Variability in Growth, Pig Weights and Hog Marketing Decisions

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Abstract

Variability in pig growth is an intrinsic characteristic of swine production. The optimal marketing strategies are identified to minimize the negative economic impact of variability for a typical all-in-all-out swine finishing facility using a recent pricing matrix and data featuring swine production in the Midwestern region. Our results show that compared with marketing all pigs from a 1,020 head barn on the same day, marketing pigs in six truckloads on different dates as groups of pigs grow to more optimal size significantly improves the profitability of production as variability increases. This finding is in line with recent producer response to new pricing matrices that prove stronger price incentives for marketing more uniform pigs. We also find that studies on optimal marketing strategies without taking into account variability in pig weights can result in exaggerated optimal marketing weights and profits of production. Growth variability management and marketing strategies continue to be essential to the economic viability of the swine industry.

Keywords: swine production, variability in growth, marketing strategy, profit maximization.

Acknowledgements

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Variability in growth and pig weights in same age pigs is an intrinsic characteristic of hog production and managing this variability is important for the pork industry. The existing literature suggests that numerous factors have an impact on variability in pig growth and bodyweight. These factors include at least genotype, disease, hygiene, nutrition, weight and age at weaning, management of lightweight pigs pre- and post-weaning, group size, stocking rate, sorting and mixing of pigs, and pig behavior (Payne et al.). Variability in pig performance may be reduced but cannot be eliminated and a coefficient of variation (CV) of 10 to 12% at market weight from a finishing barn is considered quite good and is a reasonable target for commercial production (Patience et al.).

The economic impact of variability in pig performance has been well recognized (Payne et al.). Typically, variability in pig marketing weight may entail additional economic loss to swine finishing producers because the price of lighter weight hogs and excessively heavy hogs is substantially discounted for many packer pricing matrices (Song and Miller). It also has been realized that optimized marketing strategies appear to be the most practical way of managing the herd to minimize the negative impact of variability, rather than attempting to reduce the variability per se (Patience, Gonyou, and Zijlstra). This is particularly true for systems with all-in-all-out (AIAO) management practices. For example, under a particular pricing matrix and a given CV within the herd, closing-out a finishing barn according to an optimized schedule and optimal market weights may increase the profitability of production more than any other single activity (Patience, Gonyou, and Zijlstra). A recent investigation indicates that hitting the range of carcass weights that maximize price, and avoiding penalties for light and heavy carcasses,

can increase the average price received by 4 per cent and net benefits by $5 per pig sold (Patience, Gonyou, and Zijlstra).

The influence of a packer's pricing system on hog marketing decisions has been studied previously (Roka and Hoag; Boland, Preckel, and Schnickel). Roka and Hoag found that profit maximizing producers tend to market pigs at the upper bound of the live weight range paid the highest price whenever daily gains in pork value at this price are greater than daily costs of production. Boland, Preckel, and Schnickel examined how producer profits, optimal slaughter weights, and carcass component weight change under three pricing systems. However, these studies did not account for the effect of pig weight variability in the herd, nor did they reflect recent changes in packers' pricing matrices.

The implication of variability in growth and pig weights on marketing strategies for AIAO finishing barns was also investigated in some economic studies (Deen, Skorupski, and Frey; Dritz and Tokach; Li et al.; Song and Miller). Some of these studies used a computer simulation approach to identify the profit maximizing marketing strategies for an AIAO system (Deen, Skorupski, and Frey; Li et al.). This approach involved assuming a distribution of weights of pigs or a stochastic growth function of pigs, simulating over the population in the barn, and calculating the total profit under different marketing strategies over a certain time frame to obtain the so-called "optimal" strategy that produces the highest profit. Song and Miller partitioned a pig growth curve and investigated up to four fixed shipment dates for pigs from a barn using a computer simulation model. They found that profits generated per barn increased and then decreased as the last shipment date increased, and decreased as days between batches (time needed for cleaning) increased. A limitation of this approach is that its results are

subject to possible sampling errors arising from stochastic number generation in the simulation and can be influenced by the strategy simulation design.

In this paper, we determine optimal marketing strategies that minimize the impact of the inherent growth variability in a 1,020 head barn using a recent pricing matrix given by a packer and data featuring swine production in the Midwestern region. Assume that a producer maximizes the profit of swine production per unit of time from a typical AIAO grow-finish barn given an assumed normally distributed pig weight with a known CV and pricing matrix. The pigs are assumed to be shipped to the packer in six truckloads with a capacity of 170 head and one or more trucks can be shipped on the same day. The producer chooses the optimal marketing days and optimal sorting weights for each truckload to maximize the profit in a year, which is defined as returns from hog sales less costs of production and modeled as a continuous function of time. Under the normal distribution assumption of pig weights, hog sales can be obtained by integrating the weight distribution function with corresponding prices provided in a packer's pricing matrix. The optimal marketing days and sorting weight for each batch of pigs are derived by jointly solving the first order conditions of the profit maximization problem. Analytical methods are employed to solve the nonlinear multi-variable optimization problem. The analytical results are used in conjunction with a spreadsheet to estimate annual profits from a barn as a function of CV.

This study distinguishes itself from previous studies by determining the optimal marketing strategies using an analytical approach rather than computer simulation. The computational advantage of this study enables one to conveniently explore the implications of different practices for managing pig growth variability on profitability

and on hog marketing strategies as long as the cost and effect of the strategy to control the impact of weight variability is known. The results of this research provide useful information to producers, consultants, and packers for swine production and marketing decision making.

Models of finishing hog marketing strategy

We assume that a swine finishing barn of 1,020 head is managed all-in-all-out (AIAO) and that swine producers maximize profit of production per unit time from the barn. Pigs are marketed in six truckloads with 170 pigs per truck. We assume that pig weights in the barn are normally distributed with mean μ and standard deviation σ, i.e., a distribution with a probability density function

$$(1) \qquad f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

In addition, the coefficient of variation (CV) is assumed to be constant in time, t, i.e.,

$CV = \dfrac{\sigma(t)}{\mu(t)}$ is a constant over time and independent of pig shipments from the barn as

explained later.

For simplicity, a packer's hog live market weight pricing system is approximated by a quadratic function of the following general form:

$$(2) \qquad P(x) = a_0 + a_1 x + a_2 x^2$$

where x is the live weight of a pig at market, $P(x)$ is the live weight price ($ per pound of live weight), and a_0, a_1, and a_2 are parameters ($a_0 < 0$, $a_1 > 0$, and $a_2 < 0$). Given such a hog pricing system, two marketing strategies are modeled: strategy 1: all six truckloads are shipped out on the same day; and strategy 2: truckloads are shipped out either on different dates or on the same day. Assuming that the mean of the hog live weights in the

barn μ is a function of time, t, the producers' profit maximization problem under strategy

1 is to determine the optimal mean μ at marketing or optimal days on feed t for the entire

1,020 pigs. Under strategy 2, the profit maximization problem is to determine the optimal

sort weight (weight above which all pigs will be marketed) and the optimal days on feed

for each truckload.

Model 1: marketing all pigs on the same day

The expected revenue from the entire 1,020 pigs with a weight distribution

$N(\mu, \sigma^2)$ and a pricing schedule of $P(x)$ can be expressed as

(3) $\qquad R = 1020 \int_{-\infty}^{\infty} P(x) x f(x) dx = 1020 \int_{-\infty}^{\infty} (a_0 + a_1 x + a_2 x^2) x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

Integrating equation (3) and simplifying (appendix 1), we obtain

(4) $\qquad R = 1020[a_0\mu + a_1(\mu^2 + \sigma^2) + a_2(\mu^3 + 3\mu\sigma^2)]$

The production costs per pig are assumed to fall into three categories: the cost of a

feeder pig, feed costs, and nonfeed costs. The cost of feeder pigs is the per pig purchase

price. Feed costs are determined by the unit feed price and the cumulative feed intake of a

pig after t days on feed. Nonfeed costs include building depreciations, interest and

premiums, labor costs, veterinary medical costs, transportation/marketing costs, mortality

losses, and other costs that are excluded in the other two cost items. Unlike for timeless

analyses, fixed input costs must be included in a time-dependent profit maximization

problem because the ratio of fixed costs to time is not constant for the time variable

(Dillon and Anderson, p89). For simplicity, nonfeed costs are measured in terms of $ per

pound of market weight. Let c_1 denote the unit price of a feeder pig ($/head); c_2 be the

unit price of feed ($/pound); $F(t)$ be the cumulative feed intake of a pig (pounds), which

is a function of days on feed, t; and c_3 be the nonfeed cost (\$/pound of hog live weight).

Annual profits of production, π, are then

(5)
$$\pi = \{1020[a_0\mu(t) + a_1(\mu(t)^2 + \sigma(t)^2) + a_2(\mu(t)^3 + 3\mu(t)\sigma(t)^2)$$
$$- c_1 - c_2 F(t) - c_3\mu(t)]\}\frac{365}{t}$$

Let CV = s (a constant). The first order condition of the profit maximization problem for marketing all pig on one day can be derived by differentiating equation (5) with respect to t:

(6)
$$\frac{d\pi}{dt} = [(a_0 - c_3)\mu'(t) + 2a_1(1 + s^2)\mu(t)\mu'(t) + 3a_2(1 + 3s^2)\mu(t)^2\mu'(t)$$
$$- c_2 F'(t)] - \frac{1}{t}[(a_0 - c_3)\mu(t) + a_1(1 + s^2)\mu(t)^2 + a_2(1 + 3s^2)\mu(t)^3$$
$$- c_2 F(t) - c_1] = 0$$

Assuming $\mu(t) = b_0 + b_1 t$ (i.e., a linear growth function) and $F(t) = d_0 + d_1 t + d_2 t^2 + d_3 t^3$, equation (6) can be reduced to the following cubic equation:

(7)
$$[2a_2 b_1^3(1 + 3s^2) - 2c_2 d_3]t^3 + [a_1 b_1^2(1 + s^2) + 3a_2 b_0 b_1^2(1 + 3s^2) - c_2 d_2]t^2$$
$$- (a_0 - c_3)b_0 - a_1 b_0^2(1 + s^2) - a_2 b_0^3(1 + 3s^2) + c_2 d_0 + c_1 = 0$$

Letting
$$\alpha = 2a_2 b_1^3(1 + 3s^2) - 2c_2 d_3,$$
$$\beta = a_1 b_1^2(1 + s^2) + 3a_2 b_0 b_1^2(1 + 3s^2) - c_2 d_2,$$
$$\gamma = -(a_0 - c_3)b_0 - a_1 b_0^2(1 + s^2) - a_2 b_0^3(1 + 3s^2) + c_2 d_0 + c_1,$$

we have a first order profit maximization condition of

(8) $$\alpha t^3 + \beta t^2 + \gamma = 0$$

Using $t = z - \dfrac{\beta}{3\alpha}$ (Vièta's substitution) and equation (8) and simplifying yields

(9) $$z^3 - \frac{\beta^2}{3\alpha^2}z + \frac{2\beta^3}{27\alpha^3} + \frac{\gamma}{\alpha} = 0$$

By defining $\upsilon = -\dfrac{\beta^2}{3\alpha^2}$ and $\omega = \dfrac{2\beta^3}{27\alpha^3} + \dfrac{\gamma}{\alpha}$, according to Cardano's formula (Harris and Stocker, p44), the real solution to the cubic equation (9) is

(10) $\quad z^* = \sqrt[3]{-\dfrac{\omega}{2} + \sqrt{\left(\dfrac{\omega}{2}\right)^2 + \left(\dfrac{\upsilon}{3}\right)^3}} + \sqrt[3]{-\dfrac{\omega}{2} - \sqrt{\left(\dfrac{\omega}{2}\right)^2 + \left(\dfrac{\upsilon}{3}\right)^3}}$

Hence, we obtain the optimal days on feed

(11) $\quad t^* = z^* - \dfrac{\beta}{3\alpha}$

and the optimal mean weight at marketing

(12) $\quad \mu(t^*) = b_0 + b_1 t^*$

Model 2: marketing truckloads on different dates

Given a normal weight distribution of $N(\mu,\ \sigma^2)$, the 1,020 pigs are grouped according to their weights into six 170-pig truckloads, each of which composes one sixth of the population (see figure 1) and is numbered in an order from the heaviest to the lightest in the population. The six truckloads can be separately shipped out on six different dates but any two or more of them can also be shipped on the same day. We assume that removal of pigs from the barn will not influence the growth of the remaining pigs. That is, though μ and σ are changing over time, the ratio of $\sigma(t)/\mu(t)$ (*CV*) is constant and the remaining pigs will stay in the same portion in the normal distribution of pig weights as they grow. Similar to the computation of the center of mass (Harris and Stocker, p584), the expected revenue for truckload 1 that consists of the heaviest 170 pigs in the barn is

$$(13) \quad R_1 = 170 \times \frac{\int_{\mu+0.97\sigma}^{\infty} P(x)xf(x)dx}{\int_{\mu+0.97\sigma}^{\infty} f(x)dx} = 170 \times 6 \times \int_{\mu+0.97\sigma}^{\infty} P(x)xf(x)dx$$

Using equations (1), (2), and (13) and integrating gives (see appendix 2)

$$(14) \quad R_1 = 170 \times 6[a_0(\frac{\sigma}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{\mu}{6}) + a_1(\frac{\mu^2}{6} + \frac{2\mu\sigma}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{0.97\sigma^2}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{\sigma^2}{6})$$
$$+ a_2(\frac{\mu^3}{6} + \frac{\mu\sigma^2}{2} + \frac{3\mu^2\sigma}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{3\times0.97\mu\sigma^2}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{(2+0.97^2)\sigma^3}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}})]$$

Assuming the same cost structure as discussed in model 1, the profit of production from marketing truckload 1, π_1, can be modeled as

$$(15) \quad \pi_1 = 170 \times 6[a_0(\frac{\sigma(t_1)}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{\mu(t_1)}{6}) + a_1(\frac{\mu(t_1)^2}{6} + \frac{2\mu(t_1)\sigma(t_1)}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}}$$
$$+ \frac{0.97\sigma(t_1)^2}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{\sigma(t_1)^2}{6}) + a_2(\frac{\mu(t_1)^3}{6} + \frac{\mu(t_1)\sigma(t_1)^2}{2}$$
$$+ \frac{3\mu(t_1)^2\sigma(t_1)}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}} + \frac{3\times0.97\mu(t_1)\sigma(t_1)^2}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}}$$
$$+ \frac{(2+0.97^2)\sigma(t_1)^3}{\sqrt{2\pi}}e^{-\frac{0.97^2}{2}})] - 170[c_1 + c_2F(t_1) + c_3\mu(t_1)]$$

where t_1 is the days of pigs of truckload 1 on feed. Similarly, the profit functions of truckloads 2 to 6, π_2, π_3, π_4, π_5, π_6, are obtained and their detailed representations are shown in appendix 3. Therefore, the annual profit of production, Π, can be written as:

$$(16) \quad \Pi = \frac{365}{Max\{t_1, t_2, \cdots, t_6\}} \sum_{i=1}^{6} \pi_i(t_i)$$

As noted earlier marketing times can either be all distinct or be the same for some of truckloads, letting $t_{max} = Max\{t_1, \cdots, t_6\}$ and differentiating equation (16) with respect to $t_1, ..., t_6$ yields the first order conditions for the annual profit maximization problem:

$$\frac{\partial \Pi}{\partial t_i} = \frac{365}{t_{max}} \frac{\partial \pi_i}{\partial t_i} = 0 \quad \Rightarrow \quad \frac{\partial \pi_i}{\partial t_i} = 0, \; all \; i's \; that \; t_i < t_{max}$$

(17)

$$\frac{\partial \Pi}{\partial t_{max}} = \frac{365}{t_{max}} \frac{\partial \pi_{t_{max}}}{\partial t_{max}} - \frac{365}{t_{max}^2} \sum_{i=1}^{6} \pi_i(t_i) = 0 \quad \Rightarrow \quad \frac{\partial \pi_{t_{max}}}{\partial t_{max}} = \frac{\sum_{i=1}^{6} \pi_i}{t_{max}}$$

Equation (17) shows that for the truckloads marketed prior to t_{max}, the producer's profit maximization problem is reduced to the independent determination of an optimal marketing date that maximizes the profit of each individual truckload alone and the maximum profits of these truckloads are achieved on days when marginal profits equal zero. However, for truckload(s) marketed at t_{max}, the profit maximizing marketing date is the one at which the marginal profit of the truckload(s) equals the average daily profit for the entire 1,020 pigs in the barn. This was also noted in Dillon and Anderson (p88-89) that the marginal profit per unit of time must equal the average profit per unit of time in order to maximize profit over time. Again, assuming $CV = \frac{\sigma(t_i)}{\mu(t_i)} = s$ (a constant),

$\mu(t_i) = b_0 + b_1 t_i$, and $F(t_i) = d_0 + d_1 t_i + d_2 t_i^2 + d_3 t_i^3$, $i = 1, 2, ..., 6$, equation (17) falls into a group of quadratic or cubic equations that can be algebraically solved, similar to model 1, for all t_is. The corresponding mean $\mu(t_i)$ at each optimal marketing date can be computed and the sorting weight for each truckload at marketing is also determined by the mean and variance parameters as indicated in figure 1.

Data

The growth function $\mu(t)$ and cumulative feed intake function $F(t)$ for finishing pigs are derived from Andersen and Pedersen. Assuming that gilts and barrows are each half of the herd population and that feeder pigs enter the finishing barn at a weight of 66 lbs (30 kg), the mean live weight of the pigs in the barn ($\mu(t)$, in pounds) and the mean

cumulative feed intake per pig ($F(t)$, in pounds) after t days on feed can be modeled as fourth- and third-degree polynomials in t, respectively. However, Andersen and Petersen also notice that the growth function is almost linear in days on feed. Therefore, using data generated from Anderson and Peterson ($t = 1, 2, 3, …, 120$ and associated predicted weight) and applying ordinary least squares techniques, we approximated the nonlinear growth function with the following linear equation:

(18) $\mu(t) = 63.125 + 2.096t$

The above equation assumes that the average daily gain of pigs from 66 pounds to finish is 2.096 pounds/day. The cumulative feed intake function is taken directly from Andersen and Petersen:

(19) $F(t) = 1.3436 + 2.8336t + 0.0498t^2 - 0.000193t^3$

Hog live weight base price is assumed to be $0.42 per pound of live market weight (equivalent to $0.57 per pound of carcass weight), which was the mean U.S. hog price received by farmers between 1995 and 2002 (Agricultural Statistics Board). The pricing matrix applied is shown in table 1, which represents a price schedule (adjusted to live weights) for hogs of 52% lean offered recently by a large Midwestern U.S. packer (effective at year end, 2003). Given the base price and the pricing matrix and using ordinary least squares, the step price schedule is approximated by the following continuous price function:

(20) $P(x) = -0.781 + 0.009x - 1.574*10^{-5}x^2$

where x is the live weight of a hog at marketing, in pounds; and $P(x)$ is the market price, in dollars per pound of live weight.

Feeder pig purchase price is assumed to be $50 per head (Li et al.). The price of feed and the nonfeed cost are based on data obtained from farm business records on Illinois farms. The average feed price is assumed to be $0.06 per pound (1996-2000 mean) while the average nonfeed cost for feeder-pig finishing operations is assumed to be $0.1125 per pound of live weight (Lattz, Cagley, and Raab). According to Payne et al., based on collective experience in research and commercial facilities in the United States, groups of pigs entering an AIAO facility at 20 to 25 kg (44 to 55 pound) live weight showing a coefficient of variation (*CV*) of 15 to 18% would have a *CV* of 10% when some or all of a group are first marketed. Therefore, we assume a *CV* of 10%, which is constant over the entire marketing period in this analysis.

Results and discussions

The computational results of the two optimization models are obtained using an Excel spreadsheet (table 2). With a constant *CV* of 10%, the optimal days on feed in the finishing barn for marketing all 1,020 pigs on the same day is 106 days, at a mean weight of 286 pounds. Under this marketing strategy, the annual profit is $17,103 from a barn of 1,020 head. The turns of the barn (number of batches of pigs) per year is 3.44, implying an annual output of 3,509 pigs (1,001,813 pounds) with a profit of $4.87 per market hog.

When pigs are marketed on different dates, the optimal days on feed when shipping the six truckloads from the heaviest to the lightest 170 pigs in the barn are 93, 102, 108, 113, 120, and 120 days, respectively (table 2). Under this marketing strategy, producers gain $23,928 annually producing 3.04 turns or 3,101 pigs (900,005 pounds) per year, achieving a profit of $7.72 per marketed hog. Compared with the strategy of marketing all pigs in the barn on the same day, this marketing strategy increases the profit

from the barn by $6,825, or 40%. Such significant difference in profitability between the two strategies reasonably explains the fact that marketing all pigs from a barn on the same day is generally not practiced. It is also worth noting that producers make profits from the first five truckloads (ranging between $3,459 and $423 per truckload) while they suffer an economic loss raising the last truckload of the slower growers (-$1,597 per truckload).

The impact of variability in pig growth and weight on profitability of production is further examined by varying *CV* values from zero to 32% (figure 2). As expected, figure 2 shows a strong decreasing trend in profitability as variability in pig weight increases, regardless of marketing strategy. More specifically, in the extreme case of no variability (*CV* = 0), the profit of production reaches its highest level of $30,954 per year by marketing all pigs after 109 days on feed and at an identical live weight of 291 pounds per pig. In contrast with the optimal mean market live weight of 286 pounds and the optimal annual profit of $17,103 with the same marketing strategy when *CV* is assumed to be 10%, our results suggest that the optimal market weight and profitability of production can be significantly biased upward if variability in pig weights is neglected. Furthermore, when *CV* is 15% or higher, marketing all pigs on the same day will result in no profits but only economic losses. The breakeven *CV* value for marketing pigs on six different days is about 31%, suggesting that a *CV* greater than 31% would require even smaller truckloads and more marketing days to minimize the economic loss or improve the profitability of production. Our results confirm that management of pig growth variability including marketing strategies is of great importance to the economic viability of swine production.

Conclusions

In this study, we analytically constructed and solved for two models of marketing strategies for a typical 1,020 head AIAO swine finishing barn given variability in pig weights and pricing specifications. We identified the optimal days on feed, the optimal mean (model 1), and the optimal sorting weights (model 2) at marketing for two alternative marketing strategies. Our results show that compared with marketing all pigs on the same day, marketing pigs in six truckloads and on different dates significantly improves profitability as pig weight variability increases. We also find that studies on optimal marketing strategies without taking into account variability in pig weights can result in exaggerated optimal marketing weights and profits of production. Finally, our results indicate that pig weight variability management and marketing strategies are essential to the economic viability of the swine industry.

References

Agricultural Statistics Board, NASS, USDA. *Agricultural Prices 1995-2002 Summary*. National Agricultural Statistics Service, United States Department of Agriculture. 1996-2003.

Andersen, S., and B. Pedersen. "Growth and food intake curves for group-housed gilts and castrated male pigs." *Animal Science* 63 (1996): 457-464.

Boland, M.A., P.V. Preckel, and A.P. Schnickel. "Optimal hog slaughter weights under alternative pricing systems." *Journal of Agricultural and Applied Economics* 25 (December 1993.): 148-163.

Deen, J., M.T. Skorupski, and B. Frey. "Marketing all-in-all-out grow/finish groups." *Proceedings of the 1998 Allen D. Leman Swine Conference*, University of Minnesota, Minnesota. pp. 133-135. 1998.

Dillon, J.L., and J.R. Anderson. *The Analysis of Response in Crop and Livestock Production*. Third Edition. New York: Pergamon Press, 1990.

Dritz, S.S., and M.D. Tokach. "Growth curve analysis to determine profit optimization and pig flow." *Proceedings of the 1998 Allen D. Leman Swine Conference*, University of Minnesota, Minnesota. pp. 136-141. 1998.

Harris, J.W. and H. Stocker. *Handbook of Mathematics and Computational Science*. Springer-Verlag New York, Inc. 1998.

Lattz, D.H., C.E. Cagley, and D.D. Raab. Summary of Illinois Farm Business Records for 2001. 77th Annual Report, Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign, Urbana, IL. 2002.

Li, N., A.P. Schinckel, P.V. Preckel, and B.T. Richert. "Using a stochastic model to evaluate swine production management with paylean® I: Model development and optimal management strategies." *2003 Swine Research Report*, Purdue University, IN. pp. 24-28. 2003.

Patience, J.F., A.D. Beaulieu, H.W. Gonyou, and R.T. Zijlstra. "Variation in pig performance: A checklist to improve performance." Prairie Swine Centre Inc., University of Saskatchewan. 2003. RUL:

http://adminsrv.usask.ca/psci/WhatsNew/APR03/Patience.PDF

Patience, J.F., H.W. Gonyou, and R.T. Zijlstra. "Dealing with variability in the growout." Production and Management Featured Article, Prairie Swine Centre Inc., University of Saskatchewan. 2003. RUL:

http://adminsrv.usask.ca/psci/WhatsNew/December2003/Variability.htm

Payne, H.G., B.P. Mullan, M. Trezona, and B. Frey. "A Review - Variation in pig production and performance." In P.D. Cranwell (ed.), *Manipulating Pig Production VII*. Australia Pig Science Association, Werribee, Victoria, Australia. Pp. 13-26. 1999.

Roka, F.M. and D.L. Hoag. "Manure value and liveweight swine decisions." *Journal of Agricultural and Applied Economics* 28 (July 1996.): 193-202.

Song, Y. and G.Y. Miller. "Effects of marketing decisions on net present value of pork production for independent and allied swine producers." Review of Agricultural Economics 24 (Spring/Summer 2002): 181-195.

Appendix 1. Integration of the expected revenue function in model 1

The integration of equation (3) consists of the following three separate

integrations:

$$\int_{-\infty}^{\infty} a_0 x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad \Leftarrow y = x - \mu$$

$$= a_0 \int_{-\infty}^{\infty} \frac{y+\mu}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy = a_0 \int_{-\infty}^{\infty} \frac{y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + a_0 \int_{-\infty}^{\infty} \frac{\mu}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_0 \mu$$

$$\int_{-\infty}^{\infty} \frac{a_1 x^2}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad \Leftarrow y = x - \mu$$

$$= a_1 \int_{-\infty}^{\infty} \frac{(y+\mu)^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy = a_1 \int_{-\infty}^{\infty} \frac{(y^2 + 2y\mu + \mu^2)}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_1 \int_{-\infty}^{\infty} \frac{y^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + 2a_1\mu \int_{-\infty}^{\infty} \frac{y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + a_1\mu^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_1 \int_{-\infty}^{\infty} \frac{y}{2\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy^2 + a_1\mu^2 = -\frac{a_1\sigma}{\sqrt{2\pi}} ye^{-\frac{y^2}{2\sigma^2}} \Big|_{-\infty}^{\infty} + a_1\sigma^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + a_1\mu^2$$

$$= a_1 (\mu^2 + \sigma^2)$$

$$\int_{-\infty}^{\infty} \frac{a_2 x^3}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad \Leftarrow y = x - \mu$$

$$= a_2 \int_{-\infty}^{\infty} \frac{(y+\mu)^3}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy = a_2 \int_{-\infty}^{\infty} \frac{(y^3 + 3y^2\mu + 3y\mu^2 + \mu^3)}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_2 \int_{-\infty}^{\infty} \frac{y^3}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + 3a_2\mu \int_{-\infty}^{\infty} \frac{y^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + 3a_2\mu^2 \int_{-\infty}^{\infty} \frac{y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$a_2\mu^3 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_2 (\mu^3 + 3\mu\sigma^2)$$

Summing up the above three, the expected revenue from the 1,020 pigs then

becomes:

$$R = 1020[a_0\mu + a_1(\mu^2 + \sigma^2) + a_2(\mu^3 + 3\mu\sigma^2)].$$

Appendix 2. Integration of the expected revenue function in model 2

Plugging equations (1) and (2) into (13), we have

$$R_1 = 170 \times 6 \int_{\mu+0.97\sigma}^{\infty} (a_0 + a_1 x + a_2 x^2) x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Similar to the integration in appendix 1, we proceed with the following separate integrations:

$$\int_{\mu+0.97\sigma}^{\infty} \frac{a_0 x}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = a_0 \int_{0.97\sigma}^{\infty} \frac{y+\mu}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_0 \int_{0.97\sigma}^{\infty} \frac{y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + a_0 \int_{0.97\sigma}^{\infty} \frac{\mu}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= -\frac{a_0\sigma}{\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} \Bigg|_{0.97\sigma}^{\infty} + \frac{a_0\mu}{6} = a_0 \left(\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{\mu}{6} \right)$$

$$\int_{\mu+0.97\sigma}^{\infty} \frac{a_1 x^2}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = a_1 \int_{0.97\sigma}^{\infty} \frac{(y+\mu)^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_1 \left(\int_{0.97\sigma}^{\infty} \frac{y^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + \int_{0.97\sigma}^{\infty} \frac{2\mu y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + \int_{0.97\sigma}^{\infty} \frac{\mu^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy \right)$$

$$= a_1 \left(\frac{\mu^2}{6} + \frac{2\mu\sigma}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} - \frac{y\sigma}{\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} \Bigg|_{0.97\sigma}^{\infty} + \frac{\sigma^2}{6} \right)$$

$$= a_1 \left(\frac{\mu^2}{6} + \frac{2\mu\sigma}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{0.97\sigma^2}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{\sigma^2}{6} \right)$$

$$\int_{\mu+0.97\sigma}^{\infty} \frac{a_2 x^3}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = a_2 \int_{0.97\sigma}^{\infty} \frac{(y+\mu)^3}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= a_2 \left(\int_{0.97\sigma}^{\infty} \frac{y^3}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + \int_{0.97\sigma}^{\infty} \frac{3\mu y^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + \int_{0.97\sigma}^{\infty} \frac{3\mu^2 y}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy + \int_{0.97\sigma}^{\infty} \frac{\mu^2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy \right)$$

$$= a_2 \left(\frac{\mu^3}{6} + \frac{\mu\sigma^2}{2} + \frac{3\mu^2\sigma}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{3 \times 0.97\mu\sigma^2}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{(2+0.97^2)\sigma^3}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} \right)$$

Summing them up, we get R_1 as written in equation (14). The computation of revenues of other truckloads involves integration of the same style and is omitted due to space limit.

Appendix 3. Profit functions of truckloads 2 to 6 in model 2

Profit of truckload 2

$$\pi_2 = 170 \times 6\{a_0[\frac{\mu(t_2)}{6} - \frac{\sigma(t_2)}{\sqrt{2\pi}}(e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}})] + a_1[\frac{\mu(t_2)^2}{6} + \frac{\sigma(t_2)^2}{6}$$

$$- \frac{2\mu(t_2)\sigma(t_2)}{\sqrt{2\pi}}(e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}}) - \frac{\sigma(t_2)^2}{\sqrt{2\pi}}(0.97e^{-\frac{0.97^2}{2}} - 0.43e^{-\frac{0.43^2}{2}})] + a_2[\frac{\mu(t_2)^3}{6}$$

$$+ \frac{\mu(t_2)\sigma(t_2)^2}{2} - \frac{3\mu(t_2)^2\sigma(t_2)}{\sqrt{2\pi}}(e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}}) - \frac{3\mu(t_2)\sigma(t_2)^2}{\sqrt{2\pi}}(0.97e^{-\frac{0.97^2}{2}} - 0.43e^{-\frac{0.43^2}{2}})$$

$$- \frac{\sigma(t_2)^3}{\sqrt{2\pi}}(0.97^2 e^{-\frac{0.97^2}{2}} - 0.43^2 e^{-\frac{0.43^2}{2}}) - \frac{2\sigma(t_2)^3}{\sqrt{2\pi}}(e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}})]\}$$

$$- 170[c_1 + c_2 F(t_2) + c_3\mu(t_2)]$$

Profit of truckload 3

$$\pi_3 = 170 \times 6\{a_0[\frac{\mu(t_3)}{6} - \frac{\sigma(t_3)}{\sqrt{2\pi}}(e^{-\frac{0.43^2}{2}} - 1)] + a_1[\frac{\mu(t_3)^2}{6} + \frac{\sigma(t_3)^2}{6} - \frac{2\mu(t_3)\sigma(t_3)}{\sqrt{2\pi}}(e^{-\frac{0.43^2}{2}} - 1)$$

$$- \frac{0.43\sigma(t_3)^2}{\sqrt{2\pi}}e^{-\frac{0.43^2}{2}}] + a_2[\frac{\mu(t_3)^3}{6} + \frac{\mu(t_3)\sigma(t_3)^2}{2} - \frac{3\mu(t_3)^2\sigma(t_3)}{\sqrt{2\pi}}(e^{-\frac{0.43^2}{2}} - 1)$$

$$- \frac{3\mu(t_3)\sigma(t_3)^2}{\sqrt{2\pi}}0.43e^{-\frac{0.43^2}{2}} - \frac{\sigma(t_3)^3}{\sqrt{2\pi}}0.43^2 e^{-\frac{0.43^2}{2}} - \frac{2\sigma(t_3)^3}{\sqrt{2\pi}}(e^{-\frac{0.43^2}{2}} - 1)]\}$$

$$- 170[c_1 + c_2 F(t_3) + c_3\mu(t_3)]$$

Profit of truckload 4

$$\pi_4 = 170 \times 6\{a_0[\frac{\mu(t_4)}{6} - \frac{\sigma(t_4)}{\sqrt{2\pi}}(1 - e^{-\frac{0.43^2}{2}})] + a_1[\frac{\mu(t_4)^2}{6} + \frac{\sigma(t_4)^2}{6} - \frac{2\mu(t_4)\sigma(t_4)}{\sqrt{2\pi}}(1 - e^{-\frac{0.43^2}{2}})$$

$$- \frac{0.43\sigma(t_4)^2}{\sqrt{2\pi}}e^{-\frac{0.43^2}{2}}] + a_2[\frac{\mu(t_4)^3}{6} + \frac{\mu(t_4)\sigma(t_4)^2}{2} - \frac{3\mu(t_4)^2\sigma(t_4)}{\sqrt{2\pi}}(1 - e^{-\frac{0.43^2}{2}})$$

$$- \frac{3\mu(t_4)\sigma(t_4)^2}{\sqrt{2\pi}}0.43e^{-\frac{0.43^2}{2}} + \frac{\sigma(t_4)^3}{\sqrt{2\pi}}0.43^2 e^{-\frac{0.43^2}{2}} - \frac{2\sigma(t_4)^3}{\sqrt{2\pi}}(1 - e^{-\frac{0.43^2}{2}})]\}$$

$$- 170[c_1 + c_2 F(t_4) + c_3\mu(t_4)]$$

Profit of truckload 5

$$\pi_5 = 170 \times 6 \{ a_0 [\frac{\mu(t_5)}{6} + \frac{\sigma(t_5)}{\sqrt{2\pi}} (e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}})] + a_1 [\frac{\mu(t_5)^2}{6} + \frac{\sigma(t_5)^2}{6}$$

$$+ \frac{2\mu(t_5)\sigma(t_5)}{\sqrt{2\pi}} (e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}}) - \frac{\sigma(t_5)^2}{\sqrt{2\pi}} (0.97 e^{-\frac{0.97^2}{2}} - 0.43 e^{-\frac{0.43^2}{2}})] + a_2 [\frac{\mu(t_5)^3}{6}$$

$$+ \frac{\mu(t_5)\sigma(t_5)^2}{2} + \frac{3\mu(t_5)^2\sigma(t_5)}{\sqrt{2\pi}} (e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}}) - \frac{3\mu(t_5)\sigma(t_5)^2}{\sqrt{2\pi}} (0.97 e^{-\frac{0.97^2}{2}} - 0.43 e^{-\frac{0.43^2}{2}})$$

$$+ \frac{\sigma(t_5)^3}{\sqrt{2\pi}} (0.97^2 e^{-\frac{0.97^2}{2}} - 0.43^2 e^{-\frac{0.43^2}{2}}) + \frac{2\sigma(t_5)^3}{\sqrt{2\pi}} (e^{-\frac{0.97^2}{2}} - e^{-\frac{0.43^2}{2}})] \}$$

$$- 170 [c_1 + c_2 F(t_5) + c_3 \mu(t_5)]$$

Profit of truckload 6

$$\pi_6 = 170 \times 6 [a_0 (\frac{\mu(t_6)}{6} - \frac{\sigma(t_6)}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}}) + a_1 (\frac{\mu(t_6)^2}{6} - \frac{2\mu(t_6)\sigma(t_6)}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}}$$

$$+ \frac{0.97\sigma(t_6)^2}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{\sigma(t_6)^2}{6}) + a_2 (\frac{\mu(t_6)^3}{6} + \frac{\mu(t_6)\sigma(t_6)^2}{2}$$

$$- \frac{3\mu(t_6)^2\sigma(t_6)}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}} + \frac{3 \times 0.97 \mu(t_6)\sigma(t_6)^2}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}}$$

$$- \frac{(2 + 0.97^2)\sigma(t_6)^3}{\sqrt{2\pi}} e^{-\frac{0.97^2}{2}})] - 170 [c_1 + c_2 F(t_6) + c_3 \mu(t_6)]$$

Table 1. Base price of market weight hogs and pricing matrix applied

Description	Value, $/pound
Base price of market weight hogs 259-299 lb	0.42
Pricing matrix applied, live weight range in lbs[a]	Percent of base price (%)
1-189	74
190-199	76
200-208	85
209-219	94
220-228	101
229-258	104
259-299	106
300-308	103
309-327	101
328-337	99
338-346	96
347-355	93
>356	79

[a]Estimated from the actual carcass weight pricing matrix using carcass weight/0.74=liveweight.

Table 2. Optimization results

	Model 1	Model 2: Marketing on different days					
	All out on same day	Truck-load 1	Truck-load 2	Truck-load 3	Truck-load 4	Truck-load 5	Truck-load 6
Opti. days on feed t^*, days	106	93	102	108	113	120	120
Mean of population at marketing $\mu(t^*)$, lbs	286	258	277	289	301	315	315
Sorting weight, lbs	all	>=283	>=289	>=289	>=288	>=285	<285
Average truckload weight, lbs	48,575	50,469	50,363	50,195	50,053	49,915	45,582
Profit of truckload, $	829	3459	2537	1856	1206	423	-1597
Output per batch, lbs	291,450	296,578					
Profit pre batch, $	4,972	7,871					
Annual profit, $	17,103	23,928					
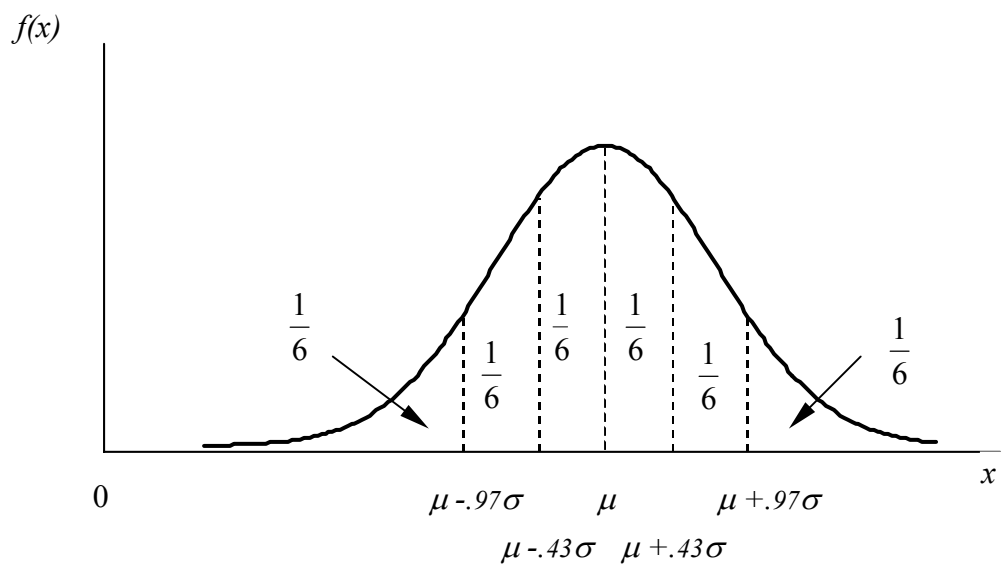
Figure 1. Distribution of pig weights and division of batches
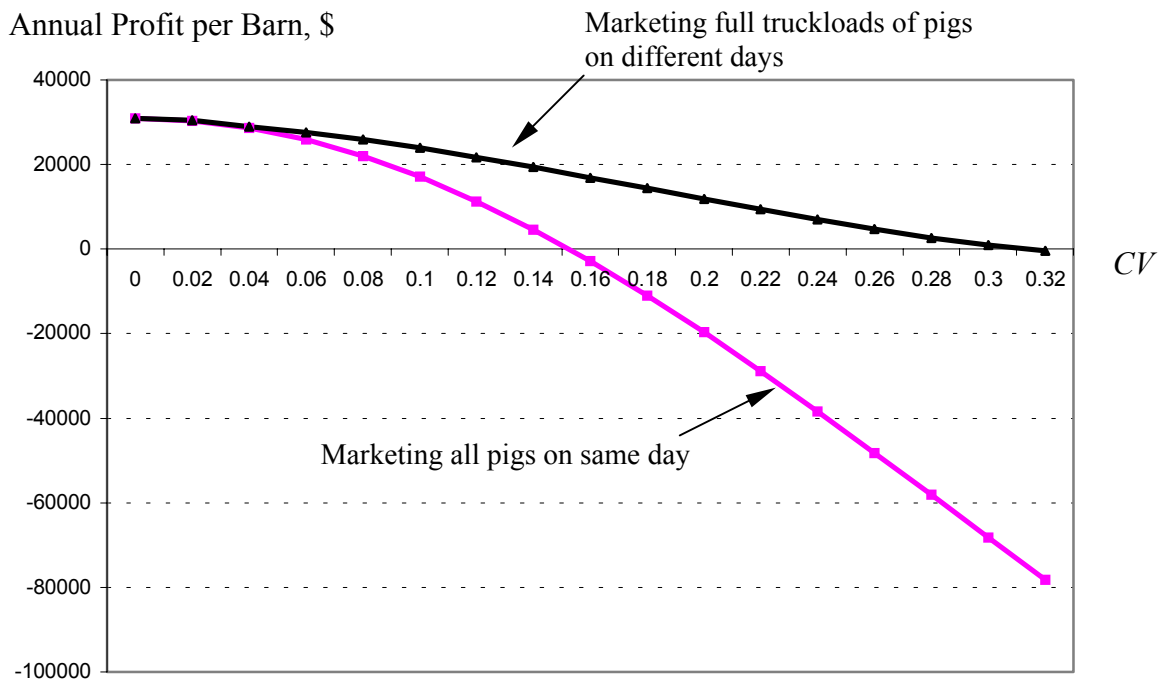
Annual Profit per Barn, $

Figure 2. Relationship between variability in pig weight and profitability