



# Political Cycles : Issue Ownership and the Opposition Advantage

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# Political Cycles : Issue Ownership And The Opposition Advantage <sup>1</sup>

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## **Abstract**

We propose a two dimensional infinite horizon model of public consumption in which investments are decided by a winner-take-all election. Investments in the two public goods create a linkage across periods. We follow the idea of issue ownership introduced by Petrocik (1996) in considering parties with different specialties. We show that the incumbent party vote share decreases the longer it stays in power. The median voter is generally not indifferent between the two parties and, when she is moderate enough, no party can maintain itself in power for ever. This result holds when the parties' main objective is to win the election and is compatible with a large range of candidates sub-objectives, that may change from one election to the next. Finally, the more parties are specialized and the more public policies have long-term effects, the more political cycles are likely to occur.

**Keywords:** Cycles, Alternation, Issue Ownership, Public goods, Opposition.

**JEL classification numbers:** D72, H41, C72

## **Résumé**

Nous proposons un modèle de consommation publique à horizon infini. Les investissements engagés dans la fourniture de deux biens publics sont déterminés par les élections. Ces investissements créent un lien entre les élections successives. Nous suivons l'idée introduite par Petrocik (1996) selon laquelle les partis "possèdent" certains thèmes, en considérant qu'ils ont des spécialités différentes. Nous montrons que la part des voix du parti au pouvoir décroît entre deux élections. L'électeur médian n'est généralement pas indifférent entre les deux partis et, lorsqu'il est suffisamment modéré, aucun parti ne peut se maintenir indéfiniment au pouvoir. Ce résultat est valide lorsque l'objectif principal des partis est de gagner l'élection et est compatible avec un grand ensemble de sous objectifs, qui peuvent changer d'une élection à l'autre. Finalement, plus les partis sont spécialisés et plus les politiques ont des effets de long terme, plus les cycles politiques sont susceptibles d'apparaître.

**Mots-clés :** Cycles, Alternance politique, Biens publics, Parti d'opposition.

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In modern democracies, the alternation of political parties in power is a frequent phenomenon. Why isn't there a greater persistence of parties in power? How can one explain the turnover of parties in government? How can one explain political cycles? We propose a theoretical model of political cycles, where the share of a party's vote decreases with the time it controls government. This effect, that we call "the opposition advantage", is different from the well known incumbent effect. Indeed, the incumbency effect measures the advantage given to the incumbent candidate competing with a challenger. The opposition effect measures the advantage of a candidate affiliated to the opposition party, when he competes against a candidate of the party in power, who is not necessarily the incumbent politician.

We propose an explanation of the opposition advantage and show that it can be a cause for political and policy cycles. We propose an infinite horizon model of elections with two parties built on two main assumptions: policies have long-term effects<sup>1</sup>, but are not irreversible, and parties have comparative advantages for the provision of two public goods (issue ownership). The two goods are imperfectly substitutable for voters. For example, citizens need good education and security at the same time. When voters are moderate, they may wish that both parties govern, but they can only elect one of them at a time. In this context, the opposition party can offer more moderate policies. Indeed, the opposition can propose to keep the incumbent party policy long-term effect and satisfy voters in focusing on the public good that it has a comparative advantage upon. On the contrary, the party in power cannot benefit from the comparative advantage of the opposition party. These two arguments suggest that the opposition party may be advantaged.

Our analysis has to be distinguished from studies focusing on politicians' careers and swings in their popularity. A large strand of this literature deals with the "Incumbency advantage"<sup>2</sup>. This theory is supported by overwhelm-

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<sup>1</sup>Many public goods have long-term effect. Important examples are national defense activities, welfare programs, environmental clean-up, building states schools, roads ....

<sup>2</sup>Ansolabehere and Snyder (2002) provide an excellent survey of the incumbency advantage literature, and an empirical contribution on state and federal elections in U.S. for

ing evidence, both in Senate elections and in elections to the House of representatives. Some of the major factors of the incumbency advantage are redistricting<sup>3</sup>, seniority systems<sup>4</sup>, and the lack of collective responsibility<sup>5</sup>.

Scholars explain political cycles with psychological arguments<sup>6</sup>, the main one being disappointment. The “Negativity effect” theory<sup>7</sup> is built on the following remark: voters’ decisions are based on the incumbent’s past performance and negative pieces of information have a greater impact than positive pieces of information. There exist two different explanations for this observation, the first one suggests that voters have a high esteem for powerful figures and are more easily disappointed than positively surprised by the government performance; the second (Abelson and Levy, 1985) states that the electorate has a strong risk aversion for potential costs of re-electing a politician who has demonstrated his bad performance. In the light of the negativity effect, Aragonés (1997) obtains a result of systematic alternation of the two parties implementing different policies. In our analysis, there is no uncertainty and electorate decisions are not based on past performance, but as usually in political models, for their preferred party at each election. Finally, the negativity and incumbency effects affect the election outcome in opposing directions. The first one leads to the defeat of the incumbent, whereas the

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the period 1942-2000. They find strong support for the incumbency advantage in state executives elections and conclude that explanations specific to the legislators incumbency advantage are not convincing.

<sup>3</sup>Cox and Katz (2002) state that redistricting caused the rise of legislators incumbency advantage after the 60s.

<sup>4</sup>McKelvey and Riezman (1992) argue that seniority tends to create a disincentive to vote for challengers.

<sup>5</sup>See Persson and Tabellini (2000, chapter 4) for a survey of the incumbents accountability literature.

<sup>6</sup>See Goertzel (2005) for a review of the American voters mood changes literature. Schlesinger (1949, 1986, 1992) consider that the electorate is inevitably disappointed by the party or the ideology that is in power. Klinberg (1952) suggests that American mood in public opinion balances between introversion and extroversion. This could explain why domestic and foreign concerns alternate through time and parties turnover in power.

<sup>7</sup>See Aragonés (1997) for a survey.

second one leads to the re-election of the incumbent. Both theories focus on individual politicians. Differently, our study does not deal with politicians but with parties.

In our model, political cycles emerge as a consequence of the opposition effect. There exists very few models considering this determinant of political cycles. Kramer (1977) and Bendor, Mookherjee and Ray (2005), study dynamic models of electoral competition between two parties with myopic behavior. Kramer (1977) suppose that the incumbent cannot change his policy whereas the challenger can locate anywhere in the policy space. He shows that candidates systematically alternate in power. Bendor, Mookherjee and Ray (2005) propose a model based on a satisficing behavior of the incumbent and a search behavior of the challenger. If the winning candidate is satisfied, then he does not change his policy until he loses the election, whereas the challenger is not satisfied, then he searches a policy that can defeat the incumbent. In our study, parties, once elected, are not constrained to keep their policy the next election. Parties behave strategically, they try to win the present election in selecting their platforms and their behavior do not change whether they are in power or not.

Another topic related to our analysis are policy cycles. Many scholars argue that policy cycles are generated by economic cycles<sup>8</sup>. We propose a different explanation; in our model, policy cycles are not generated by economic shocks but by the political structure. Since parties implement different policies<sup>9</sup>, political turnover and policy changes are clearly related. In a very different framework, Roemer (1995) shows that policy cycles arise because of stochastic changes in voters preferences in a model with policy motivated candidates with uncertainty. Our approach is different in many aspects. We suppose that parties are only office motivated and the non-

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<sup>8</sup>A huge literature studies political business cycles. See Berry (1991) for a survey.

<sup>9</sup>Hibbs (1977), Beck (1982), and Chappel and Keech (1986) show that Democrat and Republican governments have different influences on the unemployment rate. Alesina and Sachs (1988) and Tabellini and La Via (1989) show that parties are associated with different monetary policies.

convergence of platforms does not result from uncertainty but from parties multidimensional heterogeneity. Furthermore, we show that perpetual cycles (but not necessarily periodic) appear in a context with no uncertainty.

In considering an infinite number of successive elections and a dynamic link coming from public policies long-term effects, our work contributes to the literature of infinite horizon models of elections. This literature is mainly focused on the dynamic inefficiency of government <sup>10</sup>. Battaglini and Coate (2005) consider an infinite horizon model of collective spending and taxation. Public decisions are determined through a legislative bargaining process. Agents are forward looking, they take decisions in anticipating the outcomes of futures elections. The authors objective is very different from ours, because they concentrate on long-term government inefficiencies<sup>11</sup>. We do not analyze taxation and debt problems, then we suppose that the tax rate is fixed and that there is no saving and no debt.

Finally, we follow the empirical literature on the issue ownership theory of voting<sup>12</sup>, initiated by Petrocik (1996), in supposing that candidates are more able than their adversary to provide one of the two public goods. As stated by Egan (2006): "issue ownership refers to the idea that the Democrat and Republican parties "own" a set of issues which the public trusts the party

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<sup>10</sup>Baron (1996) studies a dynamic model of pork barrel policies. Gomes and Jehiel (2004) analyze the persistence of inefficiencies in a general framework of social and economic interactions that can be applied to legislative bargaining, coalition formation or exchange economies. Hassler, Storesletten and Zilibotti (2003) study public good provision in an OLG model, where an age-dependant taxation creates distortions in human capital investment. Azzimonti, Sarte and Soares (2003) focus on the role of commitment in a dynamic public spending and taxation model.

<sup>11</sup>In a close study, Azzimonti-Renzo (2005) analyzes government long-term inefficiencies when the decision maker is atomistic.

<sup>12</sup>Bélanger and Meguid (2005) show the importance of issue ownership and salience with evidence from Canada, Egan (2006) shows the nonrepresentativeness effect of issue ownership, Holian (2004) study the dynamic of issue ownership, Puglisi (2006) shows that the New York Times strategically gives emphasis to issues owned by Democrats or over which Republicans are weak, and Petrocik et al. (2003) analyse issue ownership in American presidential elections.

as substantially better able to "handle" than the other party. Democrats are generally trusted more than Republicans on issues like the environment, health care and social security; Republicans are more trusted on foreign policy and taxation". Since we consider candidates with different competences, our work also contributes to the literature on valence in politics. A growing literature deals with models where policy and quality are orthogonal dimensions<sup>13</sup>. Here, we suppose that parties' competences are different according to the different policies. As noticed by Prat (2002): "One may doubt that [voters] utility is separable in policy and valence. A left wing voter may prefer an inept right-wing politician to an effective right-wing politician because the latter is more likely to live up to his or promises and pass right-wing legislation. Still, an inept politician creates pure inefficiencies which are costly to all citizens.". Other authors analyze agency problems<sup>14</sup>, where politicians are associated to a policy-dependent competence level and voters have incomplete information on politicians type and/or actions<sup>15</sup>. We extend the assumption of heterogeneous competences to the case of two dimensions, but we suppose that they are common knowledge.

The paper is organized as follows. In section 1, we present voters behavior and parties constraints. In section 2, we derive the multiple possible outcomes of the electoral competition. In section 3, we show that the opposition party is advantaged. In section 4, we present our main results: the probability

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<sup>13</sup>This literature, initiated by Stokes (1992) focus on the problem of equilibrium existence and platforms location in spatial models when candidates have different "scores" on the quality dimension. Ansolabehere and Snyder (2000) study the unidimensional model in a world of certainty; Aragonés and Palfrey (2002) analyze the case where candidates maximize their share of votes and overcome the pure strategy equilibrium non-existence problem in studying mixed strategy equilibrium for small advantage levels. Groseclose (1999) and Aragonés and Palfrey (2004) add candidates policy concerns.

<sup>14</sup>See again Persson and Tabellini (2000, chapter 4, section 4.7) for a review of this literature.

<sup>15</sup>Rogoff and Siebert (1988) propose a model of adverse selection and Banks and Sundaram (1993, 1996) study politician accountability in models with moral hazard and adverse selection.

of winning cannot converge; when the median voter is extremist, a party can stay in power for ever, whereas when he is moderate, no party can keep power for ever; and we show that cycles are more likely to occur when the depreciation rate is low and when parties are strongly specialized. In section 5, we discuss two candidates objectives (re-election concerns and rent-seeker candidates). Finally, we conclude in section 6.

## 1 The model

We consider an infinite horizon model of elections with two opportunistic parties  $A$  and  $B$ . Each period, both parties commit themselves to implement a policy, voters elect a party and the new government implements his platform. Then, another election takes place, and so on. The government provides two durable public goods,  $a$  and  $b$ , that depreciate each period with a constant rate  $\delta$  in  $[0, 1]$ , and the government's budget is normalized to 1 at any period. A new government can either keep the existing stocks or transform one of the public good into the other. Specifically, if the level of public good  $g$  ( $g = a, b$ ) after election  $t$  is  $g_t$  and  $I_{g,t+1}$  new units are produced by the government in period  $t + 1$ , then the level in period  $t + 1$  is<sup>16</sup>:

$$g_{t+1} = (1 - \delta) g_t + I_{g,t+1}, \quad (1)$$

where  $g = a, b$ . The level  $g_{t+1}$  can be either greater or smaller than  $g_t$ . When  $g_{t+1} \geq g_t$ , this means that the government at time  $t + 1$  chooses to keep the stock of public good  $g$ . If  $g_{t+1} < g_t$ , the government either undoes or does not invest enough in good  $g$  to maintain its level. A policy  $z_t$  is a couple of public goods quantities  $(a_t, b_t)$ .

**Voters:** Voters vote sincerely and differ in the weight they place on the two public goods. Voter  $i$ 's weight for the first public good is denoted by  $\alpha_i$ ,

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<sup>16</sup>Azzimonti-Renzo (2005) and Battaglini and Coate (2005) make the same assumption on the long-term effect of public spending.



belonging to the unit interval  $[0, 1]$ . The preferences of voter  $i$  are represented by:

$$W_i(a_t, b_t) = \alpha_i \ln(a_t) + (1 - \alpha_i) \ln(b_t), \quad (2)$$

where  $a_t$  and  $b_t$  are the public goods stocks after date  $t$ . The policy after election  $t$  is noticed  $z_t = (a_t, b_t)$ .

This kind of preferences, introduced by Tabellini and Alesina (1990), allows voters to disagree about which quantities of public goods to consume. Furthermore, these preferences belong to the class of "intermediate preferences" defined by Grandmont (1978), and verify the single crossing property (Grandmont, 1978). The median voter theorem applies, i.e. the median voter's preferred policy is the unique Condorcet winner. The preferred policy of the median voter, characterized by  $\alpha_m$ , is thus the Condorcet winner in our context.

It is important to notice that the identity of the median voter  $\alpha_m$ , does not depend on the date, i.e. is independent of the dynamics of the model.

**Parties and issue ownership:** At each period, both parties propose credible platforms in order to win the election. The government's budget constraint is:

$$I_{a,t} + I_{b,t} \leq 1, \quad (3)$$

We define a party as a stable organization, which can provide the two public goods. Following Petrocik's (1996) idea of issue ownership, we suppose that the two parties are "specialized": party  $A$  "owns" issue  $a$  and party  $B$  "owns" issue  $b$ . Formally, we suppose that  $A$  has a comparative advantage in providing good  $a$  and party  $B$  a comparative advantage in providing good  $b$ . This advantage will be captured by two constants,  $\eta^A \in ]1, \bar{\eta}]$  and  $\eta^B \in ]1, \bar{\eta}]$  which are inversely related to the marginal cost of providing the public goods. Finally, we suppose that the technology for providing both public goods has constant returns to scale, with marginal costs of  $1/\eta^A$  and 1 for party  $A$  and 1 and  $1/\eta^B$  for party  $B$ . With these specifications in mind, we write the budget constraints of the two parties at an election at date  $t$  as:

Party A:

$$\frac{a_t - (1 - \delta) a_{t-1}}{\eta^A} + b_t - (1 - \delta) b_{t-1} \leq 1, \quad (\text{A})$$

Party B:

$$a_t - (1 - \delta) a_{t-1} + \frac{b_t - (1 - \delta) b_{t-1}}{\eta^B} \leq 1, \quad (\text{B})$$

where stocks of the two public goods must be positive, i.e.,  $a_t, b_t \geq 0$ . Inequality (A) defines party A's set of policy  $\mathbf{A}(t)$  and inequality (B) define party B's set of policy  $\mathbf{B}(t)$ .

## 2 Political Equilibria

### 2.1 The median voter choice

Each election has two stages. In the first stage, parties announce credible promises and in the second stage voters cast their ballot. We consider subgame perfect equilibria<sup>17</sup> of this game, given the stocks of public goods. Since voters' preferences verify the single crossing property, the median voter selects the winning party, and her choice drives the dynamics of successive elections. Indeed, this property ensures that for any pair of policies, the median voter preferred policy is also preferred by a majority of voters. Hence, the median voter behavior determines the outcome of the election. We start the analysis by deriving her preferred platform over the set of platforms. The median voter's preferred policy over  $\mathbf{A}(t)$ , denoted  $m_t^A$  is the solution to:

$$\begin{aligned} & \underset{(a_t, b_t)}{\text{Max}} [W_i(a_t, b_t)] && (\text{MA}) \\ & \text{s.t. : } (a_t, b_t) \in \mathbf{A}(t) \end{aligned}$$

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<sup>17</sup>In considering subgame perfect equilibria and then voters as players, we can conclude that the set of winning strategies is always a closed set.

and her preferred platform over  $\mathbf{B}(t)$ , denoted  $m_t^B$  is the solution to:

$$\begin{aligned} & \underset{(a_t, b_t)}{\text{Max}} [W_i(a_t, b_t)] & \text{(MB)} \\ & \text{s.t. : } (a_t, b_t) \in \mathbf{B}(t) \end{aligned}$$

Straightforward calculations allow us to characterize the median voters' preferred policies:

$$m_t^A = (\eta^A \alpha_m s_{t-1}^A, (1 - \alpha_m) s_{t-1}^A), \quad (4)$$

$$m_t^B = (\alpha_m s_{t-1}^B, \eta^B (1 - \alpha_m) s_{t-1}^B), \quad (5)$$

where  $s_{t-1}^A = 1 + (1 - \delta) \left( b_{t-1} + \frac{a_{t-1}}{\eta^A} \right)$  and  $s_{t-1}^B = 1 + (1 - \delta) \left( a_{t-1} + \frac{b_{t-1}}{\eta^B} \right)$ .

Hence, the derivation of the median voter's preferred platform depends on the public goods stocks  $a_{t-1}$  and  $b_{t-1}$ . She has to compare  $m_t^A$  and  $m_t^B$ . Let  $\Lambda_t(\cdot)$  be such that:

$$\Lambda_t(\alpha_m) = \frac{s_{t-1}^A}{s_{t-1}^B} \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}}, \quad (6)$$

The median voter weakly prefers  $m_t^A$  to  $m_t^B$  if and only if  $W_i(m_t^A) \geq W_i(m_t^B)$ . With simple computations, one can show that the median voter weakly prefers  $m_t^A$  to  $m_t^B$  if and only if  $\Lambda_t(\alpha_m) \geq 1$ . Not surprisingly, the more  $A$  is competent, the less  $B$  is competent, and the more  $\alpha_m$  is high, the higher the likelihood that the median voter chooses a policy in  $A$ 's policy set.

## 2.2 Equilibria

Each election presents two stages. In the first stage, parties select their platforms in order to win the election. Party  $A$  (respectively party  $B$ ) maximizes its probability of victory  $\pi_t^A$  (respectively  $\pi_t^B$ ). In the second stage, voters vote for their preferred candidate. Since voters preferences verify the single crossing property and voters are sincere, the second stage allow to solve the case where the median voter is indifferent between the two programs. We denote

by  $z_t^A$  party  $A$ 's platform and by  $z_t^B$  party  $B$ 's platform in the election at date  $t$ . Let  $\mathbf{M}^A(t)$  (respectively  $\mathbf{M}^B(t)$ ) be the set of party  $A$  platforms strictly preferred to  $m_t^B$  (respectively to  $m_t^A$ ). Formally:

$$\mathbf{M}^A(t) = \{z_t \in \mathbf{A}(t) : W_m(z_t) \geq W_m(m_t^B)\}, \quad (7)$$

$$\mathbf{M}^B(t) = \{z_t \in \mathbf{B}(t) : W_m(z_t) \geq W_m(m_t^A)\}, \quad (8)$$

Since parties are only interested in winning the election, a platform that the rival cannot defeat is an equilibrium strategy. This leads to a multiplicity of subgame perfect Nash equilibria, summarized in the following proposition:

**Proposition 1** *The set of subgame perfect Nash equilibria is always non empty and is given by:*

- (i)  $\mathbf{M}^A(t) \times \mathbf{B}(t)$  and the median voter votes for  $A$  if  $\Lambda_t > 1$ , and  $A$  is elected,
- (ii)  $\mathbf{A}(t) \times \mathbf{M}^B(t)$  and the median voter votes for  $B$  if  $\Lambda_t < 1$ , and  $B$  is elected,
- (iii)  $\{m_t^A\} \times \mathbf{M}^B(t)$  and the median voter votes for  $A$  (and then  $A$  wins);  $\mathbf{M}^A(t) \times m_t^B$  and the median voter votes for  $B$  (and then  $B$  wins) if  $\Lambda_t = 1$ .

(Proofs are reported in the appendix.)

These results lead to several observations. First, because parties only want to win the election and the information is complete, one party is in general certain to be elected (in cases (i) and (ii)). This party can propose many winning platforms, whereas the loser locates anywhere in his policy set. Figure 4.1 illustrates this kind of equilibrium.

Second, in very specific circumstances (in case (iii)), the median voter is indifferent between the two parties (see Figure 4.2) and the subgame perfect Nash equilibria can support both parties' victory. If this event occurs, it will dramatically change the dynamics of elections, as we discuss section 4.4.1.

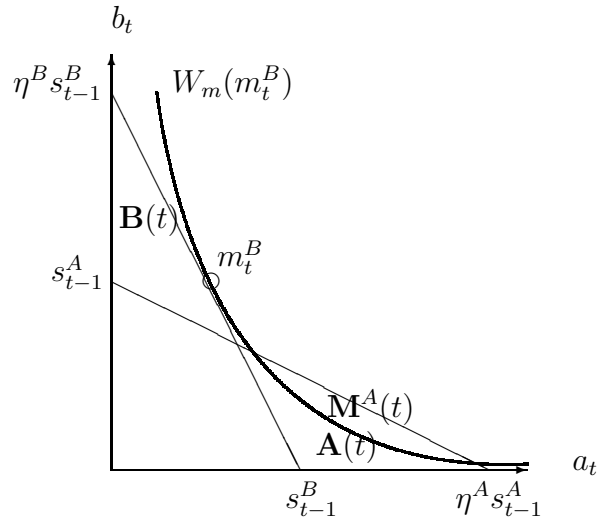


Figure 1: Candidate A winning strategies when  $\Lambda_t > 1$

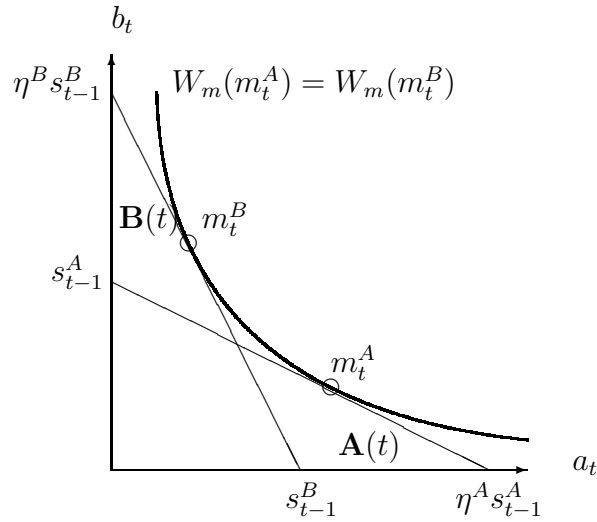


Figure 2: Equilibrium strategies when  $\Lambda_t = 1$

### 3 The opposition advantage

In this section, we discuss about the advantage conferred to the party in the opposition. Consider two elections at dates  $t$  and  $t + 1$ , and suppose that  $B$

wins the election at date  $t$ . Then  $B$  implements his policy  $z_t^B = (a_t^B, b_t^B) \in \mathbf{M}^B(t)$ , one of his equilibrium platform for election  $t$ . Since  $B$  is the winner, it is necessarily true that  $\Lambda_t \leq 1$ . First remark that  $z_t^B \notin \mathbf{A}(t)$  because of the definitions of  $\mathbf{M}^B(t)$  and  $m_t^A$ , so that  $z_t^B$  must satisfy

$$\frac{a_t^B - (1 - \delta) a_{t-1}}{\eta^A} + b_t^B - (1 - \delta) b_{t-1} > 1, \quad (9)$$

This simply means that if  $A$  would try to imitate  $B$  at election  $t$ , then he would violate his budget constraint. Furthermore, since  $B$  wins at  $t$ , then  $(a_t, b_t) = (a_t^B, b_t^B)$ . This last equation can be then rewritten as follows:

$$s_t^A - 1 > (1 - \delta) s_{t-1}^A. \quad (10)$$

By definition,  $z_t^B \in \mathbf{B}(t)$ , so that:

$$a_t^B - (1 - \delta) a_{t-1} + \frac{b_t^B - (1 - \delta) b_{t-1}}{\eta^B} \leq 1, \quad (11)$$

or, equivalently,

$$s_t^B - 1 \leq (1 - \delta) s_{t-1}^B. \quad (12)$$

Using equations 10 and 12, we obtain:

$$\frac{s_{t-1}^A}{s_{t-1}^B} < \frac{s_t^A - 1}{s_t^B - 1}, \quad (13)$$

Furthermore, it is easy to check that  $\frac{s_t^A}{s_t^B} \geq \frac{s_{t-1}^A}{s_{t-1}^B}$ , only because  $s_t^A$  and  $s_t^B$  are strictly greater than 1. Finally, the relative advantage of party  $A$  is strictly greater at election  $t + 1$  than at election  $t$ . This result is summarized in the next proposition:

**Proposition 2** *At each election, the relative advantage of the opposition party increases: for all  $t$  where  $A$  is the opposition party,  $\Lambda_{t+1} > \Lambda_t$ .*

(Proof: see the reasoning above.)

This result states that the share of votes of the opposition party generally increases from one election to the next. The intuition of this result is that

when a party is elected, since he must implement his promises, he gives the opposition party the opportunity to propose a more satisfactory platform on both dimensions. This effect drives the dynamics of elections and, when it is sufficiently large, can lead to a switch in power between the majority and the minority.

## 4 Political Cycles

In this section, we study the dynamics of elections and public good provision. The questions arising at this point are: What is the long run behavior of the dynamics of elections? May elections outcomes be durably unknown? How do cycles depend on the median voter preferences? On the parties competences? On the durability of public goods? In this section, we answer these questions and illustrate the results with simulations of the dynamic.

### 4.1 May elections outcomes be durably unknown?

We focus on the special case (iii), where the winner is unknown in election  $k$ . We have shown that the sequence  $(\Lambda_t)_t$  is increasing when  $A$  is not in power, and, by symmetry, is decreasing when  $A$  is in power. Then, the sequence is either always increasing and then for all  $t$ ,  $\Lambda_t \leq 1$ , always decreasing and for all  $t$ ,  $\Lambda_t \geq 1$ , or follows a cycle.

This sequence is not stable for  $\Lambda_t = 1$ . Indeed, suppose that there exists an election  $k$  such that  $\Lambda_k = 1$ . Then each party has one half chance of being elected in  $k$ . Without loss of generality, suppose that  $A$  is elected, then  $\Lambda_{k+1} < \Lambda_k = 1$ , and party  $B$  is elected for sure in  $k + 1$ . The following corollary of proposition 2 summarizes this result:

**Corollary 1** *If  $\Lambda_k = 1$ , the elected party in  $k$  is defeated in  $k + 1$ .*

(The proof relies on the simple argument above.)

The intuition of this result is that, when the median voter is indifferent between both platforms ( $\Lambda_k = 1$ ), he would indeed like both platforms to

be implemented in turn<sup>18</sup>. But only one party is elected, and provides a polarized platform. At the next election, the opposition party will provide a policy which uses the stock of public goods implemented by the majority, but is closer to the median voter's preferences.

## 4.2 Stable power

The following proposition provides sufficient conditions for a party to constantly remain in power.

**Proposition 3** *There exists  $0 < \underline{\alpha} < \bar{\alpha} < 1$  such that, for all  $(\alpha_m, \delta, \eta^A, \eta^B, a_0, b_0) \in [0, 1]^2 \times ]1, \bar{\eta}]^2 \times R_+^2$ :*

- (i) *If  $\alpha_m \in [0, \underline{\alpha}]$ , then party B wins all elections,*
- (ii) *If  $\alpha_m \in [\bar{\alpha}, 1]$ , then party A wins all elections.*

(Proof: see the appendix)

The intuition of this result is straightforward. If the median voter has extreme tastes, then one of the two parties is able to keep power forever by exploiting its comparative advantage in providing one of the two policies.

## 4.3 Cycles

We now analyze cycles where parties alternate in power. We wish to know when these cycles are not conjunctural, namely, when they are independent of the initial stocks of public good,  $a_0$  and  $b_0$ . We define political cycles in the following way:

**Definition 1** *A set of parameters  $(\alpha_m, \delta, \eta^A, \eta^B, a_0, b_0) \in [0, 1]^2 \times ]1, \bar{\eta}]^2 \times R_+^2$ , exhibits **political cycles** if and only if no party wins an infinite number of consecutive elections.*

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<sup>18</sup>The intuition is close to Alesina and Rosenthal (1996) at the difference that, in our model, voters cannot mix policies during a unique mandate, but they get mixed policies through successive mandates with parties turnover.



Formally, we study the case where the sequence  $(\Lambda_t)_t$  does not converge and does not diverge. Unfortunately, because there exist many equilibria at each election, we cannot give necessary and sufficient conditions on the set of parameters such that it exhibits political cycles. However, we propose a sufficient condition for the existence of political cycles:

**Proposition 4** *For all  $(\delta, \eta^A, \eta^B, a_0, b_0) \in ]0, 1[ \times ]1, \bar{\eta}]^2 \times R_+^2$ , there exist  $\alpha_1 < \alpha_2$  both in  $[0, 1]$ , such that: if  $\alpha_m \in [\alpha_1, \alpha_2]$  no party can maintain itself indefinitely in power.*

**Example 1:** Suppose (for simplicity) that the elected party implements the median voter preferred program. Consider the following numerical example:  $\eta^A, \eta^B = 1.1$ ,  $a_0 = b_0 = 0$ ,  $\delta = 0.6$ ,  $\alpha_m = 0.515$  (the median voter prefers good  $a$  to good  $b$ ). The following figure represents the dynamic of the two public goods stocks and the election winner for the 20 first elections:

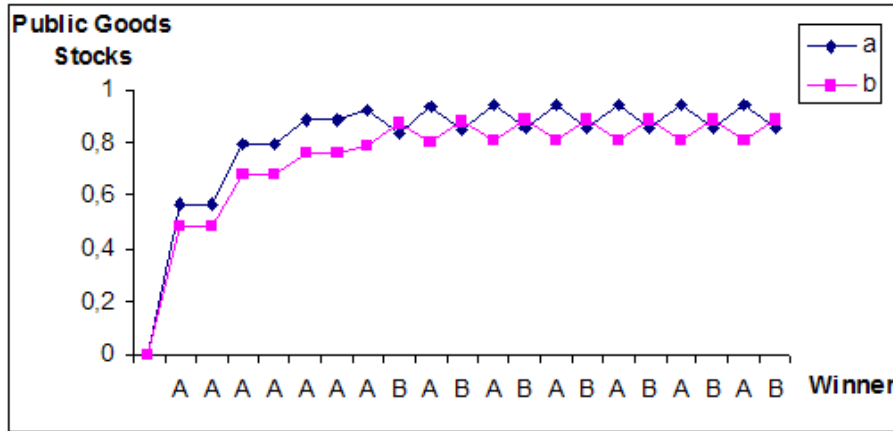


Figure 3: Political Cycles

This example illustrates well the dynamic of Political Cycles. Initially, both the public goods quantities are identical. Since the median voter prefers public good  $a$ , he elects party  $A$  until he has enough of good  $a$  (8 times in

this example). When the quantity of good  $a$  becomes high enough (relatively to the quantity of  $b$ ), he doesn't need more  $a$  and wishes to have a higher quantity of  $b$ . Hence he changes his vote and elects party  $B$ . Thereafter, he changes his vote in all election for similar reasons.

#### 4.4 Comparative statics

Since there exist many equilibria, it seems complicated to provide general comparative statics. To give an insight into the influence of the depreciation rate and the candidates competences on political cycles we suppose, for simplicity, that the winning candidate always implements the median voter preferred platform<sup>19</sup>, that is  $m_t^A$  (respectively  $m_t^B$ ) when candidate  $A$  (respectively candidate  $B$ ) wins the election  $t$ . Furthermore, we consider the simple case where  $\eta^A = \eta^B = \eta$ , i.e. when candidates are equally competent in their respective specialties. Under these assumptions, we obtain the following comparative statics results:

**Proposition 5** *The interval  $[\alpha_1, \alpha_2]$  (of maximal size) defined in Proposition 4 is unique and,*

$$\frac{\partial (\alpha_2 - \alpha_1)}{\partial \eta} > 0, \quad (14)$$

and,

$$\frac{\partial (\alpha_2 - \alpha_1)}{\partial \delta} < 0. \quad (15)$$

The higher the specialization of parties, the larger the parameter range for which political cycles occur. When parties become more specialized, they implement more extreme policies and the median voter is more willing to switch in order to see the other good provided. When the depreciation rate increases, goods have shorter effects and voters need less power turnover.

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<sup>19</sup>The median voter preferred platform is always an equilibrium platform for the winning candidate.

## 5 Extensions: parties' lexicographic preferences

The results presented in the precedent sections hold without specifying the choice of an elected party among the generally large set of winning policies. We now allow parties to select one policy in order to maximize a sub-objective function. In other words, parties of lexicographic preferences: they first want to be elected, and select among the winning platforms that platform which maximizes their subobjective. Formally, party  $A$ 's program becomes:

$$\underset{z_t^{A^*} \in \mathbf{A}(t)}{\text{Max}} \Pi_t^A(z_t^{A^*}, z_t^B), \quad (16)$$

$$s.t. : \forall z_t^A \in \mathbf{A}(t), \pi_t^A(z_t^{A^*}, z_t^{B^*}) \geq \pi_t^A(z_t^A, z_t^{B^*}),$$

and candidate  $B$ 's program is:

$$\underset{z_t^{B^*} \in \mathbf{B}(t)}{\text{Max}} \Pi_t^B(z_t^A, z_t^{B^*}), \quad (17)$$

$$s.t. : \forall z_t^B \in \mathbf{B}(t), \pi_t^B(z_t^{A^*}, z_t^{B^*}) \geq \pi_t^B(z_t^{A^*}, z_t^B),$$

### 5.1 Re-election concerns

Suppose that parties want to be re-elected, and consider the following reduced form for a long-run, non myopic behavior of political parties. At the election at date  $t$ , the winning party's subobjective is to maximize his relative advantage in the next election, that is  $\Lambda_{t+1}$  for party  $A$ , and  $\frac{1}{\Lambda_{t+1}}$  for party  $B$ . A party first wishes to be elected, and then to create the most favorable conditions for its re-election. If  $\Lambda_t = 1$ , then equilibrium programs are derived from their first objective of victory and they play  $(m_t^A, m_t^B)$ . But, if  $\Lambda_t \neq 1$ , for example  $\Lambda_t > 1$ , then party  $A$  can choose many winning programs. In this case, it chooses a platform  $z_t^A = (a_t^A, b_t^A) \in \mathbf{M}^A(t)$ . Hence, its relative advantage for the next election is  $\Lambda_{t+1} = \frac{1+(1-\delta)\left(b_t + \frac{a_t}{\eta^A}\right)}{1+(1-\delta)\left(a_t + \frac{b_t}{\eta^B}\right)} \frac{(\eta^A)^{\alpha m}}{(\eta^B)^{1-\alpha m}}$ .

Intuitively, since  $\Lambda_{t+1}$  is decreasing in  $a_t$  and increasing in  $b_t$ , party  $A$  will choose a program with a minimum of good  $a$  and a maximum of good  $b$ . The following result describe the equilibrium strategy of the winning party ( $A$  without loss of generality):

**Proposition 6**  $\Lambda_{t+1}$  admits a unique maximum over  $\mathbf{M}^A(t)$  and there exists a unique corresponding program with a minimum quantity of  $a$  and a maximum quantity of  $b$ .

(The Proof is in the appendix)

This result suggests that parties seeking re-election choose very inefficient platforms, because they do not fully exploit their comparative advantage. The intuition is that a party has to provide some of the public good that he is not competent at producing, in order to induce voters to reelect him next period. Figure 4.3 illustrates this inefficient platform, denoted  $\bar{z}_t^A$ , when  $A$  wins the election:

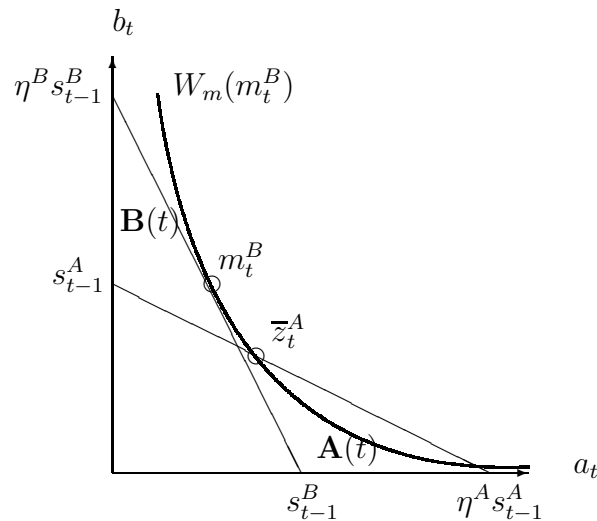


Figure 4: When candidate  $A$  has re-election concerns and  $\Lambda_t > 1$

The implemented policy is not computable for every values of  $\alpha_m$ . Fortunately, we are able to compute this program for the case where  $\alpha_m = \frac{1}{2}^{20}$ . Not surprisingly, when no party is advantaged  $\Lambda_t(\frac{1}{2}) = 1$ , the implemented policy is always the median voter preferred one:  $\bar{z}_t^A = m_t^A$  and  $\bar{z}_t^B = m_t^B$ . We propose to illustrate the elections dynamics with the following simulated example:

**Example 2:** We plot the evolution of the public goods stocks and the winning candidate (for election 13 to 44), with:  $\alpha_m = 0.5$ ,  $\eta^A = 2$ ,  $\eta^B = 2.1$ ,  $\delta = 0.69$  and  $a_0 = b_0 = 0$ .

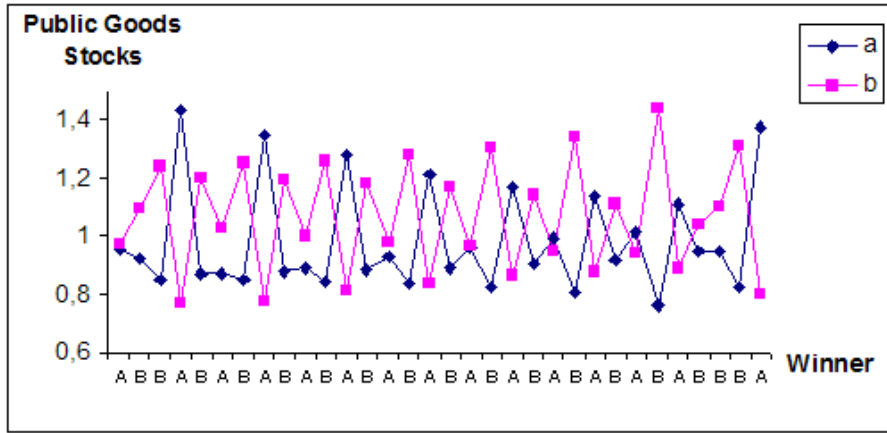


Figure 5: Dynamic with re-election seekers candidates

<sup>20</sup>Let  $\alpha_m = \frac{1}{2}$ . The winner (with re-election concerns) equilibrium program is given by:  
If  $\Lambda_t \geq 1$ :

$$\bar{z}_t^A = \left( \frac{\eta^A s_{t-1}^A}{2} \left( 1 - \sqrt{1 - \frac{1}{(\Lambda_t(\frac{1}{2}))^2}} \right), \frac{s_{t-1}^A}{2} \left( 1 + \sqrt{1 - \frac{1}{(\Lambda_t(\frac{1}{2}))^2}} \right) \right)$$

If  $\Lambda_t \leq 1$ :

$$\bar{z}_t^B = \left( \frac{s_{t-1}^B}{2} \left( 1 + \sqrt{1 - \left( \Lambda_t \left( \frac{1}{2} \right) \right)^2} \right), \frac{\eta^B s_{t-1}^B}{2} \left( 1 - \sqrt{1 - \left( \Lambda_t \left( \frac{1}{2} \right) \right)^2} \right) \right)$$

This result is obtained with the straightforward calculation of (for candidate A)  $\bar{a}_t = \arg \min_{a_t \in [0, \eta^A s_{t-1}^A]} \left( W_m \left( a_t, s_{t-1}^A - \frac{a_t}{\eta^A} \right) = W_m \left( m_t^B \right) \right)$  with  $\alpha_m = \frac{1}{2}$

In this example, candidate  $B$  is advantaged since  $\eta^B > \eta^A$ . Candidate  $B$  uses this advantage and can sometimes stay in power more than one legislature (two or three) in a row, whereas candidate  $A$  cannot (in this example) maintain himself in power longer than one legislature.

## 5.2 Rent-seeker candidates

The results of previous sections also hold when the candidates' sub-objective is to extract rents from power. Formally, if  $A$  wins the election, he chooses to maximize his rent from power:

$$\underset{r_t^A}{Max} [r_t^A], \quad (18)$$

$$s.t. : \frac{a_t^A}{\eta^A} + b_t^A + r_t^A \leq s_{t-1}^A$$

$$\text{and, } (a_t^A, b_t^A) \in \mathbf{M}^A(t)$$

In equilibrium, parties rents amount (if elected) are given by:

**Proposition 7** (i) If  $\Lambda_t \geq 1$ ,  $r_t^{A*} = \left(1 - \frac{1}{\Lambda_t}\right) s_{t-1}^A$  is the maximum of  $r_t^A$  over  $\mathbf{M}^A(t)$ ,

(ii) If  $\Lambda_t \leq 1$ ,  $r_t^{B*} = (1 - \Lambda_t) s_{t-1}^B$  is the maximum of  $r_t^B$  over  $\mathbf{M}^B(t)$ .

Then, the higher the relative advantage of candidate  $A$  ( $\Lambda_t$ ), the higher the rents he can extract. Figure 4.4 illustrates this result, where candidate  $A$ 's equilibrium platform is denoted  $\bar{z}_t^A$ :

Furthermore, notice that we know from Proposition 2, that  $\Lambda_{t+1} < \Lambda_t$ . Hence, if  $A$  wins the election at  $t$  and  $t + 1$ , we obtain that  $r_{t+1}^{A*} < r_t^{A*}$ . This would suggest that the longer a party is in power, the smaller the rents he can extract. We have to be cautious with this observation because of problems of enforceability. Indeed, if parties cannot be forced to implement their

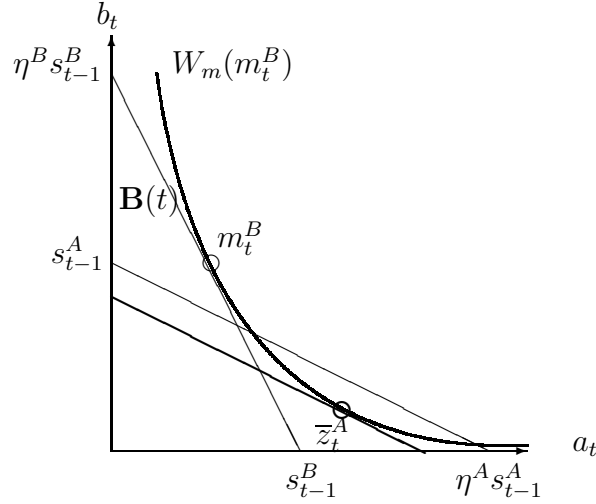


Figure 6: When candidate  $A$  is rent-seeker and  $\Lambda_t > 1$

promises, an incumbent who is certain to lose the next election will extract all the rents from power. Persson and Tabellini (2000, chapter 4) discusses this issue and provides a survey of the relevant literature.

Computing parties equilibrium programs lead to conclude that the implemented policy in election  $t$  is:

**Corollary 8** *The winner party (rent-seeker) equilibrium program is given by:*

If  $\Lambda_t \geq 1$ :

$$\bar{z}_t^A = \left( \frac{\eta^A s_{t-1}^A \alpha_m}{\Lambda_t}, \frac{s_{t-1}^A (1 - \alpha_m)}{\Lambda_t} \right)$$

If  $\Lambda_t \leq 1$ :

$$\bar{z}_t^A = (s_{t-1}^B \alpha_m \Lambda_t, \eta^B s_{t-1}^B (1 - \alpha_m) \Lambda_t)$$

(This results is directly deduced from the proof of Proposition 7)

As the policies implemented when candidates have re-election concerns, the policies implemented when parties are rent-seekers are very inefficient. Furthermore, when no party is advantaged ( $\Lambda_t = 1$ ) these policies are also identical to the median voter preferred programs.

Now, a natural issue would be to compare the two dynamics, when parties

have re-election concerns and when they are rent-seeker. First, does re-election concerned candidates implement more or less efficient policies than rent-seekers candidates? The answer is ambiguous. Indeed, the median voter is indifferent between a rent and a re-election seeker candidate, because each of them offers a sufficiently higher quantity of one of the two public goods than the other one.

Second, how does the evolution of public goods is influenced by candidates' objective? We propose to illustrate the elections dynamics with the same data as in example 2:

**Example 3:** We plot the evolution of the public goods stocks and give the winning candidate (for election 13 to 44) with the same data as in Example 2:  $\alpha_m = 0.5$ ,  $\eta^A = 2$ ,  $\eta^B = 2.1$ ,  $\delta = 0.69$  and  $a_0 = b_0 = 0$ .

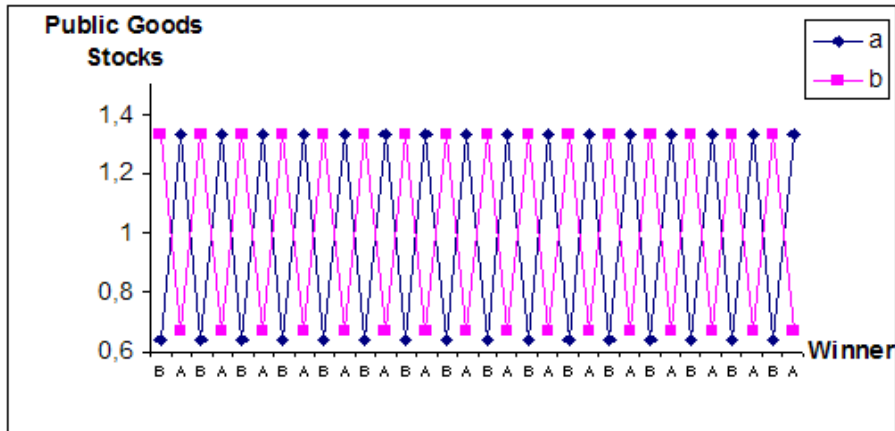


Figure 7: Dynamic with rent-seeker candidates

Not surprisingly, compared to the re-election seekers case, there seems to be more frequent alternations when candidates are rent-seekers.

## 6 Conclusion

We have considered an infinite horizon dynamic model of public consumption with durable public goods. We have shown that the longer a party keeps



power, the more the opposition is likely to come back to power. Therefore, we have been able to show that policy and political cycles can occur, when the median voter preferences are balanced enough between the public goods provided by the two parties. This result holds when the parties' main objective is to win the election and is compatible with a large range of candidates sub-objectives, that may change from one election to the next. In particular, we have shown that a candidate seeking re-election will choose a very inefficient platform, providing the minimal quantity of the public good in which he has a comparative advantage.

## 7 Appendix

### Proof of Proposition 1:

(i) If  $\Lambda_t > 1$ , by definition, the median voter strictly prefers  $m_t^A$  than  $m_t^B$ . Hence,  $m_t^A \in \mathbf{M}^A(t) \neq \emptyset$ . Let  $z_t^A \in \mathbf{M}^A(t)$  and  $z_t^B \in \mathbf{B}(t)$ , then  $W_m(z_t^A) \geq W_m(m_t^B) \geq W_m(z_t^B)$ . If the inequality is strict, then  $(\pi_t^A, \pi_t^B) = (1, 0)$  and no party has an incentive to deviate. In case of equality, the median voter is indifferent between both policies  $z_t^A$  and  $z_t^B$ . If she votes for party  $B$ , then  $A$  has an incentive to deviate, to choose, for example,  $m_t^A$  and wins then the election. If the median voter chooses to vote for party  $A$ , then  $(\pi_t^A, \pi_t^B) = (1, 0)$  and no party has an incentive to deviate.

We conclude that  $\mathbf{M}^A(t) \times \mathbf{B}(t) \subset \mathbf{E}(t)$ . Now, choose  $z_t^A \notin \mathbf{M}^A(t)$ , then  $W_m(m_t^B) > W_m(z_t^A)$ . In this case  $\pi_t^A < 1$ , then party  $A$  has an incentive to move and play, for example,  $m_t^A$ . (ii) The reasoning is the same as for point (i) in inverting  $A$  and  $B$ . (iii) If  $\Lambda_t = 1$ , by definition,  $W_m(m_t^A) = W_m(m_t^B)$ . Suppose  $z_t^A = m_t^A$ ,  $z_t^B \in B(t)$  and the median voter votes for  $A$ . Then,  $(\pi_t^A, \pi_t^B) = (1, 0)$  and no party has a strict incentive to deviate. The median voter has no strict incentive to change her vote since  $W_m(m_t^A) \geq W_m(z_t^B)$ . Hence, this is a subgame perfect equilibrium (the situation with  $A$  and  $B$  inverted is also a SPE). There is no other SPE. Indeed, suppose  $z_t^A \neq m_t^A$ ,  $z_t^B \in B(t) \setminus \{m_t^B\}$  and the median voter vote for  $A$ . If  $W_m(z_t^A) < W_m(z_t^B)$ , the median voter will change her vote. If  $W_m(z_t^A) \geq W_m(z_t^B)$ , then party  $B$  has an incentive to change its program and to choose, for example,  $m_t^B$ . Doing so, in the second stage, the median voter chooses to vote for  $B$ .

### Proof of Proposition 3:

Let us consider an election at date  $t$ . Public goods stocks are  $((1 - \delta)a_{t-1}, (1 - \delta)b_{t-1})$ , and:

$$\Lambda_t(\alpha_m) = \frac{1 + (1 - \delta) \left( b_{t-1} + \frac{a_{t-1}}{\eta^A} \right) (\eta^A)^{\alpha_m}}{1 + (1 - \delta) \left( a_{t-1} + \frac{b_{t-1}}{\eta^B} \right) (\eta^B)^{1 - \alpha_m}},$$

This is a continuous and strictly increasing function of  $\alpha_m$ . Its value is

$\frac{1+(1-\delta)\left(b_{t-1}+\frac{a_{t-1}}{\eta^A}\right)}{\eta^B+(1-\delta)(\eta^B a_{t-1}+b_{t-1})} < 1$ , and  $\frac{\eta^A+(1-\delta)(\eta^A b_{t-1}+a_{t-1})}{1+(1-\delta)\left(a_{t-1}+\frac{b_{t-1}}{\eta^B}\right)} > 1$  when  $\alpha_m = 1$ . Then, there exists a unique value of  $\alpha_m$ , denoted  $\hat{\alpha}_t$ , such that  $\Lambda_t = 1$  :

$$0 < \hat{\alpha}_t = \frac{\ln\left(\eta^B \frac{s_{t-1}^B}{s_{t-1}^A}\right)}{\ln(\eta^A \eta^B)} < 1,$$

Since this is true for all  $t$ , there exist  $0 < \underline{\alpha} < \bar{\alpha} < 1$ , such that for all  $t$  :

$$\underline{\alpha} < \hat{\alpha}_t < \bar{\alpha},$$

Finally, if  $0 \leq \alpha_m \leq \underline{\alpha}$ , then, for all  $t$ ,  $\Lambda_t < 1$ , and  $B$  wins. If  $\bar{\alpha} \leq \alpha_m \leq 1$ , then, for all  $t$ ,  $\Lambda_t > 1$ , then  $A$  wins.

#### **Proof of Proposition 4:**

In section 3, we have shown that, when  $B$  wins the election  $t$ , the two following inequalities hold:

$$s_t^A > (1 - \delta) s_{t-1}^A + 1, \quad (19)$$

and,

$$s_t^B \leq (1 - \delta) s_{t-1}^B + 1. \quad (20)$$

*Claim 1:* We claim that there exists  $k$  such that for all  $t \geq k$ ,  $B$  wins the election  $t$ . Then the two precedent inequalities hold for all  $t \geq k$ , then, for all  $t > k$ :

$$s_t^A > (1 - \delta)^{t-k} s_k^A + t - k, \quad (21)$$

$$s_t^B \leq (1 - \delta)^{t-k} s_k^B + t - k. \quad (22)$$

Combining Inequalities 21 and 22 leads to the following inequality:

$$\frac{s_t^A}{s_t^B} > \frac{(1 - \delta)^{t-k} s_k^A + t - k}{(1 - \delta)^{t-k} s_k^B + t - k},$$

Since  $B$  wins forever after  $k$ , then for all  $t > k$ ,  $\Lambda_t \leq 1$ . Furthermore  $(\Lambda_t)_t$  is increasing, then it converges to a value  $\tilde{\Lambda}$ . Remember that  $\Lambda_{t+1} = \frac{s_t^A}{s_t^B} \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}}$ . Hence, since  $(1 - \delta) < 1$ ,

$$\tilde{\Lambda} > \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}},$$

Then, there exists a real number  $0 < \varepsilon_1 < 1$ , such that a necessary condition for Claim 1 is:

$$\tilde{\Lambda} > \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}} + \varepsilon_1 > \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}}.$$

*Claim 2:* We claim that there exists  $k$  such that for all  $t \geq k$ ,  $A$  wins the election  $t$ . Then for all  $t > k$ ,  $\Lambda_t \geq 1$ . By an argument symmetric to that of Claim 1,  $(\Lambda_t)_t$  converges to  $\hat{\Lambda}$ , and there exists a real number  $0 < \varepsilon_2 < 1$ , such that a necessary condition for Claim 2 is:

$$\hat{\Lambda} < \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}} - \varepsilon_2 < \frac{(\eta^A)^{\alpha_m}}{(\eta^B)^{1-\alpha_m}}.$$

Finally, if,

$$\frac{\ln(\eta^B) + \ln(1 - \varepsilon_1)}{\ln(\eta^A \eta^B)} \leq \alpha_m \leq \frac{\ln(\eta^B) + \ln(1 + \varepsilon_2)}{\ln(\eta^A \eta^B)},$$

then  $\hat{\Lambda} < 1 < \tilde{\Lambda}$ , and Claim 1 and 2 are contradictory, so that no party can win an infinite number of consecutive elections. Then there exist  $\alpha_1 < \alpha_2$  such that no party can win an infinite number of consecutive elections.

**Proof of Proposition 5:**

*Claim 1:* There exists  $k$  such that for all  $t \geq k$ ,  $B$  wins the election  $t$ .

Then at  $t + 1$ , he implements  $m_{t+1}^B = (\alpha_m s_t^B, \eta(1 - \alpha_m) s_t^B)$  and:

$$s_{t+1}^A = 1 + (1 - \delta) \left( \eta(1 - \alpha_m) + \frac{\alpha_m}{\eta} \right) s_t^B,$$

and,

$$s_{t+1}^B = 1 + (1 - \delta) s_t^B.$$

Since  $\delta > 0$ , then  $s_t^B$  converges to  $\frac{1}{\delta}$ , and  $s_t^A$  to  $1 + \frac{1-\delta}{\delta} \left( \eta(1 - \alpha_m) + \frac{\alpha_m}{\eta} \right)$ .

Hence,  $\Lambda_t$  converges to:

$$\tilde{\Lambda}(\alpha_m) = \left( \delta + (1 - \delta) \left( \eta(1 - \alpha_m) + \frac{\alpha_m}{\eta} \right) \right) (\eta)^{2\alpha_m - 1},$$

By Proposition 2,  $(\Lambda_t)_t$  increases and we obtain that *Claim 1* is equivalent to  $\tilde{\Lambda}(\alpha_m) \leq 1$ . The inequality is weak, because by Corollary 2  $\Lambda_t$  cannot attain its limit when  $\tilde{\Lambda}(\alpha_m) = 1$ . Let  $f^B(\alpha_m) = \tilde{\Lambda}(\alpha_m) - 1$ , then *Claim 1* is equivalent to  $f^B(\alpha_m) \leq 0$ . Now we turn to the symmetric Claim for party A:

*Claim 2*: There exists  $k$  such that for all  $t \geq k$ , A wins the election  $t$ .

With the same arguments as those of *Claim 1*, we obtain that  $(\Lambda_t)_t$ , which is now decreasing, converges to:

$$\widehat{\Lambda}(\alpha_m) = \frac{1}{\delta + (1 - \delta) \left( \frac{1 - \alpha_m}{\eta} + \eta \alpha_m \right)} (\eta)^{2\alpha_m - 1},$$

And *Claim 2* is equivalent to  $\widehat{\Lambda}(\alpha_m) \geq 1$ . Let  $f^A(\alpha_m) = \frac{1}{\widehat{\Lambda}(\alpha_m)} - 1$ , then *Claim 2* is equivalent to  $f^A(\alpha_m) \leq 0$ . Furthermore,

$$f^A(\alpha_m) \propto \delta + (1 - \delta) \left( \frac{1 - \alpha_m}{\eta} + \eta \alpha_m \right) - (\eta)^{2\alpha_m - 1},$$

The right-hand term is clearly strictly concave in  $\alpha_m$  and is equal to  $\delta \left( 1 - \frac{1}{\eta} \right) > 0$  when  $\alpha_m = 0$  and  $\delta(1 - \eta) < 0$  when  $\alpha_m = 1$ . Hence,  $f^A(\alpha_m)$  as a unique root in  $]0, 1[$ , denoted  $\alpha_2$ . Furthermore,  $f^A\left(\frac{1}{2}\right) = \delta + \frac{(1 - \delta)}{2} \left( \frac{1}{\eta} + \eta \right) > 0$ , then  $\alpha_2 > \frac{1}{2}$ . Observe that  $f^A(1 - \alpha_m) = f^B(\alpha_m)$ , then  $f^B(\alpha_m)$  has a unique root  $\alpha_1 < \alpha_2$ . Finally, *Claim 1* and *Claim 2* are both contradicted if and only if  $\alpha_m \in [\alpha_1, \alpha_2]$ .

Now we can turn to the comparative statics.  $\alpha_2$  is implicitly defined as a function of  $\delta$  and  $\eta$  by:

$$\delta \eta + (1 - \delta) (1 - \alpha_2 + \eta^2 \alpha_2) - (\eta)^{2\alpha_2} = 0, \quad (23)$$

Then, differentiating this equation with respect to  $\eta$  leads to  $\frac{\partial \alpha_2}{\partial \eta} = \frac{N(\delta, \cdot)}{D(\delta, \cdot)}$  with,

$$N = 2\alpha_2 (\eta)^{2\alpha_2 - 1} - \delta - 2\alpha_2 (1 - \delta) \eta,$$

and,

$$D = (1 - \delta) (\eta^2 - 1) - 2 (\eta)^{2\alpha_2} \ln \eta,$$

It is easy to verify that  $\frac{\partial N}{\partial \delta} = 2\alpha_2\eta - 1 > 0$  because  $\alpha_2 > \frac{1}{2}$ . Since  $\eta > 1$ , we obtain:

$$N \leq 2\alpha_2 ((\eta)^{2\alpha_2-1} - \eta) < 0,$$

Furthermore,

$$\frac{\partial D}{\partial \eta} \propto (1 - \delta) (\eta)^{2(1-\alpha_2)} - (1 + 2\alpha_2 \ln \eta),$$

Let  $g(\alpha_2) = (1 - \delta) (\eta)^{2(1-\alpha_2)} - (1 + 2\alpha_2 \ln \eta)$ , then  $g'(\alpha_2) < 0$ . Since  $g(1) = -\delta - 2\alpha_2 \ln \eta$ , then  $\frac{\partial D}{\partial \eta} < 0$ . Furthermore, when  $\eta = 1$ ,  $D = 0$ , then,

$$D < 0,$$

Finally,

$$\frac{\partial \alpha_2}{\partial \eta} > 0.$$

Concerning the depreciation rate, differentiating 23 with respect to  $\delta$  leads to:

$$\frac{\partial \alpha_2}{\partial \delta} = \frac{1 + (\eta^2 - 1) \alpha_2 - \eta}{D},$$

Here, the numerator of the right-hand side is increasing in  $\alpha_2$  and is equal to  $(\eta - 1)^2$  when  $\alpha_2 = \frac{1}{2}$ , then it is always positive, hence:

$$\frac{\partial \alpha_2}{\partial \delta} < 0.$$

### Proof of Proposition 6:

We prove that  $\bar{a}_t = \arg \min_{a_t \in [0, \eta^A s_{t-1}^A]} \left( W_m \left( a_t, s_{t-1}^A - \frac{a_t}{\eta^A} \right) = W_m \left( m_t^B \right) \right)$  exists and is unique. The precedent equality is equivalent to:

$$\left( \frac{\mu}{\alpha_m} \right)^{\alpha_m} \left( \frac{1 - \mu}{1 - \alpha_m} \right)^{1-\alpha_m} = \frac{1}{\Lambda_t(\alpha_m)}, \quad (24)$$

where  $\mu = \frac{a_t}{\eta^A s_{t-1}^A} \in [0, 1]$ . Here  $\Lambda_t > 1$ , and, by proposition 3,  $\alpha_m > 0$ . The right-hand side of (24) is null when  $\mu = 0$  and equal to 1 when  $\mu = \alpha_m$ . Thus 24 admits a solution. If  $\alpha_m = 1$ , then the right-hand side is strictly decreasing in  $\mu$ , and the solution is unique. If  $\alpha_m < 1$ , then the right-hand side is concave in  $\mu$ , is null when  $\mu = 0$  or 1, and maximal when  $\mu = \alpha_m$ . Thus 24 has two different solutions. Hence, the set of solutions is finite, then the argmin exists and is unique. Now, consider the following maximization program:

$$\begin{aligned} & \underset{z_t^A \in \mathbf{A}(t)}{\text{Max}} \quad \Lambda_{t+1}, \\ \text{s.t.} \quad & W_m(z_t^A) \geq W_m(m_t^B). \end{aligned}$$

Since  $\Lambda_{t+1}$  is strictly decreasing in  $a_t$  and strictly increasing in  $b_t$ ,  $\bar{z}_t = \left(\bar{a}_t, s_{t-1}^A - \frac{\bar{a}_t}{\eta^A}\right)$  is the unique solution to this maximization problem.

**Proof of Proposition 7:** (i) It is simple to verify that the median voter's preferred program in  $\mathbf{A}(t)$  when candidate  $A$  extracts a rent  $r_t^A$  is  $\tilde{z}_t^A = \left(\eta^A \alpha_m (s_{t-1}^A - r_t^A), (1 - \alpha_m) (s_{t-1}^A - r_t^A)\right)$ . Then, the median voter weakly prefers  $\tilde{z}_t^A$  to  $m_t^B$  if and only if:

$$r_t^A \leq \left(1 - \frac{1}{\Lambda_t}\right) s_{t-1}^A.$$

(ii) Symmetrically, the median voter preferred platform in  $\mathbf{B}(t)$ , when candidate  $B$  extracts a rent  $r_t^B$ , is  $\tilde{z}_t^B = \left(\alpha_m (s_{t-1}^B - r_t^B), \eta^B (1 - \alpha_m) (s_{t-1}^B - r_t^B)\right)$ . Then, the median voter weakly prefers  $\tilde{z}_t^B$  to  $m_t^A$  if and only if:

$$r_t^B \leq (1 - \Lambda_t) s_{t-1}^B.$$

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