Efficient Estimates of a Model of Production and Cost

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Denver, Colorado, August 1-4, 2004

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Abstract

In 1944, Marschak and Andrews published a seminal paper on how to obtain consistent estimates of a production technology. The original formulation of the econometric model regarded the joint estimation of the production function together with the first-order necessary conditions for profit-maximizing behavior. In the seventies, with the advent of econometric duality, the preference seemed to have shifted to a dual approach. Recently, however, Mundlak resurrected the primal-versus-dual debate with a provocative paper titled "*Production Function Estimation: Reviving the Primal*." In that paper, the author asserts that the dual estimator, unlike the primal approach, is not efficient because it fails to utilize all the available information. In this paper we demonstrate that efficient estimates of the production technology can be obtained only by jointly estimating all the relevant primal and dual relations. Thus, the primal approach of Mundlak and the dual approach of McElroy become nested special cases of the general specification. In the process of putting to rest the primal-versus-dual debate, we solve also the nonlinear errors-in-variables problem when all the variables are measured with error.

JEL Classification: D0, C3.
Keywords: Primal, Dual, Cobb-Douglas, Nonlinear errors-in-variables.

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We thank Art Havenner and Richard Green for stimulating discussions on the subject. All errors are our responsibility. Quirino Paris is a member of the Giannini Foundation of Agricultural Economics.

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I. Introduction

Marschak and Andrews (1944), Hoch (1958, 1962), Nerlove (1963), Mundlak(1963, 1996), Zellner, Kmenta and Dreze (1966), Diewert (1974), Fuss and McFadden (1978), McElroy (1987) and Schmidt (1988) are among the many distinguished econometricians who have dealt with the problem of estimating production functions, first-order conditions, input demand functions, and cost and profit functions in the environment of price-taking firms. Some of these authors (e.g., Marschak and Andrews, Hoch, Mundlak, Zellner, Kmenta and Dreze, and Schmidt) used a primal approach in the estimation of a production function and the associated first-order necessary conditions corresponding to either profit-maximizing or cost-minimizing behavior. Their concern was to obtain consistent estimates of the production function parameters even in the case when output and inputs can be regarded as being determined simultaneously. This group of authors studied the "simultaneous equation bias" syndrome extensively. Nerlove (1963) was the first to use a duality approach in the estimation of a cost function for a sample of electricity-generating firms. After the seminal contributions of Fuss and McFadden (1978) (a publication that was delayed at least for a decade) and Diewert (1974), the duality approach seems to have become the preferred method of estimation.

In reality, the debate whether the duality approach should be preferred to the primal methodology has never subsided. As recently as 1996, Mundlak published a paper in *Econometrica* that is titled "*Production Function Estimation: Reviving the Primal*." To appreciate the strong viewpoint held by an influential participant in the debate, it is convenient to quote his opening paragraph (1996, p. 431): "*Much of the discussion on the estimation of production functions is related to the fact that inputs may be endogenous and therefore direct estimators of the production functions may be inconsistent. One way to overcome this problem has been to apply the concept of duality. The purpose of this note is to point out that estimates based on duality, unlike direct estimators of the production function, do not utilize all the available information and therefore are statistically inefficient and the loss in efficiency may be sizable*."

Our paper contributes the following fundamental point: A consistent and efficient (in the sense of using all the available information) estimation of the technical and economic relations involving a sample of price-taking firms *always* requires the joint estimation of primal and dual relations. This conclusion, we suggest, ought to be the starting point of any econometric estimation of a production and cost system. Whether or not it may be possible to reduce the estimation process to either primal or dual relations is a matter of statistical testing to be carried out within the particular sample setting. Therefore, the debate as to whether a primal or a dual approach should be preferred is moot. We will show that *all* the primal and dual relations are necessary for a consistent and efficient estimation of a production and cost system.

Section II describes the firm environment adopted in this study. Initially, we focus our attention on the papers by McElroy (1987) and Mundlak (1996) because their additive error specifications are the exact complement to each other. In order to facilitate the connection of our paper with the existing literature, we adopt much of the technological and economic environments described by them. Our model, therefore, is a general approach for the estimation of a production and cost system of relations which contains McElroy's and Mundlak's models as special cases.

Section III describes the generalized additive error (GAE) nonlinear specification adopted in our study and the estimation approach necessary for a consistent and efficient measurement of the cost-minimizing risk-neutral behavior of the sample firms. The price-taking and risk neutral entrepreneurs are assumed to optimize their cost-minimizing input decisions on the basis of their planning expectations concerning quantities and prices. Expectations are known to the decision makers but not to their accountants, let alone the outside econometrician. Measurement (i.e., observation) of the realized output and input quantities and prices, therefore, necessarily implies measurement errors. The model thus assumes the nonlinear structure of an error in variables and substantive unobservable variables model. The estimation approach to this complex (and usually unyielding) problem adopted in this study is different from the traditional approach in that we do not replace the unobservable latent variables with their observable counterpart.

Section IV discusses the consistency of the nonlinear least-squares estimator of the GAE model. Section V presents a series of nested hypotheses to test either Mundlak's or McElroys' specifications. Section VI discusses the difference in the estimation approach between traditional errors-in-variables models and the production and cost model developed in section III. Section VII uses a sample of cotton ginning cooperatives to test the hypothesis of a cost-minimizing risk-neutral behavior assuming a Cobb-Douglas technology. Section VIII concludes the paper by pointing out that the GAE model solves a vintage problem generated by the belief that a duality estimator cannot be used when all the input prices are the same.

II. Production and Cost Environments

In this paper we postulate a static context. Following Mundlak (1996), we assume that the cost-minimizing firms of our sample make their output and input decisions on the basis of expected quantities and prices and the entrepreneur is risk neutral. That is to say, a planning process can be based only upon expected information. The process of expectation formation is characteristic of every firm. Such a process is known to the firm's entrepreneur but is unknown to the econometrician. The individuality of the expectation process allows for a variability of input and output decisions among the sample firms even in the presence of a unique technology and measured output and input prices that appear to be the same for all sample firms.

Let the expected production function $f^e(\cdot)$ for a generic firm have values

$$y^e \leq f^e(\mathbf{x}), \tag{1}$$

where y^e the expected level of output for any strictly positive $(J \times 1)$ vector $\mathbf{x}$ of input quantities. After the expected cost-minimization process has been carried out, the input vector $\mathbf{x}$ will become the vector of expected input quantities $\mathbf{x}^e$ that will satisfy the firm's planning target. The expected production function $f^e(\cdot)$ is assumed to be twice continuously differentiable, quasi-concave, and non-decreasing in its arguments.

We postulate that the cost-minimizing risk-neutral firm solves the following problem:

$$c^e(y^e, \mathbf{w}^e) \overset{\text{def}}{=} \min_{\mathbf{x}}\{\mathbf{w}'^e\mathbf{x} \mid y^e \leq f^e(\mathbf{x})\}, \tag{2}$$

where $c^e(\cdot)$ is the expected cost function, $\mathbf{w}^e$ is a $(J \times 1)$ vector of expected input prices and "$'$" is the transpose operator.

The Lagrangean function corresponding to the minimization problem of the risk neutral firm can be stated as

$$L = \mathbf{w}'^{e}\mathbf{x} + \lambda[y^{e} - f^{e}(\mathbf{x})]. \tag{3}$$

Assuming an interior solution, first-order necessary conditions are given by

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{x}} &= \mathbf{w}^{e} - \lambda \mathbf{f}_{\mathbf{x}}^{e}(\mathbf{x}) = \mathbf{0}, \\ \frac{\partial L}{\partial \lambda} &= y^{e} - f^{e}(\mathbf{x}) = 0. \end{aligned} \tag{4}$$

The solution of equations (4), gives the expected cost-minimizing input demand functions $\mathbf{h}^{e}(\cdot)$, with values

$$\mathbf{x}^{e} = \mathbf{h}^{e}(y^{e}, \mathbf{w}^{e}). \tag{5}$$

In the case where equations (4) have no analytical solution, the input derived demand functions (5) exist via the duality principle.

The above theoretical development corresponds precisely to the textbook discussion of the cost-minimizing behavior of a price-taking firm. The econometric representation of that setting requires the specification of the error structure associated with the observation of the firm's environment and decisions. We regard the expected quantities and prices as non random information since the expected quantities reflect the entrepreneur's cost-minimizing decisions in which the expected prices are fixed parameters resulting from the expectation process of the individual entrepreneur.

Mundlak (1996) deals with two types of errors: a "weather" error associated with the realized (or measured) output quantity that, in general, differs from the expected (planned) level. This is especially true in agricultural firms, where expected output is determined many months in advance of realized output. Hence, measured output y differs from the unobservable expected output y^e by a random quantity u_0 according to the additive relation $y = y^e + u_0$. Furthermore, and again according to Mundlak (1996, p. 432): "As $\mathbf{w}^e$ is unobservable, the econometrician uses $\mathbf{w}$ which may be the observed input price vector or his own calculated expected input price vector." The additive error structure of input prices is similarly stated as $\mathbf{w} = \mathbf{w}^e + \boldsymbol{\nu}$. Mundlak (1996, p. 432) calls $\boldsymbol{\nu}$ "the optimization error, but we note that in part the error is due to the econometrician's failure to read the firm's decision correctly rather than the failure of the firm to reach the optimum." We will continue in the tradition of calling $\boldsymbol{\nu}$ the "optimization" error although it is simply a measurement error associated with input prices. Mundlak, however, does not consider any error associated with the measurement of input quantities.

To encounter such a vector of errors we need to refer to McElroy (1987). To be precise, McElroy (1987, p. 739) argues that her cost-minimizing model of the firm contains "… parameters that are known to the decision maker but not by the outside observer." Her error specification, however, is indistinguishable from a measurement error on the input quantities (McElroy, [1987], p. 739). In her model, input prices and output are (implicitly) known without errors. The measured vector of input quantities $\mathbf{x}$ bears an additive relation to its unobservable expected counterpart $\mathbf{x}^e$, that is $\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$. The vector $\boldsymbol{\varepsilon}$ represents the "measurement" errors on the expected input quantities.

We thus identify measurement errors with any type of sample information in the production and cost model of the firm. For reason of clarity and for connecting with the empirical literature on the subject, we maintain the traditional names of these errors, that is, u_0 is the "weather" error associated with the output actually produced, $\boldsymbol{\varepsilon}$ is the vector of "measurement" errors associated with the measured input quantities, and $\mathbf{v}$ is the vector of "optimization" errors associated with the measured input prices.

The measurable GAE model of production and cost can now be stated using the theoretical relations (1), (4), (5) and the error structure specified above. The measurable system of relations is the following set of primal and dual equations:

Primal

production function $$y = f^e(\mathbf{x} - \boldsymbol{\varepsilon}) + u_0, \tag{6}$$

input price functions $$\mathbf{w} = c_y^e(y - u_0, \mathbf{w} - \mathbf{v})\mathbf{f}_{\mathbf{x}}^e(\mathbf{x} - \boldsymbol{\varepsilon}) + \mathbf{v}, \tag{7}$$

Dual

input demand functions $$\mathbf{x} = \mathbf{h}^e(y - u_0, \mathbf{w} - \mathbf{v}) + \boldsymbol{\varepsilon}, \tag{8}$$

where $c_y^e(y - u_0, \mathbf{w} - \mathbf{v}) \overset{\text{def}}{=} \dfrac{\partial c^e}{\partial y^e}$ is the measurable marginal cost function.

Several remarks are in order. Relations (6) through (8) form a system of nonlinear equations that can be regarded as an error in variables model with substantive unobservable variables (see Zellner [1970], Theil [1971], Goldberg [1972], Griliches [1974], Klepper and Leamer [1984], Hausman and Watson [1985], Leamer [1987]). The crucial difference between these authors' models and the model presented in this paper consists in the estimation approach. The authors mentioned above replace the unobservable (expected) variables with a linear combination of exogenous variables, precisely as indicated in relations (6)-(8). In contrast, our approach consists in estimating the unobservable expected variables directly and jointly with all the technology parameters, as explained in detail in section III. In general, the production function, first-order conditions and input demand functions, convey independent empirical information in the form of errors and their probability distribution and are, therefore, necessary for obtaining efficient estimates of the model's parameters.

Although relations (7) and (8) may be regarded as containing precisely the same information, albeit in different arrangements, the measurement of their error terms requires, in general, the joint estimation of the entire system of primal and dual relations. This means that, in a general setting, all the primal and dual relations are necessary, and the debate about the "superiority" of either a primal or dual approach is confined to simplified characterizations of the error structure.

Consider, in fact, McElroys' (1987) model specification in which the "weather" and "optimization" errors are identically zero, that is, $u_0 \equiv 0$ and $\mathbf{v} \equiv \mathbf{0}$. Therefore, $y \equiv y^e$ and $\mathbf{w} \equiv \mathbf{w^e}$. In her case, the measurable GAE model (6)-(8) collapses to

$$y = f^e(\mathbf{x} - \boldsymbol{\varepsilon}), \tag{9}$$

$$\mathbf{w} = c_y^e(y, \mathbf{w})\mathbf{f}_{\mathbf{x}}^e(\mathbf{x} - \boldsymbol{\varepsilon}), \tag{10}$$

$$\mathbf{x} = \mathbf{h}^e(y, \mathbf{w}) + \boldsymbol{\varepsilon}. \tag{11}$$

McElroy (1987) can limit the estimation of her model to the dual side of the cost-minimizing problem because she implicitly assumes that the primal relations, namely the output levels and input prices, are measured without errors and, therefore, it is more convenient to estimate the dual relations (11) because the errors $\boldsymbol{\varepsilon}$ are additive in those relations while they are nonlinearly nested in equations (9) and (10).

An analogous but not entirely similar comment applies to Mundlak's (1996) specification. In his case the "measurement" errors are identically equal to zero, that is, $\boldsymbol{\varepsilon} \equiv \mathbf{0}$. Therefore, $\mathbf{x} \equiv \mathbf{x}^e$, so the measurable GAE model collapses to

$$y = f^e(\mathbf{x}) + u_0, \tag{12}$$

$$\mathbf{w} = c_y^e(y - u_0, \mathbf{w} - \mathbf{v})\mathbf{f}_\mathbf{x}^e(\mathbf{x}) + \mathbf{v}, \tag{13}$$

$$\mathbf{x} = \mathbf{h}^e(y - u_0, \mathbf{w} - \mathbf{v}). \tag{14}$$

We notice that, traditionally, Mundlak's approach to a cost-minimizing model requires the elimination of the Lagrange multiplier (equivalently, marginal cost) by taking the ratio of $(J-1)$ equations of the first-order necessary conditions to, say, the first equation (see, for example, Schmidt, 1987, p. 362). In this way the error term of the first equation is confounded into the disturbance term of every other equation. Under these conditions, it may be more convenient to follow Mundlak's recommendation and estimate the primal relations (12) and (modified) (13) because the two types of errors appear in additive form. No such a loss of information is required in the model presented here and under the more general structure of GAEM presented above (where no ratios of *(J-1)* equations to the first equation is necessary), this "advantage" no longer holds.

III. Estimation of the GAE Model of Production and Cost

We assume a sample of cross-section data on *N* cost-minimizing firms, $i = 1,...,N$. The empirical GAE model in its most general specification can thus be stated as

$$y_i = f^e(\mathbf{x}_i^e, \boldsymbol{\beta}_y) + u_{0i}, \tag{15}$$

$$\mathbf{w}_i = c_y^e(y_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_c)\mathbf{f}_\mathbf{x}^e(\mathbf{x}_i^e, \boldsymbol{\beta}_\mathbf{w}) + \mathbf{v}_i, \tag{16}$$

$$\mathbf{x}_i = \mathbf{h}^e(y_i^e, \mathbf{w}_i^e, \boldsymbol{\beta}_\mathbf{x}) + \boldsymbol{\varepsilon}_i. \tag{17}$$

The vectors of technological and economic parameters $\boldsymbol{\beta}_y, \boldsymbol{\beta}_\mathbf{w}, \boldsymbol{\beta}_\mathbf{x}, \boldsymbol{\beta}_c$ may be of different dimensions, characterize the specific relations referred to by their subscript and, in general, enter those relations in a nonlinear fashion.

The vector of error terms $\mathbf{e}_i' \overset{\text{def}}{=} (u_{0i}, \mathbf{v}_i', \boldsymbol{\varepsilon}_i')$ is assumed to be distributed according to a multivariate normal density with zero mean vector and variance matrix Σ. We thus assume independence of the disturbances across firms and contemporaneous correlation of them within a firm. If the expected quantities and prices were known, the above system of equations would have the structure of a traditional nonlinear seemingly unrelated equations (NSUR) estimation problem. In that case, consistent and efficient estimates of the parameters could be obtained using commercially available computer packages for econometric analysis. Unfortunately, the recording of planning information and decisions is not a common practice. However, if we could convince a sample of entrepreneurs to record expected quantities and prices at planning time, the direct estimation of system (15)-(17) would be feasible and efficient. Hence, lacking the "true" expected quantities and prices, the next best option is to obtain consistent estimates of them.

To confront the estimation challenge posed by the system of relations (15)-(17), we envision a two-phase procedure that produces consistent estimates of the unobservable substantive variables, represented by the expected quantities and prices and the vector of $\boldsymbol{\beta} = (\boldsymbol{\beta}_y, \boldsymbol{\beta}_{\mathbf{w}}, \boldsymbol{\beta}_{\mathbf{x}}, \boldsymbol{\beta}_c)$ parameters, in phase I and then uses those estimates of expectations in phase II to estimate a traditional NSUR model.

In phase I, the nonlinear least-squares estimation problem consists in minimizing the residual sum of squares

$$\min_{\boldsymbol{\beta}, y_i^e, x_{ij}^e, w_{ij}^e, \mathbf{e}_i} \sum_{i=1}^{N} \mathbf{e}_i' \mathbf{e}_i \,, \tag{18}$$

with respect to the residuals and all the parameters, including the expected quantities and prices for each firm, subject to equations (15), (16), (17) and the error structure postulated in section II, that is,

$$y_i = y_i^e + u_{0i}, \tag{19}$$

$$\mathbf{w}_i = \mathbf{w}_i^e + \boldsymbol{\nu}_i, \tag{20}$$

$$\mathbf{x}_i = \mathbf{x}_i^e + \boldsymbol{\varepsilon}_i, \tag{21}$$

$$\sum_{i=1}^{N} y_i^e u_{0i} = 0, \tag{22}$$

$$\sum_{i=1}^{N} w_{ij}^e \nu_{ij} = 0, \quad j = 1,...,J, \tag{23}$$

$$\sum_{i=1}^{N} x_{ij}^e \varepsilon_{ij} = 0, \quad j = 1,...,J. \tag{24}$$

The structure of the nonlinear errors-in-variables problem (15)-(24) is peculiar in that all the sample variables appear twice, once related to a nonlinear function arising from economic theory and the second time related to the linear error structure postulated by the econometrician. This double appearance does not constitute, in general, a redundant specification. In other words, it is not possible, in general, to set some vector of errors equal to zero and solve for the other remaining unknowns. The reason for this result is found in the interlocking structure of the problem. That is, every expected parameter appears in the error structure but also in at least one set of theoretically generated nonlinear functions creating thus the interlocking structure referred to above.

Constraints (22)-(24) guarantee the orthogonality of the residuals and the corresponding estimated expected quantities and prices, exactly as is dictated by the definition of an instrumental variable, a role that they play in phase II. We assume that an optimal solution of the phase I problem exists and can be found using a nonlinear optimization package such as, for example, GAMS (see Brooke *et al.* [1988]).

With the estimates of the expected quantities and prices obtained from phase I, a traditional NSUR problem can be stated and estimated in phase II using conventional econometric packages such as SHAZAM (Whistler *et al.* [2001]). For clarity, this phase II estimation problem can be stated as

$$\min_{\boldsymbol{\beta},\mathbf{e}_i} \sum_{i=1}^{N} \mathbf{e}_i' \hat{\Sigma}^{-1} \mathbf{e}_i \tag{25}$$

subject to

$$y_i = f^e(\hat{\mathbf{x}}_i^e, \boldsymbol{\beta}_y) + u_{0i}, \tag{26}$$

$$\mathbf{w}_i = c_y^e(\hat{y}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_c)\mathbf{f}_{\mathbf{x}}^e(\hat{\mathbf{x}}_i^e, \boldsymbol{\beta}_{\mathbf{w}}) + \boldsymbol{\nu}_i, \tag{27}$$

$$\mathbf{x}_i = \mathbf{h}^e(\hat{y}_i^e, \hat{\mathbf{w}}_i^e, \boldsymbol{\beta}_{\mathbf{x}}) + \boldsymbol{\varepsilon}_i, \tag{28}$$

where $(\hat{y}_i^e, \hat{\mathbf{w}}_i^e, \hat{\mathbf{x}}_i^e)$ are the expected quantities and prices of the i-th firm estimated in phase I and assume the role of instrumental variables in phase II. The matrix $\hat{\Sigma}$ can be updated iteratively to convergence.

The specification of the functional form of the production function constitutes a further challenge toward the successful estimation of the above system of production and cost functions. In the case of self-dual technologies such as the Cobb-Douglas and the constant elasticity of substitution (CES) production functions, the corresponding cost function has the same functional form and no special difficulty arises. For the general case of more flexible functional forms, however, it is well known that the functional form can be explicitly stated only for either the primal or the dual relations. The associated dual functions exist only in an implicit, latent state. The suggestion, therefore, is to assume an explicit flexible functional form for the cost function and to represent the associated implicit production function as a second-degree Taylor expansion. Alternatively, one can use an appealing approach to estimation of latent functions presented by McManus (1994) that fits a localized Cobb-Douglas function to each sample observation.

IV. Consistency of the Nonlinear Least-Squares GAE Estimator

The consistency of the NLS estimator of the GAE model follows the principle and the procedure outlined by Davidson and MacKinnon (1993, ch. 5). We simply cast the GAE model in the form that fits the assumptions stated in their theorem 5.1 (p. 148).

In order to streamline the presentation of the consistency arguments we recast the nonlinear GAE model in its simplest formulation by abstracting from the notation and structure of the more complex problem stated in the previous section. Using, in fact, the notation of Davidson and MacKinnon we now assume of dealing here with one "*Y*" variable and one "X^e" variable and write

$$\begin{aligned} y_t &= f_t(x_t^e, \beta) + \upsilon_t, \\ x_t &= x_t^e + \nu_t, \\ x_t &= g_t(y_t^e, \beta) + \nu_t, \\ y_t &= y_t^e + \upsilon_t, \end{aligned} \tag{29}$$

that we further collect into the following specification

$$q_{st} = q_{st}(z_{s't}^e, \beta) + \omega_{st}, \tag{30}$$

with $t = 1,...,T$, $s,s' = 1,...,4$, for $s \neq s'$, $z_{s't}^e \equiv 0$, and

$$q_{st} \equiv \begin{bmatrix} y_t \\ x_t \\ x_t \\ y_t \end{bmatrix}, \; q_{st}(z_{s't}^e, \beta) \equiv \begin{bmatrix} f_t(x_t^e, \beta) \\ x_t^e \\ g_t(y_t^e, \beta) \\ y_t^e \end{bmatrix}, \; \omega_{st} \equiv \begin{bmatrix} \upsilon_t \\ \nu_t \\ \nu_t \\ \upsilon_t \end{bmatrix} \text{ and } z_{s't}^e \equiv \begin{bmatrix} x_t^e & & & \\ & x_t^e & & \\ & & y_t^e & \\ & & & y_t^e \end{bmatrix}.$$

This setup corresponds to the nonlinear specification of Davidson and MacKinnon who prove consistency of the nonlinear least-squares estimator by assuming three conditions: (i) that the GAE model is asymptotically identified by the probability limit of the average sum-of-squares function of residuals; (ii) that the sequence $\{T^{-1}\sum_{t=1}^{T} q_{st}(z^{e}_{s't},\beta)\omega_{st}\}$ satisfies the Weak Uniform Law of Large Numbers with probability limit of zero; and (iii) that the probability limit of the sequence $\{T^{-1}\sum_{t=1}^{T} q_{st}(z^{e}_{s't},\beta)q_{st}(z'^{e}_{s't},\beta')\}$, for any $(z^{e}_{s't},\ \beta)$ and $(z'^{e}_{s't},\ \beta')$ is finite, continuous in $(z^{e}_{s't},\ \beta)$ and $(z'^{e}_{s't},\ \beta')$, non-stochastic, and uniform with respect to $(z^{e}_{s't},\ \beta)$ and $(z'^{e}_{s't},\ \beta')$. Under these assumptions the NLS estimator is consistent. We refer to Davidson and MacKinnon (1993, ch. 5) for further technical details.

To contribute some clarifying remarks we notice that condition (i) involves the asymptotic identification of the model. After many efforts to demonstrate (or disprove) this property for the production and cost model presented in this paper we conjecture that the condition is analytically intractable and, therefore, with no evidence to the contrary, we assume that the model is asymptotically identified. Condition (iii) involves the nonlinear function of the GAE model as stated in equation (38). As long as that function satisfies the three conditions, the NLS-GAE estimator is consistent.

V. Hypothesis Testing

The generality of the econometric model discussed in section III admits a wide spectrum of hypothesis testing. Foremost, it is of interest to test Mundlak's (1996) and McElroy's (1987) specifications against the more general model. Secondly, we wish to test whether the sample of firms might have made their decisions under cost-minimizing behavior. Thirdly, we assess the predictive ability of the three models.

5.1 Mundlak's Model Revisited

The structure of the error specification postulated by Mundlak (1996) results in the nested model given by equations (12) and (13). Mundlak's model requires both phases of the estimation procedure because the expected output and expected input prices enter relation (13). The nested nature of the hypothesis is a consequence of stating that all the disturbances associated with the input quantities are identically equal to zero, that is, $\boldsymbol{\varepsilon} \equiv \mathbf{0}$. In this case, therefore, a likelihood ratio constitutes the test statistic.

5.2 McElroy's Model Revisited

The error specification of McElroy's (1987) model assumes that $u_0 \equiv 0$ and $\mathbf{v} \equiv \mathbf{0}$. In this case, the relevant relations to estimate are given by equations (11). Under these assumptions, no additional information is contained in the other relations (9) and (10), and equation (11) represents a NSUR specification that can be estimated directly as a phase II procedure with a conventional econometric package, without the need to implement phase I of our approach since there are no expected variables to estimate. A likelihood ratio can be used to test the hypothesis that $u_0 \equiv 0$ and $\mathbf{v} \equiv \mathbf{0}$ against the more general model's structure.

5.3 Cost minimization

In the empirical application of section VI, we will adopt a Cobb-Douglas environment as explained there. In this case, the verification of the hypothesis that the sample firms have made their decisions according to a cost-minimization criterion requires the positivity of each production elasticity. We will use a Bayesian test for inequality constraints developed by Geweke (1986).

5.4 Predictions and their standard errors

Another criterion for judging the validity of the various models is to evaluate their performance in the prediction phase of the econometric analysis. Following Fuller (1980), we will obtain predictions and prediction error variances for the nonlinear models discussed above.

VI. Digression On Errors In Variables

The GAE model discussed in previous sections can be regarded as a traditional nonlinear errors-in-variables model with substantive unobservable, non-stochastic information represented by the expected quantities and prices. However, the approach to its estimation is different from the traditional approach as developed, for example, by Theil (1971, p. 608): Theil postulates an exact relation $y_\alpha^* = \beta x_\alpha^*,\ \alpha = 1,...,N$ between two "true" variables [akin to the cost-minimizing relations (1), (4) and (5)] and measurement errors on both "true" variables, that is, $y_\alpha = y_\alpha^* + v_{1\alpha}$ and $x_\alpha = x_\alpha^* + v_{2\alpha}$. He replaces y_α^* and x_α^* in the "true" model and obtains $y_\alpha = \beta x_\alpha + (v_{1\alpha} - \beta v_{2\alpha})$ as the estimable relation. Finally, he shows that the least-squares estimator of β based upon this estimable model is, in general, inconsistent. All the authors that dealt with errors-in-variables models have followed Theil's (1971) approach and replaced the unobservable "true" information with its measurable counterpart. The inconsistency and the under-identification of the traditional errors-in-variables model result from this replacement approach.

In this paper we do not replace the "true" (expected) quantities and prices of relations (15)-(17) by their measurable counterparts but, rather, produce a consistent estimate of these "true" quantities and prices jointly with all the other technological parameters of the model. This occurs in phase I via the specification and estimation of a nonlinear least-squares model. In phase II, we use these estimates of the “true” variables as instrumental variables in a NSUR model to obtain the final estimates of the parameters and all the diagnostics of the production and cost model. Our approach, therefore, is completely analogous to a three-stage least-squares estimator, since the estimation is carried out on a system basis.

VII. An Application of the GAE Model of Production and Cost

The model and the estimation procedure described in section III have been applied to a sample of 84 California cooperative cotton ginning firms. These cooperative firms must process all the raw cotton delivered by the member farmers. Hence, the level of their output is exogenous and their economic decisions are made according to a cost-minimizing behavior. This is a working hypothesis that can be tested during the analysis.

There are three inputs: labor, energy and capital. Labor is defined as the annual labor hours of all employees. The wage rate for each gin was computed by dividing the labor bill by the quantity of labor. Energy expenditures include the annual bill for electricity, natural gas, and propane. British thermal unit (BTU) prices for each fuel were computed from each gin’s utility rate schedules and then aggregated into a single BTU price for each gin using BTU quantities as weights for each energy source. The variable input energy was then computed by dividing energy expenditures by the aggregate energy price.

A gin’s operation is a seasonal enterprise. The downtime is about nine months per year. The long down time allows for yearly adjustments in the ginning equipment and buildings. For this reason capital is treated as a variable input. Each component of the capital stock was measured using the perpetual inventory method and straight-line depreciation. The rental prices for buildings and ginning equipment was measured by the Christensen and Jorgenson (1969) formula. Expenditures for each component of the capital stock were computed as the product of each component of the capital stock and its corresponding rental rate and aggregated into total capital expenditures. The composite rental price for each gin was computed using an expenditure-weighted average of the gin's rental prices for buildings and equipment. The composite measure of the capital service flow is computed by dividing total yearly capital expenditure by the composite rental price.

Ginning cooperative firms receive the raw cotton from the field and their output consists of cleaned and baled cotton lint and cottonseeds in fixed proportions. These outputs, in turn, are proportional to the raw cotton input. Total output for each gin was then computed as a composite commodity by aggregating cotton lint and cottonseed using a proportionality coefficient. For more information on the sample data see Sexton *et al.* (1989).

We assume that the behavior of the ginning cooperatives of California can be rationalized with a Cobb-Douglas production function. Hence, the system of equations to specify the production and cost environments is constituted of the following eight primal and dual relations:

Cobb-Douglas production function

$$y_i = A\prod_{j=1}^{3}(x_{ij}^{e})^{\alpha_j} + u_{0i}, \tag{31}$$

Input price functions

$$w_{ik} = \alpha_k [A\prod_{i=1}^{3} \alpha_j^{\alpha_j}]^{-1/\eta} (y_i^e)^{1/\eta} \prod_{j=1}^{3} (w_{ij}^e)^{\alpha_j/\eta} / (x_{ik}^e) + \nu_{ik}, \tag{32}$$

Input derived demand functions

$$x_{ij} = \alpha_j [A\prod_{k=1}^{3} \alpha_k^{\alpha_k}]^{-1/\eta} (y_i^e)^{1/\eta} (w_{ij}^e)^{-(\sum_{k\neq j}\alpha_k)/\eta} \prod_{k\neq j=1}^{3} (w_{ik}^e)^{\alpha_k/\eta} + \varepsilon_{ij}, \tag{33}$$

where $\eta \overset{\text{def}}{=} \sum_j \alpha_j$, $j = 1,2,3$, and $k = 1,2,3$.

The system of Cobb-Douglas relations (31)-(33) was estimated by using the two-phase procedure described in section III using the computer package GAMS (Brooke *et al.* [1988]) for phase I, and SHAZAM (Whistler *et al.* [2001]) for phase II. We must point out that with technologies (such as the Cobb-Douglas production function) admitting an explicit analytical solution of the first-order necessary conditions, either the input derived demand functions (33) or the input price functions (32) are redundant in the phase I estimation problem, and thus either set of equations can be eliminated as constraints. They are not redundant, however, in the phase II NSUR estimation problem because, as noted earlier, all the primal and dual relations convey independent information in the form of their errors and the corresponding probability distributions.

Mundlak's (1996) and McElroy's (1987) models were also estimated as nested models of the full covariance model. The results are reported in Table 1 with *t*-ratios (of the estimates) in parentheses. The values of the parameter estimates of the three nested models presented in Table 1 are rather similar but the corresponding *t*-ratios are widely different. This gain in efficiency reflects the utilization of the available information in the various specifications.

When Mundlak's (1996) and McElroy's (1987) models are tested against the more general full covariance model it turns out that Mundlak's model is soundly rejected while McElroy's model cannot be rejected. The test statistic is the traditional likelihood ratio test which exhibits 21 and 26 degrees of freedom for Mundlak's and McElroy's models, respectively. The degrees of freedom are computed as the difference between covariance parameters of the models involved in the hypothesis and the difference between prediction parameters. The chi-square critical value (at the 0.01 confidence level) for Mundlak's hypothesis is 38.88 while it is 45.64 for McElroy's hypothesis.

The cost minimization hypothesis was tested using the Bayesian approach developed by Geweke (1986). In this test, a large number of parameter samples is drawn from a suitable universe defined by the empirical estimates. The proportion of those samples that satisfy the conditions defining the given hypothesis is recorded. The higher the proportion the higher the confidence that the hypothesis is "true". The cost-minimization hypothesis is accepted unanimously in the three system models with a proportion of "successes" equal to one.

The results of the prediction analysis are presented in Table 2. We notice that the predictions generated from the full covariance model are closer to the realized values and have a consistently smaller variance than the predictions obtained with either Mundlak's or McElroy's models.

On the basis of the general performance and with special regard to the demonstrated predictive ability, we tend to favor the full covariance model for this sample of firms as the best econometric specification that rationalizes the available information.

VIII. Conclusion

We tackled the 60-years old problem of how to obtain consistent estimates of a Cobb-Douglas production function when the price-taking firms operate in a cost-minimizing environment. The simplicity of the idea underlying the model presented in this paper can be re-stated as follows. Entrepreneurs make their planning, optimizing decisions on the basis of expected, non-stochastic information. When econometricians intervene and desire to re-construct the environment that presumably led to the realized decisions, they have to measure quantities and prices and, in so doing, commit measurement errors. This background seems universal and hardly deniable. The challenge, then, of how to deal with a nonlinear errors-in-variables specification was solved by a two-phase estimation procedure. In phase I, the expected quantities and prices are estimated by a nonlinear least-squares method. In phase II, this estimated information is used in a NSUR model to obtain consistent and efficient estimates of the Cobb-Douglas technology.

In the process, the debate whether a primal or a dual approach is to be preferred for estimating production and cost relations was put to rest by the demonstration that an efficient system is composed by both primal and dual relations that must be jointly estimated. Only under special cases it is convenient to estimate either a primal (Mundlak's) or a dual (McElroy's) environment.

In connection with this either-primal-or-dual debate, it is often said (for example, Mundlak 1996, p. 433): "*In passing we note that the original problem of identifying the production function as posed by Marschak and Andrews (1944) assumed no price variation across competitive firms. In that case, it is impossible to estimate the supply and factor demand functions from cross-section data of firms and therefore (the dual estima-*

tor) $\hat{\gamma}_p$ *cannot be computed. Thus, a major claimed virtue of dual functions---that prices are more exogenous than quantities--- cannot be attained. Therefore, for the dual estimator to be operational, the sample should contain observations on agents operating in different markets*."

After many years of pondering this non-symmetric problem, the solution is simpler than expected and we can now refute Mundlak's assertion. The key to the solution is the assumption that individual entrepreneurs make their planning decisions on the basis of their expectation processes, an assumption made also by Mundlak (1996, p. 431). The individuality of such information overcomes the fact that econometricians measure a price that seems to be the same across firms. In effect we know that this uniformity of prices reflects more the failure of our statistical reporting system rather than a true uniformity of prices faced by entrepreneurs in their individual planning processes. The model proposed in this paper provides an operational dual estimator, as advocated by Mundlak, by decomposing a price that is perceived as the same across observations into an individual firm's expected price and a measurement error.

The GAE model of production and cost presented here can be extended to a profit-maximization environment and also to the consistent estimation of a system of consumer demand functions.

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TABLE 1

NSUR estimates of three production and cost models

Technological Parameters	Full Covariance Model	Mundlak's Model	McElroy's Model
Efficiency, A	0.8082	0.7488	0.7564
	(306.80)	(63.228)	(11.123)
Capital, α_K	0.4325	0.4508	0.4403
	(233.50)	(57.050)	(19.334)
Labor, α_L	0.4800	0.5081	0.5180
	(221.25)	(66.368)	(20.590)
Energy, α_E	0.2695	0.2812	0.3000
	(264.81)	(47.098)	(23.328)
Returns to Scale, $\sum\alpha_i$	1.1820	1.2401	1.2580
Loglikelihood	-301.7093	-524.7222	-318.5651
Likelihood ratio test		446.03	33.71
Degrees of freedom		21	26

t-ratios in parenthesis

TABLE 2

Predictions and predictions' *t*-ratios

Prediction of Observation 84	Actual Values	Full Covariance Model	Mundlak's Model	McElroy's Model
Output	4.3239	3.2727	2.9481	--
		(5.111)	(3.652)	
Capital price	13.740	13.304	18.426	--
		(20.55)	(8.012)	
Labor price	11.585	11.018	9.6498	--
		(23.01)	(6.089)	
Energy price	10.000	9.6800	8.6702	--
		(28.17)	(4.568)	
Capital	2.0689	2.9908	--	3.5416
		(3.963)		(2.383)
Labor	4.4532	4.0249	--	4.9418
		(7.217)		(5.654)
Energy	2.7429	2.5674	--	3.3117
		(6.551)		(5.668)