Measuring the gains from management of spatially heterogeneous resources: the case of groundwater

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Denver, Colorado, August 1-4, 2004

ABSTRACT

We develop a model for the dynamic management of spatially heterogeneous resources with multiple users. We apply our model to the case of groundwater and show that – contrary to the results of existing studies – even when externalities are highly concentrated in space, significant efficiency gains are possible over competitive outcomes.

1. INTRODUCTION

Throughout the world, groundwater resources are a major source of agricultural, potable and industrial water. Because groundwater is frequently viewed as private property, its extraction is essentially unregulated in many regions of the world. As a result of rapidly falling groundwater levels in many of these regions, the public perception is that rapid overextraction and resource depletion is occurring. This perception is used to justify regulatory intervention in the form of basin adjudication, pumping fees and quotas.

To the economist, the stylized elements of this story – a natural resource with multiple, myopic users and resultant overextraction – easily conform to a "tragedy of the commons." Indeed, for the last fifty years, economists have viewed groundwater as the archetypal common property resource (for example, among many others, [6, 15, 21, 25]). The intertemporal allocation of groundwater was one of the earliest applications of optimal control and dynamic programming techniques to economics [6, 8]. Numerous studies have analyzed the externalities that multiple resource users impose on each other by pumping water from an aquifer. A large body of work offers clear policy prescriptions – pumping taxes or quotas – that align competitive and socially optimal groundwater extraction rates.

Some major economic studies find that the quantitative difference between competitive and socially optimal groundwater management outcomes is negligible [15, 17]. Thus, even though groundwater is modeled as a common property resource, there appears to be no economic rationale for groundwater management. This clearly conflicts with management experience, which suggests impending crisis in many groundwater basins.

We suggest that most economic studies of groundwater have ignored the basic principles of hydrology. In particular, groundwater systems do not adjust instantaneously to changes in pumping rates, but diffuse gradually in response to developing pressure gradients. The response of an aquifer to pumping is complex, even under simple geological conditions. Aquifer behavior exhibits extreme heterogeneity: actions taken by one resource user have disparate effects across space and time on other users. Thus, externalities are idiosyncratic and reflect not only each user's sequence of extraction decisions and the physical properties of the aquifer, but also the explicit spatial relationship between users. Existing theoretical models of groundwater extraction in the economics literature fail to capture the spatial heterogeneity of actual groundwater resources. As a result, they overstate the degree of commonality between users of groundwater. In this paper, we argue that misspecification of the physics of groundwater flow has serious consequences and necessarily leads existing studies to inappropriate policy prescriptions. We develop a model of resource extraction with dynamic, idiosyncratic externalities that contains hydrologically correct aquifer response equations.

We show that under some circumstances, there is indeed little difference in the welfare obtained with competitive versus optimal groundwater management policies. However, there also exist realistic circumstances under which a lack of groundwater management will entail significant welfare losses. The magnitude of such losses will depend on the physical parameters of the system as well as the exact spatial relationships between resource users.

Furthermore, we argue that groundwater is not, in general, a common property resource. Nevertheless, we demonstrate that significant welfare gains can be achieved through groundwater management. The explanation of this apparent paradox is that in reality, the externalities caused by groundwater pumping are extremely concentrated in space. In other words, despite limited commonality among the global set of resource users, significant bilateral externalities exist between users located close to each other.

We present a model for the optimal management of groundwater over space and time with multiple resource users. Our model includes several major extensions to the existing literature. First, the model is spatially explicit and allows the externalities imposed by resource users to depend on their distance from each other. Second, the model allows lagged effects, permitting analysis of diffusional systems where the effects of actions taken at one point in the resource take finite time to reach other points in the resource. Third, the model incorporates hydraulic response equations, governing the behavior of groundwater over space and time, from the engineering literature.

In Section 2 of this paper, we review the existing literature on the economics of groundwater extraction. The following section presents the general dynamic optimization model used and derives optimal and competitive resource use paths. In

Section 4, we describe the integration into the economic model of equations that correctly describe the physical groundwater system. The results of welfare analyses are presented in Section 5. The last section concludes and discusses the conditions under which a role for groundwater management exists.

2. THE ECONOMICS OF GROUNDWATER EXTRACTION

The earliest economic studies of groundwater pumping date from the late 1950s and early 1960s [21, 26]. These authors understood that extraction of groundwater from a single reservoir might involve an externality. Where there were many users of a groundwater resource, each would fail to consider the present and future effects of their own actions on all other users, leading to overpumping and welfare losses. In translating this qualitative description of a dynamic common property problem into an analytical framework, later studies made a key assumption about the physical behavior of the underlying resource. The effects of each user's extraction were assumed to be transmitted identically to all users. Aquifer representations that use this assumption are commonly known as 'single-cell' or 'bathtub' models and form the analytical basis of groundwater economics.

Early studies used dynamic programming and optimal control theory to derive qualitative decision rules for the optimal intertemporal management of single-cell aquifers [6, 8]. More recent, and influential, studies have sought to quantify the welfare differences between competitive and optimally managed groundwater extraction paths using parameters from real aquifers [1, 15, 17]. These studies have suggested that the welfare differences between optimal control and competitive outcomes are negligible, obviating the need for any centralized intervention in groundwater systems. Critiques of this body of work have considered how the discount rate, possible changes in demand over time and technological adoption might affect the gains from optimal control over no management [4, 7]. However, even significant structural modifications to the groundwater extraction problem generally produce only modest welfare gains to optimal management in single-cell aquifer models. Furthermore, recent applications of dynamic game theory to the problem of groundwater extraction suggest that strategic users exploiting a single-cell aquifer can more closely approach an optimal solution by playing a non-cooperative game [11, 22, 25].

By definition, the spatial location of users is irrelevant in single-cell models, as the current state of the resource is entirely captured by a single scalar variable, usually either the depth to water or the stock of water remaining. Although this leads to analytical tractability, it is clearly incorrect to assume that two wells one hundred feet apart will have the same effect on each other as wells that are ten miles apart. A number of economic studies have attempted to capture this spatial heterogeneity of groundwater resources.

In so-called “two-cell” aquifer models, several adjacent groundwater basins are mutually connected by porous boundaries, so that water is allowed to flow between them [13, 16, 20, 34]. Each individual basin behaves as a single-cell aquifer, and may have hydrological properties and groundwater stocks that differ from its neighbors. However, in two cell models, all users within each subcell still have uniform effects on each other and are not resolved spatially.

Conjunctive use models, where users simultaneously exploit surface and subsurface reservoirs, are analogous to two-cell models [9, 10, 18, 30, 31, 32]. The porous boundary between ‘cells’ of a conjunctive use model is the canal system linking a surface water reservoir and an aquifer. Surface water reservoirs, by definition, are single-cell systems; in conjunctive use models, the aquifer is also modeled as a single cell. Thus, as with standard two-cell models, all water users, irrespective of location, have the same marginal effect on the underlying aquifer.

Finally, aquifers may be modeled as multi-cell systems where the flow between cells is dictated by finite difference approximations based on the partial differential equations governing the flow of groundwater [2, 3, 23, 24]. Typically, such analyses are case studies of specific groundwater basins and are carefully calibrated using hundreds of parameters. Because of this, many of these studies do not involve any economic optimization, but instead compare simple rule-of thumb management policies [2, 3]. The few studies that do involve dynamic optimization have used regression on simulation outputs in order to linearize relevant parameters [23, 24]. Because of their complexity, multi-cell aquifer models may provide management guidelines for specific groundwater basins, but do not provide general welfare conclusions.

The model we present in this paper is spatially explicit and hydrology-based. Rather than uniformity throughout the resource – as in a single-cell model – groundwater levels are allowed to vary continuously across space in response to local conditions. In our dynamic model, each user’s resource use has an idiosyncratic effect on other users. Externalities vary across space and time as a function of the explicit spatial relationship between users as well as their extraction decisions. We fully integrate hydraulic response equations from the groundwater hydrology literature [12, 14, 29, 33] into this analytical model in order to capture the spatially heterogeneous behavior of aquifers. This approach allows us to analyze how both the spatial relationship of users to each other and underlying resource heterogeneity affect the welfare gains from groundwater management.

Our analytical approach is similar to several recent models in the fishery economics literature [27, 28]. In these spatially explicit models, the fishery resource is composed of heterogeneous "patches" of fish population with varying biomass and dispersal properties. However, these bioeconomic models use steady-state analyses and do not obtain closed-form solutions for relevant optimality conditions [27, 28]. Conversely, in our analysis, we are able to develop closed-form solutions to the dynamic optimization problem.

3. THE BASIC MODEL

Consider a groundwater resource with *J* separate users that are spatially distributed above the resource with fixed, known locations. Assume that each user $j = 1, \ldots, J$ operates a single well and must decide how much water to extract from the aquifer during each decision period. Users derive per-period benefits from the use of water given by the function $f_j\left(u_{jt}\right)$, where u_{jt} is the amount pumped by user *j* in period *t*. Assume that there are decreasing marginal returns to pumping groundwater, so that $f_j'\left(u_{jt}\right) > 0$ and that $f_j''\left(u_{jt}\right) < 0$.

Because water must be lifted from the top of the aquifer to the ground surface, pumping is costly. The pumping lift for user *j* at time *t* is given by x_{jt}. Defining the average and marginal cost for pumping a unit of groundwater through a unit of vertical distance as C, the total per-period benefit net of extraction costs for each user j is given by

$$\pi_{jt} = f_j\left(u_{jt}\right) - C u_{jt} x_{jt} \tag{0.1}$$

The aggregate net benefit is given by the sum of individual benefits, appropriately discounted over the time horizon of interest.

Pumping water from a well will result in the water surface in that well decreasing, or being drawn down. Thus, as a result of ongoing resource extraction by groundwater users, the pumping lifts through which each unit of water must be raised will change. The evolution over time of water levels in the aquifer, and hence the net benefit of resource use, is determined by users' extraction decisions. These will depend on the assumptions that each user makes about both the physical behavior of the resource and the role of other users in its exploitation. Below, we consider three different solution concepts, corresponding to different user behavior, for the multi-user groundwater extraction problem: optimal, competitive and myopic, and competitive with limited foresight.

3.1. Optimal extraction

The optimal extraction decisions by all users maximize aggregate net benefit and form a baseline from which to compare alternative user behaviors. If the per-period discount factor is β and the planning horizon spans N periods, then the vector of welfare-maximizing groundwater extraction paths, $\mathbf{u}^*$, is defined by

$$\mathbf{u}^* = \begin{bmatrix} u_{11}^* \\ \vdots \\ u_{JN}^* \end{bmatrix} = \underset{u_{jt} \in \mathbb{R}_+}{\arg\max} \sum_{j=1}^{J} \sum_{t=1}^{N} \beta^t \left\{ f_j\left(u_{jt}\right) - C u_{jt} x_{jt} \right\} \quad (0.2)$$

Recharge of the aquifer, assumed to be constant and defined for user *j* as R_j in each period, will tend to reduce pumping lifts. Conversely, ongoing pumping will increase the distance through which each unit of water must be lifted: current extraction will adversely affect all users in future periods. In every period the distance to water in each well, x_{jt}, will be a function of the previous extraction history of all users. If θ_{ijs} is the drawdown (decrease in water level) imposed on well *i* by user *j* pumping a unit of water *s* time periods ago, then the equation of motion for the depth to water for each user is given by

$$x_{jt} = \sum_{s=0}^{t-1} \left[\sum_{j=1}^{J} \left\{ u_{is} \theta_{ij(t-s)} \right\} - R_j \right] + x_0 \quad (0.3)$$

where x_0 is the initial water depth. Note that the sign of $\theta_{ij(s+1)} - \theta_{ijs}$ has not been defined. If $\theta_{ij(s+1)} - \theta_{ijs} = 0$, all effects of any single period's resource use occur immediately and do not change over time. Alternatively, if $\theta_{ij(s+1)} - \theta_{ijs} > 0$, there are cumulative effects from past actions and impacts on the resource accrue and increase over time. Finally, if $\theta_{ij(s+1)} - \theta_{ijs} < 0$, the system exhibits reduced impacts on the resource and recovery from previous actions over time. Almost all existing resource economics studies make the implicit assumption that $\theta_{ij(s+1)} - \theta_{ijs} = 0$. This means that the full effect on the resource of any user's resource extraction decisions is immediate. However, there is no underlying physical necessity for this restriction. In particular, for resources which are diffusional in character, and therefore exhibit slow adjustment to extraction, the sign of $\theta_{ij(s+1)} - \theta_{ijs}$ may be positive or negative. As discussed in more detail in Section 4 below, groundwater systems exhibit diffusional properties. For example, if pumping starts at a new well, the drawdown resulting from this change will not be observed at a distant point for some time. In the model described here, this corresponds to $\theta_{ij(s+1)} - \theta_{ijs} > 0$. Conversely, after pumping from a well ceases, groundwater flow from surrounding areas will reduce drawdown over time. This corresponds to a value of $\theta_{ij(s+1)} - \theta_{ijs} < 0$.

Whereas many dynamic optimization models use difference or differential equations to describe the evolution of the state variable, equation (0.3) is a simple summation. If and only if $\theta_{ij(s+1)} - \theta_{ijs} = 0$, equation (0.3) can be expressed as a first order difference equation. If this is not the case, past actions will produce time-variant effects on the state variable, which will exhibit path-dependency.

Using equations (0.2) and (0.3), the appropriate Lagrangian for the benefit-maximizing dynamic optimization problem is

$$L = \sum_{j=1}^{J}\sum_{t=1}^{N}\left\{\beta^t\left\{f_j\left(u_{jt}\right) - Cu_{jt}x_{jt}\right\} + \lambda_{jt}\left\{\sum_{s=0}^{t-1}\left[\sum_{j=1}^{J}\left\{u_{is}\theta_{ij(t-s)}\right\} - R_j\right] + x_0 - x_{jt}\right\}\right\} \quad (0.4)$$

where λ_{jt} is the adjoint variable for user *j* and time period *t*. The relevant first order conditions for this problem are

$$\begin{aligned} &\beta^k\left\{f_l'\left(u_{lk}^*\right) - Cx_{lk}\right\} + \sum_{i=1}^{J}\sum_{s=k+1}^{N}\lambda_{is}\theta_{il(s-k)} = 0, \quad k = 1,\ldots,N-1 \\ &\beta^N\left\{f_l'\left(u_{lN}^*\right) - Cx_{lN}\right\} = 0 \end{aligned} \quad (0.5)$$

and

$$-\beta^k Cu_{lk}^* - \lambda_{lk} = 0 \quad (0.6)$$

The terminal value of the adjoint variable, $\lambda_{jN} = 0$. Condition (0.6) may be used to define the adjoint variable for $k < N$ as $\lambda_{lk} = -\beta^k C u_{lk}^*$. Substituting for both the adjoint variable and for the equation of motion from (0.3) allows first order conditions (0.5) to be rewritten as

$$f_l'\left(u_{lk}^*\right) - C\left\{\sum_{s=0}^{t-1}\left[\sum_{i=1}^{J}\left\{u_{is}^*\theta_{il(k-s)}\right\} - R_l\right] + x_0\right\} - C\left\{\sum_{i=1}^{J}\sum_{s=k+1}^{N}\beta^{s-k}u_{is}^*\theta_{il(s-k)}\right\} = 0, \quad k = 1,\ldots,N-1$$
$$f_l'\left(u_{lN}^*\right) - C\left\{\sum_{s=0}^{N-1}\left[\sum_{i=1}^{J}\left\{u_{is}^*\theta_{il(N-s)}\right\} - R_l\right] + x_0\right\} = 0$$

(0.7)

Concavity of $f_j\left(u_{jt}\right)$ is both necessary and sufficient for a unique interior solution. Thus, as shown by (0.7), for each user it is optimal to pump water in each period until the marginal benefit of pumping is equal to the sum of the marginal cost and the discounted marginal damage imposed on all users in all future periods.

3.2. Competitive, myopic extraction

Competitive groundwater extraction by multiple users is an alternative to the optimal solution discussed above. In this paper we shall not consider non-cooperative extraction as a strategic game. Instead we assume that competitive groundwater users behave myopically. Game-theoretic studies of groundwater extraction from single-cell and two-cell aquifers suggest that strategic behavior can ameliorate some of the welfare losses of non-cooperative extraction [11, 13, 22, 25]. However, we provide three reasons why strategic behavior is not of concern in the model presented in this study. First, as shown by Karp [19] and Brooks *et al*. [5], for large numbers of resource users, myopic and strategic behaviors converge. Our simulations assume that there are dozens of users; this is enough to ensure that strategic and myopic behavior will in any case be very similar. Second, limited survey evidence from the Central Valley of California suggests that actual groundwater users do not view the extraction problem strategically or take into account their neighbors' behavior in deciding their own resource use [11]. Finally, in the single-cell model, users are symmetric and the aggregate groundwater stock is a clear signal of behavior. In more complicated models with idiosyncratic, lagged externalities, it is not possible to uniquely ascertain other users' behavior. Moreover, the state of the resource at any point in time varies across space and depends, in a complex manner, on the entire extraction history before that time. Because of this, we suggest that there is no aggregate measure that can be used as a meaningful signal by non-cooperative, strategic users. If continuous monitoring and reporting of all users is not feasible, it is difficult to see how users would construct appropriate reaction functions. Hence, in this section we limit ourselves to myopic behavior by groundwater users.

Assuming that the net benefit function (0.1) and groundwater equation of motion (0.3) are unchanged, the vector of myopic extraction paths $\mathbf{u}^m$ is defined as

$$\mathbf{u}^m = \begin{bmatrix} u_{11}^m \\ \vdots \\ u_{JN}^m \end{bmatrix} = \underset{u_{jt} \in \mathbb{R}_+}{\arg\max} \left\{ f_j\left(u_{jt}\right) - C u_{jt} x_{jt} \right\} \tag{0.8}$$

Each user j will maximize their own single-period benefits without regard for the future. From equation (0.1), we can see that the appropriate first order condition will be

$$f_j'\left(u_{jt}^m\right) - C x_{jt} = 0 \tag{0.9}$$

Equation (0.9) states that myopic groundwater users will equate their marginal benefit with the marginal cost of groundwater extraction in each period. Substituting from equation (0.3) for x_{jt}, the system of equations defining the competitive, myopic solution is given by

$$f_l'\left(u_{lk}^m\right) - C\left\{\sum_{s=0}^{t-1}\left[\sum_{i=1}^{J}\left\{u_{is}^m \theta_{il(k-s)}\right\} - R_l\right] + x_0\right\} = 0 \qquad (0.10)$$

3.3. Competitive extraction with limited foresight

A third solution concept that we consider is competitive extraction with limited personal foresight. By this we mean that individual users take into account the effects of their own pumping on their future groundwater levels, but consider neither the effects on other users nor how other users' present extraction decisions will affect them in the future. Although this kind of limited foresight is near-rational, it provides a useful comparison to optimal and myopic extraction as it allows decomposition of the welfare effects of the pumping externality into own-effects and other users' effects. Limited foresight-type behavior has been analyzed in single-cell aquifer models [22], where it is sometimes described as "competitive", but not myopic. In the case of a single-cell aquifer, externalities are equally imposed on all users and users are assumed to be identical, so that the solution exhibits symmetry not observed in this model. Because of this symmetry, there is no reason to expect that resource users will ever behave with limited foresight in a single-cell model: if users can calculate the impact of their own extraction on groundwater levels, they have by definition also calculated the effect of every other user on groundwater levels. This is not the case in a spatially explicit model, and given the complexity of calculating other users' impacts on the resource, it is much more likely that users will not consider them, even if they do take into account their own impacts.

The vector of personal foresight extraction paths, $\mathbf{u}^p$ is defined as

$$\mathbf{u}^p = \begin{bmatrix} u_{11}^p \\ \vdots \\ u_{JN}^p \end{bmatrix} = \underset{u_{jt} \in \mathbb{R}_+}{\arg\max} \sum_{t=1}^{N} \beta^t \left\{ f_j\left(u_{jt}\right) - C u_{jt} x_{jt} \right\} \tag{0.11}$$

Once again, substituting for x_{jt} from equation (0.3) gives the system of equations describing the personal foresight solution:

$$\begin{aligned} & f_l'\left(u_{lk}^p\right) - C\left\{ \sum_{s=0}^{t-1} \left[\sum_{i=1}^{J} \left\{ u_{is}^p \theta_{il(k-s)} \right\} - R_l \right] + x_0 \right\} - C \sum_{s=k+1}^{N} \beta^{s-k} u_{ls}^p \theta_{ll(s-k)} = 0, \quad k = 1, \ldots, N-1 \\ & f_l'\left(u_{lN}^p\right) - C\left\{ \sum_{s=0}^{N-1} \left[\sum_{i=1}^{J} \left\{ u_{is}^p \theta_{il(N-s)} \right\} - R_l \right] + x_0 \right\} = 0 \end{aligned}$$

(0.12)

From a comparison of equations (0.7), (0.10) and (0.12) it is clear that the personal foresight pumping trajectory will fall between the myopic and optimal trajectories at every point in space and time. This follows directly from the observation that the marginal benefit for each user under personal foresight will be between the marginal benefits for myopic and optimal solutions.

4. AQUIFER RESPONSE TO GROUNDWATER PUMPING

The drawdown function, θ_{ijs}, describes how the groundwater resource responds to pumping over space and time. Thus, it provides the link between the physical aquifer system and the economic system of resource users. For example, the single-cell aquifer model used in existing groundwater economics studies corresponds to the restriction that drawdown functions for all users and time intervals are equal, so that $\theta_{ijs} = \overline{\theta}$ for all values of i, j, and s. This leads to analytical simplicity at the expense of hydrologic realism. In order to capture the correct physical nature of the groundwater resource, the drawdown function must be based on the appropriate physics of fluid flow. The derivation of the unit drawdown of an aquifer caused by ongoing pumping is a well-known result in hydrology based on solution of the partial differential equations describing diffusional processes, and can be found in many hydrology texts [12, 14, 33].

Consider an ideal aquifer, namely one that is horizontal and of infinite areal extent, homogeneous and isotropic, of constant thickness, and confined above and below by impermeable layers. The classic result of Theis [29] is that the drawdown a distance r from a well pumping at constant rate u at a time s after the start of pumping is given by

$$x_0(r) - x_s(r) = \frac{u}{4\pi T} \int_{r^2 S/4Ts}^{\infty} \frac{e^{-z}}{z} dz = \frac{u}{4\pi T} W\left(\frac{r^2 S}{4Ts}\right) \tag{0.13}$$

The exponential integral with the particular lower bound of integration in (0.13) is known as the well function, $W(\cdot)$. The constants S and T in equation (0.13) describe the key physical properties of the aquifer, namely the storativity and transmissivity of the aquifer, respectively. Storativity is a measure of the impact on groundwater levels in the aquifer of extracting one unit of water. It is a dimensionless parameter, defined for a confined aquifer as the volume of water released from storage per unit of surface area per unit decrease in the hydraulic head [12, 14]. The transmissivity of an aquifer is a measure of the speed and extent to which the impacts of any changes to the aquifer pass through it. Aquifer transmissivity is defined as the hydraulic conductivity of the aquifer multiplied by its thickness, where the hydraulic conductivity is a constant of proportionality relating specific discharge from a region to the hydraulic gradient across it [12, 14]. Taken together, the transmissivity and storativity may be thought of as describing the diffusional characteristics of a particular groundwater resource. In a high transmissivity, low storativity aquifer, the effects of pumping at any well will quickly be observed throughout the resource, and each unit of water withdrawn will cause significant drawdown. Conversely, in a low transmissivity, high storativity aquifer, extracting a unit of water will cause much less drawdown, and even this will be limited in extent to areas close to the pumping well.

The incremental drawdown between two time periods during which the pumping rate remains constant is thus given by the difference

$$x_s(r) - x_{s-1}(r) = \frac{u}{4\pi T}\left\{W\left(\frac{r^2 S}{4T(s-1)}\right) - W\left(\frac{r^2 S}{4Ts}\right)\right\} \tag{0.14}$$

Note that because $\partial W(\cdot)/\partial s < 0$ and $W(\cdot) \geq 0$, $x_s(r) - x_{s-1}(r) > 0$ if $u > 0$. The Theis solution assumes a single pumping well and constant pumping rates. However, it can easily be extended to include both multiple wells and pumping rates that vary through time. The well-known principle of superposition in hydrology states that the drawdown caused by multiple wells is linearly separable [12, 14]. Thus, equation (1.14) holds for each well in an aquifer irrespective of the total number of pumping wells and their relationship to each other. Hence the drawdown caused by multiple wells is simply the sum of drawdowns caused by individual wells. Similarly, changes in pumping rate at a single well site can be modeled by assuming that there exist multiple wells at the same point in space, that each start pumping at different points in time. Hence, a comparison of equations (0.3) and (0.14) shows that the drawdown function θ_{ijs} is defined as

$$\theta_{ijs} = \frac{1}{4\pi T}\left\{W\left(\frac{r(i,j)^2 S}{4T(s-1)}\right) - W\left(\frac{r(i,j)^2 S}{4Ts}\right)\right\} \tag{0.15}$$

where $r(i, j)$ is the distance between users i and j. Substitution of expression (0.15) into equations (0.7), (0.10) and (0.12) allows the realistic hydrologic system to be embedded directly into the dynamic economic framework.

5. COMPARISON OF GROUNDWATER MANAGEMENT POLICIES

Having developed a theoretical model of optimal groundwater management over space and time, it is necessary to consider the quantitative gains to possible management policies. In particular, there is much debate over the magnitude of welfare gains to groundwater management based on calibrations of single-cell aquifer models. As a baseline, using parameters for the Pecos aquifer of New Mexico, Gisser and Sanchez [17] found a negligible welfare difference (0.004%) between optimal control and no control scenarios. Clearly, these and similar results [1, 15] suggest that regulation of groundwater extraction is inappropriate. To ease comparison between existing literature and our study, we have used the same parameters as the studies of Gisser and Sanchez [17] and Gisser [15]. We have added several parameters as necessitated by our explicit spatial modeling (Table 1). In particular, we use a value for storativity of 8.6×10^{-5} and a transmissivity of 57,600 gallons/day/foot. Note that these parameters do not have clear analogs in single-cell aquifer models. However, the values used are in the range found in groundwater basins suitable for extraction [12].

The first order conditions derived in (0.7), (0.10) and (0.12) are systems of simultaneous equations. Following Gisser and Sanchez [17], we assume that the individual benefit $f_j(u_{jt})$ is adequately approximated by a quadratic function. In this case, the marginal benefit function $f_j'(u_{jt})$ is linear and conditions (0.7), (0.10) and (0.12) reduce to the systems of nonhomogeneous linear equations

$$\begin{aligned} &\textit{Optimal} && \mathbf{A}\mathbf{u}^* = \mathbf{b} \\ &\textit{Competitive, myopic} && \tilde{\mathbf{A}}\mathbf{u}^m = \mathbf{b} \\ &\textit{Competitive, personal foresight} && \hat{\mathbf{A}}\mathbf{u}^p = \mathbf{b} \end{aligned} \tag{0.16}$$

The square matrix $\mathbf{A}$ holds coefficients for the set of optimal extraction paths. It has (*NJ* x *NJ*) elements, and contains the slope of each user's marginal benefit function, $f_j''(u_{jt})$, as well as the idiosyncratic drawdown functions, θ_{ijs}. The elements of the vector $\mathbf{b}$ are the sums of relevant constant terms, in this case the initial depth to water, intercepts of the marginal benefit function, and the aggregated sum of recharge.

From a comparison of equations (0.7) and (0.10), it is clear that the matrix $\tilde{\mathbf{A}}$, containing coefficients for the set of myopic extraction paths, is simply the lower triangular portion of the matrix $\mathbf{A}$. Similarly, equation (0.12) shows that the matrix $\hat{\mathbf{A}}$, containing coefficients for the set of limited foresight extraction paths, may be obtained from $\tilde{\mathbf{A}}$ by adding the appropriate drawdown functions for own-effects, namely θ_{jjs}, for each time period.

Pumping trajectories for each solution concept will depend on the exact spatial relationship between groundwater users. Thus, in order to analyze the welfare differences between optimal, myopic and limited foresight solutions, the number of resource users and their specific locations relative to each other must be specified. The welfare differences derived for any specific set of well locations are of little interest by themselves, as any other set of well locations will result in different welfare measures.

Instead, we analyzed the mean welfare characteristics from a large number of repeated trials, each with a different set of randomly assigned well locations.

In this paper, we report results using twenty five users and seventy time periods. Similar results were obtained with different numbers of users, time periods, and hydrological parameters. Each trial proceeded as follows. Users were randomly located on the surface above the aquifer. The mean user spacing was calculated and used as a measure of the spatial distribution of users for each trial run. Each user was assigned a marginal benefit function whose slope was a random variable following

$$f_j'' = f'' \cdot \varepsilon_j, \quad \varepsilon \sim U[0.7, 1.3] \tag{0.17}$$

The constant f'' was chosen so that the expected marginal benefit function aggregated across all users had the same slope as the marginal benefit function used by Gisser and Sanchez [17]. Disaggregation of the marginal benefit function from that used by Gisser and Sanchez [17] was necessary because their original study assumed a single representative user. Whereas this may be justifiable for a single-cell aquifer, individual users must be separately and uniquely identified in a spatially explicit model. We chose to allow marginal benefit functions to vary slightly between users in our analyses to reflect heterogeneity between users in the real world. However, this variability in the slope of the marginal benefit function does not influence our basic results. The variance of the slope of the marginal benefit function in (1.17) is 0.03; resource users are still relatively homogeneous in their benefit characteristics. Moreover, the same intercept on the marginal product axis was used for all marginal benefit functions, and again, this was taken from Gisser and Sanchez [17].

Once user locations and marginal benefit functions were assigned, the relevant drawdown functions θ_{ijs} were calculated and the optimal, myopic and limited foresight coefficient matrices ($\mathbf{A}$, $\tilde{\mathbf{A}}$ and $\hat{\mathbf{A}}$, respectively) constructed. Using (0.16), the pumping trajectories $\mathbf{u}^*$, $\mathbf{u}^m$ and $\mathbf{u}^p$ were then calculated, and the welfare under each trajectory was found. Finally, the welfare differences between the optimal solution and the other two solutions were calculated. Repeated trials were performed for new spatial locations of groundwater users and a range of aquifer surface areas (parameter values used are shown in Table 1).

By definition, the optimal extraction trajectory $\mathbf{u}^*$ will be welfare-maximizing, the myopic trajectory $\mathbf{u}^m$ will have the lowest aggregate welfare, and the personal foresight trajectory $\mathbf{u}^p$ will attain an intermediate level of welfare (Figure 1). This is because, as noted previously, with personal foresight, pumping levels in each period will be intermediate between the other two concepts. Our analysis shows that there can be significant differences between the aggregate welfare with optimal, myopic, and personal foresight trajectories (Figure 1). Additionally, the differences observed vary as a function of mean well spacing.

As can be seen from Figure 1, mean welfare differences are highest for small mean user spacing and decrease as user spacing increases. When the distance between groundwater users is generally small, each user has a significant ability to impact their neighbors. Because of this the own-effect is only a small fraction of the total externality at low mean user spacing. The relative contribution of the own-effect on the externality can be seen in Figure 1, where the own-effect is represented by the difference between the filled diamonds and open circles at any particular mean well spacing. At low mean user spacing, most of the welfare difference between optimal and competitive policies is due to the aggregated externality effect of all users, of which the own-effect is only a small fraction. Conversely, at high mean user spacing, the own-effect dominates the externality, and much of the myopic welfare loss is caused by the failure to take into account one's own actions. However, even at high mean user spacing, there can still be a significant welfare difference between optimal, myopic, and personal foresight trajectories.

For any given interval of mean well spacing, there is a high variance of welfare differences within any given user behavior (Figure 1). For example, in the spacing interval 200-300 feet, the difference between optimal and myopic trajectories is between 9% and 21%. In the spacing interval from 30,000-40,000 feet, the difference ranges from 1% to 6%. This variability reflects the position of the resource users relative to each other. In particular, the degree of clustering will play a major role in determining welfare differences, but is not resolved well with a single measure of mean spacing.

Recall that in the original study of Gisser and Sanchez [17], the welfare difference between optimal and myopic trajectories was found to be 0.004%. Most extensions of Gisser and Sanchez's study modify economic variables and functions whilst maintaining the single-cell physical model [4, 7]. However, unless drastic changes, such as exponential demand growth with time, are introduced, the welfare differences between optimal and myopic paths in these extensions are still at most a few percent. Conversely, in this study, we use the same parameter values as Gisser and Sanchez wherever possible. Nevertheless, our results show that a large divergence in welfare between optimal and competitive, myopic extraction trajectories is possible. In particular, note that following

Gisser and Sanchez [17] we use a high annual discount rate, 10%, in our analysis and assume static demand.

Our results demonstrate that there exist conditions under which significant gains from optimal groundwater management are possible. The difference in results between this study and previous work is driven by the physical behavior of the underlying resource. In particular, we resolve an implicit paradox that arose in previous studies. In common property resources with a large number of competitive resource extractors imposing externalities on each other, we would expect to see significant gains from optimal management. Single-cell aquifers are common property resources with multiple competitive users, yet previous work (counterintuitively) suggests that gains from optimal management are negligible. If groundwater is modeled as a spatially explicit system, it is no longer strictly a common property resource. Instead, resource users will have the largest effect on themselves, and their ability to impose externalities will be spatially limited and concentrated in a neighborhood around them. However, when two wells are situated close to each other, their bilateral externalities can be extremely large, so that there will be a significant gain from regulation. If all wells are far enough apart, the ability of any user to impact their neighbors is limited. The groundwater that each user extracts is then effectively private property, and there will be no significant gains from regulation.

6. CONCLUSIONS

The optimal management of groundwater resources over space and time, and with multiple resource users, is studied. We argue that a good economic model of resource use is underpinned by good science, and show that in a correctly specified groundwater system, spatial considerations matter. The relative locations of resource users, as well as their spacing, are key determinants of the externalities produced by each user's extraction. In turn, the spatial distribution of pumping externalities determines the potential gain from an optimal system of groundwater management, however this is implemented.

For parameter values that occur in the real world, the welfare gains of optimal extraction of groundwater over no regulation can be significant – greater than twenty percent in some of our analyses. Large gains from groundwater regulation are possible when water users are clustered together in space. As the mean distance between pumping wells increases, the potential gains from optimal regulation decrease. Our analysis leads to a seemingly paradoxical result: we show that groundwater should not be modeled as a common property resource, but at the same time, there may be significant gains to regulation because of externalities. This is because the externalities caused by groundwater pumping are highly concentrated in space, and decrease rapidly with distance away from a pumping well.

The results of this paper are significantly different to those in most existing economic studies of groundwater extraction. In general, these studies model groundwater as a common property resource (the single-cell aquifer), where the externality from each user's pumping is uniformly distributed to all users and all points across the areal extent of the aquifer. Hence, for a large aquifer, the marginal effect of a unit of pumping will be negligible, and the optimal and myopic competitive solutions will necessarily be very similar. As shown in this paper, such single-cell models fail to capture adequately important aspects of the behavior of real aquifers. Because of this, policy recommendations based on such models, even when they provide both apparently robust and intuitively appealing results, should be viewed with caution.

We have demonstrated that location is important in groundwater management. Thus, not only how much water is pumped, but where this pumping occurs, must be considered when evaluating water resources. Because of this, the role of land ownership in determining the location of wells is critical, and will have efficiency as well as equity implications. In the framework presented here, we have not considered the optimal location and density of wells, but this is an area for future research. Similarly, a more realistic model of groundwater extraction and use allows improved analysis of groundwater management policies, both extant and hypothetical. In particular, our analysis highlights the importance of well spacing in determining the potential gains from groundwater regulation. Well spacing regulations are quite common regulatory tools in real-world groundwater management. Whereas previous studies were unable to analyze such spatial regulations, the framework described here can provide an economic rationale for these zoning restrictions, and allow a comparison between spatial policy instruments and more traditional instruments such as pumping taxes and quotas.

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VARIABLE	VALUE
Decision period length, days	30
Annual discount rate, % (*)	10
Initial water depth, x_0, feet (*)	170
Unit pumping cost, $/AF/foot (*)	0.035
Per-period recharge, R_j, feet (*)	0.11
Expected slope of aggregate marginal benefit function, $/AF (*)	-3.733×10^{-3}
Intercept of aggregate marginal benefit function, $ (*)	144
Number of users	25
Number of periods	70
Lengthscale of aquifer, feet	500 – 50 000
Aquifer storativity	8.6×10^{-5}
Aquifer transmissivity, gal/day/ft	57 600

TABLE 1. Parameter values used in the groundwater extraction simulation. Variables denoted with a (*) are taken directly from the studies of Gisser and Sanchez [17] and Gisser [15].

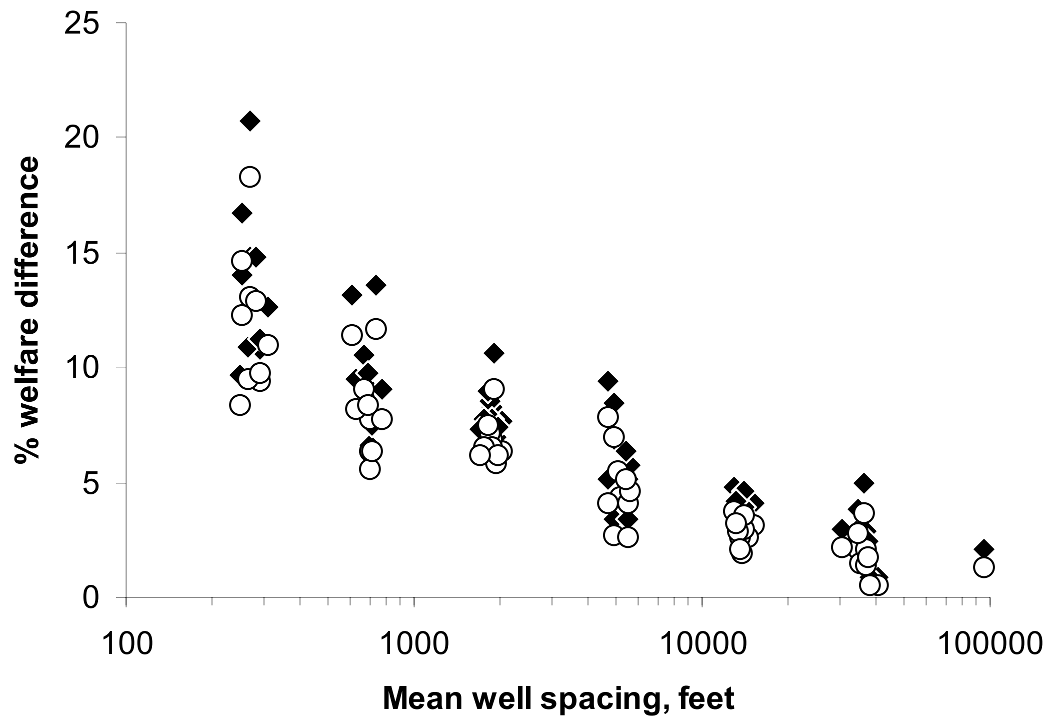

FIGURE 1. Welfare differences between optimal, competitive and myopic pumping trajectories.
Results are shown for multiple trial runs using 25 users and 70 time periods. A transmissivity parameter of 57,600 gal/day/ft was used, together with a storativity of 8.6 x 10^{-5}. The percentage difference in welfare between optimal, myopic and personal foresight pumping trajectories for each simulation run is shown. Filled diamonds correspond to the welfare difference between optimal and myopic trajectories. Open circles correspond to the welfare difference between optimal and personal foresight trajectories.