = $[\sum_{i=1}^{15}(C_i^c - C^F)^2\ /\ 15]^{1/2}\ /\ C^F$ where C_i^c is the mean CV for draw i, case c (L_{15} or R_{15}) and C^F is the corresponding full choice set estimate.
Scenario 1: Close Point Lookout.
Scenario 2: Close Sandy Point.
Scenario 3: Double WP at Breezy Point.
Scenario 4: Double WP at all sites.
L_F: Full set logit.
R_F: Full set RPL.
L_{15}: Mean of 15 logit with sampling.
R_{15}: Mean of 15 RPL with sampling.

In the previous analysis, the sampled choice sets are composed of 4 alternatives: 3 of 9 sampled sites and the chosen site. We account for the potential sampling variability by repeating the process 15 times and averaging over the repetitions. To test whether our results are dependent on this configuration of 4 alternatives and 15 repetitions, we extend the sampling of alternatives to two cases: 5 and 7, which together with the chosen site makes a choice set with 6 and 8 alternatives. Then to test the effect of 15 repetitions, we repeat the experiments (for four alternatives) 100 times. We show the impact of these different experiments in table 6, where we go to the welfare results only, skipping the parameter estimates. This table has the same structure as table 5, except the mean square is not provided.

The correct measure of losses from the full set for the scenarios is repeated in columns 1 and 2. For comparison, consider scenario 1, for 6 sites, for the logit and RPL. Compared to 4 sites, the ratio of L_{15}/L_F goes from 1.26 to 1.11 and the ratio of R_{15}/R_F goes from 1.14 to 0.98. In both cases they get closer. If we compare the repetitions result, we look at columns 7 and 8 in table 6 and compare with the appropriate quantities from table 5. For 100 repetitions, the logit ratio of sampled to full goes from 1.26 to 1.25 while the RPL ratio goes from 1.14 to 1.12. Hence the number of repetitions appears to be only a small contributor to variation in welfare estimates. Further, there is no indication in the changing repetitions results that the RPL behaves differently from the logit. While there may be some variation due to sampling of alternatives, it is fairly rapidly eliminated by averaging several draws, at least in the 10 alternatives case considered here.

Conclusion

The attraction of the random parameters model is its greater generality. It allows the marginal utility of attributes to vary across individuals. Logit models with random parameters do not have the independence of irrelevant alternatives characteristic.

Table 6
Comparison of Welfare Measurements

	Losses in Dollars		Ratios of Welfare Estimates, 6 sites		Ratios of Welfare Estimates, 8 sites		Ratios of Welfare Estimates, 4 sites	
	(1) L_F	(2) R_F	(3) L_{15}/L_F	(4) R_{15}/R_F	(5) L_{15}/L_F	(6) R_{15}/R_F	(7) L_{100}/L_F	(8) R_{100}/R_F
Scenario 1	–$1.87	–$2.25	*1.11*	*0.98*	*1.14*	*0.99*	*1.25*	*1.12*
Scenario 2	–$3.55	–$3.33	*1.04*	*0.98*	*1.03*	*0.96*	*1.12*	*1.04*
Scenario 3	–$0.64	–$1.48	*1.69*	*1.13*	*2.33*	*1.36*	*1.47*	*1.12*
Scenario 4	–$4.91	–$9.43	*1.40*	*1.07*	*1.73*	*1.21*	*1.25*	*1.09*

Notes: The scenarios are the same as in table 5.
L_F: Full set logit.
R_F: Full set RPL.
L_{15}: Mean of 15 logit with sampling.
R_{15}: Mean of 15 RPL with sampling.
L_{100}: Mean of 100 logit with sampling—4 alternatives.
R_{100}: Mean of 100 RPL with sampling—4 alternatives.

The greater generality comes at the cost of more time and computer space in the estimation of RPL's. These imply a greater need to sample alternatives in the common situation in recreation economics where there are many alternatives.

We have investigated two issues bearing on the use of the RPL. First, are RPL parameters and welfare estimates substantially or systematically different from logit models? If the answer to both questions is no, then the greater generality of the RPL may be of little practical value, because the models give the same results. In the two cases we have studied the RPL tends to give higher welfare losses from the constructed scenarios. The estimated parameter values tend to differ, too. Whether these results stem from the log-normal distribution of the random parameters, or are the consequence of the greater generality bears more study.

Concerning the behavior of the RPL under sampling of alternatives, our most instructive results stem from comparing estimating sampled alternative models with a full choice set for the Chesapeake Bay beach data. There we find that the sampling of alternatives does not systematically or substantially alter the RPL results. In fact, in terms of welfare estimates, the RPL is less dispersed than the logit. For all four welfare scenarios, the mean welfare estimate for the sampled RPL is no more than 14% different from the full set RPL, compared with the logit, which differs by as much as 47% from the full set logit. Consequently there is no evidence of systematic inconsistency in sampling alternatives with the RPL. From sensitivity analysis of RPL with sampling with respect to the number of repetitions, we found 15 repetitions is enough to reduce the sample variation. With 15 repetitions, most parameter estimates and welfare measurements do not deviate more than 1% and none deviate more than 7% from their counterparts using the full set.

Attempting to answer basic questions about sampling with the RPL has led to a variety of other questions. One issue that is evident and needs considerable exploration is the effect of the choice of distributions for the random parameters. We have chosen only log-normal distributions for our random parameters. It is evident that in several cases the results are sensitive to this choice. Further, we have not introduced correlation among parameters. More study of the impacts of these distributional assumptions will broaden our understanding of the random parameter logit.

References

Ben-Akiva, M., and S.R. Lerman. 1985. *Discrete Choice Analysis: Theory and Application to Travel Demand*. Boston, MA: MIT Press.

Bockstael, N.E., M. Hanemann, I.E. Strand. 1986. Measuring the Benefits of Water Quality Improvement Using Recreation Demand Models. Environmental Protection Agency. Co-operative Agreement CR-81143-01-1.

Haab, T., and R. Hicks. 1997. Accounting for Choice Set Endogeneity in Random Utility Models for Recreational Demand. *Journal of Environmental Economics and Management* 34(2):127–47.

Herriges, J.A., and D.J. Phaneuf. 1999. Controlling Correlation across Choice Occasions and Sites in a Repeated Mixed Logit Model of Recreation Demand. Working paper, pp. 32.

Hicks, R., S. Steinback, A. Gautam, and E. Thunberg. 1999. Volume II: The Economic Value of New England and Mid-Atlantic Sportfishing in 1994. NOAA Technical Memorandum pp. 1–26. NMFS Office of Science and Technology, Silver Spring, MD.

Hicks, R., and I.E. Strand. 1999. The Extent of Information: Its Relevance for Random Utility Models. *Land Economics* In press.

McFadden, D. 1978. Modeling the Choice of Residential Location. *Spatial Interaction Theory and Planning Models,* A. Karlqvist, L. Loudqvist, F. Snickars, and J.W. Weibull, eds. pp. 75–96. Amsterdam: North-Holland.

Parsons, G.R., and M.J. Kealy. 1992. Randomly Drawn Opportunity Sets in a Random Utility Model of Lake Recreation. *Land Economics* 68(1):93–106.

Parsons, G.R., M. Massey, and T. Tomasi. 1999. Familiar and Favorite Sites in Random Utility Models of Recreation Demand. Working paper, pp. 1–16.

Train, K.E. 1998. Recreation Demand Models with Taste Difference over People. *Land Economics* 74(2):230–39.

___. 1999. Mixed Logit Models for Recreation Demand. *Valuing Recreation and the Environment: Revealed Preference Methods in Theory and Practice,* J.A. Herriges and C.L. Kling eds. pp. 121–40. Northampton, MA: Edward Elgar Publishing Ltd.