OPTIMAL CONTRACTS FOR EXPLORATION WITH COST RECOVERY OF AN EXHAUSTIBLE NATURAL RESOURCE UNDER ASYMMETRIC INFORMATION

by

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Abstract

Exploration of an exhaustible resource with cost recovery under asymmetric information about cost is modeled and analyzed employing Principal-Agent theory. Allocation of lower than full information level of effort for the high-cost firms is found socially optimal. However, distortion is less in a two-stage process of exploration and extraction

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I. Introduction

The economic theory of exhaustible resources, beginning with the seminal work of Hotelling (1931), has been primarily concerned with the optimal extraction of a fixed reserve base over time. Although a considerable amount of research has been dedicated to the optical extraction of exhaustible resource, theoretical studies on the exploration process are relatively few in number. Among these handful contributions we might mention Uhler (1976, 1979), Pindyck (1978, 1980), Gilbert (1979,1981), Deshmukh and Pliska (1980,1985), Arrow and Chang (1982), Pesaran (1990) and Quyen (1989,1991). However, the question of reserve discoveries under asymmetric information has largely been neglected. Gaudet, Lasserre, and Long (1995) have studied optimal nonrenewable resource royalty contracts when the extracting agent has private information on costs assuming that the private information is not correlated over time. Osmundsen (1998) has developed a model of dynamic exhaustible natural resource taxation, subject to private information about reserves when the private information parameter is inter-temporally correlated. But, none of the above studies has taken the exploration issue in consideration. The objective of this study is to develop a theoretical model for exploration of exhaustible resource under asymmetric information.

An exploration program is a search for mineral deposits whose number, sizes, and locations are all uncertain. The objective of exploration is to find new reserves in order to meet the demand for minerals. Typically, in the developed countries, the exclusive right to search for and extract minerals takes the form of lease over the life of the mine to a private firm or individual (Gaudet, Lasserre, and Long, 1995) and the leases are given out through auctions. However, there are many instances, especially in developing countries, where there is no well-behaved auction market and leases are given out in some form of contracts. For example, the government of Bangladesh invites International Oil Companies (IOCs) for the exploration and extraction of its natural gas. Typically the government signs Product Sharing Contracts (PSC) with a firm for exploration and extraction of the natural gas. A typical PSC recovers firm's exploration and extraction costs and shares the profit gas. In most of the cases exploration and extraction take place in a two-stage process. This paper considers only the first stage of the two-stage process.

In the case of exploration with cost recovery, if the exploration costs are perfectly known to both the owner of the resource and the exploring firms, then the solution to the problem is not very complicated. In practice, however, exploration costs are private information of the exploring firms. This gives rise to the problem of adverse selection. In this paper, we attempt to develop a model for exploration under cost recovery contract employing the Principle-Agent Theory. Defining the discovery and exploration cost functions the next sections outlines the basic framework of the model. Section three discusses the optimal contracts under symmetric as well as asymmetric information cases assuming non-stochastic stock of reserves. The case of stochastic stock of reserves is analyzed in section four. The final section draws conclusions and gives directions for future studies of optimal exploration and extraction contracts.

II. The Basic Theoretical Framework

Let us begin by first considering a non-stochastic stock of discoverable reserves, which is a function of exploration effort only. The exploring firm incurs cost by exerting exploration effort. It reports the stock of newly discovered stock of reserves and the total exploration cost to the government. The government, the owner of the resources, covers the cost of the firm in the form of a transfer payment. So, the firm's objective is to maximize profit from exploration, which is the difference between the transfer payment received from the government and its total cost. On the other hand, the government maximizes social welfare, which is the value of the stock of discovered reserves plus the profit of the exploring firm. If the resource price, the stock of reserves and the cost of exploration are perfectly observable by both the government and the exploring firm, then the solution takes the form of a static optimization problem. The government therefore offers a transfer payment to the firm for a given level of effort such that the firm is left with zero profit.

In practice, however, the owner of the resources or the government does not know the true cost of exploration. This asymmetry of information creates a situation where adverse selection may occur, which should be solved by employing the well-known principal-agent theory tools. Throughout this paper we will think of the government being the principal and the exploring firm being the agent. But we need to define the stock discovery function and the cost function first.

The Discovery Function

While explaining his model of petroleum exploration, Uhler (1979) states that current exploration effort and the size of undiscovered reserves are important factors in explaining the discovery rate although the geological knowledge is also important. The greater the geological knowledge the higher will be the discovery rate for any given amount of effort and undiscovered reserves. On the other hand, for any given level of current search effort and geological knowledge, the larger the stock of undiscovered reserves the higher is the rate of discovery. But, the initial stock of undiscovered reserves is not observable. Uhler (1979) proposes an observable variable, which can be used as a substitute for the undiscovered resource stock. He specifies that current exploration effort has four primary effects. First, it directly affects the rate at which reserves are discovered. Second, it reduces the stock of undiscovered reserves to the extent that it results in discoveries. Third, it adds to the geological knowledge. Finally, current effort increases the stock of cumulative effort, by definition. Thus, the stock of cumulative effort is inversely related to the stock of undiscovered reserves and may act as a substitute for it in the specification of the production function.

Using the arguments above, Uhler (1979) establishes the relationship between the discovery rate and the stock of exploration effort. He argues that, for initial values of the stock of exploration effort, x, the effect of geological knowledge dominates the effect of reduced reserves and the discovery rate rises with increases in x. But as x increases, the effect of reduced reserves begins to dominate the effect of additional geological knowledge and the discovery rate begins to fall. Thus the marginal product of cumulative effort is at first positive but becomes negative after some critical value of x is reached. Uhler (1979) specifies the functional relationship as follows:

$$S(x,v) = Av^{\rho} \exp[-\beta(x-k)^2] \tag{1}$$

where $v = \dot{x}$, and A, ρ, β and k are parameters. This relationship states that for a given level of current effort, the reserves discovery rate increases until x reaches k, at which point it declines asymptotically toward the horizontal axis. Figure 1 illustrates the relationship between the discovery rate and the stock of exploration effort.

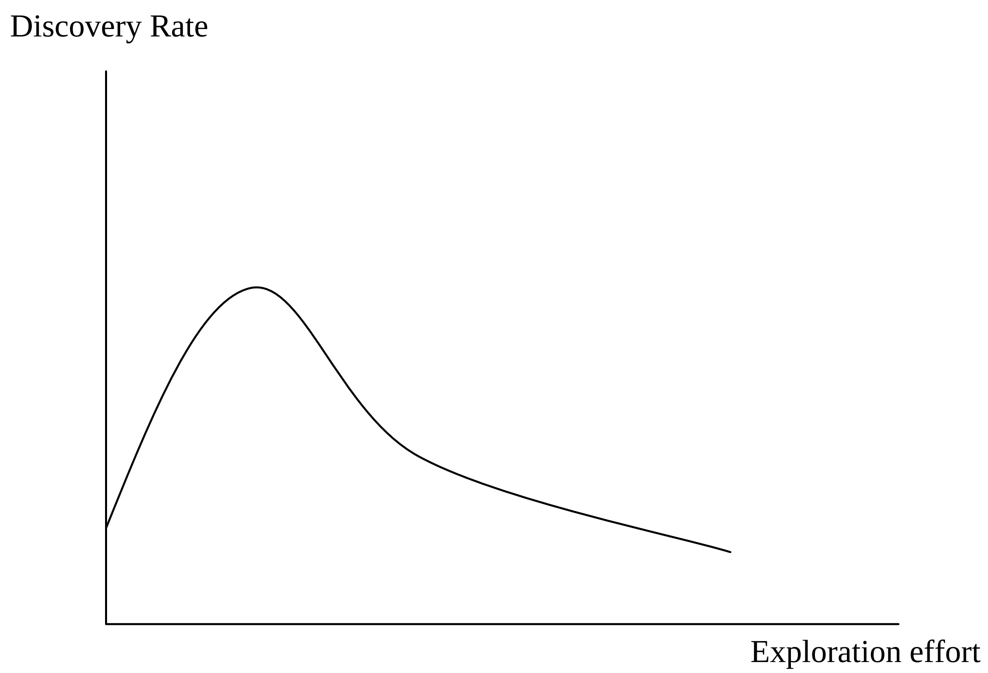

Figure 1

Equation (1) has the advantage that it is reasonably flexible since its centrality and spread change with variations in the parameters. On the other hand, it tends to overstate the discovery rate for very low values of x since at $x = 0$, $S(x, v) > 0$. However, for a single period case we can simplify the discovery function as:

$$S(x) = Ax^{\rho} \;; \quad \text{for} \quad 0 < \rho < 1 \tag{2}$$

The single period discovery function in (2) represents a generalization of the relationship between the stock of newly discovered reserves and the level of current exploratory effort which satisfies the following conditions:

$$\frac{\delta S(x)}{\delta x} > 0 \qquad \text{and} \qquad \frac{\delta^2 S(x)}{\delta x^2} < 0$$

The Exploration Cost Function

Following Uhler (1979), we assume that the exploring firm's single period cost function is given by some function $C(x_i, \theta_i)$, where x_i is the level of effort exerted by firm i and θ_i is a cost parameter known only to the exploring firm. The parameter θ_i can reflect various aspects of the firm's efficiency. Uhler assumes the cost function to be of the following quadratic form:

$$C(x_i, \theta_i) = \theta_i x_i + \frac{b}{2} x_i^2, \qquad b \geq 0. \tag{3}$$

Hence the marginal cost is

$$MC(x_i, \theta_i) = \frac{\delta C(x_i, \theta_i)}{\delta x_i} = \theta_i + b x_i$$

The fact that total and marginal cost move in the same direction when θ_i changes guarantees that the static single crossing property is satisfied. However, to keep things simple we will assume that $b = 0$. So the cost function in this analysis takes the form of

$$C(x_i, \theta_i) = \theta_i x_i \tag{4}$$

We assume that both θ_i and x_i are known to the firm and only x_i is known to the government. However, the government knows the density of θ_i which is given by $f(\theta_i) > 0$. For simplicity we assume that $\theta \in [\theta_1, \theta_2]$, i.e. there are only two types of firm, θ_1 and θ_2 with $\theta_2 > \theta_1$. We also assume that π being the probability that the firm is low-cost type and $1 - \pi$ being the probability that the firm is high-cost type.

Let us assume that the government makes the transfer payment T_i to the firm for exerting effort x_i. So, an exploring firm earns profit, $\Pi_i = T_i - \theta_i x_i$. Since the principal does not know the true θ_i, she is facing a mechanism design problem by which she can extract the true information about the θ_i. Here, we intend to model the problem through direct mechanism design so that the government can set a transfer payment schedule T_i, $i = 1, 2$, that maximizes expected social welfare. Social welfare can be taken as a weighted sum of government revenue minus transfer payment and producer's surplus. To simplify the problem, we assume that the country is a price taker in the resource market which is exogenously given and $P_s = 1$. Thus we may write the social welfare as

$$W = S(x) - T + \alpha\Pi \tag{5}$$

where

$$\Pi = T - C(x,\theta) \tag{6}$$

Following Gaudet et al., we adopt the standard assumption that $0 < \alpha < 1$: a dollar in government revenue is valued more highly than a dollar that remains as profits in the hand of the firm.

III. The Model of Exploration with Cost Recovery

The Symmetric Information Case

Before going to solve for the optimal contract scheme under asymmetric information, we first derive the properties of cost recovery schedule, which maximizes social welfare in the case where both the principal and the agent have perfect information about the cost structure. This symmetric information case is a useful benchmark, as it yields the first best solution to the problem.

Assume that before the exploration takes place, the firm reveals the exact value of θ_i to the government. The government then wishes to maximize

$$\underset{x}{Max} \quad W = S(x) - T + \alpha\Pi = Ax^{\rho} - T + \alpha\Pi$$

subject to

$$\Pi = T - C(x,\theta) = T - \theta x$$

In this case, the government will bind the firm on zero profit since there is no asymmetry about the cost information. Hence, $\Pi = T - C(x,\theta) = 0$, as a result $T = C(x, \theta) = \theta x$. Assuming $\rho = \frac{1}{2}$, and substituting for the constraint, the maximization problem takes the form

$$\underset{x}{Max} \quad W = Ax^{\frac{1}{2}} - \theta x .$$

For an interior solution the first order condition is

$$\frac{A}{2} x^{-\frac{1}{2}} - \theta = 0 .$$

The first order condition is just the value of marginal product equals marginal cost condition, where marginal cost is replaced by the marginal exploration cost. This condition gives the optimal efforts, transfer receipts, and stock of newly discovered reserves for both of the firm as follows. For firm 1:

$$x_1^* = \frac{A^2}{4\theta_1^2}, \qquad T_1^* = \frac{A^2}{4\theta_1}, \qquad S_1^* = \frac{A^2}{2\theta_1}. \tag{7}$$

For firm 2:

$$x_2^* = \frac{A^2}{4\theta_2^2}, \qquad T_2^* = \frac{A^2}{4\theta_2}, \qquad S_2^* = \frac{A^2}{2\theta_2}. \tag{8}$$

And, for $\theta_1 < \theta_2$,

$$x_1^* > x_2^*, \qquad T_1^* > T_2^*, \quad \text{and} \quad S_1^* > S_2^*. \tag{9}$$

The government will, therefore, maximize social welfare by offering the first best contracts (x_1^*, T_1^*) and (x_2^*, T_2^*).

The Asymmetric Information Case

Let us now turn to the second best situation in which the efficiency parameter, θ_i, is not known to the government. The government now only knows that the probability of a farm being low-cost type is π. If the government offers the first best contracts (x_1^*, T_1^*), (x_2^*, T_2^*), the low-cost type firm will not choose (x_1^*, T_1^*) but (x_2^*, T_2^*) since

$$T_2 - C(x_2, \theta_1) = \theta_2 \frac{A^2}{4\theta_2^{\,2}} - \theta_1 \frac{A^2}{4\theta_2^{\,2}} = \frac{A^2}{4\theta_2^{\,2}}(\theta_2 - \theta_1) > 0$$

But the high-cost type firm will still choose (x_2*, T_2*) since

$$T_1 - C(x_1, \theta_2) = \theta_1 \frac{A^2}{4\theta_1^{\,2}} - \theta_2 \frac{A^2}{4\theta_1^{\,2}} = \frac{A^2}{4\theta_1^{\,2}}(\theta_1 - \theta_2) < 0 .$$

Thus the two types of firms are not separated anymore. The government, however, will anticipate this opportunistic behavior of the low-cost type firm and will try to separate the firms by designing a contract schedule such that the firms find it in their best interests to reveal the true value of θ_i. In order to design a direct truthful mechanism where the firms reveal their true θ, the government solves the following problem.

$$\underset{x_1, x_2}{Max} \quad EW = \pi[S(x_1) - T_1 + \alpha\Pi_1] + (1-\pi)[S(x_2) - T_2 + \alpha\Pi_2]$$

subject to

$$T_1 - \theta_1 x_1 \geq 0 \qquad (\text{IR}_1)$$

$$T_2 - \theta_2 x_2 \geq 0 \qquad (\text{IR}_2)$$

$$T_1 - \theta_1 x_1 \geq T_2 - \theta_1 x_2 \qquad (\text{IC}_1)$$

$$T_2 - \theta_2 x_2 \geq T_1 - \theta_2 x_1 \qquad (\text{IC}_2)$$

The first two constraints are the individual rationality or participation constraints, which guarantee that each type of firm accepts the contract designated for it. The latter two constraints are the incentive compatibility constraints, which states that each firm prefers the contract designed for it. It can be shown easily that only (IR_2) and (IC_1) are active constraints and we can neglect (IR_1) and (IC_2). Thus from (IR_2) we have

$$T_2 = \theta_2 x_2,$$

and, from (IC_2) we have

$$T_1 = T_2 + \theta_1 x_1 - \theta_1 x_2 = (\theta_2 - \theta_1)x_2 + \theta_1 x_1 = \Pi_1 + \theta_1 x_1$$

where $\Pi_1 = T_1 - \theta_1 x_1 = (\theta_2 - \theta_1)x_2 + \theta_1 x_1 - \theta_1 x_1 = (\theta_2 - \theta_1)x_2$. Moreover, adding up ($\text{IC}_1$) and ($\text{IC}_2$) and solving for x_1 and x_2 we can show that $x_1 \geq x_2$. Thus, the monotonicity condition also holds. Substituting (IR_2) and (IC_1) in the objective function the maximization problem takes the form of

$$\underset{x_1, x_2}{Max} \quad EW = \pi[Ax_1^{\frac{1}{2}} - (1-\alpha)\Pi_1 - \theta_1 x_1] + (1-\pi)[Ax_2^{\frac{1}{2}} - \theta_2 x_2]$$

$$= \pi[Ax_1^{\frac{1}{2}} - (1-\alpha)(\theta_2 - \theta_1)x_2 - \theta_1 x_1] + (1-\pi)[Ax_2^{\frac{1}{2}} - \theta_2 x_2]$$

The first order conditions are

$$\frac{\delta EW}{\delta x_1} = \pi(\frac{1}{2}Ax_1^{-\frac{1}{2}} - \theta_1) = 0 \text{ , and}$$

$$\frac{\delta EW}{\delta x_2} = -\pi(1-\alpha)(\theta_2 - \theta_1) + (1-\pi)(\frac{1}{2}Ax_2^{-\frac{1}{2}} - \theta_2) = 0 .$$

Solving for x_1 and x_2 we have

$$x_1^{\circ} = \frac{A}{4\theta_1^{\,2}} \qquad (10)$$

$$X_2^{\circ} = \frac{A^2(1-\pi)^2}{4[\pi(1-\alpha)(\theta_2 - \theta_1) + \theta_2(1-\pi)]^2} \qquad (11)$$

We see that $x_1^* = x_1^{\circ}$ but $x_2^* > x_2^{\circ}$. It follows that $T_2^{\circ} < T_2^*$ and $S_2^{\circ} < S_2^*$. So there is a distortion in the contract for the high-cost type firm. Compared to the symmetric case, lower level of exploring effort is allowed here for the high-cost type firm. High-cost type allocation is lowered to restrict the rent accrues to the low-cost type. The high-cost type allocation is distorted in such a way that the expected social gain from increasing one unit of effort for the high-cost type is exactly offset by the resulting social welfare loss. For the low-cost type firm, however, there is no distortion in the contract. Thus, in the case of adverse selection, the low-cost (more efficient) firm gets an efficient allocation and the high-cost (less efficient) firm gets a sub-efficient allocation. As a result, the low cost type firm earns a positive surplus (informational rent) and the high-cost type firm gets zero surplus.

It should also be noted here that if $\alpha = 1$, then the optimal contracts take the form of those in the symmetric case. That is if a dollar in government revenue is valued as the same as a dollar that remains as profits in the hand of the firm, then it is optimal to offer symmetric information case contracts even if there exists asymmetry about the cost information. Thus, α plays a significant role in this analysis.

IV. The Case of Stochastic Stock of Reserves

So far our analysis is based on the assumption that the stock of reserves is non-stochastic. In reality, however, the number and sizes of the stocks in a region are most likely to be uncertain. Arrow and Chang (1982) as well as Quyen (1991) assumed that the number of deposits existing in any unit area of the region follow a Poission distribution with a known parameter. Here we are concerned about the uncertain size of the stock only. Let us assume that the probability of discovering a stock of size S is λ $(0<\lambda<1)$, which is a function of the effort level such that probability of discovering S increases with exerted effort. Given a specific functional form of $\lambda(x)$ we can solve for the optimal contracts following the procedure above. Since the specific functional form of $\lambda(x)$ is not known, the best we can do is try to solve for the optimal contracts following the general functional form of all of the functions. In that case, the government maximizes social welfare solving the following problem.

$$\underset{x_1, x_2}{Max} \quad EW = \pi[\lambda(x_1)S(x_1) - T_1 + \alpha\Pi_1] + (1-\pi)[\lambda(x_2)S(x) - T_2 + \alpha\Pi_2]$$

subject to

$$T_1 - C(x_1,\theta_1) \geq 0 \qquad (\text{IR}_1)$$

$$T_2 - C(x_2,\theta_2) \geq 0 \qquad (\text{IR}_2)$$

$$T_1 - C(x_1,\theta_1) \geq T_2 - C(x_2,\theta_1) \qquad (\text{IC}_1)$$

$$T_2 - C(x_2,\theta_2) \geq T_1 - C(x_1,\theta_2) \qquad (\text{IC}_2)$$

Again, it can be shown that only (IR_2) and (IC_1) are active constraints and we can neglect (IR_1) and (IC_2). Thus from (IR_2) we have

$$T_2 = C(x_2, \theta_2),$$

and, from (IC_2) we have

$$T_1 = T_2 + C(x_1,\theta_1) - C(x_2,\theta_1) = C(x_2,\theta_2) - C(x_2,\theta_1) +_1 C(x_1,\theta_1)$$

where $\Pi_1 = T_1 - C(x_1,\theta_1) = C(x_2,\theta_2) - C(x_2,\theta_1)$. Moreover, adding up (IC_1) and (IC_2) and solving for x_1 and x_2 we can show that $x_1 \geq x_2$, i.e., the monotonicity condition also holds. Substituting T_1 and T_2 in the objective function the maximization problem can be stated as:

$$\underset{x_1, x_2}{Max} \quad EW = \pi[\lambda(x_1)S(x_1) - (1-\alpha)\Pi_1 - C(x_1,\theta_1)] + (1-\pi)[\lambda(x_2)S(x_2) - C(x_2,\theta_2)]$$

$$= \pi[\lambda(x_1)S(x_1) - (1-\alpha)\{C(x_2,\theta_2) - C(x_2,\theta_1)\} - C(x_1,\theta_1)] + (1-\pi)[\lambda(x_2)S(x_2) - C(x_2,\theta_2)]$$

The first order conditions are:

$$\frac{\delta EW}{\delta x_1} = \pi[\lambda(x_1)S_{x_1}(x_1) + S(x_1)\lambda_{x_1}(x_1) - C_{x_1}(x_1,\theta_1)] = 0, \text{ and}$$

$$\frac{\delta EW}{\delta x_2} = -\pi(1-\alpha)[C_{x_2}(x_2,\theta_2) - C_{x_2}(x_2,\theta_1)] + (1-\pi)[\lambda(x_2)S_{x_2}(x_2) + \lambda_{x_2}(x_2)S(x_2) - C_{x_2}(x_2,\theta_2)] = 0$$

Again, the second first order condition implies that the allocation for the high-cost type firms should be distorted in order to restrict the rent accrues to the low-cost type. Compared to the case of symmetric information about the cost structure, for the high-cost type firms the level of effort should be lowered in such a way that the marginal social benefit is exactly equal to the marginal social loss. There is no distortion in the contract for the low-cost type firms. However, in the case of stochastic stock of reserves, optimal allocation of efforts also depends on $\lambda(x)$.

V. Conclusions

A model for optimal exploration contract with cost recovery under asymmetric information is developed both for non-stochastic and stochastic stock of reserves. In both of the cases it is found that a sub-efficient allocation for the high-cost type firms is optimal from the social point of view. However, for a clear understanding of the stochastic case we need a specific structure of the probability distribution of the stock, which is a function of exploration effort.

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