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PRISMA, The Size-Class Module

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Summary

PRISMA - an acronym of Policy Research Instrument for Size-aspects in Macro-economic Analysis – is an economic macro-sector model. It has been designed in such a way that it produces results consistent with those produced by the current macro-sector model of CPB, Netherlands Bureau for Economic Policy Analysis. PRISMA is used for forecasting, scenario building and what-if analyses with respect to government policies and exogenous shocks. Its time horizon is 3-25 years.

PRISMA consists of a kernel and a number of modules. Its kernel is documented in De Wit (2001). PRISMA's most important module is the size-class module. In this report the present version of this module is documented.

In the kernel of PRISMA the business sector is subdivided into eighteen industries. In the size-class module the non-primary private industries (thirteen in total) are further subdivided into three size-classes: small (0-9 employees), medium-sized (10-99 employees), and large (100 or more employees) businesses. Hence, the following two types of model exercises become possible. First, economic effects derived by PRISMA's kernel – for example when forecasting, building a scenario, or evaluating the consequences of changes in policy or the economic environment – can be “translated” into the prospects for SMEs. Second, when circumstances change differently for SMEs compared to large businesses – for example due to a policy measure that focuses particularly on SMEs – the consequences can be evaluated by using the size-class module.

1 Introduction

Description of PRISMA

PRISMA - an acronym of Policy Research Instrument for Size-aspects in Macro-economic Analysis – is an economic macro-sector model, which has been developed within the research programme SCALES.¹ It has been designed in such a way that it produces results consistent with those produced by the current macro-sector model of CPB, Netherlands Bureau for Economic Policy Analysis. PRISMA is used for forecasting, scenario building and what-if analyses with respect to government policies and exogenous-shocks. Its time horizon is 3-25 years.

PRISMA consists of a kernel and a number of modules. Its kernel is documented in De Wit (2001). PRISMA's most important module is the size-class module. In this report the present version of this module is documented.²

Overview of chapter

Section 1.1 provides a short description of the size-class module. Subsequently, section 1.2 points out the surplus value of the development of the module, while section 1.3 gives some examples of applications in which the module was used. Section 1.4 concludes with an overview of the remainder of this report.

1.1 Short description of module

Division into size-classes

In the kernel of PRISMA the business sector is subdivided into eighteen industries. In the size-class module the non-primary private industries (thirteen in total) are further subdivided into three size-classes: small (0-9 employees), medium-sized (10-99 employees), and large (100 or more employees) businesses. Hence, it becomes possible to analyse size-class effects.

Top-down and bottom-up mode

The module has been built in such a way that size-class effects can be analysed in two ways. In the top-down mode of the module results for industry totals, which are determined by PRISMA's kernel, are merely subdivided consistently into results for the three size-classes. Alternatively, in the bottom-up mode the module calculates size-class results directly from first principles. Subsequently, industry and macro results are found by aggregation. Hence, in the bottom-up mode industry and macro results are influenced by the explicit modelling of the size-class structure. Thus, these industry and macro results can be different from those produced by PRISMA's kernel.

1.2 Surplus value

Consequences for size-classes

As far as we know, the explicit modelling of a size-class structure into a macro-sector model is unique. Such modelling makes the following analyses possible.

First, economic effects derived by PRISMA's kernel – for example when forecasting, building a scenario, or evaluating the consequences of changes in policy or the economic environment – can be “translated” into the prospects for SMEs. The top-down mode of the size-class module (see the previous section) is especially appropriate for these sorts of applications.

¹ The research programme SCALES – an acronym of SCientific Analysis of Entrepreneurship and SMEs – is financed by the Dutch Ministry of Economic Affairs.

² The present version is that of 2001, April 26th

Consequences of size-classes

Second, when circumstances change differently for SMEs compared to large businesses – for example due to a policy measure that focuses particularly on SMEs – the consequences can be evaluated by using the size-class module. For these sorts of applications the bottom-up mode of the module is appropriate.

Independent size-class effects?

Third, in some respects small and/or medium-sized businesses behave differently compared to large businesses. Thus, introducing a size-class dimension to a model might result in more realistic model analyses because independent size-class effects can be taken into account. In fact, in this report we will investigate this hypothesis by comparing results of the size-class module in the bottom-up mode – in which explicit size-class effects are possible – to results of the module in the top-down mode – in which the size-class structure cannot have an independent effect on industry and macro results. It will appear (section 4.2) that – in the present module - independent size-class effects are not very dominant in explaining industry and macro results.

1.3 Examples of applications

In this section we give some examples of applications in which the size-class module of PRISMA has been used together with its kernel.

The first example concerns a medium-term what-if analysis. In Peeters (2002) it is analysed with the help of PRISMA what the macro and sector consequences are if the world economy develops differently than CPB assumes. With the aid of the size-class module it is analysed what these alternative developments mean with respect to the prospects of small, medium-sized, and large businesses.

To give another example, Kwaak (2001a) investigates, on behalf of the employers' organisation "MKB-Nederland", the economic effects of decreasing the employers' social benefits contributions – for the economy as a whole and especially for SMEs.

Finally, Kwaak and Vroonhof (1998) used the model to analyse the impacts of various forms of energy taxes, which were proposed by government at the time. The size-class module of PRISMA was essential in these analyses, because these energy taxes were levied merely on SMEs and consumers.

1.4 Overview of the report

In chapter 2 the general structure of the module is described. Furthermore, it is discussed (i) why developments for SMEs might be different from average and (ii) what influence can be expected from explicitly modelling a size-class structure. Subsequently, chapter 3 describes the behavioural equations of the module that are modelled in a size-class specific manner. Finally, chapter 4 offers some results from simulation analyses.

2 General structure and its consequences

Overview of chapter	In this chapter we discuss the general structure of the size-class module (section 2.1). ¹ Subsequently, we discuss which effects can be expected as a result of this structure. Thus, we describe in section 2.2 why developments for SMEs might be different from average and in section 2.3 what influence we can expect from explicitly modelling a size-class structure.
Parallel with sector models versus macro models	The inclusion of a size-class structure into a model is analogous to the inclusion of a sector structure into a macro model. Hence, the analysis of the differences between the PRISMA model with or without the size-class module (sections 2.1, 2.3, and 4.2) contribute to the scientific discussion with respect to the relative merits of sector models compared to macro models.

2.1 General structure

	<i>Size-classes and their interpretation</i>
Three size-classes	In the size-class module industries in the business sector are subdivided into three size-classes: small (0-9 employees), medium-sized (10-99 employees), and large (100 or more employees) businesses. Developments of these size-classes are analysed.
Interpretation of size-classes	Size-classes make up the aggregation of all relevant businesses. Note however, that number and identity of businesses in a size-class may change over time due to underlying business dynamics - entry, exit, growth, decline- that are not modelled themselves. Thus, the development of the <i>size-class</i> is analysed, not that of the underlying businesses.
Example	For example, if in a certain industry the employment of the size-class "small businesses" has grown, this does not necessarily or only mean that the employment of the current small businesses has grown. On the contrary, as it stands to reason that average firm size in the size-class "small businesses" (having 0-9 employees) is more or less constant over time, it probably means that the <i>number</i> of small businesses has grown.
	<i>Base year</i>
Size-class specific base year values	In the kernel of PRISMA the business sector is subdivided into eighteen industries. In the size-class module the non-primary private industries (thirteen in total) are further subdivided into three size-classes: small, medium-sized, and large businesses. See Table 2.1. Hence, base year values of all level variables relating to the thirteen non-primary private industries must be subdivided into three size-class values. In Kwaak (2001b) it is described how this is done exactly.
	<i>Construction of size-class equations</i>
Point of departure	Point of departure for specifying the size-class equations in a certain industry are the industry equations that are described and motivated in De Wit (2001). Hence, we choose the structure of the size-class equations in an industry the same as that for the

¹ The notational conventions adhered to in this chapter can be found in annex I. For expositional reasons some of the formulas in this chapter are relatively simple approximate ones. The exact equivalents can be found in annex IV.

industry total from De Wit (2001). Parameters and determining variables are allowed to be size-class specific, though.

table 2.1 Business sector: division into size-classes^a

<i>nr</i>	<i>industry</i>
<i>non-primary private industries: three size-classes</i>	
1	food industry
2	chemical industry
3	metallurgic industry
4	other manufacturing
5	construction
6	wholesale trade
7	retail trade
8	hotels, restaurants
9	car trade and repair
10	transport and communication
11	banking and insurance
12	other commercial services
15	oil industry ^b
<i>other industries: no size-classes</i>	
13	agriculture
14	mining
16	public utilities
17	housing
18	medical and social care

a See Annex 3 for the precise classification of industries.

b In the oil industry there are no small businesses. Hence only medium-sized and large businesses are distinguished.

Constraint on size-class parameters

We intend to assure that the industry behaviour that results from the aggregation of the size-class equations resembles the behaviour of the industry total following from De Wit (2001) as closely as possible. This can be accomplished by imposing a constraint on the parameters of the size-class equations. We will now show which constraint is needed.

Take, for example, the relationship:

$$\overset{\circ}{lemp}_{ind} = \mathbf{a}_{ind} \overset{\circ}{ygf}_{ind} \quad (2.1)$$

from De Wit (2001), which states that for a certain industry the relative change in the labour volume of employees (*lemp*) is related to the relative change in value added (*ygf*) by a fixed elasticity α . We postulate the same relationship for the underlying size-classes *k*:

$$\overset{\circ}{lemp}_k = \mathbf{a}_k \overset{\circ}{ygf}_k \quad (2.2)$$

These size-class equations implicitly imply an average elasticity for the industry as a whole (α_{avs}), which may and shall be different from the constant elasticity (α_{ind}) for the industry. It is easily derived that this implied average elasticity equals:

$$\mathbf{a}_{avs} \equiv \frac{\overset{\circ}{lemp}_{ind}}{\overset{\circ}{ygf}_{ind}} = \frac{\sum_k (\overset{\circ}{lemp}_k / \overset{\circ}{lemp}_{ind})^{-1} \mathbf{a}_k \overset{\circ}{ygf}_k}{\sum_k (\overset{\circ}{ygf}_k / \overset{\circ}{ygf}_{ind})^{-1} \overset{\circ}{ygf}_k} \quad (2.3)$$

It follows that the average elasticity implied by the size-class equations will not be constant over time in general. However, if the relative changes in value added do not differ between size-classes and the employment shares of the size-classes are constant over time, then the implied average elasticity is constant over time and equal to:

$$\mathbf{a}_{avs} = \sum_k (\overset{\circ}{lemp}_k / \overset{\circ}{lemp}_{ind}) \mathbf{a}_k \quad (2.4)$$

Therefore, we demand that the size-class elasticities (α_k) are chosen such that their weighted average is equal to the elasticity of the adjoining equation for the industry total (α_{ind}), the weights being the employment shares of the base year:

$$\sum_k (\overset{\circ}{lemp}_{k,t=0} / \overset{\circ}{lemp}_{ind,t=0}) \mathbf{a}_k = \mathbf{a}_{ind} \quad (2.5)$$

Influence of the constraint

The above constraint on the size-class elasticities assures that, in the first simulation year, the employment development that results from the aggregation of the size-class equations, is the same as the employment development according to De Wit (2001), as long as the relative change in value added is not size-class specific. However, if (i) the latter restriction is violated or (ii) later simulation years are considered, this may not be the case.

Top-down and bottom-up mode

The way, in which size-class values are determined in behavioural equations, is dependent on whether the top-down or bottom-up mode is used.

Bottom-up and top-down mode

In the bottom-up mode size-class values are determined fully independently from each other. Subsequently, industry totals are determined by aggregating the size-class values. In the top-down mode things are the other way round. First, the industry total is determined without using any information about the underlying size-class structure. Second, preliminary size-class values are calculated in exactly the same way as would have been done in the bottom-up mode. In general, these preliminary size-class values will not add up to the industry total that was determined in the first step. Therefore, in a third step, these preliminary size-class values are rescaled in such a way, that this adding-up consistency is established.

Rescaling principle

There is a - more or less arbitrary - choice to be made how to design this rescaling process. We have chosen to do this rescaling in such way that the ratios between the preliminary size-class levels remain the same during the rescaling process.¹ In our view,

¹ For example, for a certain industry the preliminary size-class levels for employment are 20.000, 30.000 and 50.000 for small, medium-sized, and large businesses, respectively. However, the industry total should add up to 90.000. Then the levels are rescaled by multiplying the preliminary levels by 0.9, resulting in 18.000, 27.000, and 45.000 employees for small, medium-sized, and large businesses, respectively.

this is the most reasonable and robust way to establish the necessary adding-up consistency.

Only positive values are to be rescaled

The above rescaling process only makes sense if values are positive. Hence, we set up the model in such a way that only variables which values are necessarily positive are rescaled. For example, net investments are not rescaled: instead, the stock of capital and depreciation are rescaled.

Many variables have to be rescaled

There are more variables that have to be rescaled than one would think at first glance. For example, in our model the number of self-employed individuals (lse) follows the development of the volume of employees ($lemp$), at the industry level as well as for all size-classes k .¹

$$\begin{aligned} lse_{ind}^{\circ} &= lemp_{ind}^{\circ} \\ lse_k^{\circ} &= lemp_k^{\circ} \end{aligned} \tag{2.6}$$

It is easily shown that even for such a trivial relationship between variables the preliminary size-class values for lse_k will not add up – in general - to the industry value lse_{ind} . Only if for all size classes the ratio between lse and $lemp$ would be the same the adding-up problem would not exist.

2.2 Why results for SMEs may be different from average

One of the reasons for developing the size-class module is the translation of macro and industry developments into developments of SMEs. Such a translation is only necessary if there is reason to believe that developments of SMEs will in general be different from the average macro development. Hence, we sum up some mechanisms that could cause such a difference.

- Industry effects. In some industries (for example, the retail trade) SMEs are more dominant than in other. Hence, if output grows relatively fast in the retail trade, this will result *ceteris paribus* in a higher growth for the total SME sector than the macro average.
- Structure effects. The economic structure of SMEs may be different from those of large businesses. For example, in manufacturing SMEs are less export oriented than large businesses. Hence, if exports grow relatively fast this will favour large manufacturing businesses more than it favours small ones, so that *ceteris paribus* the output growth of SMEs will lag behind.
- Behavioural effects. The economic behaviour of SMEs may be different from those of large businesses. For example, in the export market small businesses in manufacturing tend to react more to price changes of competitors than large businesses (see the next chapter). Hence, if foreign competitors lower their price, real exports of small businesses will decrease more than real exports of large businesses.

In chapter 4 we will show to what extent the above effects actually show up in simulation analyses. It will appear there that in the present module especially industry effects are responsible for differences between SMEs and large businesses.

¹ This is the basic assumption in the kernel of PRISMA. The Entrepreneurship module of PRISMA contains a more sophisticated way of modelling the number of self-employed individuals. See Bosma and Fris (2002).

2.3 Why bottom-up results may be different from top-down ones

In section 2.1 we explained that we constrain our size-class equations in such a way that their aggregated behaviour resembles the equation for the industry total as closely as possible. Nevertheless, macro and industry results in the bottom-up mode will be different from those in the top-down mode. Clearly, these differences arise from the explicit modelling of a size-class structure. Hence we will label them as independent size-class effects. In this section we will analyse under what conditions bottom-up results differ from top-down ones.

The general case

Let us first explore the general case. Let variable y be dependent on variable x with elasticity α :

$$\dot{y} = \mathbf{a} \dot{x} \quad (2.7)$$

For this specific relationship, we can reformulate our research question as: "under what conditions does the bottom-up elasticity at the industry level differ from the top-down one?" In the bottom-up case the above relationship holds at all times t for the three size-classes k :

$$\dot{y}_{k,t} = \mathbf{a}_k \dot{x}_{k,t} \quad (2.8)$$

As derived in section 2.1, this implies the following time-dependent elasticity at the industry level:

$$\mathbf{a}_{ind,bottom-up,t} = \frac{\sum_k (y_{k,t-1} / y_{ind,t-1}) \mathbf{a}_k \dot{x}_{k,t}}{\sum_k (x_{k,t-1} / x_{ind,t-1}) \dot{x}_{k,t}} \quad (2.9)$$

In the top-down mode we postulate a time-independent elasticity at the industry level. As derived in section 2.1, this elasticity is equal to:

$$\mathbf{a}_{ind,top-down} = \sum_k (y_{k,t=0} / y_{ind,t=0}) \mathbf{a}_k \quad (2.10)$$

Hence, by comparing the above expressions for the bottom-up and top-down elasticities at the industry level it becomes clear under what conditions these elasticities differ. However, it is instructive to discuss two typical special cases that lead to such differences.

Size-class specific elasticities

Size-class specific elasticities may lead to differences between the bottom-up and top-down elasticity at the industry level. As an example, we take the relationship:

$$s_x = \mathbf{g} p_{sx} \quad (2.11)$$

which states that the volume of exports (s_x) is related to its price (p_{sx}) by a fixed (negative) elasticity γ . If we assume that for all three size-classes prices of export products

develop in the same way, the expressions for the bottom-up and top-down elasticities at the industry level boil down to:

$$\mathbf{g}_{ind, bottom-up, t} = \sum_k (sx_{k, t-1} / sx_{ind, t-1}) \mathbf{g}_k \quad (2.12a)$$

$$\mathbf{g}_{ind, top-down} = \sum_k (sx_{k, t=0} / sx_{ind, t=0}) \mathbf{g}_k \quad (2.12b)$$

in this case. It is known that small businesses' exports are more sensitive for price developments than exports of large businesses, that is: for small businesses the elasticity γ is larger (more negative) than for large businesses. Hence, if for one reason or another the export share of small businesses increases, the bottom-up elasticity at the industry level becomes larger (more negative) than the top-down one. Thus, in such a case exports at the industry level become more sensitive to the export price in the bottom-up mode as compared to the top-down mode.

Size-class specific
production structure

Differences in production structure between the size-classes may also lead to differences between the bottom-up and top-down elasticity at the industry level. Look, as an example, at the familiar Leontieff relationship:

$$\dot{dm} = \dot{q} \quad (2.13)$$

which states that the relative change in the demand for intermediate goods/services (dm) equals the relative change in production (q). As in this relationship the elasticity is equal to 1 for all size-classes, the expressions for the bottom-up and top-down elasticities at the industry level boil down to:

$$\mathbf{a}_{ind, bottom-up, t} = \frac{\sum_k (dm_{k, t-1} / dm_{ind, t-1}) \dot{q}_{k, t}}{\sum_k (q_{k, t-1} / q_{ind, t-1}) \dot{q}_{k, t}} \quad (2.14a)$$

$$\mathbf{a}_{ind, top-down} = 1 \quad (2.14b)$$

Let us assume that for the industry under consideration the production structure between the size-classes differs such that small businesses have a lower input/production ratio than large businesses. Then, if for some reason or other production of small businesses grows faster than that of large businesses, at the aggregate industry level the growth in demanded intermediate goods/services will fall behind the growth in production. In other words, in such a case the bottom-up elasticity becomes less than the top-down elasticity of 1.

Why size-classes develop differently

In both special cases above one of the essential elements for the occurrence of independent size-class effects, was the assumption that "for some reason or other" size-classes develop differently. Thus, one may ask whether such different developments necessarily occur. The answer is: yes. Take, for example, value added which - by definition - is equal to sales minus intermediate goods/services. It is easily shown that - even

if relative changes in sales and intermediate goods/services would not be size-class specific - in general the relative change in value added will be size-class specific.¹

Size of differences

In this section we have explained theoretically how independent size-class effects might occur. It is a matter of the actual structure and behaviour of SMEs to which extent these effects actually occur. We will investigate this issue in chapter 4. It will appear there that – given the structure and assumed behaviour of Dutch SMEs – independent size-class effects are small.

¹ This is due to the fact that in general the ratio value added / sales is size-class specific.

3 Description of equations

In this chapter, the behavioural equations that are modelled in a size-class specific manner are described.¹ The underlying assumptions are explained. As only in the non-primary private industries size-classes are distinguished, only these thirteen industries show up in this chapter. In one of these, the oil industry, only two instead of three size-classes are distinguished. For this industry, the size-class “small” is not defined. The differences in elasticities (parameter values) between the size-classes are described in terms of deviations per size-class from the industry average, as expressed by a multiplication factor. For example, if for a certain parameter the industry average equals 0.5 and the multiplication factor for the size-class small equals 1.1, then the actual parameter value for the size-class small is equal to $0.5 * 1.1 = 0.55$. The multiplication factors have been chosen in such a way that for the base year the weighted average is equal to one (consistent with the industry-average value of the parameter). One has to bear in mind that in doing so, the factors have been rounded, so that there are cases which are, at first sight, inconsistent (for example if the multiplication factors are 1; 1; and 0.9 for the three respective size-classes). Industries that, for a certain equation, are not listed in the parameter tables are not modelled in a size-class specific manner. The structure of this chapter is as follows. In sections 3.1 and 3.2, the influence of foreign countries is described. This concerns exports and the import penetration on markets for consumption goods/services. In sections 3.3 and 3.4, the factor demands are dealt with. This concerns employment and capital (investments). In sections 3.5 and 3.6, finally, two sorts of prices are described, viz. the price setting behaviour of businesses (domestic prices) and the prices of competitors. Each section within this chapter has a part in which the general framework of the equation concerned is described (“General”, mainly derived from De Wit, 2001), and a part in which the differences per size-class are described (“Size-class specifics”).

3.1 Sales of exported goods/services

3.1.1 General

For most industries sales of exports follow:

$$\dot{sx} = \alpha \dot{twld} + \beta (\dot{p}_{twld} - \dot{p}_{twld,av})_{-1} + \gamma (\dot{p}_{sx} - \dot{p}_{twld})_{-1} + d \quad (3.1)$$

where:

sx	sales of exported goods/services of domestic origin
$twld$	world trade
p_{twld}	price of foreign goods/services that compete with exports
$p_{twld,av}$	weighted average of foreign manufacturing prices
p_{sx}	price of domestically produced export goods/services

Values of the parameters α , β , and γ , are presented in Table 3.1G (G denotes general).

¹ The notational conventions adhered to can be found in Annex 1. Annex 2 contains a list of variables.

table 3.1G Parameter values of equation (3.1): exports

industries	a	b	g
food industry	1.00		-1.12
chemical industry	1.41	-0.175	-2.04
metallurgic industry	1.28	-0.175	-1.98
other manufacturing	1.23	-0.175	-1.64
construction	0.9		-0.17
wholesale trade	0.8		-0.34
transport and communication ^a	0.127		-0.30
banking and insurance	0.7		-0.50
other commercial services	0.7		-0.50
oil industry	1	-0.308	

a For this industry, the export equation is modelled slightly different from (3.1).

Exports follow world trade (first term of (3.1)). Elasticities α are calibrated in such a way that the weighted average over all exported goods is 1. The second term of (3.1) reshuffles exports between manufacturing industries dependent on price developments on the world market. Furthermore, exports depend on the price differential between domestic and foreign suppliers (third term). Finally, there is a constant term (δ) that reshuffles exports between size-classes.

3.1.2 Size-class specifics

The deviations per size-class with regard to the parameter values in Table 3.1G are listed in Table 3.1S (S denotes size-class specifics). The reshuffling mechanism between manufacturing industries is not modelled in a size-class specific manner, so parameter β is left out of the table.

World trade elasticity a

For the manufacturing industries there are two opposite effects of world trade on exports. First, small businesses' exports are more sensitive to the business cycle than the exports of large businesses, implying a bigger impact for small businesses. The second effect originates from the fact that the vast share of exports by small and medium-sized businesses goes to Belgium and Germany. Because import growth of these two countries is generally somewhat smaller than the growth of world trade (CPB, 2000), we assume a smaller impact for small and medium-sized businesses. These effects appear to cancel each other out so that no size-class dependency is modelled in effect.¹

For the transport and communication industry, the impact of world trade is largest for large businesses, because the vast share of large businesses' exports in this industry relates to navigation and aviation, whereas small businesses' exports relate to inland navigation and transport over way.

Price elasticity g

To compensate for scale disadvantages small businesses' export prices are relatively high. As we assume a positive relationship between the level of prices and the price

¹ The separate size-class patterns are 1.2; 1.1; 1 (business cycle effect) and 0.9; 0.95; 1.05 (effect of lower imports Belgium and Germany compared to world trade).

elasticity, it follows that the price elasticity of exports for small businesses is relatively high (see table 3.1S).

table 3.1S Size-class multipliers of equation (3.1): exports^a

industry	size-class	a	g	d (%) ^b
1. food industry; chemical industry; metallurgical industry; other manufacturing; oil industry	small	no size-class de- pendency	1.3	1
	medium-sized		1.1	0.5
	large		0.9	-0.1
2. construction; wholesale trade	small	no size-class de- pendency	same as group 1	no size-class de- pendency
	medium-sized			
	large			
3. transport and communication	small	0.7	same as group 1	no size-class de- pendency
	medium-sized	0.7		
	large	1.2		
4. banking and insurance	small ^c	no size-class de- pendency	same as group 1	<i>no exports</i>
	medium-sized			1
	large			0
5. other commercial services	small	no size-class de- pendency	same as group 1	-0.5
	medium-sized			1
	large			-0.5

a Parameter β in (3.1) is not size-class specific.

b Average over size-classes of parameter δ is 0.

c There are no exports for the size-class small of this industry.

Trend term d

Finally, there is a trend term that reshuffles exports between size-classes, based on annual changes in the numbers of exporting businesses. For example, for manufacturing industries, almost all large businesses export, while only a small part of the small and medium-sized businesses export. However, the numbers of small and medium-sized businesses that export are increasing, and hence, the share of total exports accounted for by small and medium-sized businesses is increasing relative to the share of exports accounted for by large businesses.¹

3.2 Imports-sales ratio consumption goods/services

3.2.1 *General*

Not imports themselves are modelled but the ratio between imports and sales of domestic origin. The imports-sales ratios with respect to investment goods and intermediate goods/services are not modelled in a size-class specific manner. They will not be described here.

¹ Because parameter δ represents a reshuffling mechanism between size-classes, the industry-averages of δ are 0 instead of 1. This is also the reason why δ is not in Table 1G.

For each industry producing consumption goods/services we have:

$$\dot{m}_c = a \dot{dc} + b (\dot{p}_{mc} - \dot{p}_{sc})_{-1} + g \quad (3.2)$$

where:

\dot{m}_c imports-sales ratio with respect to consumption goods/services
 \dot{dc}_i demand for consumption goods/services produced by the i th industry
 \dot{p}_{mc} price of imported consumption goods/services
 \dot{p}_{sc} price of sales of domestically produced consumption goods/services
 Values of the parameters α , β , and γ are presented in Table 3.2G.

table 3.2G Parameter values of equation (3.2): imports-sales ratio for consumption goods/services

industries	a	b	g
food industry	1.1	0.001 ^a	
chemical industry	1.36	-0.5	0.021
metallurgic industry	0.8	-1.32	0.031
other manufacturing	1.32	-1.3	
oil industry	0.25	-0.3	

a As we calibrate our model on the CPB model Athena, we use this figure. It seems more reasonable, however, to round this figure to precisely zero.

Import penetration increases when markets expand. Furthermore, import penetration depends on the price differential between domestic and foreign suppliers. Finally, for the chemical and metallurgic industries import penetration increases autonomously as well.

3.2.2 Size-class specifics

The size-class dependency is reflected in Table 3.2S. The effect of consumption demand (parameter α) and the constant γ are not size-class specific and hence, are not in the table.

table 3.2S Size-class multipliers equation (3.2): imports-sales ratio for consumption goods/services^a

industry	size-class	b
all industries from table 3.2G	small	1.3
	medium-sized	1.1
	large	0.9

a Parameters α and γ in (3.2) are not size-class specific.

The impact of the price differential on domestic markets between domestic and foreign suppliers is biggest for small businesses. Small businesses' prices are usually higher than large businesses' prices. Also, price elasticity is assumed to be bigger when prices are higher. Consequently, a change in prices has a bigger impact on import penetration for small businesses than for large businesses.

3.3 Long-run employment

3.3.1 General

Employment is determined in two stages. First, for each industry the long-run equilibrium level is determined. Subsequently, actual employment is determined in such a way that it is guaranteed that in the long run actual employment indeed follows the long-run equilibrium level. Since actual employment is not modelled in a size-class specific manner, this variable is not described here.

For most industries the long-run equilibrium level of employment is determined by:

$$\overset{\circ}{lemp}^* = \mathbf{a}_1 \overset{\circ}{ygf} + \mathbf{a}_2 \overset{\circ}{ygf}_{-1} + \mathbf{b} (\overset{\circ}{p}_{wc} - \overset{\circ}{p}_{ygf}) + \mathbf{g} \overset{\circ}{clt} + \mathbf{d} \quad (3.3)$$

where:

$\overset{\circ}{lemp}$	labour volume, employees
$\overset{\circ}{ygf}$	value added, gross/factor costs
$\overset{\circ}{p}_{wc}$	rate of wage costs
$\overset{\circ}{p}_{ygf}$	price of value added, gross/factor costs
$\overset{\circ}{clt}$	contractual labour time

Values of the parameters $\alpha_1, \alpha_2, \beta, \gamma$, and δ are presented in Table 3G.

table 3.3G Parameter values equation (3.3): long-run employment

industry	α_1	α_2	β	γ	δ (%)
food industry	0.986		-0.394		-2.396
chemical industry	0.702		-0.218		-2.466
metallurgical industry ^a	0.822		-0.229	-0.494	-3.151
other manufacturing	0.962		-0.510		-1.866
construction	0.987		-0.361	-0.667	-1.079
wholesale trade	0.330	0.566	-0.595		-1.040
retail trade	0.330	0.566	-0.595		-1.040
hotels, restaurants	0.736		-0.466		-0.136
car trade and repair	0.330	0.566	-0.595		-1.040
transport and communication	0.660		-0.410	-0.471	-0.693

a For the metallurgical industry, parameter α relates to the growth of production capacity instead of actual production growth.

Thus, the equilibrium value of employment follows value added (first and second term), be it with an elasticity below 1. Furthermore, real wage costs appear influential: if labour becomes more costly businesses substitute away from labour (third term). The contractual labour time is influential as well (fourth term). Finally, autonomous technological development is modelled by adding a constant (fifth term).

3.3.2 Size-class specifics

The size-class dependency is reflected in Table 3S. The impact of contractual labour time is not modelled differently per size-class, so parameter γ is not in the table.

table 3.3S Size-class multipliers equation (3.3): long-run employment^a

industry	size-class	a ₁ and a ₂	b	d
1. food industry chemical industry; metallurgical industry; other manufacturing	small	1.25	1.5	0.8
	medium-sized	1	1.2	1
	large	1	0.8	1.1
2. construction; transport and communication	small	same as group 1	1.2	same as group 1
	medium-sized		1	
	large		0.8	
3. wholesale trade	small	no size-class dependency	same as group 2	same as group 1
	medium-sized			
	large			
4. retail trade; hotels, restaurants; car trade and repair	small	no size-class dependency	1.1	same as group 1
	medium-sized		1	
	large		0.9	

a Parameter γ in (3.3) is not size-class specific.

Output elasticity a

In the manufacturing industries, the construction industry, and the transport and communication industry, the effect of value added is biggest for small businesses. This reflects the presence of economies of scale. In these industries, labour productivity is generally lower in small businesses than in large businesses. The bigger value of α for small businesses reflects the fact that also *annual changes* in labour productivity are generally lower for small businesses (due to diseconomies of scale).

Wage elasticity b

The impact of real labour costs is biggest for small businesses. This is due to the labour-intensive way the production process is organized in small businesses compared to large businesses. In other words, because for small businesses wage costs form a relatively large part of total costs, small businesses are more sensitive to changes in the rate of wage costs.

These differences are particularly substantial in the manufacturing industries. Hence differences in wage elasticity are highest in these industries.

Trend term d

The absolute value of the constant δ is biggest for large businesses. This reflects the higher level of labour saving technological progress in large businesses.

3.4 Net growth of capital

3.4.1 *General*

Net capital growth is not modelled uniformly for all industries. In fact, we distinguish five groups of industries for which net capital growth is modelled differently. Two groups are not modelled in a size-class specific manner, so these groups are not described here.

The first group of industries comprises four PRISMA industries, viz. the metallurgic industry, other manufacturing, transport and communication, and the oil industry. For these industries we have the following equation:

$$\begin{aligned} \dot{k} = & l \dot{k}_{-1} + a ygf_{-1} + bq_{-1} + g_1(r_{cap} - \underline{r})_{-1} + g_2(r_{cap} - \underline{r})_{-2} + \\ & + d_1 \dot{clt}_{-1} + d_2 \dot{clt}_{-2} + e \end{aligned} \quad (3.4a)$$

where:

k	stock of capital
ygf	value added, gross/factor costs
q	utilisation rate
r_{cap}	return on capital
r	interest rate
clt	contractual labour time

Values of the parameters λ , α , β , γ , δ , and ϵ are presented in Table 3.4G.

table 3.4G Parameter values of (3.4a) – (3.4c): net growth of capital

industry	l	a	b	g_1	g_2	d_1	d_2	e (%)
oil industry	0.521	0.124	0.115		0.121			-7.560
metallurgical industry	0.569	0.201	0.172	0.123	-0.058	-0.042	-0.042	-12.950
other manufacturing	0.444	0.167	0.187	0.266	-0.146	-0.144	-0.144	-14.860
transport and communication	0.526	0.095	0.127	0.213	0.125			-8.381
food industry	0.470	0.135	0.233	0.034		-0.176	-0.176	-16.420
chemical industry	0.542	0.090	0.236		0.060			-17.090
construction	0.855	$1-\lambda$	0.099	0.080	-0.055	-0.012	-0.012	0.082
wholesale trade	0.852	$1-\lambda$	0.188	0.052	-0.052	-0.050		0.528
retail trade	0.852	$1-\lambda$	0.188	0.052	-0.052	-0.050		0.528
hotels, restaurants	0.631	$1-\lambda$	0.230	0.210	-0.029			1.025
car trade and repair	0.852	$1-\lambda$	0.188	0.052	-0.052	-0.050		0.528
banking and insurance	0.631	$1-\lambda$	0.230	0.210	-0.029			1.025
other commercial services	0.631	$1-\lambda$	0.230	0.210	-0.029			1.025

There are two mechanisms causing capital to follow value added in the long run. The first two terms in (3.4a) combined comprise the first mechanism. However, capital does not follow value added completely for the industries under consideration because $\alpha+\lambda < 1$. Furthermore, the dependence on the utilisation rate (third term) also implies that capital follows value added in the long run. Capital growth is also dependent on the return on capital (fourth and fifth term). Here, the (exogenous) interest rate serves as a benchmark. The contractual labour time is influential as well (sixth and seventh term). Finally, autonomous technological development is modelled by adding a constant (eighth term).

The second group of industries comprises the food and chemical industry. For these two PRISMA industries capital growth is modelled in nearly the same way as for the first group. The only difference is that gross capital growth is modelled instead of net growth. Thus, we have:

$$\dot{k} + \mathbf{d}_{cap} = \mathbf{I} (\dot{k} + \mathbf{d}_{cap})_{-1} + \text{same as 3.4a} \quad (3.4b)$$

where:

\mathbf{d}_{cap} depreciation rate of capital

The third group of industries comprises seven PRISMA industries with a size-class dimension, viz. construction, wholesale trade, retail trade, hotels and restaurants, car trade and repair, banking and insurance, and other commercial services. For these industries capital growth reads:

$$\dot{k} = \mathbf{I} \dot{k}_{-1} + \mathbf{a} \mathbf{ystr}_{-1} + \mathbf{b} \mathbf{q}_{-1} + \text{same as 3.4a} \quad (3.4c)$$

where:

\mathbf{ystr} structural production

Capital growth is modelled slightly different for these industries because the level of the utilisation rate is not defined for these industries. Only the *change* in the utilisation rate is defined. Note that because for all these industries $\alpha + \lambda = 1$, capital follows exactly structural production in the long run.

3.4.2 Size-class specifics

The size-class dependency is reflected in Table 3.4S. Only the effect of the return on capital is modelled in a size-class specific manner so there is only one column with size-class multipliers in the table.

table 3.4S Size-class multipliers equations (3.4a)-(3.4c): net growth of capital^a

industry	size-class	g_1 and g_2
all 13 industries with a size-class dimension	small	1.2
	medium-sized	1
	large	0.8

^a Parameters λ , α , β , δ_1 , δ_2 , and ϵ in (3.4) are not size-class specific.

The effect of capital return on investments is bigger for small businesses. This is because small businesses are usually more dependent on high returns to obtain the required loans from banks.

3.5 Prices of domestically produced goods/services

3.5.1 General

The price setting behaviour of domestic businesses is dependent on the sales category of the goods/services sold. Thus, an industry can set as many as five different prices, viz. prices for private and public consumption goods/services, investments, exports, and intermediate goods/services.

Prices of public goods/services are not modelled in a size-class specific manner. Therefore, they are not described here.

Price setting equations for the other four sales categories all have the same structure. For example, prices for private consumption goods/services are governed by:

$$\overset{\circ}{p}_{sc} = \mathbf{a} \overset{\circ}{uc}_{-1} + (1 - \mathbf{a}) \overset{\circ}{pcc}_{-1} + \mathbf{b} \overset{\circ}{q} \quad (3.5a)$$

where:

$\overset{\circ}{p}_{sc}$	price of private consumption goods/services
$\overset{\circ}{uc}$	unit costs
$\overset{\circ}{pcc}$	price of goods/services that compete with domestically produced consumption goods/services
$\overset{\circ}{q}$	utilisation rate

Thus, price development is a weighted average of developments in unit costs and the price development of the goods/services of competitors. The weighting parameter α differs between industries and sales categories. A typical value is 0.75.

Furthermore, the utilisation rate plays a role. The utilisation rate is an indicator for market demand. If it increases, prices increase as well. The utilisation rate elasticity of the price differs also between industries and sales categories. A typical value of β is 0.25.

The price setting equations for investment goods, exported goods/services, and intermediate goods/services are, in respective order:

$$\overset{\circ}{p}_{si} = \mathbf{a} \overset{\circ}{uc}_{-1} + (1 - \mathbf{a}) \overset{\circ}{pci}_{-1} + \mathbf{b} \overset{\circ}{q} \quad (3.5b)$$

$$\overset{\circ}{p}_{sx} = \mathbf{a} \overset{\circ}{uc}_{-1} + (1 - \mathbf{a}) \overset{\circ}{pcx}_{-1} + \mathbf{b} \overset{\circ}{q} \quad (3.5c)$$

$$\overset{\circ}{p}_{sm} = \mathbf{a} \overset{\circ}{uc}_{-1} + (1 - \mathbf{a}) \overset{\circ}{pcm}_{-1} + \mathbf{b} \overset{\circ}{q} \quad (3.5d)$$

3.5.2 Size-class specifics

The size-class dependency is modelled uniformly across the four sales categories consumption goods/services, investment goods, exported goods/services, and intermediate goods/services. See table 3.5S.

table 3.5S Size-class multipliers equations (3.5a)-(3.5d): prices of domestically produced products

industry	size-class	a	b
all 13 industries with a size-class dimension	small	0.9	0.9
	medium-sized	0.9	0.9
	large	1.1	1.1

In general, small and medium-sized businesses operate with relatively high sales prices, to compensate for scale disadvantages with respect to their larger counterparts. Therefore, they have fewer possibilities than large businesses to increase their prices even further in case of higher unit costs or higher utilisation rates (parameters α and β , respectively).

3.6 Prices of competitors

3.6.1 General

The prices of competitors that affect the price setting behaviour of businesses (equation (3.5a) until (3.5d)), differ between industries and sales categories, just as the price setting behaviour itself.

3.6.2 Size-class specifics

For some industries and some sales categories there is a size-class dependency. This concerns the competitors' prices for small and medium-sized businesses. Three types of size-class specific competitors' prices are identified. The industries concerned with the type of competitors' price per sales category are listed in Table 3.6S.

table 3.6S Type of competitors' price per industry and sales category ^a

industry	consumption goods/services	investment goods	intermediate goods/services
food industry			type III
other manufacturing	type I	type I	type I
construction		type II	
wholesale trade	type II	type I	type II
retail trade	type I		
other commercial services			type I

^a For empty cells there is no size-class dependency

Type I: influence large businesses

For some industries and sales categories, small and medium-sized businesses primarily compete with their larger counterparts. Therefore the competitors' price is set equal to the sales price of large businesses. For the category consumption goods, this reads:

$$\overset{\circ}{p}_{CC, small, medium} = \overset{\circ}{p}_{sc, large} \quad (3.6a)$$

Type II: influence large businesses and average consumption price

For some (sheltered) industries and sales categories, the size class dependency is somewhat less strong than type I. Here, the competitors' price for small and medium-sized businesses is equal to the average of the sales price of large businesses on the one hand, and the average consumption price (which is the default competitors' price for sheltered industries) on the other hand. For the category consumption goods, this reads:

$$\overset{\circ}{p}_{CC, small, medium} = 0.5 * \left(\overset{\circ}{p}_{sc, large} + \overset{\circ}{p}_c \right) \quad (3.6b)$$

Type III: influence large businesses and import price

For the food industry, the competitors' price for the size-classes small and medium-sized is modelled analogously to (3.6b), albeit that the average consumption price in the formula is replaced by the import price (see (3.6c) for the category intermediate

goods/services). This is because the food industry is an exposed industry instead of a sheltered one.

$$\dot{pcm}_{small,medium} = 0.5 * \left(\dot{p}_{sc,large} + \dot{p}_{mmc} \right) \quad (3.6c)$$

4 Simulation analysis

In this chapter we discuss the results from some simulation analyses. In section 4.1 we investigate to what extent and why results for SMEs and large enterprises differ. In section 4.2 we investigate to which extent the explicit modelling of a size-class structure affects industry and macro results, or in other words: to which extent the bottom-up mode produces different results from the top-down mode.

4.1 Differences between SMEs and large businesses

In this section we analyse the differences in impacts between SMEs and large businesses of a 1% increase in world trade.

More specifically, in this analysis the base run relates to the CPB scenario European Coordination. In the simulation run world trade is increased by 1% in 2000. Reported differences with respect to the base run are evaluated after a period of eight years (i.e. in 2007) and expressed as percentages of the base run level (2007). In both the base run and the simulation run we use the top-down mode.

We distinguish two subsections. First, we focus on size-class differences at the macro level for a number of key variables. Second, for the variables gross value added and labour volume of employees we provide a more detailed picture, which includes results by size-class *and* industry.

4.1.1 Macro key variables

In Table 4.1 we see that there are six variables at the macro level for which the cumulative effect in the eighth year differs relatively much between size-classes. These are sales of consumption goods/services, sales of exports, price of sales, real wage rate, labour volume of self-employed and gross investments. These will be discussed in turn.

sales of consumption goods/services

At the macro level, sales of consumption goods and services increase more for small businesses than for large businesses. This is mainly an *industry* effect. Consumption sales develop differently between industries. Because consumption sales of small businesses are distributed differently across industries compared to those of large businesses, the difference in development at the macro level arises. In particular, small businesses' consumption sales are concentrated in the industry "other commercial services" and - to a lesser extent - in retail trade and hotels & restaurants, all experiencing relatively large increases in consumption sales in this simulation. Large businesses' consumption sales are concentrated in the food industry, transport & communication, retail trade, and banking & insurance. Of these four industries, the increase in consumption sales is particularly small for the former two industries.

Two main mechanisms account for the differences across industries. First, the increase in world trade leads to higher exports and hence higher incomes and higher consumption. However, not every consumption category increases to the same extent. For example, due to low income elasticities the increase is relatively small for the various food categories and for transport & communication services, explaining the relatively small effect on consumption sales for the food industry and the transport & communication industry, respectively. On the other hand, consumption sales for the other commercial services are dominated by the consumption categories other services and recreation &

culture. Both categories have high-income elasticities, and hence the increase is relatively large for other commercial services.

The second mechanism is that of increasing import penetration in manufacturing industries when consumption demand increases (see the values for elasticity α in Table 3.2G). This has a negative impact on the sales of consumption goods/services of domestic origin for the manufacturing industries.

table 4.1 Impact of a 1% increase in world trade on the private enterprise sector by size-class, selected variables *

	small	medium-sized	large	total
volume of sales				
- consumption goods/services	0.45	0.34	0.24	0.34
- investment goods	0.35	0.39	0.37	0.37
- intermediate goods/services	0.37	0.37	0.36	0.36
- exports	0.54	0.64	0.69	0.67
- total sales	0.40	0.42	0.46	0.44
gross value added at factor costs	0.41	0.41	0.41	0.41
price of sales	0.20	0.16	0.13	0.15
labour costs per employee	0.29	0.28	0.29	0.29
real wage rate	0.27	0.32	0.34	0.31
employment				
- employees	0.22	0.22	0.22	0.22
- self-employed	0.19	0.19	0.09	0.19
- total employment	0.21	0.22	0.22	0.21
gross investments	0.72	0.51	0.53	0.58

* Cumulative effect after eight years, in % of baseline.

sales of exports

At the macro level, exports of small businesses increase less in eight years than exports of large enterprises. Two effects account for this: an *industry* effect and a *behavioural* effect. Both effects work in the same direction but the industry effect is stronger than the behavioural effect.

The industry effect is due to different export developments across industries. As we can see from Table 3.1G, the direct effect of the increase in world trade is biggest for the manufacturing industries (elasticity α in equation (3.1)). For small businesses, the exports of the manufacturing industries comprise a smaller share of total exports than for large businesses. Therefore, the effect on exports at the macro level is smaller for small businesses than for large businesses.

The behavioural effect involves the negative impact of the increased export prices on the volume of exports (see the third explaining variable in equation (3.1)). First, export price increases are higher for small businesses than for large businesses (this is explained below). Second, the price elasticity for small businesses is higher than for large businesses (see table 3.1S). As a result, exports of small businesses reduce more by the increased export prices than exports of large businesses.

price of sales

At the macro level we see that sales prices (all sales categories) of small businesses increase more than sales prices of large businesses. This is because, in general, exports comprise a smaller share of total sales for small businesses than for large businesses. Therefore, production of small businesses increases less and as a consequence unit costs increase more because of scale disadvantages. As changes in unit costs are passed on to prices, sales prices of small businesses increase more than large businesses' prices. This holds in spite of the fact that unit costs are passed on to prices to a smaller extent for small businesses (see Table 3.5S). So, in terms of the types of effects defined in chapter 2, a behavioural effect is dominated by a structure effect.

real wage rate

The real wage rate is defined as the nominal wage rate deflated by the price of value added (factor costs). As the nominal wage rate is not size-class specific (and not even industry specific), size-class differences in the real wage rate are entirely accounted for by the price of value added. Apart from some deviating developments in the price of intermediate deliveries, the price of value added follows the sales prices (see above). Hence, the size-class pattern is exactly the opposite of the pattern for the price of sales: small businesses have the lowest increase in the real wage rate, while large businesses have the highest increase.

labour volume of self-employed

At the macro level, the increase of self-employed persons is larger in the small business sector than in the large business sector. This is mainly an industry effect. In the industry "other commercial services" the increase in the number of self-employed is relatively small in the large business sector, while only in this industry we find self-employed people in large businesses.

gross investments

Besides replacements of depreciated capital, there are three important determinants of investments: value added, the utilisation rate and the return on capital (see equation 3.4a). In the manufacturing industries and construction, investments increase less for small businesses, because of the lower increases in value added (see Table 4.2 in the next section), in the utilisation rate and in the return on capital.¹ Furthermore, depreciation is smaller for small businesses in manufacturing industries, contributing to lower gross investments for small businesses in these industries.

For the service industries, the picture is the other way round: investments of small businesses increase more than those of large businesses, though the differences are smaller than in manufacturing. The increase in investments is especially high in the industry "other commercial services". This is caused in particular by the persistent positive impact of the return on capital. Furthermore, the high return on capital is passed on to investments in a stronger way than for most industries (see parameter γ in Table 3.4G). Finally, the effect is even stronger for small businesses relative to large businesses because of the size-class specific way in which the effect of the return on capital is modelled (see Table 3.4S).

¹ This is at least the case in the first simulation years. The latter two variables are in equation (3.4a) in levels instead of relative changes so that the early impact has a relatively large effect on cumulative growth of investments.

How do these different trends count up at the macro level? Because small businesses' investments are dominated by other commercial services (35% of total investments in the base year) and large businesses' investments by industries with lower investments, viz. transport & communication and banking/insurance (together 50% of total investments), we find higher investment increases for small businesses than for large businesses at the macro level. So, the industry effect is dominant.

Based on the above analysis of six macro key variables, we conclude that differences between SMEs and large businesses at the macro level are mainly caused by industry effects.

4.1.2 *Detailed results gross value added and labour volume of employees*

In this section we provide results by size-class and industry for the variables gross value added factor costs and labour volume of employees.

gross value added at factor costs

Because demand for intermediate deliveries follows production, and because production is mainly determined by total sales (apart from stock building), value added developments follow total sales. As we see in table 4.1, at the macro level the effect on sales of consumption goods is bigger for small businesses while the effect on exports is bigger for large businesses. There are no size-class differences for value added, as these differences cancel each other out.

However, at the industry level, size-class differences do emerge. See table 4.2. There are two types of effects possible: a *structure* effect and a *behavioural* effect.

To give an example of the former type of effect, in the chemical industry value added of small businesses increases less compared to large businesses. The reason is the composition of value added. For small businesses the share of exports in value added is smaller and it is this sales category that increases most.

For some other industries, the size-class difference for value added is entirely a behavioural effect. For example, value added of small businesses increases more in the industries "car trade and repair" and "other commercial services" compared to large businesses. In the former case this is due to higher sales of investment goods and in the latter case this is due to higher sales of consumption goods and services.

labour volume of employees

From Table 4.2 it appears that within industries, size-class differences with respect to the development of the labour volume follow those with respect to value added. This is not surprising as value added is an important determinant of employment. The magnitude of the size-class differences may differ somewhat from those of value added. This can be explained from the fact that there are two (opposite) behavioural effects that influence the labour volume. First, the increase in value added is passed on to the labour volume more strongly for small businesses: see parameter α in Table 3.3S. This effect relatively favours the labour volume of small businesses. Second, the increase in the real wage rate has a bigger negative effect for small businesses: see parameter β in Table 3.3S. This effect relatively favours the labour volume of large businesses. Furthermore, there may also be size-class differences concerning the real wage rate itself (see section 4.1.1).

table 4.2 Impact of a 1% increase in world trade on production and employment, by industry and size-class *

	small	medium-sized	large	total
gross value added at factor costs				
food industry	0.28	0.40	0.44	0.41
chemical industry	0.55	0.75	0.89	0.86
metallurgical industry	0.23	0.36	0.41	0.38
other manufacturing	0.26	0.38	0.45	0.40
construction	0.34	0.37	0.37	0.36
wholesale trade	0.44	0.46	0.44	0.45
retail trade	0.37	0.37	0.37	0.37
hotels, restaurants	0.36	0.36	0.37	0.36
car trade and repair	0.54	0.55	0.43	0.53
transport and communication	0.35	0.35	0.29	0.31
banking and insurance	0.40	0.39	0.32	0.35
other commercial services	0.45	0.39	0.35	0.40
oil industry	-	0.59	0.78	0.78
labour volume, employees				
food industry	0.21	0.17	0.34	0.28
chemical industry	0.31	0.36	0.52	0.47
metallurgical industry	0.26	0.35	0.61	0.49
other manufacturing	0.13	0.15	0.32	0.22
construction	0.32	0.25	0.29	0.28
wholesale trade	0.29	0.30	0.25	0.29
retail trade	0.07	0.12	0.16	0.13
hotels, restaurants	0.11	0.12	0.16	0.13
car trade and repair	0.30	0.31	0.16	0.28
transport and communication	0.11	0.07	0.09	0.09
banking and insurance	0.16	0.16	0.14	0.14
other commercial services	0.23	0.17	0.14	0.17
oil industry	-	0.00	0.00	0.00

* Cumulative effect in eighth year, in % of baseline

4.2 Differences between bottom-up and top-down mode

The explicit modelling of a size-class structure might and shall have influence on industry and macro results. See section 2.3 for the theoretical reasons for this phenomenon. However, the extent to which the size-class structure actually has influence depends on the extent in which the structure and behaviour of SMEs differ from large businesses in reality. We have investigated this by comparing results of the size-class module in the bottom-up mode – in which explicit size-class effects are possible – to results of the module in the top-down mode – in which the size-class structure cannot have an independent effect on industry and macro results.

More specifically, we have compared top-down and bottom-up results for:

- the cumulative effects after 25 years in the base-run;

- the cumulative effects after 8 years for the following seven standard variants that were also analysed when testing the kernel of PRISMA: (i) an increase in world trade by 1%, (ii) an increase in competitive export prices by 1%, (iii) an increase in the interest rate by 1% point, (iv) an increase in the wage rate by 1%, (v) an increase in labour supply by 20000 labour years, (vi) a reduction in direct taxes by 1% of GDP, and (vii) a reduction in social security contributions of employers by 1% of GDP.

All simulations were compared with respect to the key macro data and the developments of gross value added and employment at the industry level.

We found that differences between bottom-up results and top-down results were very moderate indeed. For almost all variables bottom-up results did not differ more than 15% from the original top-down result. Whenever the results differed more than 15%, invariably the original top-down effect itself was very small. Hence, the absolute change because of the bottom-up mode was still small in those cases.

We conclude that – given the structure and assumed behaviour of Dutch SMEs – independent size-class effects are small.

List of literature

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- Peeters, 2002, *Het midden- en kleinbedrijf in de jaren 2003-2006*, A200201, EIM, Zoetermeer.
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Annex I Notational conventions

The following notational conventions are adhered to:

- small characters in italics (up to 5) denote real variables
- the same characters in capitals denote the nominal equivalent
- Greek characters denote ratios of variables, or parameters
- subscripts i, j indicate industries. In many cases these subscripts are suppressed if the intention is clear from the context
- subscripts k, l indicate categories of goods/services
- a subscript of -1 (-2) denotes a one (two) year lagged variable
- a subscript of $-1/2$ ($-3/2$) denotes the average of a non- (1 year) lagged and 1 (2) year(s) lagged variable
- p_x denotes the price of x
- $\Delta x \equiv x - x_{-1}$ that is: absolute change
- $\overset{\circ}{x} \equiv \Delta \ln x = \ln(x/x_{-1}) \approx \Delta x / x_{-1}$ that is: relative change
- underscores denote exogenous variables
- x^* denotes the long-run equilibrium value of x

Annex II List of variables

In this annex a complete list of the variables of PRISMA is given. As a consequence, some variables are not present elsewhere in this report.

The first column gives the symbol with which the variable is denoted. The second column gives a description of the variable. The third column gives for some variables the relation of the variable to others. The fourth column gives the internal code of the variable and is for internal use only.

About the sequence of ordering. First, variables denoted by normal characters are listed in alphabetic order. Subsequently, variables denoted by Greek characters are listed, again in alphabetic order. Finally, some remaining variables – not denoted by symbols at all – are listed in an arbitrary order.

Symbol	Description	Relation to other variables	Code
<i>binc</i>	balance of macro income account, = disposable income		YDX
	balance of other income entries in macro accounts		SYO
<i>bsp</i>	balance of macro spending account		SYX
<i>c</i>	macro consumption	$dc + tvac + mcnc$	CGX
c_k	consumption expenditures on category <i>k</i>		CGK
<i>cap</i>	production capacity		CAP
<i>clt</i>	contractual labour time		ATC
<i>dc</i>	demand for consumption goods/services	$sc + mc$	CDE
<i>deb</i>	debt of government at end of the year		DEB
<i>del</i>	deliveries of businesses to government in macro accounts		LEV
<i>dep</i>	depreciation of capital stock		DBX
<i>di</i>	demand for investment goods	$si + mi$	IDE
<i>dm</i>	demand for intermediate goods/services	$sm + mm$	INT
<i>dstat</i>	statistical difference in balance of payments		STT
<i>g</i>	government consumption	$sg + mg + tvag = gadm + gki + gstr$	GXX

<i>gadm</i>	part of government consumption: administration costs of social security funds		GAD
<i>gki</i>	part of government consumption: payments of social security funds in kind		GKI
<i>gstr</i>	part of government consumption: government consumption in strict sense		GST
<i>h</i>	labour productivity	ygf / l	HAT
<i>i</i>	gross investments	$di + tvai$	IBB
i_k	investments of <i>k</i> th category		IBK
<i>k</i>	stock of capital		KBZ
<i>l</i>	labour volume, total	$lemp + lse$	ALZ
<i>lemp</i>	labour volume, employees		ALX
<i>lse</i>	labour volume, self-employed		AZX
<i>lsup</i>	supply of labour		ASX
<i>m</i>	expenditures on intermediate goods/services	$dm + tvam + mmnc$	VBR
<i>m</i>	imports	$mc + mcnc + mg + mi + mm + mn + mx$	
<i>mc</i>	imports of consumption goods/services		MCX
<i>mcnc</i>	imports of consumption goods/services that do not compete with domestically produced goods/services		MCN
<i>mg</i>	imports of public goods/services at the macro level		MCO
<i>mi</i>	imports of investment goods		MIX
<i>mm</i>	imports of intermediate goods/services, total	$mmc + mmnc$	MGH
<i>mmc</i>	imports of intermediate goods/services that compete with domestically produced goods/services		MGC
<i>mmnc</i>	imports of intermediate goods/services that do not compete with domestically produced goods/services		MGN
<i>mn</i>	imports resulting in stock building		MNX
<i>mx</i>	imports that are re-exported, = re-exports		BWE
<i>n</i>	stock building	Δstk	NXX
p_{wcon}	real wage rate		LBC
p_{wdr}	rent		LBI
<i>pcc</i>	price of goods/services that compete with domestically produced consumption goods/services		CCG

<i>pci</i>	price of goods/services that compete with domestically produced investment goods		CIB
<i>pcx</i>	price of goods/services that compete with domestically produced exported goods/services	$= p_{twld}$	
<i>pcm</i>	price of goods/services that compete with domestically produced intermediate goods/services		CGO
<i>q</i>	production	$s + n = ygm + m$	BPR
<i>r</i>	interest rate		RXX
<i>r_{av}</i>	average interest rate paid by government		RMX
<i>r_{real}</i>	real interest rate		RRX
<i>RGOV</i>	interest paid by government		ROV
<i>rpp</i>	real purchasing power of employees		LBDH
<i>s</i>	total sales of domestic origin		AFZ
<i>sc</i>	sales of consumption goods/services of domestic origin		CGT
<i>sfin</i>	final sales of domestic origin	$sc + sg + si + sx$	FAF
<i>sg</i>	sales of public goods/services of domestic origin		COT
<i>si</i>	sales of investment goods of domestic origin		IBH
<i>sm</i>	sales of intermediate goods/services by supplying and by receiving industry		GOL
<i>ssb</i>	social security benefits	$ssbu + ssbo = ssbgv + ssbfd$	YUB
<i>ssbfd</i>	social security benefits paid by social security funds	$(1-c) * ssb$	UIS
<i>ssbgv</i>	social security benefits paid by government	$c * ssb$	UIR
<i>ssbo</i>	social security benefits, other		YUO
<i>ssbu</i>	social security benefits, for unemployed		YUU
<i>sscbs</i>	social security contributions paid by businesses due to gross wages	$s_{bus} * W$	SOC
<i>sscgv</i>	social security contributions paid by government		PRO
<i>sscem</i>	social security contributions paid by employees due to gross wages	$s_{hs} * W$	PRG
<i>sscpr</i>	social security contributions paid by private sector		PRS
<i>stk</i>	stocks	$s s$	SXX
<i>sub</i>	subsidies		SUX
<i>sx</i>	sales of exported goods/services of domestic origin	$sxgds + sxen + sxsrv$	BTX
<i>sxgds</i>	sales of exported goods excluding energy		BTG

<i>sxen</i>	sales of exported energy		BTE
<i>sxsrv</i>	sales of exported services		BTS
<i>t</i>	taxes, total	$t_{dir} + t_{cor} + t_{ind} + t_{va}$	TAX
<i>tc</i>	total costs	$m + (t_{ind} - sub) + dep + wc + wcse$	TCX
<i>tcor</i>	corporate taxes		VBX
<i>tdir</i>	direct taxes paid by private households		DBG
<i>tinc</i>	income taxes on gross wages	$t_{inc} * W$	LBX
<i>tind</i>	indirect taxes excluding VAT		BIX
<i>tva</i>	VAT, total	$t_{vac} + t_{vag} + t_{vai} + t_{vam}$	BVX
<i>tvac</i>	VAT on consumption goods/services		VAC
<i>tvag</i>	VAT on public goods/services at the macro level		BVO
<i>tvai</i>	VAT on investment goods		VAI
<i>tvam</i>	VAT on intermediate goods/services		BVV
<i>twld</i>	world trade		CWH
<i>u</i>	unemployment	$l_{sup} - l$	WKL
<i>uc</i>	unit costs	TC / q	TCXQ
<i>W</i>	gross wages	$l_{emp} * p_w$	LBB
<i>WC</i>	wage costs		LBT
<i>WCSE</i>	imputed wage costs of self-employed	$l_{se} * p_{wc}$	TLZ
<i>x</i>	exports	$sx + mx$	
<i>ydssb</i>	disposable income due to social security benefits		YUD
<i>ydw</i>	disposable income due to wages		LBD
<i>ydz</i>	disposable income due to non-wage income		WMD
<i>ygf</i>	value added, gross/factor costs	$y_{nf} + dep$	YBF
<i>ygm</i>	value added, gross/market prices	$y_{gf} + (t_{ind} - sub)$	YBM
<i>y_{nf}</i>	value added, net/factor costs	$wc + z$	YNF
<i>y_{nm}</i>	value added, net/market prices	$y_{nf} + (t_{ind} - sub)$	YNM
<i>ystr</i>	structural production		YST
<i>z</i>	non-wage income	$y_{nf} - wc$	

	Greek characters		
d_{cap}	depreciation rate of capital		DBXQ
d_{disp}	disposable income as share of income		LUDQ
c	share of social security benefits paid by government		UIRQ
L	wedge between wage costs and net wages		WDGQ
m_c	imports-sales ratio with respect to consumption goods/services	mcc / sc	MCQ
m_i	imports-sales ratio with respect to investment goods	mi / si	MIQ
m_m	imports-sales ratio with respect to intermediate goods/services	mmc / sm	MGQ
q	utilisation rate	ygf / cap	YBFQ
r_{cap}	return on capital	$(z-wcse) / k_1$	WMRQ
r_{rep}	net replacement ratio, = minimum benefit / 0.8 * average wage		UITQ
s	stocks-sales ratio		SXXQ
s_{bus}	social security contributions paid by businesses as a fraction of gross wages		SOCQ
s_{emp}	social security contributions paid by employees as a fraction of gross wages		PRGQ
t_{inc}	average income tax rate		LBXQ
t_{vac}	VAT tariff on consumption goods/services		VACQ
t_{vai}	VAT tariff on investment goods		VAIQ
u	unemployment rate	$u / lsup$	WKLQ
	No special symbol because variables can be expressed easily in other symbols if necessary:		
	sales of intermediate goods/services by receiving industry	$sm_j = \sum_i sm_{ij}$	OOT
	sales of intermediate goods/services by supplying industry	$sm_i = \sum_j sm_{ij}$	GOT
	price of imported consumption goods/services of k th category	p_{mc_k}	MCK
	price of imported investment goods of k th category	p_{mi_k}	MIK
	VAT on consumption goods/services, macro level	$\sum_k tvac_k$	BVC
	VAT on investment goods, macro level	$\sum_k tvai_k$	BVI
	balance of payments	$x - m$	BOP

	wage costs including imputed wage costs of self-employed	$WC + WCSE$	LBZ
	non-wage income excluding imputed wage costs of self-employed	$Z - WCSE$	WMR
	imports excluding re-exports	$m - mx$	MXX
	balance of payments as share of GDP	$bpay / ygf$	BOPO
	labour income share	$(wc+wcse) / ynf$	LBZQ
	Balance of macro spending account as share of GDP	bsp / ygf	SYXQ
	total of taxes as share of GDP	t / ygf	TAXQ
	indirect taxes (excluding VAT) minus subsidies	$tind - sub$	TKS
	sum of non-wage disposable income and corporate taxes	$ydz + tcor$	WMM
	Technical variables		
	tracks position of switches		SWI
	switch: if SIZBN=1 size-classes are calculated bottom-up		SIZ
	switch: if BTXBN=1 extra export mechanisms are activated		BTXB
	check variable		VBRO

Annex III Classification of industries

In this annex the classification of the eighteen industries in the business sector and the government sector is given. In the table below, the first column gives the PRISMA industry numbering together with the PRISMA description of the industry. The second column gives the definition of the industry in SBI codes. The third column gives the description of these SBI industries in Dutch. The fourth column gives the internal code of these SBI industries.

Below the table this annex concludes with describing the relationship between PRISMA industries and the industries distinguished by the CPB model Athena.

PRISMA industries	SBI code	Description in Dutch	code
1 food industry	151, 155	dierlijke voedingsmiddelenindustrie	idv
	15 ex 151,155,159	overige voedingsmiddelenindustrie	ioy
	159, 16	genotmiddelenindustrie	igm
2 chemical industry	24, 25	chemie, rubber- en kunststofindustrie	ich
3 metallurgic industry	27	basismetalaalindustrie	iba
	28	metaalproductenindustrie	imp
	29, 30	machine-industrie	ima
	31, 32, 33	electrotechnische-, instrumenten- en optische industrie	iet
	34, 35	transportmiddelenindustrie	itr
4 other manufacturing	17, 18, 19	textiel-, kleding- en lederindustrie	itk
	20, 26, 36 (excl. 36631), 37	hout- en bouwmaterialenindustrie, meubel en ov. industrie	ihb
	21	papier- en papierwarenindustrie	ipa
	22	grafische industrie	igu
5 construction	45111, 45211, 4522, 45253, 4550	burgerlijke- en utiliteitsbouw	but
	45112, 4512, 45212, 45213, 45231, 45232, 4524, 45251, 45252, 45254	grond-, weg- en waterbouw	bgw
	454	afwerkers	bsa
	4531, 4532, 4533, 4534	bouwinstallatiebedrijven	bin

6 wholesale trade	511	handelsbemiddeling	grt
	512	groothandel agrarische producten en levende dieren	grl
	513	groothandel voedings- en genotmiddelen	grv
	514	groothandel consumptiegoederen	grc
	516	groothandel investeringsgoederen	gri
	515 (excl. 5153), 517	groothandel intermediaire goederen	gin
	5153	groothandel bouwmaterialen	grb
7 retail trade	5211, 522	detailhandel food	dfo
	5244, 5245, 5246, 52494, 52495, 52497, 52498	detailhandel woninginrichting, DHZ, verfen huishoud. art.	dwd
	5241, 5242, 5243	detailhandel textiel, kleding en schoeisel	dtk
	5232, 5233, 5212, 5247, 5248, 52491 - 52493, 52496, 52499, 525	detailhandel overige non-food	don
	526, 527	detailhandel niet in winkel, reparatie	dnw
8 hotels, restaurants	551	hotel en pensions	hhp
	552	vakantiecentra en jeugdherbergen	hvj
	553	restaurants, cafetaria's etc.	hre
	554	cafés e.d.	hca
	555	besloten horeca-inrichtingen	hbi
9 car trade and repair	50101 -50103, 50105, 50301, 50302, 50401	autogroothandel	agr
	50104, 50205, 50402, 50201, 50203, 50303, 50204, 505	autodetailhandel	are

10	transport and communication	6021, 6022, 6023	personenvervoer	tpv
		6024, 603	goederenvervoer over land	tgw
		611, 612, 62	scheep- en luchtvaart	tsl
		63	dienstverlening t.b.v. vervoer	tvv
		64	post- en communicatie	com
11	banking and insurance	65, 66, 67	bank- en verzekering	zbv
12	other commercial services	741	juridische- en economische dienstverlening	zpr
		742	architect- en ingenieurbureaus	zte
		744	marketingbureaus	zma
		71, 72, 743, 746, 747, 748	overige zakelijke dienstverlening	zco
		8041, 93	overige diensten particulier	oov
		73, 853, 91, 95	overige diensten niet-particulier	onp
		745	uitzendbureaus	zau
		92	cultuur, sport, RTV	ocr
13	agriculture	01, 02, 05	landbouw	lan
14	mining	10, 11, 14	delfstoffenwinning	del
15	oil industry	23	olie-industrie	iao
16	public utilities	40, 41	openbaar nut	nut
17	housing	701, 703	makelaars	zog
		702	exploitatie onroerend goed	won
18	medical and social care	5231, 851, 852	zorg	zrg
19	government	75, 80 (excl. 8041), 90, 99, 36631	overheid	ovh

Relationship between PRISMA industries and Athena industries

Below only non 1-1 relationships are indicated.

- The 3 PRISMA industries ‘wholesale trade’, ‘retail trade’, and ‘car trade and repair’ combine to the Athena industry ‘handel en reparatie’.
- The PRISMA industry ‘transport and communication’ comprises the two Athena industries ‘transport’ and ‘post en communicatie’.
- The 2 PRISMA industries ‘other commercial services’ and ‘hotels, restaurants’ combine to the 2 Athena industries ‘tertiair overig’ and ‘uitzendbureaus’.

Annex IV Exact equations

For expositional reasons we used in chapter 2 in some places relatively simple approximate equations that only hold for small relative changes. In this annex we give the exact equivalents (marked by a prime) of these approximate equations.

In PRISMA x circle denotes the delta log of a variable, which is only by approximation equal to the relative change of x :

$$\overset{\circ}{x} \equiv \Delta \ln x = \ln(x/x_{-1}) \approx \Delta x/x_{-1}$$

Hence, in section 2.1 the exact equation for the average industry elasticity (2.3) reads:

$$\mathbf{a}_{avs} \equiv \frac{\overset{\circ}{lemp}_{ind}}{\overset{\circ}{ygf}_{ind}} = \frac{\ln \left[\sum_k (lemp_k / lemp_{ind})_{-1} \exp(\mathbf{a}_k \overset{\circ}{ygf}_k) \right]}{\ln \left[\sum_k (ygf_k / ygf_{ind})_{-1} \exp(\overset{\circ}{ygf}_k) \right]} \quad (2.3)'$$

If the relative changes in value added do not differ between size-classes and the employment shares of the size-classes are constant over time, this expression for the average industry elasticity boils down to:

$$\mathbf{a}_{avs} = \ln \left[\sum_k (lemp_k / lemp_{ind}) \exp(\mathbf{a}_k \overset{\circ}{ygf}) \right]^{1/\overset{\circ}{ygf}} \quad (2.4)'$$

Hence, this average industry elasticity is approximately constant over time if the relative changes in value added are small.

In the general case of section 2.3, the exact expression for the time-dependent elasticity at the industry level in the bottom-up case (2.9) reads:

$$\mathbf{a}_{ind, bottom-up, t} = \frac{\ln \left[\sum_k (y_{k,t-1} / y_{ind,t-1}) \exp(\mathbf{a}_k \overset{\circ}{x}_{k,t}) \right]}{\ln \left[\sum_k (x_{k,t-1} / x_{ind,t-1}) \exp(\overset{\circ}{x}_{k,t}) \right]} \quad (2.9)'$$

In the specific case of the price dependency of exports of section 2.3, this expression boils down to (if we assume that for all three size-classes prices of export products develop in the same way):

$$\mathbf{g}_{ind, bottom-up, t} = \ln \left[\sum_k (sx_{k,t-1} / sx_{ind,t-1}) \exp(\mathbf{g}_k \overset{\circ}{p}_{sx,t}) \right]^{1/\overset{\circ}{p}_{sx,t}} \quad (2.12a)'$$

In the specific case of the Leontieff relationship of section 2.3, the expression for the time-dependent elasticity at the industry level in the bottom-up case boils down to:

$$\mathbf{a}_{ind, bottom-up, t} = \frac{\ln \left[\sum_k (dm_{k,t-1} / dm_{ind,t-1}) \exp(q_{k,t}^\circ) \right]}{\ln \left[\sum_k (q_{k,t-1} / q_{ind,t-1}) \exp(q_{k,t}^\circ) \right]} \quad (2.14a)'$$