# Bootstrapping Your Fish or Fishing for Bootstraps?: Precision of Welfare <br> Loss Estimates from a Globally Concave Inverse Demand Model of Commercial Fish Landings in the U.S. Great Lakes. 

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#### Abstract

: We extend Holt and Bishop's (2002) normalized quadratic inverse demand function by obtaining measures of precision of elasticities and consumer welfare loss estimates due to reductions in commercial landings in the U.S. Great Lakes. Even with curvature constraints imposed, confidence intervals are obtained by using the bootstrap, subsample bootstrap, and subsample jackknife.


Key words: quadratic inverse demand function, bootstrap, subsample bootstrap, subsample jackknife

June 28, 2004

# Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Denver, Colorado, August 1-4, 2004 

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## 1. Introduction

A primary concern in the resource economics literature is the estimation of welfare effects associated with implementing policies that conserve natural resources. Much research has been devoted to supplying policy makers with estimates of welfare gains and losses associated with quotas of fisheries and similar policies. The typical information set provided by the researcher includes point estimates of welfare effects, which in turn offers no opportunity to test hypotheses about the welfare measures. Point estimates offer information about a specific point on the distribution but they do not offer information about how close the estimate is to the true population parameter. Therefore, a point estimate that is measured imprecisely may induce a policy maker to incorrectly conclude that the proposed policy will have a significant effect on consumers or producers, when in fact the resultant welfare effect may not be statistically different from zero. A measure of precision such as a confidence interval or standard error should always be included in addition to a point estimate to provide additional information about the underlying distribution of the welfare measure [Kling and Sexton (1990); Kling(1991)].

Another important concern when calculating welfare measures is that the models they are derived from are generally specified to conform to theoretical economic restrictions such as monotonicity, homotheticity, and concavity/convexity. In the context of welfare analysis the later is particularly important-a measure of compensating variation, for example, derived from an upward sloping compensated demand function is likely meaningless. More generally, welfare estimates obtained for a demand system in which, say, the quasi concavity of the expenditure function is not satisfied are generally
suspect. Therefore, restrictions that impose downward sloping (compensated) demand curves and upward sloping supply curves are often imposed a priori, typically by using parametric restrictions. A complicating factor when imposing restrictions of this form is that they involve inequality constraints. Standard error estimates and confidence bounds have traditionally been difficult to compute using classical statistical inference because the parameter space is truncated; traditional distributional assumptions no longer apply because the asymptotic distribution is no longer normal. The existing literature generally relies on Baysian techniques to obtain measures of precision when inequality constraints are imposed [see, e.g., Chalfant et al. (1991); Terrell (1996); and Piggott (2003)]. The researcher, however, has another option. Andrews (1999) shows that resampling techniques such as the subsample bootstrap and subsample jackknife are consistent methods to obtain measures of precision in the presence of inequality constraints. These methods are relatively easy to apply and rely on classical statistical inference. Even so, application of these methods in empirical work has, to date, been extremely limited.

In the present paper we use the semiflexible normalized quadratic inverse demand system (SNQIDS) developed by Holt and Bishop (2002), which in turn is an adaption of the normalized quadratic expenditure function of Diewert and Wales (1988a). Specifically, this model will be used to obtain measures of precision on estimates of compensating and equivalent variation for consumer welfare losses associated with a reduction total allowable catch for commercial fisherman in the U.S. Great Lakes region. The normalized quadratic inverse distance function is a way to estimate a globally concave, locally flexible distance function. Holt and Bishop (2002) also show that along with maintaining theoretical consistency, the imposition of curvature within the inverse
distance function framework allows consistent estimation of money metric welfare losses associated with quantity restrictions. They utilized their model to obtain short-run demands for six different kinds of fish landings from 1971-1991 for U.S. Great Lakes ports and obtain estimates of compensating and equivalent variation welfare measures associated with a $10 \%$ reduction of fishing stocks for six different types of fish; Holt and Bishop (2002), however, did not provide measures of precision associated with their welfare estimates.

The current paper differs from that of Holt and Bishop (2002) in several fundamental ways. First, commercial fish landings and price data associated with U.S. Great Lakes ports are now available for the 1971-2001 period; we therefore update and re-estimate the SNQID models originally reported on by Holt and Bishop (2002). Second, and more importantly, we obtain confidence intervals on the estimates of welfare losses to fish consumers associated with reducing catch quotas for commercial fishermen by utilizing the bootstrap, subsample bootstrap, and subsample jackknife. This is the first known application where measures of precision are obtained in a classical statistics framework for welfare estimates when concavity is imposed on the model. These measures of precision will allow policy makers to obtain a more accurate picture of the welfare losses associated with catch restrictions than is allowed by point estimates alone.

The remainder of the paper is organized as follows. In the next section we review the specification of the SNQIDS; in section 3 money-metric measures of welfare loss measures in quantity space are briefly reviewed. In section 4 we discuss the simulation methodology used to obtain confidence intervals in the case where curvature restrictions (inequality constraints) are imposed on the model's parameters. In section 5 the data are
discussed, the final model specification used in estimation is presented, and the econometric results are reviewed. The final section concludes.

## 2. A Globally Concave Inverse Demand System: The SNQIDS

This paper uses Holt and Bhsiop's (2002) semiflexible normalized quadratic inverse demand system (SNQIDS) as the basic modeling framework for estimating inverse demand systems for fish landed in Great Lakes ports, U.S. In this section we briefly describe the specification and derivation of the SNQIDS-additional details may be found in Holt and Bishop (2002).

Le $q_{i t}$ denote the quantity landed in time period $t, t=1, \ldots T$, of fish species $i, p_{i t}$ the corresponding vessel-level price, $\boldsymbol{q}_{t}=\left(q_{1 t}, \ldots, q_{n t}\right)^{T}$ the $n$-vector of quantities in time period $t, \boldsymbol{p}_{t}$ the corresponding price vector, and $m_{t}=\boldsymbol{p}_{t}^{T} \boldsymbol{q}_{t}$ is group expenditure in time $t$. Also, let $\boldsymbol{q}^{*}>\mathbf{0}_{n}$ denote some (possibly arbitrary) base-period or reference quantity vector. Ignoring for the moment the time subscripts, the SNQIDS is derived from the following normalized quadratic distance function:

$$
\begin{equation*}
D(u, \boldsymbol{q})=\boldsymbol{c}^{T} \boldsymbol{q}+\left[\boldsymbol{b}^{T} \boldsymbol{q}+\frac{1}{2}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-1} \boldsymbol{q}^{T} A \boldsymbol{q}\right] u^{-1} \tag{1}
\end{equation*}
$$

where $\alpha=\left(\alpha_{1}, \ldots, \alpha_{n}\right)^{T}$ is a pre-determined parameter vector, $\boldsymbol{c}=\left(c_{1}, \ldots, c_{n}\right)^{T}$ and $\boldsymbol{b}=\left(b_{1}, \ldots, b_{n}\right)^{T}$ are vectors of estimable parameters, $A=\left[a_{i j}\right]$ is a $n \times n$ parameter matrix, $u$ is an unobservable utility index, and a superscripted $T$ denotes vector(matrix)
transposition. As Holt and Bishop (2002) discuss, the distance function in (1) must also satisfy the following conditions:

$$
\begin{gather*}
\boldsymbol{\alpha}^{T} \boldsymbol{q}^{*}=1, \quad \boldsymbol{\alpha}>\mathbf{0}_{n}  \tag{2a}\\
\boldsymbol{c}^{T} \boldsymbol{q}^{*}=0, \text { and }  \tag{2b}\\
A \boldsymbol{q}^{*}=\mathbf{0}_{n}, \quad A=A^{T} . \tag{2c}
\end{gather*}
$$

Applying the Shephard-Hanoch lemma to (1) gives a system of compensated inverse demands: the Antonelli demands. Specifically,
(3) $\pi_{i}^{a}(u, \boldsymbol{q})=\frac{\partial D(u, \boldsymbol{q})}{\partial q_{i}}=c_{i}+\left[b_{i}+\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-1} \sum_{j=1}^{n} a_{i j} q_{j}-\frac{1}{2}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-2} \boldsymbol{q}^{T} A \boldsymbol{q}\right] u^{-1}, i=1, \ldots, n$,
where $\pi_{i}=p_{i} / m$ denotes the $i$ th normalized price. By construction the Antoneelli demands in (3) are homogeneous of degree zero in quantities.

Of course Antonelli demands are not directly estimable because the utility index $u$ is not observed. Uncompensated inverse demands that are, in fact, estimable, may be obtained in the following manner. First, as Deaton (1979) notes, the distance function implicitly defines the consumer's utility function. Specifically, $D(u, \boldsymbol{q})=1$ at the optimum, which implies that (1) may be solved explicitly for the utility index $u$ as

$$
\begin{equation*}
U(\boldsymbol{q})=\frac{\left[\boldsymbol{b}^{T} \boldsymbol{q}+\frac{1}{2}\left(\alpha^{T} \boldsymbol{q}\right)^{-1} \boldsymbol{q}^{T} A \boldsymbol{q}\right]}{1-\boldsymbol{c}^{T} \boldsymbol{q}} \tag{4}
\end{equation*}
$$

Utility function (4) may then be used to substitute for $u$ in (3), giving

$$
\begin{equation*}
\pi_{i}(\boldsymbol{q})=c_{i}+\frac{\left[b_{i}+\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-1} \sum_{j=1}^{n} a_{i j} q_{j}-\frac{1}{2} \alpha_{i}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-2} \boldsymbol{q}^{T} A \boldsymbol{q}\right]\left[1-\boldsymbol{c}^{T} \boldsymbol{q}\right]}{\left[\boldsymbol{b}^{T} \boldsymbol{q}+\frac{1}{2}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}\right)^{-1} \boldsymbol{q}^{T} A \boldsymbol{q}\right]}, i=1, \ldots, n, \tag{5}
\end{equation*}
$$

a system of observable inverse demands.
Several additional restrictions on the parameters of the system in (5) are required in estimation. To start, the system in (5) is homogeneous of degree zero in $\boldsymbol{b}$ and $A$. To achieve identification we simply require that

$$
\begin{equation*}
\boldsymbol{b}^{T} \boldsymbol{q}^{*}=1, \tag{6}
\end{equation*}
$$

an additional set of parameter restrictions used along with those in (2). Holt and Bishop (2002) also show that the matrix $A$ must be negative semi-definite for distance function (1) to be (globally) concave in quantities. If this requirement is not automatically satisfied it may be imposed in the following manner. Let $\tilde{A}$ denote a $(n-1) \times(n-1)$ obtained from A by deleting the last row and column; these terms may be recovered by using the restrictions in (2a). The implication is that if $\tilde{A}$ is negative semi-definite than $A$ will also be negative semi-definite. We may then redefine $\tilde{A}$ as:

$$
\begin{equation*}
\tilde{A}=-S S^{T}, \quad S=\left[s_{i j}\right], \quad s_{i j}=0 \forall i>j . \tag{7}
\end{equation*}
$$

In other words, $S$ is the $(n-1) \times(n-1)$ Cholesky decomposition of $\tilde{A}$. In model implementation the $s_{i j}$ parameters are estimated in lieu of the $a_{i j}$ parameters (Diewert and Wales, 1998a).

As a practical matter, if negativity must be imposed using the Cholesky decomposition, it is typically the case that the positive eigenvalues associated with the unrestricted estimates of $\tilde{A}$, while now negative, will be very close to zero. It may therefore be desirable to further reduce the rank of $\tilde{A}$. Following Diewert and Wales (1988b), let $K \leq(n-1)$ be the rank of $\tilde{A}$. In the case where $K<(n-1), \tilde{A}$ is associated with a $K$-column Cholesky decomposition. That is, $S$ is defined according to

$$
\begin{equation*}
S=\left[s_{i j}\right], \quad s_{i j}=0 \text { for } 1 \leq i \leq j \leq n-1 \text { and for } j=K+1, \ldots, n-1 . \tag{8}
\end{equation*}
$$

In other words, S is a lower triangular ( $n-1$ ) $\mathrm{x}(n-1)$ matrix with zeros in its final $(n-1)-K$ columns. The combination of (1)-(2), and (6)-(8) yields the SNQIDS.

## 3. Measuring Welfare Losses with a Distance Function

As previously noted, an advantage of the SNQIDS is that concavity of the distance function may be maintained globally, and therefore consistent welfare loss estimates associated with varying catch limits (restrictions) may be obtained. Palmquist (1988) and Kim (1997) develop the basic framework for obtaining measures of welfare loss in quantity space. Following Kim (1997), a measure of (normalized) compensating variation $(C V)$ associated with changing the quantity vector from $\boldsymbol{q}^{0}$ to $\boldsymbol{q}^{1}$ is given by

$$
\begin{equation*}
\overline{C V}=D\left(u^{0}, \boldsymbol{q}^{1}\right)-D\left(u^{0}, \boldsymbol{q}^{0}\right), \tag{9}
\end{equation*}
$$

where $D($.$) denotes a distance function. Here u^{0}$ denotes the base-period utility level, defined implicitly by the condition $D\left(u^{0}, \boldsymbol{q}^{0}\right)=1$. In (9) $\overline{C V}$ denotes the amount of additional (normalized) outlay necessary for a representative consumer to attain $u^{0}$ when confronted with quantity vector $\boldsymbol{q}^{1}$. Positive values for $\overline{C V}$ imply the consumer is made worse off with $\boldsymbol{q}^{1}$ relative to $\boldsymbol{q}^{0}$.

A measure of (normalized) equivalent variation ( $E V$ ) may be obtained in a similar way. That is, with a change in quantities from $\boldsymbol{q}^{0}$ to $\boldsymbol{q}^{1}, E V$ is given by

$$
\begin{equation*}
\overline{E V}=D\left(u^{1}, \boldsymbol{q}^{1}\right)-D\left(u^{1}, \boldsymbol{q}^{0}\right), \tag{10}
\end{equation*}
$$

where $u^{1}$ is implicitly defined by $D\left(u^{1}, \boldsymbol{q}^{1}\right)=1$. As specified in (10), $\overline{E V}$ is the amount of additional (normalized) expenditure necessary for the consumer to maintain utility level $u^{1}$ when facing quantity vector $\boldsymbol{q}^{1}$. Again, a positive value for $\overline{E V}$ indicates the consumer is worse off with $\boldsymbol{q}^{1}$ as compared to $\boldsymbol{q}^{0}$. As Kim (1997) shows, for nonhomothetic preferences, $C V$ will be less than $E V$ for a single quantity decrease.

Of course it is useful to have money-metric (i.e., non-normalized) measures for $C V$ and $E V$. Following Palmquist (1988), such measures may, in turn, be obtained by simply re-scaling $\overline{C V}$ and $\overline{E V}$ by total outlay. That is,

$$
\begin{equation*}
C V=m^{0}\left[D\left(u^{0}, \boldsymbol{q}^{1}\right)-D\left(u^{0}, \boldsymbol{q}^{0}\right)\right], \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
E V=m^{0}\left[D\left(u^{1}, \boldsymbol{q}^{1}\right)-D\left(u^{1}, \boldsymbol{q}^{0}\right)\right], \tag{12}
\end{equation*}
$$

where $m^{0}$ denotes total expenditure (sales) before any quantity changes occur. The welfare measures in (11) and (12) in conjunction with the SNQIDS in (1) and (2) are used in the subsequent empirical application to obtain welfare loss estimates associated with imposing more stringent catch restrictions.

## 4. Bootstrap and Subsampling Methodology

Econometricians frequently estimate models in which parameters are constrained to be on a boundary of the parameter space. The need to do so usually arises when $a$ priori theoretical restrictions require a certain estimated parameter to be of a specific sign. Examples of these types of restrictions include traditional demand analysis where the income effect for a normal good is constrained to be positive while the own-price effect is constrained to be negative; cost function analysis where curvature constraints imply that second-order price terms satisfy concavity conditions; and time series models for conditional heteroskedasticity where the GARCH parameters are constrained to be non-negative. Traditionally, inequality constraints have been problematic for the researcher because standard error estimates and confidence bounds are difficult to compute using classical statistical inference.

Recent theoretical work by Andrews (1999) explores the use of resampling techniques to calculate confidence intervals when a parameter (or set of parameters) is constrained to be on a boundary. At first glance the natural solution might appear to be to use the traditional bootstrap method pioneered by Efron (1979). Since its development, the bootstrap has become a popular method to calculate confidence intervals. As Andrews (1999) demonstrates, however, this procedure is not asymptotically correct to the first order when parameters are on a boundary. This is because the bootstrap puts too much mass below the cutoff point for the parameter and therefore does a poor job of mimicking the true population distribution. For this reason, Andrews (1999) proposes using subsample bootstrap jackknife methods in lieu of the traditional bootstrap.

The subsample bootstrap and subsample jackknife are similar to their standard counterparts except that a subset of the data is used to estimate the model. The subsample jackknife differs from the standard jackknife in that more than one observation is deleted. Specifically, to perform the subsample jackknife, $d$ (greater than 1) observations are dropped, parameter estimates are calculated using the remaining $m$ (where $m=T-d, T$ being the sample size) observations, and the process is repeated until all possible samples of size $m$ have been drawn. Because the potential number of subsamples to be drawn is likely far too large to allow for an efficient calculation of each of the possible subsamples, the researcher typically only takes a random sample of the possible subsamples to create subsample jackknife estimates. The subsample bootstrap differs from the standard bootstrap by drawing, with replacement, repeated samples of size $q$ (where $q$ is less than $T$ ) from the initial sample of size $T$. Andrews (1999)
demonstrates that the subsample bootstrap yields a consistent asymptotic distribution, unlike the standard bootstrap, by basing the bootstrap on these smaller samples.

The two subsampling methods work when the bootstrap is inappropriate because their requirements for consistency are weaker than the consistency requirements for the bootstrap. The intuition behind these techniques is that by using a subsample instead of the entire sample, the rate of convergence is slowed down relative to the bootstrap. To formalize this result, assume $F_{m T}$ is the estimate of the empirical distribution under one of the two subsampling techniques and $F_{T}$ the estimate of the empirical distribution under the bootstrap. If $m$ goes to infinity, $T$ goes to infinity, and $m / T$ goes to 0 , then the random sampling error of the bootstrap estimator is smaller than the random sampling error of the subsampling estimator. This makes the subsampling method less sensitive to the behavior of the mapping of the asymptotic distribution of the statistic in a neighborhood of the true distribution. While Andrews' (1999) work demonstrates the theoretical advantages of the subsample jackknife and subsample bootstrap, it is important to note that he does not investigate the empirical or finite sample practicality of the alternative subsampling approaches.

## 5. An Application With Great Lakes Fish Data

To illustrate the subsampling techniques described above and to compare them to results from the theoretically inconsistent bootstrap, the SNQIDS is estimated for fish landed in the U.S. Great Lakes region. The raw data are compiled from the Great Lakes Fishery Laboratory of the U.S. Fish and Wildlife Service and consist of monthly figures on amounts landed in pounds and average monthly prices in dollars per pound. The categories included are Whitefish, Laketrout, Yellow Perch (Perch), Lake Herring, Chub,
and Smelt. The sample period has been expanded from the original Holt and Bishop (2002) paper to include 1971-2001. Although the original data are monthly, the data are aggregated to bi-monthly because of small landings during certain months, leaving 186 observations. All quantity data are divided by total U.S. population, and are therefore expressed in per capita terms and, as well, are further normalized to have unit means. The data are summarized in Table 1.

The model used in estimation is a semiflexible normalized quadratic inverse demand system (SNQIDS) with concavity imposed. The system we estimate differs somewhat from that utilized by Holt and Bishop (2002), and is given by

$$
\begin{equation*}
w_{i t}=c_{i t} q_{i t}+\frac{\left[b_{i}+\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}_{t}\right)^{-1} \sum_{j=1}^{n} a_{i j} q_{i t} q_{j t}-\frac{1}{2} \alpha_{i} q_{i t}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}_{t}\right)^{-2} \boldsymbol{q}_{t}^{T} A \boldsymbol{q}_{t}\right]\left[1-\boldsymbol{c}_{t}^{T} \boldsymbol{q}_{t}\right]}{\left[\boldsymbol{b}^{T} \boldsymbol{q}_{t}+\frac{1}{2}\left(\boldsymbol{\alpha}^{T} \boldsymbol{q}_{t}\right)^{-1} \boldsymbol{q}_{t}^{T} A \boldsymbol{q}_{t}\right]}+v_{i t}, \tag{13}
\end{equation*}
$$

where

$$
c_{i t}=c_{i 1}+\sum_{j=2}^{6} c_{i j} D_{j t}+c_{i 7} t
$$

and where $i=1, \ldots, 6, n=6$, and $t=1, \ldots, 186$. In (9) $w_{i t}=\pi_{i t} q_{i t}$, which is the share in total value of sales of the $i$ th fish category. As well, $D_{j t}$ are bi-monthly dummy variables that equals one when the current period corresponds to bi-month $j$, zero otherwise. In (13) $v_{i t}$ is an iid mean-zero stochastic error term. Let $\boldsymbol{v}_{t}=\left(v_{1 t}, \ldots, v_{n t}\right)^{T}$. It then follows that $E\left(\boldsymbol{v}_{t}\right)=\mathbf{0}_{n}$ and $E\left(\boldsymbol{v}_{t} \boldsymbol{v}_{t}^{T}\right)=\Omega$, where $\Omega$ is the model's contemporaneous covariance matrix. The adding up condition implies, of course, that $\Omega \boldsymbol{i}_{n}=\mathbf{0}_{n}$.

As is customary for normalized quadratic demand systems, we let the reference bundle equal a unit vector, that is, $\boldsymbol{q}^{*}=\boldsymbol{i}_{n}$. We also follow prior research [e.g., Diewert and Wales (1988a)] and set $\alpha=(1 / n, \ldots, 1 / n)^{T}$. By using these definitions the restrictions in (2) are simply

$$
\begin{gather*}
\boldsymbol{\alpha}^{T} \boldsymbol{i}_{n}=1  \tag{14a}\\
\boldsymbol{c}^{T} \boldsymbol{i}_{n}=0, \text { and }  \tag{14b}\\
A \boldsymbol{i}_{n}=\mathbf{0}_{n}, \quad A=A^{T} . \tag{14c}
\end{gather*}
$$

In estimation we also follow Holt and Bishop (2002) and include first- and sixthorder system-wide autocorrelation matrices; this is done, moreover, by using the framework developed by Holt (1998). The primary difference between the specification in (13)-(14) and that utilized originally by Holt and Bishop (2002) is that bi-monthly dummy variables and a trend term have now been included in the model specification as these were found to be statistically important.

Maximum likelihood parameter estimates are obtained by deleting the equation for Smelt and then using nonlinear iterated SUR estimation techniques as implemented in version 7.10 of GQOPT. An unrestricted version of the model was estimated, wherein it was found that two eigenvalues associated with the $A$ matrix were positive. Alternate versions of the SNQIDS were then estimated by varying the rank $K$ between 1 and 5. As with the original Holt and Bishop (2002) application, the rank 2 SNQIDS model is preferred to rank 3,4 , and 5 models on the grounds of a likelihood ratio test and the SBC
and HQC criterions. Parameter estimates are not presented to conserve space; they are, however, available upon request. For more details on the development of the model and price and scale flexibility computation, refer to Holt and Bishop (2002).

Confidence intervals for price and scale flexibility and welfare measures are computed using bootstrap, subsample bootstrap, and subsample jackknife techniques. These results are reported in Tables 2-6. The confidence intervals are computed using either bootstrapped or jackknifed $t$-statistics and standard errors from 100 resampled standardized errors while the estimates are obtained using the original 186 observations. The sample size used to compute the subsample bootstrap and subsample jackknife confidence intervals is 150 observations.

Table 2 presents own-quantity Antonelli or compensated elasticities estimated at the mean values with the corresponding confidence intervals from the bootstrap, subsample bootstrap, and subsample jackknife techniques. Looking first at the elasticity estimates, in spite of the additional data, they are all similar in magnitude to the results reported by Holt and Bishop (2002). Turing to the confidence intervals, all of the estimates are statistically significant except for Lake Herring own-quantity elasticity for the subsample bootstrap and subsample jackknife and own-quantity Chub for the subsample bootstrap. The boostraped confidence intervals are generally more precise than the confidence intervals resulting from the other two methods, although as Andrew's (1999) notes, these estimates are potentially biased.

Tables 3 and 4 follow similar patterns to the results reported in Table 2. Ownquantity uncompensated flexibilities found in Table 3 are similar in magnitude to Holt and Bishop's (2002) results, and the flexibilities are generally significant except for the
bootstrapped chub and smelt confidence intervals. Table 4 presents estimated consumption scale elasticities, which are all statistically significant using the three methods to compute confidence intervals. As with previous results, the bootstrapped confidence intervals are generally more precise than are those for the other two methods. The confidence intervals also allow us to test whether the fish are necessity goods ( $\mathrm{f}_{i}<-1$ ) or luxuries $\left(\mathrm{f}_{i}>-1\right)$. The point estimates indicate that Whitefish, Lake Trout, and Chub are candidates to be necessities although the confidence intervals from the three differ as to whether the scale elasticities and statistically less than -1 . Specifically, the bootstrapped confidence interval has Whitefish and Chub statistically less than -1 , the subsample bootstrap has Whitefish and Lake Trout statistically less than -1 and the subsample jackknife only has Lake Trout statistically less than -1. Perch, Lake Herring and Smelt are all possibilities for luxury goods but the confidence interval results yet again obtain different conclusions. The bootstrap and subsample bootstrap both have Perch and Lake Herring greater than -1 but the subsample jackknife only has Lake Herring as statistically greater than -1 . In this case, the conclusion a researcher draws depends on the method used to compute the confidence interval.

The final application of the resampling techniques is to obtain confidence intervals on the estimates of welfare losses to fish consumers associated with reducing catch quotas for commercial fishermen. This is the first known application wherein measures of precision are obtained in a classical statistics framework for welfare estimates from a system of demand equations when concavity is imposed on the model. By using (11) and (12) in conjunction with the estimated rank 2 SNQIDS, estimates compensated variation $(C V)$ and equivalent variation $(E V)$, evaluated at the sample means
and corresponding to a ten percent reduction in catch, are derived. The point estimates, along with confidence intervals, are presented in Tables 5 and 6 .

Comparing the present results with those reported by Holt and Bishop (2002), the estimates obtained here based on the longer data set are generally larger in magnitude (ranging from approximately $\$ 190,000$ more for Whitefish to $\$ 8,400$ more for Lake Herring). An exception to this pattern is the mean estimate of Chub, which is approximately $\$ 40,000$ smaller in the expanded data set. Differences between estimates of $C V$ and $E V$ are in general small. Lake Herring, at a difference of $\$ 98$ is the smallest, while Whitefish has the largest difference at $\$ 25,063$. These results follow the same pattern found in Holt and Bishop (2002).

Regarding the 95-percent confidence interval results for $C V$ and $E V$, the ordinary bootstrap confidence intervals indicate that these estimates are all statistically significant. Alternatively, the subsample bootstrap suggests that the CV estimate for Lake Herring is not statistically different from zero. Statistical significance of welfare estimates are most problematic when the subsample jackknife is used. In this case $C V$ measures for Lake Herring and Chub are not significantly different from zero. Likewise, EV measures for Perch, Lake Herring, and Smelt are not statistically significant at the 95-percent level. Therefore-as with price and scale flexibilities-any determination about the statistical significance of implied welfare measures depends crucially upon the method used to compute confidence intervals.

## 6. Conclusions

In this paper, we re-estimate the SNQID models originally reported on by Holt and Bishop with an expanded data set. More importantly, we obtain confidence intervals
for the estimates of own-quantity price and scale flexibilities in addition to welfare losses to fish consumers associated with reducing catch quotas for commercial fishermen. These confidence intervals are obtained, in turn, by utilizing the standard bootstrap as well as the subsample bootstrap and subsample jackknife. We report here the first known application where measures of precision are obtained in a classical statistics framework for welfare estimates when concavity is imposed in a demand systems framework. The measures of precision, especially for welfare measures, will allow policy makers to more accurately gauge the welfare losses associated with catch restrictions than is allowed by point estimates alone. It is important to note that the conclusions drawn about the statistical significance of an estimate is sometimes dependant on the resampling technique used to compute the confidence interval. Further investigation into the small sample properties of these various simulation techniques is therefore called for.

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Table 1. Descriptive statistics for shares in total sales of fish landed in the U.S. Great Lakes 1971-2001.

| Fish Category | Average | Standard <br> Deviation | Minimum | Maximum |
| :--- | :---: | :---: | :---: | :---: |
| 1. Whitefish | 0.482 | 0.152 | 0.083 | 0.788 |
| 2. Lake Trout | 0.028 | 0.018 | 0.006 | 0.103 |
| 3. Yellow Perch | 0.210 | 0.107 | 0.001 | 0.470 |
| 4. Lake Herring | 0.018 | 0.019 | 0.001 | 0.117 |
| 5. Chub | 0.195 | 0.137 | 0.020 | 0.732 |
| 6. Smelt | 0.067 | 0.089 | 0.000 | 0.605 |

Table 2. Mean estimated own-quantity Antonelli (compensated) flexibilities with 95\% confidence intervals.

|  |  |  |  | Subsample <br> Bootstrap |  | Subsample <br> Jackknife |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Point |  |  |  |  |  |  |
| Fish Category | Estimate | Lower | Upper | Lower | Upper | Lower | Upper |
|  | 1. Whitefish | -0.022 | -0.029 | -0.014 | -0.044 | -0.022 | -0.049 |
| 2. Lake Trout | -0.031 | -0.078 | -0.011 | -0.235 | -0.023 | -0.259 | -0.014 |
| 3. Yellow Perch | -0.076 | -0.093 | -0.064 | -0.163 | -0.038 | -0.183 | -0.029 |
| 4. Lake Herring | -0.058 | -0.070 | -0.037 | -0.060 | 0.002 | -0.067 | 0.013 |
| 5. Chub | -0.019 | -0.025 | -0.011 | -0.116 | 0.015 | -0.105 | -0.052 |
| 6. Smelt | -0.003 | -0.029 | -0.001 | -0.129 | -0.010 | -0.104 | -0.022 |

Table 3. Mean estimated own-quantity uncompensated flexibilities with $95 \%$ confidence intervals.

|  |  |  |  | Subsample <br> Bootstrap |  | Subsample <br> Jackknife |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Point |  |  |  |  |  |  |
| Fish Category | Estimate | Lower | Upper | Lower | Upper | Lower | Upper |
| 1. Whitefish | -0.554 | -0.567 | -0.523 | -0.556 | -0.519 | -0.561 | -0.511 |
| 2. Lake Trout | -0.063 | -0.107 | -0.044 | -0.269 | -0.055 | -0.293 | -0.008 |
| 3. Yellow Perch | -0.241 | -0.265 | -0.226 | -0.361 | -0.224 | -0.349 | -0.206 |
| 4. Lake Herring | -0.072 | -0.087 | -0.052 | -0.074 | -0.013 | -0.079 | -0.011 |
| 5. Chub | -0.230 | -2.762 | 1.627 | -0.183 | -0.094 | -0.236 | -0.182 |
| 6. Smelt | -0.049 | -1.380 | 0.814 | -0.206 | -0.041 | -0.258 | -0.018 |

Table 4. Mean estimated consumption scale elasticities with $95 \%$ confidence intervals.

| Fish Category | Point Estimate | Bootstrap |  | Subsample <br> Bootstrap |  | Subsample Jackknife |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lower | Upper | Lower | Upper | Lower | Upper |
| 1. Whitefish | -1.090 | $-1.135$ | -1.041 | -1.101 | -1.002 | -1.120 | -0.994 |
| 2. Lake Trout | -1.094 | -1.312 | -0.933 | -1.298 | -1.091 | -1.320 | -1.051 |
| 3. Yellow Perch | -0.776 | -0.828 | -0.741 | -0.984 | -0.683 | -1.106 | -0.673 |
| 4. Lake Herring | -0.880 | -0.936 | -0.867 | -0.893 | -0.742 | -0.937 | -0.611 |
| 5. Chub | -1.143 | -1.225 | -1.027 | -1.250 | -0.839 | -1.253 | -0.450 |
| 6. Smelt | -0.661 | -1.136 | -0.072 | -1.737 | -0.525 | -1.721 | -0.491 |

Table 5. Annualized mean compensated variation for a 10 percent reduction in catch with $95 \%$ confidence intervals.

| Fish Category | Point Estimate | Bootstrap |  | Subsample <br> Bootstrap |  | Subsample Jackknife |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lower | Upper | Lower | Upper | Lower | Upper |
| 1. Whitefish | 776,182 | 731,352 | 808,943 | 729,655 | 1,056,655 | 96,834 | 3,588,379 |
| 2. Lake Trout | 73,221 | 60,029 | 79,149 | 62,941 | 191,751 | 35,447 | 1,612,582 |
| 3. Yellow Perch | 294,816 | 282,212 | 303,491 | 267,138 | 536,437 | 201,360 | 1,336,740 |
| 4. Lake Herring | 24,645 | 21,786 | 26,317 | -576 | 202,096 | -44,138 | 576,748 |
| 5. Chub | 190,640 | 165,065 | 215,159 | 142,770 | 335,186 | -39,295 | 1,008,154 |
| 6. Smelt | 63,169 | 38,460 | 95,374 | 51,227 | 295,923 | 34,195 | 445,604 |

Table 6. Annualized mean equivalent variation for a 10 percent reduction in catch with $95 \%$ confidence intervals.

| Fish Category | Point Estimate | Bootstrap |  | Subsample <br> Bootstrap |  | Subsample Jackknife |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lower | Upper | Lower | Upper | Lower | Upper |
| 1. Whitefish | 801,245 | 752,019 | 836,407 | 764,892 | 1,070,607 | 73,403 | 5,821,624 |
| 2. Lake Trout | 75,966 | 62,543 | 81,862 | 57,914 | 117,432 | 15,808 | 1,396,285 |
| 3. Yellow Perch | 300,432 | 287,386 | 309,484 | 223,792 | 369,259 | -14,549 | 715,860 |
| 4. Lake Herring | 24,743 | 21,875 | 26,426 | -17,197 | 27,868 | -37,006 | 183,448 |
| 5. Chub | 194,415 | 167,869 | 219,631 | 182,746 | 235,863 | 159,650 | 556,931 |
| 6. Smelt | 63,578 | 38,803 | 96,097 | 19,105 | 87,178 | -30,838 | 132,598 |


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