



Does a Disadvantaged Candidate Choose an Extremist Position ?

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Abstract

Does a disadvantaged candidate always choose an extremist program? When does a less competent candidate have an incentive to move to extreme positions in order to differentiate himself from the more competent candidate? Recent works answer by the affirmative (Grosseclose 1999, Ansolabehere and Snyder 2000, Aragonès and Palfrey 2002, 2003). We consider a two candidates electoral competition over public consumption, with a two dimensional policy space and two dimensions of candidates heterogeneity. In this setting, we show that the conclusion depends on candidates relative competences over the two public goods and distinguish between two types of advantages (an absolute advantage and comparative advantage in providing the two public goods).

Keywords:

candidate quality, extremism, public goods consumption.

Titre: Un candidat désavantagé choisit-il une position extrémiste?

Résumé :

Un candidat désavantagé choisit-il toujours une position extrémiste? Quand est-ce qu'un candidat a intérêt à se déplacer vers une position extrémiste pour se différencier d'un candidat plus compétent? Des travaux récents répondent positivement à cette question (Grosseclose 1999, Ansolabehere et Snyder 2000, Aragonès et Palfrey 2002, 2003). Nous considérons une compétition électorale à deux candidats sur des biens publics, avec un espace politique à deux dimensions et deux dimensions dans l'hétérogénéité des candidats. Nous montrons que les résultats dépendent des compétences relatives des candidats pour la fourniture des deux biens publics, en distinguant deux types d'avantages (avantage relatif ou absolu).

Mots clefs:

qualité des candidats, extrémisme, biens publics

JEL: C72, D72

Does a disadvantaged candidate choose an extremist position?*

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Abstract

Does a disadvantaged candidate always choose an extremist program? When does a less competent candidate have an incentive to move to extreme positions in order to differentiate himself from the more competent candidate? Recent works answer by the affirmative (Grosseclose 1999, Ansolabehere and Snyder 2000, Aragonès and Palfrey 2002, 2003). We consider a two candidates electoral competition over public consumption, with a two dimensional policy space and two dimensions of candidates heterogeneity. In this setting, we show that the conclusion depends on candidates relative competences over the two public goods and distinguish between two types of advantages (an absolute advantage and comparative advantage in providing the two public goods).

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1 Introduction

Does a disadvantaged candidate always choose an extremist program? When does a less competent candidate have an incentive to move to extreme positions in order to differentiate himself from the more competent candidate?

Our objective is to answer these questions, and in so doing, to reexamine the results obtained in the recent literature on the competence of politicians. We consider a two candidates electoral competition over public consumption, with a two dimensional policy space and two dimensions of candidates heterogeneity. In this setting, we show that the conclusion depends on candidates relative competences over the two public goods and distinguish between two types of advantages (an absolute advantage and comparative advantage in providing the two public goods).

The closest works to this paper are Ansolabehere and Snyder (2000), Aragonés and Palfrey (2002, 2003), Groseclose (1999). These papers focus on variations of the spatial model of election, introduced by Downs (1957), where two candidates have to choose a position on the unit interval. In all these works, candidates have an unidimensional personal characteristic that determines their (dis)advantage. In these analyzes, voters utility is separable in policy and politician personal characteristic. They study the existence of the equilibrium and conclude that the advantaged candidate locates more centrally than the disadvantaged one.

Ansolabehere and Snyder (2000) show that, in the absence of uncertainty, the advantaged candidate locates at the center, and that the disadvantaged candidate always loses and locates anywhere on the unit interval. As noticed by Aragonés and Palfrey (2002), the existence of equilibrium becomes a problem when there is uncertainty or when candidates maximize their share of votes. In this last case, the advantaged candidate always wants to choose the same program as the disadvantaged candidate to get all the votes, whereas the disadvantaged candidate has an incentive to differentiate his platform in order to get at least some votes. Aragonés and Palfrey (2002) examine the existence of mixed strategy equilibria in this electoral competition. They consider a discrete unit interval, and show that, when the advantage is small enough, the advantaged candidate chooses a probability distribution with a single peak in the center, whereas the disadvantaged candidate chooses a probability distribution with two peaks, one on each side of the center. In the present work, as in these two papers, voters utility function can be written as additively separable in policy and valence, but candidates scores on

the valence dimension differs among voters. If a candidate benefits from an absolute advantage, our results are close to Ansolabehere and Snyder (2000); when an equilibrium exists, a candidate with an absolute advantage generally locates centrally, and the disadvantaged candidate locates anywhere in his policy set.

Groseclose (2001) and Aragones and Palfrey (2003) show that the existence problem can disappear when candidates have policy preferences. Groseclose (1999) shows that when candidates put sufficiently high weight on policy, a pure strategy equilibrium may exist and the advantaged candidate chooses a more moderate position than the disadvantaged candidate. Aragones and Palfrey (2003) consider two candidates who privately know their ideal point and their tradeoffs between policy preferences and winning and show that a pure strategy equilibrium always exists. They also show that the result of Aragones and Palfrey (2002) is the limit case when policy preferences goes to zero.

One stream of the political economy literature, reviewed by Persson and Tabellini (2000, chapter 4, section 4.7), assumes that candidates differ in their ability to deliver services to citizens¹. These papers investigate electoral accountability when voters have incomplete information on politicians. In our model, candidates differ in their competences but they are common knowledge.

Other scholars consider different asymmetries between the candidates². Several analyzes show that Republican and Democrat have different effects on the economy³, and study the impact of real or perceived economic perfor-

¹Rogoff and Siebert (1988) study a model of adverse selection; Rogoff (1990) and Banks and Sundaram (1993, 1996) study politicians accountability in models with moral hazard and adverse selection.

²See Ansolabehere and Snyder (2000) and Groseclose (2001) for a review of this literature.

³Hibbs (1977), Beck (1982), and Chappel and Keech (1986) show that Democrat and Republican governments have different influences on the unemployment rate. Alesina and Sachs (1988) and Tabellini and La Via (1989) show that parties are associated with different monetary policies.

mance on elections outcomes⁴.

However, none of these papers considers candidates with a two dimensional competence. In section 2, we propose a political competition model where the candidates propose two public goods. The two opportunistic candidates have different competences to provide two public goods. They share the same beliefs on the median voter preferences and maximize their probability of winning. We define two kinds of advantages in this model, the absolute advantage (one candidate is better in the provision of both goods) and the comparative advantage (each candidate is better in the provision of one of the two goods). In section 3, we focus on the case where one candidate has an absolute advantage; our results are similar to those of spatial valence models, that is, an equilibrium exists if and only if the advantage is large enough, the advantaged candidate wins with certainty, and he generally locates more centrally than the disadvantaged candidate. In section 4, we analyze the situation of comparative advantages; the results are sensibly different: candidates specialize in the provision of one of the public goods. We show that a pure strategy equilibrium generally exists. Finally, candidate's equilibrium probability of winning increases with the candidate competences. We then propose some discussions in section 5 and conclude in section 6

2 The model

The model is inspired by the "Multidimensional Public Consumption Model" introduced in Tabellini and Alesina (1990). We first define the two types of agents, voters and candidates:

Voters: Let assume a population of voters of mass 1. The government provides two public goods, $x \geq 0$ and $y \geq 0$. Citizens disagree on the importance of the two public goods and citizen i 's preferences are parametrized by the weight $\alpha_i \in [0, 1]$ he places on public good x . If $1 < \alpha_i < 0$, his preferences

⁴Fiorina (1981) and Austen-Smith and Banks (1989) assume that citizens vote retrospectively conditioned to the difference between platforms and performance. Aragoes (1997) surveys and contributes to the literature on the "negativity effect" where voters vote on past performances and weight more negative than positive informations. See also Kernell (1977), Lau (1982), Klein (1991), Abelson and Levi (1985), Mueller (1973), Bloom and Price (1975), and Key (1966).

are summarized in the following utility function:

$$\begin{aligned} W_i() &= u(c) + \alpha_i \ln(x) + (1 - \alpha_i) \ln(y) \text{ if } x, y > 0, \\ &= -\infty \text{ if } xy = 0. \end{aligned} \tag{1}$$

If $\alpha_i = 0$,

$$\begin{aligned} W_i() &= u(c) + \ln(y) \text{ if } y > 0, \\ &= -\infty \text{ if } y = 0. \end{aligned} \tag{2}$$

And, if $\alpha_i = 1$,

$$\begin{aligned} W_i() &= u(c) + \ln(x) \text{ if } x > 0, \\ &= -\infty \text{ if } x = 0. \end{aligned} \tag{3}$$

These preferences belong to the set of intermediate preferences defined by Grandmont (1978), and satisfy the single crossing property. Hence, a Condorcet winner exists and it is given by the preferred policy of the median voter α_m .

Candidates: We consider two office motivated candidates A and B . When a candidate is elected, he gets an exogenous ego-rent normalized to 1. In the seminal model of multidimensional public consumption, the two candidates have the same competencies to provide both public goods. And, when the government budget is fixed (as in our model), both candidates platforms converge to the median voter preferred policy.

We relax this assumption and suppose that each candidate has different competencies associated to each public good. Candidates are heterogeneous on two dimensions. Let (η_x^C, η_y^C) be candidate C competencies to provide x and y (for $C = A, B$). These competencies determine the candidates' efficiency in providing each public good, and are inversely related to the cost of providing each public good. With these assumptions, candidates face different budget constraints when they are in power. We consider linear costs to provide both public goods and normalize the government budget to 1. Hence, if candidate C is elected, his budget constraint is given by ⁵:

$$\frac{x}{\eta_x^C} + \frac{y}{\eta_y^C} = 1, \tag{4}$$

⁵Since rents from power are exogenous, candidates have an incentive to exhaust their entire budget.

for $C = A, B$, with $\eta_x^C, \eta_y^C > 0$ and $x, y \geq 0$.

Since we suppose that platforms must be credible and there is no debt, candidates have different policy sets. Let $z^C = (x^C, y^C)$ denote one candidate C platform, $C = A, B$.

Uncertainty: Candidates share the same beliefs over the distribution of voters. They suppose that $F(\alpha)$ is the probability that α_m is lower than α , i.e. $F(\alpha) = \Pr(\alpha_m \leq \alpha)$ for all $\alpha \in [0, 1]$. Moreover, we suppose that the two candidates maximize their probability of winning. However, the model would be unchanged if we suppose that there is no uncertainty, F is the cumulative distribution of α_i on $[0, 1]$ and the two candidates maximize their number/share of votes. Indeed, in both cases, the payoff function of candidate A is:

$$\pi^A(z^A, z^B) = \int_{\{\alpha_i \in [0, 1]: W_i(z^A) \geq W_i(z^B)\}} dF(\alpha_i). \quad (5)$$

Remark that if we put all the competencies to 1, then the model is exactly identical to the multidimensional public consumption model. The policy set becomes unidimensional and there exists a unique equilibrium where both platforms converge to the expected median voter preferred program. Now we show that results are affected when competencies differ among goods and candidates.

2.1 Definitions

We define absolute and comparative advantages in the context of public goods consumption. A candidate has an absolute advantage when he outperforms his opponent over the two policy dimensions. A natural definition of an absolute advantage is the following:

Definition 1 *Candidate A has an absolute advantage on another candidate B to provide both public goods, if and only if $\eta_x^A \geq \eta_x^B$ and $\eta_y^A \geq \eta_y^B$, with at least one strict inequality.*

We define the comparative advantages situation where each candidate is relatively better than his opponent in providing one of the public goods. Formally,

Definition 2 *Candidate A has a comparative advantage to provide x and B has a comparative advantage to provide y if and only if $\frac{\eta_x^A}{\eta_x^B} > 1 > \frac{\eta_y^A}{\eta_y^B}$.*

2.2 Payoff functions

In this section, we derive the candidates payoff functions. Candidates maximize their probability of victory. Let π^A and π^B denote candidate A and candidate B 's payoff. Furthermore as $\pi^B = 1 - \pi^A$, it is sufficient to compute candidate A 's payoff function. If all quantities are strictly positive⁶, voter i prefers z^A to z^B if and only if:

$$\alpha_i \ln \left(\frac{x_A y_B}{x_B y_A} \right) \geq \ln \left(\frac{y_B}{y_A} \right). \quad (6)$$

Let $\hat{\alpha}$ be the type of the voter indifferent between z^A and z^B :

$$\hat{\alpha} \ln(x_A) + (1 - \hat{\alpha}) \ln(y_A) = \hat{\alpha} \ln(x_B) + (1 - \hat{\alpha}) \ln(y_B). \quad (7)$$

We deduce from this expression:

$$\hat{\alpha} = 1 - \frac{\ln \left(\frac{x_A}{x_B} \right)}{\ln \left(\frac{x_A y_B}{x_B y_A} \right)}. \quad (8)$$

Hence, candidate A gets votes from left (small α_i) or votes from right (high α_i), depending on the candidates' relative positions. Formally, if $\frac{x_A y_B}{x_B y_A} > 1$, candidate A 's payoff is given by:

$$\pi^A(z^A, z^B) = 1 - F(\hat{\alpha}). \quad (9)$$

If $\frac{x_A y_B}{x_B y_A} = 1$, then all voters prefer z^A to z^B if and only if $y_A \geq y_B$:

$$\begin{aligned} \pi^A(z^A, z^B) &= 1 \text{ if } y_A > y_B, \\ &= \frac{1}{2} \text{ if } y_A = y_B, \\ &= 0 \text{ if } y_B > y_A. \end{aligned} \quad (10)$$

And, if $\frac{x_A y_B}{x_B y_A} < 1$, candidate A 's payoff is given by:

$$\pi^A(z^A, z^B) = F(\hat{\alpha}). \quad (11)$$

We now turn to the determination of equilibrium when one of the candidates has an absolute advantage.

⁶Cases where candidates propose only of one good are considered in the proofs.

3 Absolute advantage of one of the candidates

3.1 Equilibria

The situation of an absolute advantage is similar to the unidimensional spatial model when one candidate has a valence advantage, and our results are comparable to those of spatial models with uncertainty over the median voter preferences. When the advantage is small, as in spatial models⁷, there is no pure strategy equilibrium.

Proposition 1 *Suppose that A has an absolute advantage. If $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} > 1$, then there does not exist a pure strategy equilibrium.*

The intuition of this result is the following. The advantaged candidate gets all votes when he imitates the disadvantaged candidate. Since the advantage is small, the disadvantaged candidate can differentiate himself from the advantaged candidate and get a positive share of votes. There is thus no pure strategy equilibrium. Now, when the advantage is large enough, the advantaged candidate can provide large quantities of both public goods so that the disadvantaged candidate gets no vote, whatever his policy choice⁸:

Proposition 2 *Suppose that A has an absolute advantage. If $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} \leq 1$, then there exists a continuum of pure strategy equilibria where payoffs are $\pi^{A*} = 1$ and $\pi^{B*} = 0$, and platforms are given by:*

$$z^{A*} = \left(\mu, \left(1 - \frac{\mu}{\eta_x^A} \right) \eta_y^A \right),$$

with $\mu \in \left[\eta_x^B, \left(1 - \frac{\eta_y^B}{\eta_y^A} \right) \eta_x^A \right]$, and z^{B*} is any candidate B feasible program.

In this situation, the advantaged candidate is always certain to win the election, because he always provides more of both goods than the disadvantaged candidate. We now analyze the relation between absolute advantage and the symmetry of the electoral platform.

⁷See Groseclose (1999), Ansolabehere and Snyder (2000) and Aragonés and Palfrey (2002) for similar results in spatial models.

⁸See Ansolabehere and Snyder (2000) for a similar result in a spatial model with no uncertainty about the voters distribution.

3.2 Absolute advantage and location on the policy space

In our context, we need to specify what we call a symmetric platform in the public goods consumption model. We suppose from now on that F is the cumulative of the uniform distribution on $[0, 1]$.

Definition 3 A platform $z = (x, y) \in [0, 1]^2$ is **symmetric** if and only if $x = y$.

Now, we define the following order relation to compare candidates positions:

Definition 4 A platform $z = (x, y)$ is (weakly) **more symmetric** than a platform $z' = (x', y')$ if and only if $I(z) = \left| \frac{x}{x+y} - \frac{1}{2} \right| \leq I(z') = \left| \frac{x'}{x'+y'} - \frac{1}{2} \right|$.

We call $I(z)$ the position index of policy z . The more a platform is asymmetric, the higher the position index. We use this index to compare the candidates equilibrium platforms.

In the case where candidate A has an absolute advantage, these definitions do not allow to make a clear comparison, because of the multiplicity of equilibria. For example, suppose that $\eta_x^B = \eta_y^B = 10$ and $\eta_x^A = \eta_y^A = 30$. The condition $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} \leq 1$ holds, then proposition 2 ensures that (z^A, z^B) and $(z^A, z^{B'})$ with $z^A = (10, 20)$, $z^B = (1, 9)$ and $z^{B'} = (5, 5)$ are two equilibria. The position indices are: $I(z^{B'}) = 0 < I(z^A) = \frac{1}{6} < I(z^B) = \frac{2}{5}$. Hence $z^{B'}$ is more symmetric than z^A which is more symmetric than z^B .

We thus consider the average candidates equilibrium positions of the candidates. Let \mathbf{S}^{C^*} be the set of candidate C equilibrium platforms.

Definition 5 If the equilibrium payoffs are identical for every equilibrium, the set of candidate C ' equilibrium platforms, \mathbf{S}^{C^*} , is said to be (weakly) **generally more symmetric** than the set of candidate C' ' equilibrium platforms, $\mathbf{S}^{C'^*}$, if:
$$\int_{z \in \mathbf{S}^{C^*}} I(z) dz \leq \int_{z \in \mathbf{S}^{C'^*}} I(z) dz.$$

When a candidate has an absolute advantage, he always wins with probability 1, and his opponent always loses. Our definitions suppose that each candidate plays one of the equilibrium strategies with equal probability. We obtain the following result:

Proposition 3 *If candidate A has an absolute advantage, the set of equilibrium platforms for candidate A is generally more symmetric than the set of equilibrium platforms for candidate B.*

This result is similar to Ansolabehere and Snyder (2000). They show, in a unidimensional spatial model, that the set of equilibrium platforms is generally more central for the advantaged candidate than for the disadvantaged candidate. We focus now on the situation where candidates have comparative advantages.

4 Comparative advantage

In this section, we derive the unique equilibrium when candidates have comparative advantages, and provide necessary and sufficient conditions for existence.

4.1 Equilibrium

Suppose A has a comparative advantage to provide x and B has a comparative advantage to provide B . Let $\theta_x = \frac{\eta_x^A}{\eta_x^B}$ and $\theta_y = \frac{\eta_y^B}{\eta_y^A}$ be the respective strength of candidate A and candidate B comparative advantage (in this case, definition 2 states that $\theta_x, \theta_y > 1$). The following result holds:

Proposition 4 *Suppose that candidate A has a comparative advantage in good x and candidate B a comparative advantage in good y . Then, there exists at most one pure strategy equilibrium, where the equilibrium payoffs are:*

$$\pi^{A*} = 1 - \hat{\alpha}^*, \quad \pi^{B*} = \hat{\alpha}^*,$$

with $\hat{\alpha}^* = \frac{\ln \theta_y}{\ln(\theta_x \theta_y)}$, and the equilibrium platforms are:

$$z^{A*} = (\eta_x^A \hat{\alpha}^*, \eta_y^A (1 - \hat{\alpha}^*)),$$

$$z^{B*} = (\eta_x^B \hat{\alpha}^*, \eta_y^B (1 - \hat{\alpha}^*)).$$

The intuition for the proof is as follows. Candidates cannot both choose platforms specializing in one of the public goods. If it were true, one of them would have an absolute advantage, and by the same reasoning as in the

previous section, a pure strategy equilibrium may fail to exist. Candidates cannot specialize in the public good for which they don't have a comparative advantage, since they would then have an incentive to use their advantage and provide more of both good than their opponent. Hence, candidates must be specializing in the public good for which they have a comparative advantage.

However, when the comparative advantage of a candidate is not high enough, the other candidate may want to imitate it. As in the case of a small absolute advantage, one cannot guarantee existence of a pure strategy equilibrium. This leads to the following result (here, $\theta_x, \theta_y > 1$ is always true).

Proposition 5 *The equilibrium exists if and only if $\theta_x \ln(\theta_x) \geq \frac{\ln(\theta_y)}{\theta_y}$ and $\theta_y \ln(\theta_y) \geq \frac{\ln(\theta_x)}{\theta_x}$.*

Figure 1 represents the area where a pure strategy equilibrium exists:

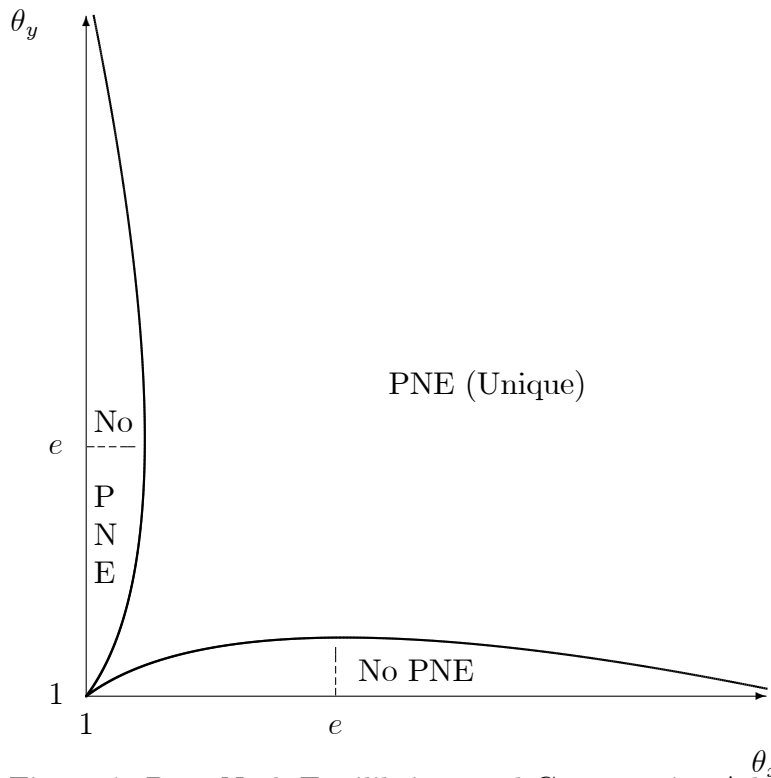


Figure 1: Pure Nash Equilibrium and Comparative Advantages

We now present two comparative statics results on the equilibrium. First we show that a candidate who has a higher comparative advantage, obtains a higher payoff.

Corollary 1 *A candidate payoff increases with his comparative advantage:*

$$\frac{\partial \pi^{A*}}{\partial \theta_x} > 0, \text{ and } \frac{\partial \pi^{B*}}{\partial \theta_y} > 0.$$

However, we also obtain the less obvious result that, when candidate A becomes better at providing x , his equilibrium quantity of x does not necessarily increase:

Corollary 2

- (i) *The sign of $\frac{\partial x^{A*}}{\partial \eta_x^A}, \frac{\partial y^{B*}}{\partial \eta_y^B} \propto \ln(\theta_x \theta_y) - 1$ can be positive or negative*
- (ii) $\frac{\partial y^{A*}}{\partial \eta_y^A}, \frac{\partial x^{B*}}{\partial \eta_x^B} > 0$.

Corollary 2 shows that an increase in a candidate's competence does not necessarily translate into an increase in the public good provision in the equilibrium platform. This result stems from two countervailing effects. On the one hand, when η_x^A increases, candidate A substitutes public good x to public good y (a substitution effect). But, on the other hand, he has an incentive to increase his provision of public good y^{A*} (an income effect which may dominate the substitution effect).

4.2 Comparative advantage and platform symmetry

In this section, we provide a sufficient condition under which candidate B chooses a more symmetric platform than candidate A when both candidates have comparative advantages in one of the public goods (remember $\theta_x = \frac{\eta_x^A}{\eta_x^B} > 1$ and $\theta_y = \frac{\eta_y^B}{\eta_y^A} > 1$).

Proposition 6 *If A has a comparative advantage in x and B a comparative advantage in y then z^{B*} is always more symmetric than z^{A*} if and only if*

$$\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B} \geq \left(\frac{\ln \frac{\eta_x^A}{\eta_x^B}}{\ln \frac{\eta_y^B}{\eta_y^A}} \right)^2.$$

Proposition 6 provides a necessary and sufficient condition for the platform of candidate B to be more balanced than that of candidate A . This condition holds when $\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B}$ is large enough. The natural question arising at this point can be, does there exist a link between competencies symmetry and candidate's platform symmetry? Formally, does $|\eta_x^A - \eta_y^A| \geq |\eta_x^B - \eta_y^B|$ means that $\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B} \geq \left(\frac{\ln \frac{\eta_x^A}{\eta_x^B}}{\ln \frac{\eta_y^A}{\eta_y^B}} \right)^2$? The answer is no. Indeed, consider the following numerical example; let $\eta_x^A = 10$, $\eta_y^A = 5$, $\eta_x^B = 6$ and $\eta_y^B = 6$, then $|\eta_x^A - \eta_y^A| \geq |\eta_x^B - \eta_y^B| = 0$ and $\frac{\eta_x^A \eta_x^B}{\eta_y^A \eta_y^B} = 2 \leq \left(\frac{\ln \frac{10}{6}}{\ln \frac{5}{6}} \right)^2$. Then B has more balanced competencies but his program is more asymmetric than candidate A 's one.

5 Discussions

In this section, we discuss two points. The first remark highlights the link between our model and valence models. The second point we discuss relates to the voters utility function form.

5.1 Link with valence models

In valence models, there are two orthogonal dimensions, one being exogenous (valence) and the other being endogenous (policy). The log form of the voters utility function makes the model close to valence models. Recall that when C proposes $z^C = (x^C, y^C)$, the platform must respect:

$$\frac{x^C}{\eta_x^C} + \frac{y^C}{\eta_y^C} = 1, \quad (12)$$

for $C = A, B$. To compare the public consumption model to valence models, we propose two variable changes. Let $s^C = \frac{x^C}{\eta_x^C}$ denote the share invested in good x by candidate C , $C = A, B$. After this transformation, strategy s^C belongs to $[0, 1]$. With the budget constraints, we can redefine voter i utility function as follows:

$$V_i(s^C) = u_i(s^C) + \delta_i^C, \quad (13)$$

for $C = A, B$; $u_i(s^C) = \alpha_i \ln(s^C) + (1 - \alpha_i) \ln(1 - s^C)$ and $\delta_i^C = \alpha_i \ln(\eta_x^C) + (1 - \alpha_i) \ln(\eta_y^C)$ ⁹.

This is a non-spatial valence model. Indeed, voters utility functions are separable in the policy and valence dimensions. We will now consider the equivalent of the absolute advantage in a valence model. Say that a candidate has a Unanimity Valence Advantage (UVA) when all voters consider him best on the valence dimension:

Definition 6 *Candidate A has a Unanimity Valence Advantage (UVA) if and only if: $\forall i, \delta_i^A \geq \delta_i^B$ with, for at least one voter j , $\delta_j^A > \delta_j^B$.*

The following proposition confirms the intuition that the UVA and the absolute advantage are, in our context (log utility), two similar definitions:

Proposition 7 *Candidate A has a UVA if and only if he has an absolute advantage.*

Note that this comparison is only possible because the voters utility functions have a log form.

5.2 Extension to other utility functions

Let consider a more general class of utility functions:

$$W_i() = \alpha_i G(x) + (1 - \alpha_i) H(y),$$

where $G', H' > 0$. It seems not possible to make the same comparison with valence models anymore. However, the results of propositions 1 to 3 still hold because the proofs only rely on the monotonicity of the utility functions and the budget constraints. It seems more difficult to extend the results of the model when candidates have comparative advantages. Indeed, the proof of proposition 4 relies on the log-form since it allows to characterize the unique possible equilibrium. We can conjecture that (if an equilibrium exists) both candidates will still specialize. It seems difficult to determine the situations where an equilibrium exists, since the payoff functions are not continuous.

⁹Notice that δ_i^C may be negative. The important argument is the difference between both candidates images $\delta_i^A - \delta_i^B$. If the latter is positive, then i prefers A to B on the non-policy dimension.

6 Conclusion

We have shown that when candidates have two-dimensional competences, two kinds of advantages can be defined. When one the candidates has an absolute advantage, he generally adopts a more symmetric equilibrium platform than the disadvantaged candidate. The conclusion is ambiguous when the candidates have comparative advantages. Candidates provide different quantities of public goods and their probability of winning increases with their competencies. Furthermore, we have given necessary and sufficient conditions for the existence of a (unique) pure strategy equilibrium.

Appendix

Proof of Proposition 1: $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} > 1$. We distinguish two cases. Suppose $((x_A^*, y_A^*), (x_B^*, y_B^*))$ is an equilibrium:

Case 1 *If not $\frac{x_A^*}{x_B^*} \geq 1, \frac{y_A^*}{y_B^*} \geq 1$, with at least one inequality being strict and $(x_A^*, y_A^*) \neq (x_B^*, y_B^*)$: A can propose $x'_A = x_B^*$ and $y'_A = \eta_y^A \left(1 - \frac{x_B^*}{\eta_x^A}\right) > \eta_y^B \left(1 - \frac{x_B^*}{\eta_x^B}\right) = y_B^*$ because he has an absolute advantage. Then, it is not an equilibrium.*

Case 2 *If $\frac{x_A^*}{x_B^*} \geq 1, \frac{y_A^*}{y_B^*} \geq 1$, with at least one inequality being strict and $(x_A^*, y_A^*) \neq (x_B^*, y_B^*)$. Candidate B' payoff is null ($\pi_B = 0$), because he proposes smaller quantities of both public goods than his adversary. We distinguish the following subcases:*

If $x_A^ > \eta_x^B$, then $y_A^* = \eta_y^A \left(1 - \frac{x_A^*}{\eta_x^A}\right) < \eta_y^A \left(1 - \frac{\eta_x^B}{\eta_x^A}\right) < \eta_y^B$. B can propose $y'_B > y_A^*$, hence $\pi_B = F(\hat{\alpha}) > 0$.*

If $y_A^ > \eta_y^B$, then $x_A^* = \eta_x^A \left(1 - \frac{y_A^*}{\eta_y^A}\right) < \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A}\right) < \eta_x^B$. B can propose $x'_B > x_A^*$, hence $\pi_B = [1 - F(\hat{\alpha})] > 0$.*

If $x_A^ < \eta_x^B$ and $y_A^* < \eta_y^B$, then B can move to (x''_B, y''_B) with $y''_B > y_A^*$ and $x''_B > x_A^*$ and he gets a strictly positive payoff. Finally, it cannot be an equilibrium.*

Proof of Proposition 2: $\frac{\eta_x^B}{\eta_x^A} + \frac{\eta_y^B}{\eta_y^A} \leq 1$: The proof is in two steps. In the first step, we show that the situations described in proposition 2 are equilibria. In the second step, we show that there is no other equilibrium.

Step 1: Let us prove that $((x_A^*, y_A^*), (x_B^*, y_B^*)) = \left(\left(\mu, \eta_y^A \left(1 - \frac{\mu}{\eta_x^A} \right) \right), (x_B, y_B) \right)$ with $\mu \in \left[\eta_x^B, \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A} \right) \right]$ is an equilibrium. Here, $x_A^* \geq \eta_x^B \geq x_B$, $\forall x_B \in [0, \eta_x^B]$ and $y_A^* \geq y_B$, $\forall y_B \in [0, \eta_y^B]$, with at least one inequality being strict. Hence, candidate B cannot be strictly better. Furthermore, A gets the maximum payoff, $\pi_A^* = 1$.

Step 2: Now, let us show that $((x_A^*, y_A^*), (x_B^*, y_B^*)) = \left(\left(\mu, \eta_y^A \left(1 - \frac{\mu}{\eta_x^A} \right) \right), (x_B, y_B) \right)$ with $\mu \notin \left[\eta_x^B, \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A} \right) \right]$ is not an equilibrium. Since $\mu < \eta_x^B$ or $\mu > \eta_x^A \left(1 - \frac{\eta_y^B}{\eta_y^A} \right)$, B can not receive a strictly positive payoff. Finally, it cannot be an equilibrium.

Proof of Proposition 3: Candidate A 's mean equilibrium position index is:

$$\bar{I}_A = \frac{\left| \frac{\eta_x^B}{\left(1 - \frac{\eta_x^B}{\eta_x^A} \right) \eta_y^A + \eta_x^B} - \frac{1}{2} \right| + \left| \frac{\left(1 - \frac{\eta_y^B}{\eta_y^A} \right) \eta_x^A}{\eta_y^B + \left(1 - \frac{\eta_y^B}{\eta_y^A} \right) \eta_x^A} - \frac{1}{2} \right|}{2},$$

and, candidate B 's mean index is $\bar{I}_B = \frac{1}{2}$. Furthermore, by definition of an absolute advantage, $\frac{\eta_x^B}{\eta_x^A} \leq 1$ and $\frac{\eta_y^B}{\eta_y^A} \leq 1$ with at least one strict inequality, so that $\bar{I}_A < \frac{1}{2} = \bar{I}_B$.

Proof of Proposition 4: Up to a change of variable ($s^C = \frac{x^C}{\eta_x^C}$), the model is not modified when the utility of voter i is given by:

$$V_i(s^C) = u_i(s^C) + \delta_i^C,$$

with $s^C \in [0, 1]$, $u_i(s^C) = \alpha_i \ln(s^C) + (1 - \alpha_i) \ln(1 - s^C)$ and $\delta_i^C = \alpha_i \ln(\eta_x^C) + (1 - \alpha_i) \ln(\eta_y^C)$, $C = A, B$ (see the discussion section for a detailed explanation).

The indifferent voter is given by (if $s^C \neq 0, 1$; $C = A, B$):

$$\hat{\alpha}(s^A, s^B) = \frac{N(s^A, s^B)}{D(s^A, s^B)},$$

where $N(s^A, s^B) = \ln \theta_y + \ln \frac{1-s^B}{1-s^A}$ and $D(s^A, s^B) = \ln \theta_x \theta_y + \ln \frac{s^A}{s^B} \frac{1-s^B}{1-s^A}$.

Suppose $0 < \hat{\alpha}(s^A, s^B) < 1$, then in an interior equilibrium (s^{A*}, s^{B*}) , the first order conditions are:

$$\frac{\partial \hat{\alpha}(s^{A*}, s^{B*})}{\partial s^A} \propto s^{A*} D(s^{A*}, s^{B*}) - N(s^{A*}, s^{B*}) = 0,$$

and,

$$\frac{\partial \hat{\alpha}(s^{A*}, s^{B*})}{\partial s^B} \propto N(s^{A*}, s^{B*}) - s^{B*} D(s^{A*}, s^{B*}) = 0,$$

then,

$$s^{A*} = s^{B*} = \hat{\alpha}(s^{A*}, s^{B*}).$$

Hence,

$$\hat{\alpha}(s^{A*}, s^{B*}) = \frac{\ln \theta_y}{\ln \theta_x \theta_y},$$

with $\frac{\ln \theta_y}{\ln \theta_x \theta_y} \in [0, 1]$, because the definition of comparative advantages ensures that $\theta_x, \theta_y > 1$. To complete the proof, we have to show that situations where $\hat{\alpha}(s^A, s^B)$ is not defined or does not belong to $]0, 1[$ cannot correspond to an equilibrium.

First remark that all situations where one candidate gets a null payoff cannot be an equilibrium. Indeed, this candidate can always imitate his opponent and then $\hat{\alpha}(s^A, s^B) = \frac{\ln \theta_y}{\ln \theta_x \theta_y}$ and both players payoffs become strictly positive.

Now suppose that $\hat{\alpha}(s^{A*}, s^{B*})$ is not defined, i.e., either $D(s^{A*}, s^{B*}) = 0$ (equivalent to $\frac{x_A^* y_B^*}{x_B^* y_A^*} = 1$), or s^{A*} or s^{B*} is in $\{0, 1\}$. If $D(s^{A*}, s^{B*}) = 0$, then candidate A's payoff is given by:

$$\begin{aligned} \pi^A(s^{A*}, s^{B*}) &= 1 \text{ if } s^{A*} < 1 - \theta_y(1 - s^{B*}), \\ &= \frac{1}{2} \text{ if } s^{A*} = 1 - \theta_y(1 - s^{B*}), \\ &= 0 \text{ otherwise.} \end{aligned}$$

Suppose (s^{A*}, s^{B*}) such that $s^{A*} \leq 1 - \theta_y(1 - s^{B*})$ is an equilibrium. Then $\pi^B(s^{A*}, s^{B*}) \in \{0, \frac{1}{2}\}$, whereas $\pi^B(s^{A*}, s^{B*}) = 1$ until $0 \leq s^B \leq \frac{\theta_y - 1 + s^{A*}}{\theta_y} \leq 1$. Hence B has an incentive to deviate, this is a contradiction. If s^{A*} or s^{B*} is in $\{0, 1\}$, but not both of them. Then one of the candidate gets a null payoff and this cannot be an equilibrium. Now, if s^{A*} and s^{B*} are in $\{0, 1\}$, then $\pi^A(s^{A*}, s^{B*}) = \pi^B(s^{A*}, s^{B*}) = \frac{1}{2}$. If one of the candidate deviates and locates in $]0, 1[$, he gets all the votes, then this is not an equilibrium.

Suppose that $\hat{\alpha}(s^A, s^B) \leq 0$ or $\hat{\alpha}(s^A, s^B) \geq 1$, then one of the two players gets a null payoff. We have already proved that this cannot be an equilibrium.

Proof of Proposition 5: We first prove the following lemma (remember that $\theta_x, \theta_y > 1$ here):

Lemma 1 $\frac{\ln \theta_y}{\ln \theta_x \theta_y} < \frac{\theta_x(\theta_y - 1)}{\theta_x \theta_y - 1}$.

Proof of Lemma 1: Let $\theta_x = \theta$ and $\theta_y = \lambda\theta$ with $\frac{1}{\theta} < \lambda$. Then the inequality can be written as follows:

$$h(\lambda) = \lambda\theta^2 \ln \theta - (\theta - 1) \ln \lambda - (2\theta - 1) \ln \theta > 0,$$

The differentiate of h is $h'(\lambda) = \theta^2 \ln \theta - \frac{(\theta - 1)}{\lambda} > \theta l(\theta) = \theta^2 \ln \theta - \theta(\theta - 1)$. The function l is increasing ($l'(\theta) = \ln \theta$) and $l(1) = 0$, then $h'(\lambda) > 0$. Furthermore, $h(1) = 0$, then the inequality is always true.

Without loss of generality, we focus on candidate A incentives to deviate from $(s^{A*}, s^{B*}) = \left(\frac{\ln \theta_y}{\ln \theta_x \theta_y}, \frac{\ln \theta_x}{\ln \theta_x \theta_y} \right)$. There are many situations where A may obtain a higher payoff. Straightforwardly, candidate A has no incentive to play $s^A \in \{0, 1\}$, otherwise, $\pi^A(s^A, s^{B*}) = 0$.

Case 1 *If A can deviate by playing s^A such that its payoff is given by equation 9, i.e. $D(s^A, s^{B*}) > 0$ (equivalent to $\frac{x_A y_B^*}{x_B^* y_A} > 1$). Suppose $\hat{\alpha}(s^A, s^{B*}) \leq 0$, then his payoff $\pi^A(s^A, s^{B*}) = 1$. The two conditions imply that $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1 - s^{B*}}{1 - s^A} > 1$ and $\theta_y \frac{1 - s^{B*}}{1 - s^A} \leq 1$ (it means that $N(s^A, s^{B*}) < 0$). This is equivalent to $\frac{s^{B*}}{s^{B*} + (1 - s^{B*})\theta_x \theta_y} < s^A \leq 1 - \theta_y(1 - s^{B*})$. Such a value of s^A exists if and only if $\frac{\theta_x(\theta_y - 1)}{\theta_x \theta_y - 1} < s^{B*} < 1$. Since $s^{B*} = \frac{\ln \theta_x}{\ln \theta_x \theta_y}$, lemma 1 ensures that this cannot be true. Then candidate A cannot play this kind of deviation. Now, suppose $0 < \hat{\alpha}(s^A, s^{B*}) < 1$, then $\pi^A(s^A, s^{B*}) = 1 - F(\hat{\alpha}(s^A, s^{B*}))$. Here, the second order derivative of candidate A 's payoff is:*

$$\begin{aligned} \frac{\partial^2 \pi^A(s^A, s^{B*})}{(\partial s^A)^2} &= -\frac{\partial^2 \hat{\alpha}(s^A, s^{B*})}{(\partial s^A)^2} \\ &= \frac{1 - 2s^A}{(s^A(1 - s^A))^2} [s^A D(s^A, s^{B*}) - N(s^A, s^{B*})] \\ &\quad - \frac{1}{s^A(1 - s^A)} D(s^A, s^{B*}). \end{aligned}$$

Hence,

$$\frac{\partial^2 \pi^A (s^{A*}, s^{B*})}{(\partial s^A)^2} \propto (\hat{\alpha} (s^{A*}, s^{B*}))^2 - \hat{\alpha} (s^{A*}, s^{B*}) < 0.$$

Then s^{A*} maximizes the payoff of candidate A in that case.

Case 2 Suppose candidate A deviates such that its payoff is given by equation (10), i.e. $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1-s^{B*}}{1-s^A} = 1$ (equivalent to $\frac{x_{Ay_B^*}}{x_{By_A^*}} = 1$). Then $\frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y} = s^A$. In this case,

$$\begin{aligned} \pi^A (s^A, s^B) &= 1 \text{ if } s^A < 1 - \theta_y (1 - s^{B*}), \\ &= \frac{1}{2} \text{ if } s^A = 1 - \theta_y (1 - s^{B*}), \\ &= 0 \text{ if } s^A > 1 - \theta_y (1 - s^{B*}). \end{aligned}$$

In the previous case, we have seen that $1 - \theta_y (1 - s^{B*}) < \frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y}$, then this deviation is not profitable ($\pi^A (s^{A*}, s^{B*}) > \pi^A (s^A, s^{B*}) = 0$).

Case 3 If A can deviate by playing s^A such that its payoff is given by equation 11, i.e. $D (s^A, s^{B*}) < 0$ (equivalent to $\frac{x_{Ay_B^*}}{x_{By_A^*}} < 1$). Suppose that A can deviate by playing s^A such that $\hat{\alpha} (s^A, s^{B*}) \geq 1$, then his payoff $\pi^A (s^A, s^{B*}) = 1$. The two conditions imply that $\theta_x \theta_y \frac{s^A}{s^{B*}} \frac{1-s^{B*}}{1-s^A} < 1$ and $\frac{s^{B*}}{\theta_x} \leq s^A$ (it means that $N (s^A, s^{B*}) \leq D (s^A, s^{B*})$). These two conditions are equivalent to $\frac{s^{B*}}{\theta_x} \leq s^A < \frac{s^{B*}}{s^{B*} + (1-s^{B*})\theta_x \theta_y}$. Such a deviation exists if and only if $s^{B*} > \frac{\theta_x (\theta_y - 1)}{\theta_x \theta_y - 1}$, and lemma 1 states this cannot be true. Now suppose that A deviates by playing s^A such that $0 < \hat{\alpha} (s^A, s^{B*}) < 1$ (then $D (s^A, s^{B*}) < N (s^A, s^{B*}) < 0$). Then $s^A < \frac{s^{B*}}{\theta_x}$ and $s^A < 1 - \theta_y (1 - s^{B*})$. It is easy to show that $1 - \theta_y (1 - s^{B*}) < \frac{s^{B*}}{\theta_x}$ with lemma 1, then $s^A < 1 - \theta_y (1 - s^{B*})$. The first derivative of candidate A's payoff is:

$$\begin{aligned} \frac{\partial \pi^A (s^A, s^{B*})}{\partial s^A} &= \frac{\partial \hat{\alpha} (s^A, s^{B*})}{\partial s^A} \\ &= \frac{1}{1-s^A} D (s^A, s^{B*}) - \frac{1}{s^A (1-s^A)} N (s^A, s^{B*}), \end{aligned}$$

The roots of this equation are given by $\hat{\alpha} (\bar{s}^A, s^{B*}) = \bar{s}^A$. The second order derivative verifies:

$$\frac{\partial^2 \pi^A (\bar{s}^A, s^{B*})}{(\partial s^A)^2} \propto (\hat{\alpha} (\bar{s}^A, s^{B*}))^2 - \hat{\alpha} (\bar{s}^A, s^{B*}) < 0,$$

Finally, $\bar{s}^A = \hat{\alpha}(\bar{s}^A, s^{B*})$ with $\theta_x \theta_y \frac{\bar{s}^A}{s^{B*}} \frac{1-s^{B*}}{1-\bar{s}^A} < \theta_y \frac{1-s^{B*}}{1-\bar{s}^A} < 1$ is the only remaining possible deviation. Candidate A has an incentive to deviate if and only if $\pi^A(\bar{s}^A, s^{B*}) > \pi^A(s^{A*}, s^{B*})$, i.e. if and only if $\bar{s}^A > 1-s^{A*}$. Let $\tilde{s}^A = 1-s^{A*}$, then A has an incentive to deviate iff $\tilde{s}^A D(\tilde{s}^A, s^{B*}) > N(\tilde{s}^A, s^{B*})$ and $\tilde{s}^A < 1-\theta_y(1-s^{B*})$. These inequalities are equivalent to:

$$\begin{aligned} \ln \frac{\theta_x}{\theta_y} \ln \left[\theta_x \theta_y \frac{\ln \theta_x}{\ln \theta_y} \right] &> 0, \text{ and,} \\ \ln \theta_x &< \frac{\ln \theta_y}{\theta_y}, \end{aligned}$$

By a symmetry argument, candidate B has an incentive to deviate iff:

$$\begin{aligned} \ln \frac{\theta_y}{\theta_x} \ln \left[\theta_x \theta_y \frac{\ln \theta_y}{\ln \theta_x} \right] &> 0, \text{ and,} \\ \ln \theta_y &< \frac{\ln \theta_x}{\theta_x}, \end{aligned}$$

Suppose $\theta_x \geq \theta_y$, then the equilibrium exists iff $\theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$ and $(\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x \theta_y}$ or $\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x}$), i.e. iff $\frac{1}{\theta_x \theta_y} \leq \frac{\ln \theta_y}{\ln \theta_x}$. If $\theta_y \geq \theta_x$ the equilibrium exists iff $\frac{\ln \theta_y}{\ln \theta_x} \geq \frac{1}{\theta_x}$ and $(\theta_x \theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$ or $\theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$), i.e. iff $\theta_x \theta_y \geq \frac{\ln \theta_y}{\ln \theta_x}$. Finally, the equilibrium exists iff $\theta_x \ln \theta_x \geq \frac{\ln \theta_y}{\theta_y}$ and $\theta_y \ln \theta_y \geq \frac{\ln \theta_x}{\theta_x}$.

Proof of Proposition 6: First notice that $\hat{f}(X) = \frac{X\hat{\alpha}^*}{X\hat{\alpha}^*+1-\hat{\alpha}^*} - \frac{1}{2} \geq 0$ if and only if $X \geq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}$ and is a strictly increasing function of X , because $\hat{\alpha}^* \in]0, 1[$. Since $\frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A}$, we consider three cases:

Case 1 Suppose $\frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A} \leq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}$, then $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^B}{\eta_y^B}\right) - \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) < 0$.

Case 2 Suppose $\frac{1-\hat{\alpha}^*}{\hat{\alpha}^*} \leq \frac{\eta_x^B}{\eta_y^B} < \frac{\eta_x^A}{\eta_y^A}$, then $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) - \hat{f}\left(\frac{\eta_x^B}{\eta_y^B}\right) > 0$.

Case 3 Suppose $\frac{\eta_x^B}{\eta_y^B} \leq \frac{1-\hat{\alpha}^*}{\hat{\alpha}^*} \leq \frac{\eta_x^A}{\eta_y^A}$, then $I(z^{A*}) - I(z^{B*}) = \hat{f}\left(\frac{\eta_x^A}{\eta_y^A}\right) + \hat{f}\left(\frac{\eta_y^B}{\eta_x^B}\right) - 1$. With simple computations, we find that this last expression is positive if and only if $\frac{\eta_x^A}{\eta_y^A} \frac{\eta_x^B}{\eta_y^B} \geq \left(\frac{1-\hat{\alpha}^*}{\hat{\alpha}^*}\right)^2$.

Proof of Proposition 7:

The necessary condition is straightforward: if Candidate A has an absolute advantage, then $\eta_x^A \geq \eta_x^B$ and $\eta_y^A \geq \eta_y^B$, with at least one strict inequality, and it directly follows that:

$$\forall \alpha_i \in]0, 1[, \alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) > \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B),$$

and, for $\alpha_i \in \{0, 1\}$,

$$\alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) \geq \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B).$$

Regarding the sufficient condition, suppose that Candidate A has a UVA, then:

$$\forall \alpha_i \in [0, 1], \alpha_i \ln(\eta_x^A) + (1 - \alpha_i) \ln(\eta_y^A) \geq \alpha_i \ln(\eta_x^B) + (1 - \alpha_i) \ln(\eta_y^B).$$

Notice that for $\alpha_i = 0$, the inequality becomes $\eta_y^A \geq \eta_y^B$, and, for $\alpha_i = 1$, it becomes $\eta_x^A \geq \eta_x^B$.

Now, we claim that $\eta_y^A = \eta_y^B = \eta_y$ and $\eta_x^A = \eta_x^B = \eta_x$. By definition of the UVA, there exists α in $[0, 1]$ such that:

$$\alpha \ln(\eta_x) + (1 - \alpha) \ln(\eta_y) > \alpha \ln(\eta_x) + (1 - \alpha) \ln(\eta_y),$$

this is impossible.

References

- [1] Abelson, R.P. and A. Levi (1985), "Decision making and decision theory", in G. Lindzey and E. Aronson eds., *The Handbook of Social Psychology* (3rd ed. vol.1). New York: Random House.
- [2] Alesina, A. and J. Sachs (1988), "Political parties and the business cycle in the United States, 1948-1984", *Journal of Money, Credit and Banking* (February): 63-84.

- [3] Ansolabehere, S. and J.M. Snyder (2000), “Valence politics and equilibrium in spatial election models”, *Public Choice* 103, 327-336.
- [4] Aragonés, E. (1997), “Negativity effect and the emergence of ideologies”, *Journal of Theoretical Politics* 9 (2): 198-210.
- [5] Aragonés, E. and T. Palfrey (2002), “Mixed equilibrium in a Downsian model with a favored candidate”, *Journal of Economic Theory* 103: 131-161.
- [6] Aragonés, E. and T. Palfrey (2003), “Spatial competition between two candidates of different quality: the effects of candidate ideology and private information”, *Working Paper*, California Institute of Technology.
- [7] Austen-Smith, D. and J.S. Banks (1989), “Electoral accountability and incumbency”, in P.C. Odgershook eds., *Models of Strategic Choice in Politics*. Ann Arbor: University of Michigan Press.
- [8] Banks, J. and R. Sundaram (1993), “Adverse selections and moral hazard in a repeated election model”, in W. Barnett, M. Hinich, and N. Schofield, eds., *Political economy: institutions, information, competition and representation*. New York: Cambridge University Press.
- [9] Banks, J. and R. Sundaram (1996), “Electoral accountability and selection effects”, University of Rochester, Rochester, N.Y. *Mimeographed*.
- [10] Beck, N. (1982), “Parties, administrations, and American economic outcomes”, *American Political Science Review* 76: 83-94.
- [11] Bloom, H.S., and H.D. Price (1975), “Voter response to short-term economic conditions: the asymmetric effect of prosperity and recession”, *American Political Science Review* 59: 7-28.
- [12] Chappel, H., and W. Keech (1986), “Party differences in macroeconomic policies and outcomes”, *American Economic Review* (may): 71-74.
- [13] Downs, A. (1957), *An economic theory of democracy*, New York, Harper.
- [14] Fiorina, M. (1981), *Retrospective voting in American elections*, New Haven, Conn.: Yale University Press.

- [15] Grandmont, J.M. (1978), “Intermediate preferences and the majority rule”, *Econometrica* 46: 317-330.
- [16] Groseclose, T. (1999), “Character, charisma, and candidate locations: Downsian models when one candidate has a valence advantage”, *Working Paper*, Stanford University.
- [17] Groseclose, T. (2001), “A model of candidate location when one candidate has a valence advantage”, *Working Paper*, Stanford University.
- [18] Hibbs, D.A (1977), “Political parties and macroeconomic policy”, *American Political Science Review* 71: 1467-78.
- [19] Kernell, S. (1977), “Presidential popularity and negative voting”, *American Political Science Review* 71: 44-66.
- [20] Key, V.O. (1966), *The responsible electorate*. New York, Vintage.
- [21] Klein, J.G. (1991), “Negativity effects in impression formation: a test in the political arena”, *Personality and Social Psychology Bulletin* 17 (4): 412-418.
- [22] Lau, R.R. (1982), “Negativity in political perception”, *Political Behavior* 4: 353-378.
- [23] Mueller, J.E. (1973), *War, presidents and public opinion*. New York: Wiley.
- [24] Persson T. and G. Tabellini (2000), *Political economics: explaining economic policy*, The MIT Press.
- [25] Rogoff, K. (1990), “Equilibrium political budget cycles”, *American Economic Review* 80: 21-36.
- [26] Rogoff, K., and A. Siebert (1988), “Elections and macroeconomic policy cycles”, *Review of Economic Studies* 55: 1-16.
- [27] Tabellini, G., and A. Alesina (1990), “Voting on the budget deficit”, *American Economic Review* 80: 37-39.
- [28] Tabellini, G., and V. La Via (1989), “Money, debt, and deficits in the U.S.”, *Review of Economics and Statistics* (February).