Is Equity a Constraint? Applications to Block Rate and Other Pricing Schemes with Heterogeneous Users*†

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1 Introduction

In choosing any policy, whether it be a regulation or a pricing scheme, it is important to consider the social goals of the policy. Social goals might include things like equity, giving incentives for conservation, or economic efficiency. Considering the scarcity of water resources in many parts of the world, an important area of research is the effect of different water pricing schemes. The water-pricing scheme can be chosen to affect the total quantity of water demanded by consumers. However, other concerns may be addressed via the chosen pricing scheme as well. Equity issues between different water users, and economic efficiency of water use can be addressed. Historically many water systems have used average cost pricing as a way to recover costs. Increased water scarcity leads us to consider water reform systems that move away from average cost pricing towards systems aimed to promote conservation and increased water use efficiency.

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If the goal of water pricing policy is economic efficiency, then water markets are a first-best solution. There is an extensive literature on the optimality of water markets to achieve the most efficient allocation of water use. This literature began with the work of Burness and Quirk [1]. They argue that if water rights are tradable between heterogeneous water users, the most efficient outcome will result. In this literature, there isn't a need to mention the role of a water agency, because bargaining between parties will achieve the economically efficient (first-best) outcome. This argument relies on a lack of transaction costs for water trading. Because of the high costs involved with water storage and conveyance, transaction costs may be prohibitive to trading in water, especially between non-adjacent land areas. When the required assumptions do not hold, there is a role for a water agency in determining water prices and allocations to users.

A first-best pricing policy is to price water at the marginal cost of supply. If budget balance is required, this should be achieved through non-distortionary (lump sum) transfers. We will derive the first-best pricing outcome and compare it to several second-best pricing policies, including tiered pricing and average pricing. This analysis will be done for agricultural water supplied to land that is heterogeneous in quality, and will require budget balance of the water utility. Water agencies are often constrained to a zero profit condition. This is particularly true of a state-owned water utility, or in a place like California where many water districts are run for the benefit of their members.

Despite the fact that the economic efficiency of water trading or marginal cost pricing has been well-established, the number of water systems throughout the world that use such systems is close to zero. Some may argue that equity considerations are important in this choice. This paper develops a framework to analyze the equity and efficiency of various water pricing options, including tiered pricing. It recognizes that water users are heterogeneous in their characteristics and demand, and this heterogeneity will have to be incorporated in the design and assessment of alternative water pricing mechanisms.

2 Empirical Model

Let y denote output per acre; x, applied water per acre; and γ, land quality. Land quality varies from $\underline{\gamma}$ to $\overline{\gamma}$ with density $g(\gamma)$. The production function per acre is $y = f(x, \gamma)$ with $f_x > 0$, $f_{xx} < 0$, and $f_\gamma > 0$, where the subscripts denote partial derivatives. We normalize the price of output to equal one, and all other prices are relative to the price of output.

There is a fixed cost per acre, denoted by k. This includes the costs of preparing the land for planting and controlling pests. For simplicity, we assume that this cost does not depend on the size of a farm or on the quality of the land. The parameter $\delta(\gamma)$ indicates the proportion of land with quality γ used for a specific crop by the farmer. Aggregate water use is denoted by $\overline{X}$, and the cost of providing water is composed of a fixed cost, denoted by F and a variable cost, denoted by $V(\overline{X})$, with $V' > 0$ and $V'' > 0$. It is generally assumed that the marginal cost of supplying water increases as the total quantity supplied increases. This is because the least expensive water sources are generally developed first. Increasing the supply of water requires deeper pumping of groundwater, or the delivery of surface water from further away.

3 Measuring Efficiency and Equity

A component that is crucial to the work we present in this paper is the choice of an appropriate measure for both efficiency and equity.

3.1 Measurements of Inequality

There are several choices available for a measure of inequality. The first is the percentage of the population facing a certain input price who are priced out of the market entirely (the percentage without access to the resource). In a similar vein, we could define the measure of inequality as the percentage of the population whose input use is less than x_L, where x_L is defined as a 'lifeline quantity', or a minimum subsistence level. For example, estimates of the minimum necessary quantity of water per capita for consumption and sanitation range from 20 to 40 liters ([4]). While these inequality measures are somewhat crude, the benefits of using them is that they are relatively easy to understand and calculate. Previous work had used a similar type of measure of inequity. In a study of a government run irrigation system in the Philippines, Ferguson [3] uses the amount of land that does not receive sufficient irrigation water as a measure of inequity.

Another method that we use to measure inequality is a comparison of the

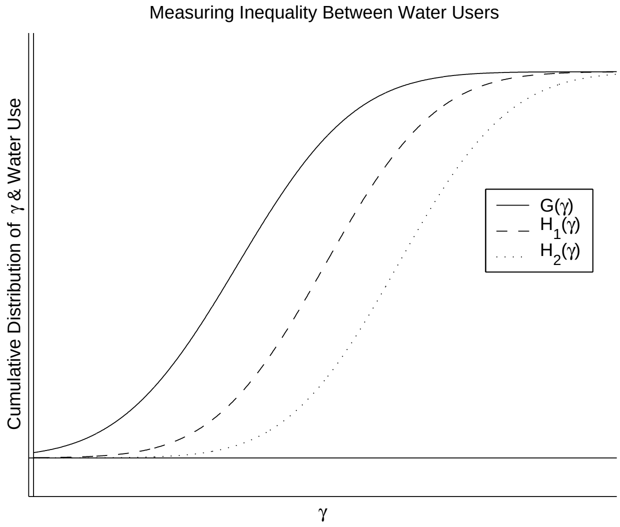

Figure 1: Measuring Inequality

cumulative distribution functions of the resource use patterns by population. We define $h(\gamma)$ as the density function of the percentage of total resource use, and $H(\gamma)$ as the cumulative distribution function of total resource use, with $H(\underline{\gamma}) = 0$ and $H(\overline{\gamma}) = 1$. We compare distributions of resource use and consider one to be more equitable than another if it second-order stochastically dominates the other.

An example of this is in Figure 1. In Figure 1, we graph the distribution of the population ($G(\gamma)$), along with two possible distributions for cumulative water use. We denote these with $H_1(\gamma)$ and $H_2(\gamma)$. Using the second-order stochastic dominance criteria, we consider $H_1(\gamma)$ to achieve a higher level of equity than $H_2(\gamma)$. If some proportion of the population has zero consumption of the resource, the distribution function will be flat over a range of values of γ, and then will increase.

3.2 Measurements of Inefficiency

In existing water pricing systems throughout the world, inefficiencies can result from many sources. These include a lack of appropriate pricing and subsidies on other costs associated with water use, such as electricity, among many others ([5], [6]). However, in this paper we assume that producers do not have market power to influence input prices, and that other inputs to production, such as electricity, fertilizers, or labor are priced without market distortions.

When a water utility charges the same price for each unit consumed, as with marginal or average cost pricing, the measurement of inefficiency is fairly straightforward. We consider the measure of inefficiency as the deadweight loss resulting from a suboptimal allocation of resources. By definition, this is the area between the marginal cost and marginal benefit functions above the optimal level of resource use. However, we make a slight simplification and approximate this using the area of a triangle, and denote the measure of inefficiency as $\frac{1}{2}(\overline{X}^{actual} - \overline{X}^{*})(V'(\overline{X}^{actual}) - w)$.

When a water utility charges different prices based on the level of consumption, the measurement of inefficiency is a little more complicated. Examining the amount of inefficient water use in this scenario requires us to consider the proportion of the population that do not buy their **last** unit at marginal cost, instead of the total amount of the resource purchased below marginal cost.

4 Optimal Allocation Rule

In this section we develop the optimal pricing rule for a utility that sells water at a constant price to its customers. We assume that the amount of water demanded at a given price increases as γ (land quality) increases.

We model the rent per acre (r(γ)) as the following:

$$r(\gamma) = (f(x(\gamma, w), \gamma) - k - wx(\gamma, w)) \tag{1}$$

4.1 Individual Profit Maximization

We begin by modeling a profit-maximizing farmer, who has to decide whether to produce, given the quality of his land and the price of water. We model the optimization problem as follows:

$$\max_{x(\gamma,w),\delta(\gamma)} (f(x(\gamma, w), \gamma) - k)\delta(\gamma) - wx(\gamma, w) \tag{2}$$

Farmers will decide to plant on an acre ($\delta(\gamma) = 1$) if the total revenues exceed the total costs. If total costs exceed total revenues, the land will be left fallow or used for some other crop and $\delta(\gamma) = 0$. There exists a critical land quality (γ_L) where the rent per acre is zero. At this point, $f(x(\gamma_L, w), \gamma_L) - k = wx(\gamma_L, w)$. At this land quality, total revenue per acre equals total cost. This separates a region of higher quality lands that will be fully farmed from lower quality lands that will not be farmed. The lowest land quality in use, γ_L is a function of k, and w. From this result, we find the following:

$$\delta(\gamma) = 1 \quad \text{and} \quad r(\gamma) > 0 \quad \text{for} \quad \gamma > \gamma_L(k, w)$$

$$\delta(\gamma) = 0 \quad \text{and} \quad r(\gamma) \leq 0 \quad \text{for} \quad \gamma \leq \gamma_L(k, w)$$

On any acre with positive production, water use will be determined by the price of output (p), the price of water (w), and the land quality (γ). A farmer will choose the optimal amount of water (x^*) to apply so that the following condition holds:

$$\frac{\partial f(x^*(\gamma, w), \gamma)}{\partial x^*(\gamma, w)} = w \tag{3}$$

In subsequent sections, we define $x^*(\gamma, w)$ as the profit maximizing level of input use for an individual with land quality γ facing input price w, and $y^*(\gamma, w) = f(x^*(\gamma, w), \gamma)$ as the profit maximizing choice of output.

4.2 Social Optimality

Taking the behavior of an individual farmer as given by equations 2 and 3, we consider the optimization problem facing a social planner who charges the same price for each unit of water consumed:

$$\max_{w} \int_{\gamma_L(k,w)}^{\overline{\gamma}} (y^*(\gamma, w) - k)\delta(\gamma)g(\gamma)d\gamma - F - V(\overline{X}) \tag{4}$$

Deriving the first order conditions for a social optimum shows that, as expected, the price of the input (water) should equal the marginal cost of provision. Therefore, we define $w^* = V'(\overline{X})$.

Aggregate water use will be the following:

$$\overline{X} = \int_{\gamma_L(k,w^*)}^{\overline{\gamma}} x^*(\gamma, w^*)g(\gamma)d\gamma$$

This equilibrium is likely to violate the no-profit condition for a utility. If $\overline{X}w^* > F + V(\overline{X})$ the utility accumulates profits. If a zero profit condition is imposed, then any profits or losses will have to be distributed in a non-distortionary manner. For example, each acre of land may receive a share of the profits, or pay a per-acre fee to cover the losses. In practice, this type of mechanism is unrealistic due to the difficulty in implementation. For this to be non-distortionary, these rebates/fees have to be independent of the amount of water a farmer uses, the crop he/she grows, or if he/she leaves land fallow.

5 Average Cost Pricing

Average cost pricing is often used when a utility is constrained to zero profits. However, the use of average cost pricing is economically inefficient, because the marginal cost of providing water to the user doesn't equal the price the user pays for water. If fixed costs (F) are small relative to variable costs (V), the average cost will be below the marginal cost. It is usually assumed that this is the case. There are a few reasons for this. One reason is that the conveyance and storage costs (variable costs) of supplying water are high. Also, the government or an outside agency often subsidize the fixed costs of developing a water project. This has been true in both developed and developing countries. For example, in the United States, most of the water projects in the Western United States were either paid for or highly subsidized by the federal government. In many developing countries, water projects have been built from external funding received by agencies such as the World Band or the International Monetary Fund. When fixed costs aren't a concern, those managing the water simply want to recover the variable costs of water provision.

In the following, we use a superscript A to denote behavioral choices under average-cost pricing, and a superscript * to denote behavior under marginal-cost pricing. We denote the water price by w^A, which is determined so that total revenues just cover the total costs of water provision, as shown below:

$$w^A = \frac{F + V(\overline{X}^A)}{\overline{X}^A}$$

As above, we define the lowest quality of land in operation by γ_L^A. Given these conditions, if it is profitable to operate, a farmer will choose to apply water so that the following condition holds:

$$\frac{\partial f(x^*(\gamma, w^A), \gamma)}{\partial x^*(\gamma, w^A)} = w^A$$

Total water demanded will be the integral of the individual demands:

$$\overline{X}^A = \int_{\gamma_L^A(k,w^A)}^{\overline{\gamma}} x^*(\gamma, w^A) g(\gamma) d\gamma$$

Average cost pricing can lead to inefficient water use. This is because water users are not choosing quantities at the margin efficiently. When w^A is less than w^*, too much water will be demanded at the margin. The change in total water use will be both at the intensive and at the extensive margins, as shown below.

$$\overline{X}^A = \overline{X} + \underbrace{\int_{\gamma_{L(k,w^*)}}^{\overline{\gamma}} (x^*(\gamma, w^A) - x^*(\gamma, w^*)) g(\gamma) d\gamma}_{\text{change at the intensive margin}} + \underbrace{\int_{\gamma_{L(k,w^A)}}^{\gamma_{L(k,w^*)}} x^*(\gamma, w^A) g(\gamma) d\gamma}_{\text{change at the extensive margin}} \tag{5}$$

Proposition 1 *The welfare loss under average cost pricing is increasing in the slope of the variable cost function ($V(\overline{X})$), and decreasing in the rate of change of yield ($f(x, \gamma)$) to greater levels of inputs.*

Proof of Proposition 1: See appendix.

6 Tiered Pricing Solution

A tiered pricing system is a way to provide budget balance of the water utility, but to also have marginal cost pricing at higher levels of water use. This will provide the incentives for efficient water use at the margin, as well as addressing equity concerns.

The pricing system is an increasing block rate system. This type of system is commonly used in electricity markets. However, it has been largely ignored in the water literature. With tiered pricing, there will be a certain amount of water allocated to each acre of land in production at a low price. This quantity will be denoted by x_0, and it is available at a price of w_0. For any water above x_0, the price will equal the marginal cost of provision, w^M. There are several reasons that an increasing block rate pricing system would be used in water pricing. The first reason is for economic efficiency - it changes the price of the last unit of water used, so that at the margin a user chooses water use efficiently. Another reason could be for equity reasons. If it is a priority of agricultural policy to keep small family farms in business, it might motivate a tiered-pricing policy. If small farms that require less water pay a reduced price, it would help to keep those farmers in business (essentially, this would be a transfer from large farmers to small farmers).

The model here is of an allocation per production acre. There are other ways that a tiered pricing system could be implemented. For example, a water user could be allocated some percentage of their historical usage at a low rate. In this case, the allocated quantity will be $x_0(\gamma)$, instead of having the allocation constant among users. Another possible scenario is that each grower receives a certain allocation of water for a low price, regardless of the number of acres the grower farms. However, we argue that if there is large heterogeneity across farm sizes and land quality, the most feasible system is a per acre allocation.

Since the quantity of water demanded at a given price increases as land quality (γ) increases (due to greater input-use efficiency), lower quality lands will buy at the lower price w_0, while the higher quality land will receive their allocation, and will buy more water at the higher price w^M. Mathematically, this assumption can be stated as if $\gamma_1 > \gamma_2$, then $x^*(\gamma_1, w) > x^*(\gamma_2, w) \forall w$. Using the tiered pricing system will create up to 4 categories of water users, based on land quality. Some of these categories may be empty for certain land distributions or for certain values of the parameters.

1. The first category includes those who cannot farm profitably at water price w_0. As with the optimal solution, there will be some value of land quality that separates the land that is in production from land kept fallow. We define $\gamma_L(k, w_0)$ as the lowest quality land in production.

2. The second category is those who set marginal product equal to price w_0. Here we define $\gamma_L^S(x_0, w_0)$ as the highest quality land that uses water at price w_0 efficiently.

3. The third category is those who use water inefficiently - they buy x_0 at price w_0, but in doing so they are not setting the value of marginal product equal to the input price. They would be willing to buy more water, but not at price w^M.

4. The fourth category is those who use water efficiently at the high price of w^M. The lowest level of land quality in this group is $\gamma_H^S(x_0, w^M)$, which we define as the lowest quality land that uses water at price w^M efficiently.

An example of this is in Figure 2. In this figure, there are only three types of user characteristics that shape the demand for water. We define these three types as γ_L, γ_M, and γ_H. When faced with the same level of input price, each type demands a different quantity of water. Demand from each type is shown with its respective marginal benefit curve, denoted by $MB(\gamma_i)$. Given the choice of x_0 and w_0 shown in the figure, the only group of users that purchase water at an efficient level is γ_H. Therefore, γ_H is in the fourth category of users defined above. The first group, γ_L is in the second category, while the second group, γ_M, is in the third category. There are no users in the first category, since all buy some amount of water at the low price (w_0).

The land qualities that separate these categories are functions of the parameters of the problem, as shown below. The exogenous variables in the problem are k, and F. Continuing the notation from earlier, we use a superscript T to denote behavioral choices under tiered pricing.

Using this notation, the second-best optimization problem is:

$$\max_{x_0, w_0} \int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} (f(x^*(\gamma, w_0), \gamma) - k) g(\gamma) d\gamma + \int_{\gamma_L^S(x_0,w_0)}^{\gamma_H^S(x_0,w^M)} (f(x_0, \gamma) - k) g(\gamma) d\gamma$$
$$+ \int_{\gamma_H^S(x_0,w^M)}^{\overline{\gamma}} (f(x^*(\gamma, w^M)) - k) g(\gamma) d\gamma - F - V(\overline{X}^T) \quad (6)$$

Subject to the following two conditions:

$$F + V(\overline{X}^T) = \int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} w_0 x(\gamma, w_0) g(\gamma) d\gamma + \int_{\gamma_L^S(x_0,w_0)}^{\gamma_H^S(x_0,w_M)} w_0 x_0 g(\gamma) d\gamma +$$
$$+ \int_{\gamma_H^S(x_0,w_M)}^{\overline{\gamma}} (w_0 x_0 + w^M(x(\gamma, w^M) - x_0)) g(\gamma) d\gamma \quad (7)$$

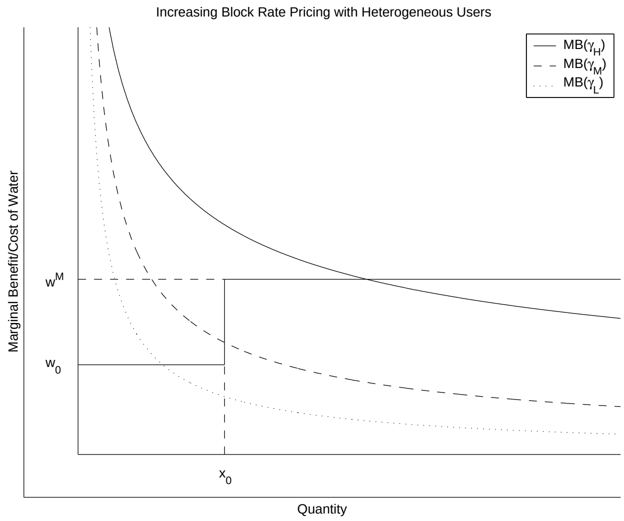

Figure 2: Example of Increasing Block Rate Pricing with Heterogeneous Users

$$w^M = V'(\overline{X}^T) \tag{8}$$

The following also hold, based on the definition of the variables:

$$\overline{X}^T = \int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} x(\gamma, w_0)g(\gamma)d\gamma + \int_{\gamma_L^S(x_0,w_0)}^{\gamma_H^S(x_0,w_M)} x_0 g(\gamma)d\gamma + \\ + \int_{\gamma_H^S(x_0,w_M)}^{\overline{\gamma}} x(\gamma, w^M)g(\gamma)d\gamma \tag{9}$$

$$x^*(\gamma_L^S, w_0) = x_0 \tag{10}$$

$$x^*(\gamma_H^S, w_M) = x_0 \tag{11}$$

$$f'(\gamma, w_0) = w_0 \quad \forall \quad \gamma \quad s.t. \quad \gamma_L \leq \gamma \leq \gamma_L^S \tag{12}$$

$$f'(\gamma, w_M) = w_M \quad \forall \quad \gamma \quad s.t. \quad \gamma_H^S(x_0, w_M) \leq \gamma \leq \overline{\gamma} \tag{13}$$

Setting λ_1 as the shadow value of the budget constraint, and λ_2 as the shadow value of marginal cost pricing at the top tier, the first order conditions of the problem are as follows:

$$\frac{\partial L}{\partial w_0} = \underbrace{\frac{-\partial \gamma_L}{\partial w_0}((x^*(\gamma_L, w_0)w_0)g(\gamma_L) +}_{\Delta \text{ output value from tail}} \underbrace{\int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} w_0 \frac{\partial x^*}{\partial w_0} g(\gamma) d\gamma}_{\Delta \text{ output value from low tier}} -$$
$$- \underbrace{V'(\overline{X}^T)\frac{\partial \overline{X}^T}{\partial w_0}}_{\Delta \text{ variable costs}} + \lambda_1 (\underbrace{V'(\overline{X}^T)\frac{\partial \overline{X}^T}{\partial w_0}}_{\Delta \text{variable costs}} + \underbrace{\frac{\partial \gamma_L}{\partial w_0} w_0 x^*(\gamma_L, w_0) g(\gamma_L)}_{\Delta \text{ revenue from tail}} -$$
$$- \underbrace{\int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} (x^*(\gamma, w_0) + w_0 \frac{\partial x^*}{\partial w_0}) g(\gamma) d\gamma}_{\Delta \text{ revenue from low tier}} - \underbrace{x_0(1 - G(\gamma_L^S))}_{\Delta \text{ revenue from other tiers}}) +$$
$$+ \lambda_2 \underbrace{(V''(\overline{X^T})\frac{\partial \overline{X^T}}{\partial w_0})}_{\Delta \text{ marginal costs}} = 0 \quad (14)$$

$$\frac{\partial L}{\partial x_0} = \underbrace{\int_{\gamma_L^S(x_0,w_0)}^{\gamma_H^S(x_0,w_M)} p \frac{\partial f}{\partial x_o} g(\gamma) d\gamma}_{\Delta \text{ output value of middle tier}} + \underbrace{(w^M - w_0)(1 - G(\gamma_H^S))}_{\Delta \text{ subsidy to high tier}} +$$
$$+ \lambda_1 (\underbrace{V'(\overline{X}^T)\frac{\partial \overline{X}^T}{\partial x_0}}_{\Delta \text{ variable costs}} - \underbrace{w_0(G(\gamma_H^S) - G(\gamma_L^S))}_{\Delta \text{ revenue from the middle tier}} +$$
$$+ \underbrace{(w_M - w_0)(1 - G(\gamma_H^S))}_{\Delta \text{ subsidy to the high tier}}) + \lambda_2 \underbrace{V''(\overline{X}^T)\frac{\partial \overline{X}^T}{\partial x_0}}_{\Delta \text{ marginal cost}} = 0 \quad (15)$$

An important factor is the shape of the land density function $g(\gamma)$. The tiered pricing solution will be very different if most of the density is located near the center of the distribution or if most of the density is near the upper and lower limits. For certain land density functions, tiered pricing can achieve the first-best solution, while still satisfying the balance budget requirement. If the tier is set so that every water user is in the top tier, water use will reproduce the first-best solution. There could be multiple sets of (x_0, w_0) that satisfy the first order conditions of the problem.

Proposition 2 *If the pair* (x_0, w_0) *is chosen so that* $0 < x_0 < x^*(\underline{\gamma}, w^M)$, *then the first-best outcome can be achieved through tiered pricing.*

Proof of Proposition 2: See appendix.

Proposition 3 *if* $0 < x_0 < x^*(\underline{\gamma}, w^M)$, *the choice of* (x_0, w_0) *does not affect the measure of inequality.*

Proof of Proposition 3: See appendix.

Proposition 4 *If* x_0 *is determined by other considerations, such as a lifeline quantity (chosen by a measure of a minimal need) and* $x_0 > x^*(\underline{\gamma}, w^M)$, *then*

- *the inefficiency resulting from tiered pricing is increasing in* x_0.
- *the inefficiency is exacerbated when a large proportion of the land is of moderate quality.*
- *the inefficiency is decreasing in the responsiveness of the yield function* $(f(x, \gamma))$ *to higher input use.*

Proof of Proposition 4: See appendix.

7 Conclusion

In this paper, we have presented a framework for analyzing equity and efficiency measures of resource use when users of that resource are heterogeneous in their characteristics and demand. One result that we show is that the tail end of the distribution of users will determine whether tiered pricing is efficient, while the moments of the distribution will determine the measure of loss when there is inefficiency. We identify two sources of inefficiency with tiered pricing - a level of entry that is too high and excessive use by existing users. These are effects at the intensive and extensive margins, and both will create inefficiency. The parameters of the distribution of users and their demand will determine the relative importance of each source of distortion.

The next step is to analyze certain production and distribution functions. This can be done both analytically and with a simulation program. One production function that has been used to look at agriculture is a quadratic production function, such has been done by Caswell and Zilberman [2] in modeling the California cotton industry.

Other production functions that are often used in analyzing agricultural production are a Von-Liebig (fixed proportion) technology, or a Cobb-Douglas production function.

These extensions, along with an examination of other water-pricing policies are the next step in this research. The question of choosing an appropriate water-pricing policy is extremely important and timely. As the number of people in water scarce regions and the environmental needs for clean water grow, disputes over water use and water rights are certain to be a part of our future.

A Appendix

A.1 Proof of Proposition 1

Proposition 1: *The welfare loss under average cost pricing is increasing in the slope of the variable cost function* ($V(\overline{X})$)*, and is decreasing in the rate of change of yield* ($f(x,\gamma)$) *to greater levels of inputs.*

In this proposition, we make two claims. To show they are correct, we first remind the reader of the measurement of inefficiency we use when users pay the same price for each unit consumed. We consider the traditional measure of deadweight loss, and approximate this with a triangular area for ease of calculation. Accordingly,

$$DWL \approx \frac{1}{2}(\overline{X}^A - \overline{X}^*)(V'(\overline{X}^A) - w^A)$$

For the first claim, we consider how a change in the slope of the variable cost function (mathematically a change in $V'(\overline{X})$) changes the measure of inefficiency. The proof is straightforward, since

$$\frac{\partial(DWL)}{\partial V'(\overline{X})} \approx \frac{1}{2}(\overline{X}^A - \overline{X}^*)$$

As $\overline{X}^A > \overline{X}^*$, this expression is positive. Therefore, deadweight loss increases as the slope of the variable cost function increases. It is important to note that since consumers respond to the price charged (w^A), the total quantity used ($\overline{X}^A$) does not depend on the cost function.

For the second claim, we first note that the aggregate demand ($\overline{X}$) for a resource at a single price is simply the sum of all individual demands. Therefore, we consider the impact of the shape of the production function on an individual's demand. We use the result of the individual profit maximization that for any input price w_i, an individual will set $\frac{\partial f(x,\gamma)}{\partial x_i} = w_i$. Taking a first-order Taylor series expansion around $\frac{\partial f(x,\gamma)}{\partial x^A}$ yields the following:

$$\frac{\partial f(x,\gamma)}{\partial x^A} \approx \frac{\partial f(x,\gamma)}{\partial x^*} + \frac{\partial^2 f(x,\gamma)}{\partial x^{*2}}(x^A - x^*)$$

Substituting in the condition for profit maximization and rearranging terms yields the following:

$$\frac{(w^A - w^*)}{\frac{\partial^2 f(x,\gamma)}{\partial x^{*2}}} \approx (x^A - x^*)$$

The term $\frac{\partial^2 f(x,\gamma)}{\partial x^{*2}}$ is negative by assumption, validating the fact that input price and quantity demanded are inversely related. This term measures the rate of change of the yield function in response to changes in input quantities, and the larger this rate of change, the smaller the change in quantity demanded after an input price change. The intuition behind this is that if the change in yield responds quickly to changes in inputs, a smaller change in inputs is necessary to satisfy the profit maximization condition.

A.2 Proof of Proposition 2

Proposition 2: *If the pair (x_0, w_0) is chosen so that $0 < x_0 < x^*(\underline{\gamma}, w^M)$, then the first-best outcome can be achieved through tiered pricing.*

The intuition is that this pricing policy puts all water users in the top tier - those who buy water efficiently at marginal cost. As discussed earlier, we define the level of inefficiency as the difference between the optimal level of aggregate resource use and the actual level, as denoted by $\overline{X}^T - \overline{X}^*$. Using the categories defined earlier, if $x_0 < x^*(\underline{\gamma}, w^M)$, the first three categories are empty, and the entire population is in the fourth category. This is the group that uses water efficiently at price w^M. Under these assumptions, the total quantity used under tiered pricing is the following:

$$\overline{X}^T = \int_{\underline{\gamma}}^{\overline{\gamma}} x^*(\gamma, w^M) g(\gamma) d\gamma$$

Since w^M is equal to the marginal cost of water ($V'(\overline{X})$), aggregate demand is equivalent to demand under marginal cost pricing. However, a water utility earns a lower level of profits than under marginal cost pricing, since not all units are priced at marginal cost. More specifically, total revenues are equal to the following expression:

$$\text{Total Revenue} = (x_0 w_0) + \int_{\underline{\gamma}}^{\overline{\gamma}} w^M (x^*(\gamma, w^M) - x_0)$$

Therefore, the utility can be held to a zero-profit condition, while efficiency in water use among users is still achieved.

A.3 Proof of Proposition 3

Proposition 3: *if* $0 < x_0 < x^*(\underline{\gamma}, w^M)$*, the choice of* (x_0, w_0) *does not affect the measure of inequality.*

To show that this is true, we consider the decision from each individual of how much water to use under tiered pricing and under marginal cost pricing. As shown in the proof of proposition 2, each individual uses the same amount of water under both tiered pricing and marginal cost pricing. Therefore, the distribution of water use under each pricing scheme is identical. Therefore, while customers pay a lower total price for their water use, the distribution of quantity used does not change.

A.4 Proof of Proposition 4

Proposition 4: *If* x_0 *is determined by other considerations, such as a lifeline quantity (chosen by a measure of a minimal need) and* $x_0 > x^*(\underline{\gamma}, w^M)$*, then*

- the inefficiency resulting from tiered pricing is increasing in x_0.
- the inefficiency is exacerbated when a large proportion of the land is of moderate quality.
- the inefficiency is decreasing in the responsiveness of the yield function ($f(x, \gamma)$) to higher input use.

We denote the level of inefficiency by the difference between aggregate water use under marginal cost pricing and under tiered pricing. This difference is $\overline{X}^T - \overline{X}^*$. Inefficiency results from those users who are in the first and second category defined earlier - those that set the marginal product of water equal to w_0 and those that use exactly x_0. First, we show the following:

$$\overline{X}^T - \overline{X}^* = \int_{\gamma_L(k,w_0)}^{\gamma_L^S(x_0,w_0)} (x^*(\gamma, w_0) - x^*(\gamma, w^M))g(\gamma)d\gamma$$

$$+ \int_{\gamma_L^S(x_0,w_0)}^{\gamma_H^S(x_0,w^M)} (x_0 - x^*(\gamma, w^M))g(\gamma)d\gamma$$

Differentiating this expression with respect to the variable x_0 and simplifying gives the following expression:

$$\frac{\partial(\overline{X}^T - \overline{X}^*)}{\partial x_0} = \frac{\partial \gamma_H^S}{\partial x_0} x_0 g(\gamma_H^S) + G(\gamma_H^S) - G(\gamma_L^S)$$

Both components of the expression are non-negative. The first shows the increased inefficiency resulting from a greater proportion of the population using water inefficiently. The second component shows how those who are already using water inefficiently exacerbate that inefficient use when the level of x_0 is increased.

For the last part of this statement, refer to the proof of proposition 1, where we show that a greater responsiveness of the yield function to higher water use decreases inefficiency. The same argument holds for the proportion of the population who buy water below marginal cost under tiered pricing.

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