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The Complexity Approach to Economics: a Paradigm Shift

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The Complexity Approach to
Economics: a Paradigm Shift

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Summary: The paper evaluates the impact of the complexity approach on economics from two viepoints. On the one hand, the dissolution of the shared notion of mainstream, which began in the mid '80s. Until then, the Samuelsonian paradigm had been able to absorb those contributions that critically challenged its assumptions and results. This process took place at the expense of gradually blurring the boundaries of the paradigm itself, which has lead to speak of its dissolution. The emergence of complexity theory raises the need to either extend further the boundaries of the old paradigm so as to close the gap between apparently irreconcilable differences or to define a new one. On the other hand, it is legitimate wonder to what extent complexity theory qualifies as a 'paradigm' and thus whether it can be a candidate for substituting the Samuelsonian paradigm.

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I. INTRODUCTION

The last two decades have witnessed a growing influence of complex system analysis on the physical, biological and social sciences. Economics has not remained indifferent to the power of nonlinear interactions to generate complex structures and an astonishing range of potential behaviours. Indeed, the possibility of explaining, in theoretical terms and by means of models, complex and sometimes erratic economic phenomena has sparked a rapidly widening interest: the theoretical and analytical tools of complex systems analysis have been seen by an increasing number of scholars as a promising route towards overcoming important weaknesses of the traditional approach to the representation and understanding of economic facts.

The complexity approach has developed through different lines of thought and across diverse disciplines[2] in ways that make it quite difficult to evaluate its impact on economics. The complexity perspective implies foundations that clash with the received ones and a methodological stance that appears in contrast with the traditional economic theory-making. The latter aspect is rather unexplored within the approach, and scholars dealing with it often make discordant claims.

One of the most important contributions to the development of complexity theory is the foundation in 1984 of the Santa Fe Institute devoted to promoting interdisciplinary studies of complex systems. Its members included physicists, biologists, computer scientists and economists. Among them was Kenneth Arrow, one of the most representative scholars in the mainstream. The Santa Fe Institute has generated many of the crucial advances brought about by complexity theory (e.g., agent-based modelling, generative science and the SWARM project). However, as far as economics is concerned, some of the major complexity scholars seem reluctant to admit that the complexity view is a departure from the route traced by the mainstream. An interesting example of such attitude is provided by the three volumes (1988, 1997, 2005) published in the Santa Fe Institute series of Studies in the Science of Complexity, with the title *The Economy as an Evolving Complex System.* The first volume (Anderson, Arrow and Pines 1988) reflected the contrasting vocation of researchers: Anderson insisted on the path-breaking features of the science of complexity, while Arrow claimed that the complexity endeavour could be absorbed in the body of traditional economic thought. In the second volume, Arthur, Durlauf and Lane (1997) admit that there is still a need to define what are, in this new viewpoint, the problems under study and what kind of solution is sought: 'it is premature to talk about methods for generating and assessing understanding when what is to be understood is still under discussion' (1997, p.14). A few years later, Blume (Blume and Durlauf 2001) seems convinced that a discussion on method is no longer necessary, since the

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[2] For a survey of the origins of the science of complexity see Barkley Rosser (1999) and Fontana (2006).

complexity approach is only a language for scientific research whose usefulness resides in the ability to simplify the analysis under some specific and particular conditions. The third volume (Blume and Durlauf 2005) still preserves the ambiguity between innovation and tradition. The editors dedicate the book to Kenneth Arrow as the 'intellectual leader of the SFI [Santa Fe Institute] Economics Program ever since its inception' and maintain that 'the models here presented do not represent any sort of rejection of neoclassical economics […] that was able to absorb SFI type advances without changing its fundamental nature'. Hanappy (2007) points out that there is a contradiction between form and contents since the models presented in the volume undeniably run counter to the neoclassical framework and assumptions[3].

Further examples of the resistance in admitting the revolutionary aspects of complexity science in economics come from other outstanding scholars within this field of research: the complexity approach is confined to an ancillary role also in Kirman (2005, p. 18) and Lesourne (2002), for whom complexity theory is of no use in the elaboration of a new methodology, but it is simply a way to explain those phenomena that resist to standard analysis.

Economists who explicitly speak in terms of methodological change are few and their statements are sometimes vague. According to Barkley Rosser (2004, p. IX) 'awareness of the ubiquity of complexity is transforming the way that we think about economics', while B. Arthur highlights a change in the reference point for theorisation by stressing that the inclusion of heterogeneity in economics makes it more similar to biology than to nineteenth century physics (1994a). As for the purport of this change, he agrees that it implies more than a mere extension of standard theory: it is a different way of thinking. He also observes that 'there are signals everywhere these days in economics that the discipline is loosing its rigid sense of determinism, that the long dominance of positivist thinking is weakening and that economics is opening itself to a less mechanistic, more organic approach'(1994a, p. 1).

S. Markose, in the introductory paper to an issue of the *Economic Journal* devoted to *Computability and evolutionary complexity: markets as complex adaptive systems,* states that 'these principles mark a paradigm shift from earlier ways of viewing such phenomenon [the market]' (2005, p. 159). However, while Markose gives a comprehensive and interesting survey of the state of the art in complexity theory, its origins and application to economics, in the paper the issue of the paradigm shift remains untackled. Evaluating the impact of the complexity approach on economic science is

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[3] A similar contradiction can be found in Epstein (1999). He first states that reproducing the known dynamics of a system in an equation is 'devoid of explanatory power in spite of its descriptive accuracy' (p. 51). However, in the course of the paper he speculates on the possibility of reconciling a generative economic science with a deductive one: 'it would be wrong to claim that Arrow-Debreu general equilibrium theory is devoid of explanatory power because it is not generative. It addresses different questions that those of primary concern here' (p. 56).

important for - at least - two reasons. On the one hand, the dissolution of the shared and demarcated notion of mainstream, which began in the mid '80s. Until then, the Samuelsonian paradigm, which had been the prevalent one since post WWII, had been able to absorb those contributions, such as bounded rationality and information theory, that critically challenged its assumptions and results. This sweeping process took place at the expense of gradually blurring the boundaries of the paradigm itself, which has lead to speak of its dissolution. The emergence of complexity theory raises *de facto* the need to either extend further the boundaries of the old paradigm so as to close the gap between apparently irreconcilable differences (as was the case for K. Arrow and the Santa Fe Institute), or to define a new one. On the other hand, it is legitimate investigate to what extent complexity theory qualifies as a 'paradigm' and thus whether it can be a candidate for substituting the Samuelsonian paradigm in unifying economics or whether it is just another of the numerous, important but not structural, criticism to the mainstream.

The discussion of these topics which is the object of this paper, allows to formulate the claim that the impact of the complexity approach on economic science goes beyond the novelty of its particular bringing to the explanation of economic phenomena.

In contrast with Blume and Durlauf (2001), I feel that complexity science is more than a mere language allowing a simplification of analysis. In the paper, I will argue that complexity theory is a scientific paradigm whose characteristics imply a methodological revolution. The work is organized as follows: Section II provides a definition of complex systems and complexity. Section III analyses economic science as seen from the complexity perspective; Section IV describes the elements of the complexity paradigm; Section V summarizes the main achievements of complexity theory; Section VI relates the complexity view to the thought of some XIX and XX century economists highlighting common roots and possible lines of research. Section VII reports some concluding remarks.

II. COMPLEXITY

1. Complexity: its features

Complexity in economics has mainly come to be identified with the Santa Fe Institute perspective, according to which a complex system is characterised by the presence of a high number of interacting heterogeneous agents, the absence of any global controller, the presence of adaptation by learning and evolution, and the dominance of out-of-equilibrium dynamics.

A more detailed description implies five essential features. First, complex systems are comprised of many morphologically diverse parts. Economies consist of a huge number of individual agents, organised in a great variety of groups and institutional structures. These parts are morphologically diverse. Removal of one part leads the system to self-reorganise and to a series of changes aimed at compensating for the gap in the system.

Second, complex systems exhibit a variousness of nonlinear dynamics. This is due to the fact that the different components operate on different temporal and spatial scales. In turn, it implies that aggregate behaviour cannot simply be derived from the summation of the behaviour of individual components. Even when a catalogue of the activities of most of the participating sub-components is available, an understanding of the effect of changes on the whole system is far from achieved.

Third, complex systems are open, dissipative systems that maintain themselves away from thermodynamic equilibrium. Indeed the large fluctuations we observe in economic time series seem to indicate that economies tend to operate in a critical state, way out of balance, where minor disturbances may lead to events (avalanches) of all dimensions. Such state is what the literature on complexity refers to as self-organized criticality (Bak, 1997).

Fourth, complex systems can respond adaptively to change, in ways that tend to increase their probability of persisting. Their interacting parts adapt by changing their behaviour (even in innovative ways) as conditions change and experience accumulates. In turn, the environment of any adaptive element largely consists of other adaptive parts. Therefore, a portion of any individual's efforts at adaptation is spent adapting to other adaptive individuals. This feature is a major source of complex temporal patterns.

Fifth, complex systems have irreversible histories. In nature, each individual organism is the unique result of the interaction between its genetic code and the environment, in social phenomena each epiphany is the product of individual actions, under a given institutional setting, in precise circumstance of time and space (see 2.1). (Brown, 1994; Kauffman, 1993; Holland, 1995; Gell-Mann, 1995).

2. Complexity: typology

As suggested by Foster (2005), economists have developed these features along three distinct but interrelated paths: those who look at complexity as an inherent property of the dynamical behaviour of the system (or 'dynamic' complexity), those who that use the term 'complexity' to refer to systems whose analysis requires computationally heavy procedures ('logical' or 'computational' complexity) and those that intend complexity as the study of the connections in a system ('connective complexity') aiming at exploring its evolutionary properties.

2.1 Dynamic complexity

The most uncontroversial definition of dynamic complexity is essentially a mathematical one: an economic system is dynamically complex if its deterministic endogenous processes do not lead it asymptotically to a fixed point, a limit cycle, or an explosion (Day, 1994). All systems that fit this definition have some degree of nonlinearity within them. At the same time, there are nonlinear

systems that are not complex, such as standard exponential growth models. Non-linear systems may, for example, generate periodic fluctuations (or limit cycles), that is fluctuations that are regularly cyclical, with specific configurations repeated at fixed intervals. Non-linearity is a necessary but not sufficient condition for complexity: complex dynamics are processes that involve non-periodic fluctuations and switches in regime or structural changes, such as those implied by bifurcations and transitions to chaos.

The latter are object of two important branches in the theory of complex dynamics. Bifurcation theory is the study of points in a system at which the qualitative behaviour of the system changes – the critical thresholds that may trigger drastic change. Bifurcation theory makes it possible to study the behaviour of a nonlinear system over time. This kind of analysis is referred to as comparative dynamics.[4] It enables us to see if a qualitative kind of behaviour persists when the initial conditions are perturbed. It also provides hints on the type of change that can occur when some crucial parameters, instead of being assumed constant for analytical convenience, are allowed to vary. It allows us to study, for example, the conditions that can bring about irreversible or slowly reversible changes. Therefore, it also makes it possible to study the extent to which the behaviour of a system could be influenced or controlled by policy. Chaotic dynamics are another important kind of complex behaviour. The term chaos in its present meaning was first used by Li and Yorke (1975). However, the significance of such behaviour in the natural sciences had already been recognised by Lorenz (1963) in meteorology, and May (1974, 1975) in population biology.

There exist several different mathematical definitions of chaos.[5] The common underlying concept, however, is randomness or irregularity that arises in a deterministic system. The intuitive notion is that of a time evolution with sensitive dependence on initial conditions – that is, arbitrarily close initial conditions display independent evolution as time proceeds. The two other basic properties are topological transitivity (indecomposability), and density of periodic points.

All such features have strong implications from the point of view of the analysis of economic systems. Sensitivity to initial conditions implies unpredictability; the system shows path-dependence and although in principle it should be possible to predict future dynamics as a function of time, this is in fact impossible because any error in specifying the initial condition, no matter

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[4] Samuelson (1947) first used the expression. An extensive treatment of comparative dynamics in economics is in Day (1994).

[5] For a formal definition see, for example, Devaney (1998) and Ott (1993). The latter offers an in-depth and broad treatment of the subject of chaos in dynamical systems, whereas the former is in essence a mathematics text. More accessible introductions are Devaney (1992), Lorenz (1993), Peak and Frame (1994), and Ruelle (1991) (in order of increasing reader-friendliness). A fascinating application of chaos theory to the study of biological systems and physiological rhythms is Glass and Mackey (1988).

how small, leads to an erroneous prediction at some future time. Indecomposability implies that it is not possible to derive implications on the behaviour of the system's subcomponents by analysing them separately. Yet, the presence of elements of regularity (periodic points) leaves room for a deeper understanding based on the accumulation of experience and data, as we keep perturbing the system and studying its reactions.

In economics, chaotic dynamics have received a great attention, particularly since the 1990s.[6] An analysis of this literature reveals at least three major theoretical implications of chaos for economic models. First, the existence of chaos offers a further plausible explanation for the pervasive irregularity of economic time series – an endogenous one – in addition to the conventional theory of exogenous shock (Goodwin 1991). A second implication is that the possibility of chaotic dynamics would render the rational expectations hypothesis untenable even if all its underlying assumptions were satisfied, e.g. if all agents had perfect information on the functioning of the economy and on the behaviour of the other agents (Kelsey 1988, pp. 682-83; Chiarella 1990, pp. 124-125; Medio and Gallo 1992, pp. 17-18). Perfect foresight out of steady states would be impossible, in economics as in physics and biology. A third implication of chaos concerns the irreversibility of time. Although dissipative systems could, - in principle - be integrated either forward or backward in time, in practice a correct computation of the path of the system in the past is generally not possible: it would require an absolute precision in determining the current position. In the presence of chaotic dynamics, however time reversibility becomes impossible in theory. In fact, if we consider a unimodal map f, for example, the existence of a turning point makes f non--invertible because the inverse of f is set-valued: the inverse of f is no longer a function – it maps not onto a point but into a set of points. Functions that display chaotic dynamics can therefore only be integrated forward in time.[7]

2.2 Computational Complexity

Computational complexity refers to the computational and cognitive skills of the decision makers. In deciphering the environment in order to make a decision, the decision maker may face two kinds of computational complexity (Albin 1998). The first one concerns problems that are undecidable, so that no mind or computer can devise a computational procedure able to solve them in a finite time. A typical example of such a problem is self-reference: an agent has to form conjectures about the

6 Examples include Chiarella (1999), Brock et al. (1991), and Medio and Gallo (1992). A number of representative papers are collected in Benhabib (1992), Creedy and Martin (1994), Dechert (1996), Barnett et al (1996), and Hamouda and Rowley (1999).

[7] For a more thorough discussion of the implications of chaos for economic modelling, see Gandolfo (1997), pp. 528-32.

conjectures of the other agent(s): assuming that each decision maker tries to foresee what the others' conjecture would be and to adjust her own on that basis, this leads to a procedure that never settles on a solution. Undecidability is not overcome by perfect rationality[8] and foresight since they could lead to infinite regress[9]. The agent should resort to some other aids to settle on a solution. These could be procedural rationality (Simon 1969), animal spirits (Keynes 1973, ch. XII), or focal points (Schelling 1960, p. 57). In economic literature, there are many examples of self-reference[10], but it is with the development of the complexity approach that they have become central to choice theory (Arthur 1994b).

The second type of computational complexity regards problems that are in theory decidable but for which the cost of an optimal solution can be so high as to deprive the optimal choice of any possible advantage for the decision maker (Albin 1998, p. 46).

Undecidability and computational costliness shed light on the controversy regarding the epistemological or ontological nature of complexity. McIntire (1998) posed the question whether complexity is merely a revival of the old debate concerning indeterminism versus hidden variables. Undecidability suggests that there are limits to our knowledge that do not depend on us but on the nature of the problem; in these cases complexity is ontological. In the context of decidable problems, the presence of computational costs and limited cognitive skills suggests (Simon 1969) that there are limits on our side that prevent us from deciphering complexity; in these cases complexity is epistemological.

As will be shown, computational complexity implies that mathematical maximisation techniques cannot be applied to modelling economic decision making. Furthermore, studies in rationality have shown that human computational skills are inadequate to support heavy computations, and therefore that the optimal choice is often unattainable. Thus, computational complexity compels us to search for new models and metaphors for decision-making.

2.3 Connective Complexity

Connective complexity refers to the links existing between the elements forming a system. In the presence of connective complexity, it is the kind of relationship that links the elements of the

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[8] Actually, it may be worsened by it (Koppl and Barkley Rosser, 2002).

[9] This is a merely illustrative analysis of non decidability. For a more technical treatment see, for instance, Binmore (1987) and (Albin 1988).

[10] Cournot's duopoly (1838), Morgenstern's Holmes- Moriarty problem (1928-1935), Keynes' beauty contest (1936, p. 156), Hayek's arguments against prediction (1967), Arthur's (1994b) El Farol Bar Problem.

system to one another that shapes their behaviour, and it is the changes in such relationships that cause the system to evolve. The hallmark of this kind of complexity is the emphasis on forces that act to maintain the order of the system and on countervailing forces that drive it towards disorder. The struggle between the two generates novel elemental kinds and relations, and leads to the disappearance of some (say, old or unfit) structures. It is this process of creative destruction that fosters selection and, thus, evolution (Foster - Metcalfe 2001). Connective complexity has been recently elaborated by Foster (2005), but its features closely recall Hayek's theory of spontaneous orders. A spontaneous order has a degree of complexity which depends on 'the minimum number of elements of which an instance of the pattern must consist in order to exhibit all the characteristic attributes of the class of patterns in question' (Hayek 1967, p. 76).

The elements of a spontaneous order (agents), in obeying autonomously to their own laws of motion (action), arrange themselves into a system characterised by regularities that can be explained in terms of those laws[11]. The overall result depends on the feedback between individual action and the environment. Other features that are attributed to such an order are 'redundancy' (the functioning of the overall order is not much affected by the malfunctioning of some of its parts because of the high number of links existing among the elements, e.g. the market) and path-dependency (the consequences of stochastic events determine situations that once established follow a certain path). Moreover, within spontaneous orders, evolution (in terms of rules that are more o less favourable to the survival and prosperity of the group) takes place both at the individual and group level.

In connective complexity (and spontaneous orders) - in opposition to the traditional assumptions of economic theory - macro regularities do not result from homogeneity of means and ends, nor from linearity in interaction; rather they derive from the relationship that links the elements to one another. This is more than sharing some common property such as Olympic rationality and perfect knowledge. Sugden (1996) provides a helpful example. He compares the mainstream view of agents and their interaction to bricks. He observes that all bricks when dropped from the top of a building fall downwards. Once the brick's law of motion is known, 'we can form correct expectations about the behaviour of all bricks, but the regularity we observe is nothing but the common property itself'. In contrast, Hayek's (and Foster's) view implies that the macro regularities are generated by the properties of the links that connect the agents. Sugden (1996) exemplifies this case with the gradient of a scree slope which 'is a property that the rocks have in relation to one another'. Connective complexity appears as a criticism to the reductionism of the traditional approach. It is impossible to study the properties of complex systems by observing its elements in isolation.

Connections between elements are relevant since they determine the 'goodness' of a complex system. That is to say that a given system can be more or less beneficial for its elements depending

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[11] Hayek is considered a precursor of generative science (Vriend 1999).

on the configuration of the network of connections. For instance, a market can be more or less efficient depending on its rules of exchange, or an innovation can spread more or less rapidly depending on links among adopters.

III. THE WAY WE THINK ABOUT ECONOMICS

From the above analysis, it emerges that the complexity perspective implies, as compared to the mainstream view a radically different perception of the nature of economic phenomena that rely on heterogeneity, processes and evolution. This attitude, as far as theorising is concerned, results in the rejection of the pervasiveness of linearity, of the perfect rationality postulate, of the centrality of the equilibrium, and of reductionism; when it comes to modelling, it results in the refusal of the view of economics as a purely mathematical science.

Theorising and modelling are only apparently separated in the mainstream view. As Schumpeter emphasized (1954), the introduction of mathematics in economics with the Marginalist revolution triggered a change in economic methods. The adoption of the mathematical method of reasoning changed: 'one's whole attitude to the problems that arise from theoretical schemata of quantitative relations between things' (1954, p. 955). Economics progressively tended to became a mathematical science. The process was completed in Vienna in the 1930's with the axiomatization of the general equilibrium theory accomplished by K. Schlesinger and J. von Neumann. In von Neumann's perspective, the concern for a realistic interpretation of economic models disappears. Von Neumann proposed mathematical solutions to theoretical problems. Solutions aimed at elegance were characterised by the elegance of the solution itself, logical completeness, concision and rigour even if obtained under extremely abstract assumptions. Eventually, and for about fifty years, mathematics[12] predominates on the economic content (Debreu 1986). Economic propositions embedded in theorems and proofs have geometric precision and a general[13] and abstract applicability, disregard realism and derive their validity from correctness of logical deduction. As Debreu writes: 'as a formal model of an economy acquires a mathematical life of its own, it becomes the object of an inexorable process in which rigor, generality, and simplicity are

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[12] 'An axiomatized theory first selects its primitive concepts and represents each one of them by a mathematical object. […] Next assumptions on the objects representing the primitive concepts are specified, and consequences are mathematically derived from them. The economic interpretation of the theorems so obtained is the last step of the analysis. According to this schema, an axiomatized theory has a mathematical form that is completely separated from its economic content. If one removes the economic interpretation of the primitive concepts, of the assumptions, and of the conclusions of the model, its bare mathematical structure must still stand' (Debreu, 1986, p.1265)

[13] 'The pursuit of generality in a formalized theory is no less imperative than the pursuit of rigor' (Debreu 1986, p. 1267).

relentlessly pursued' (Debreu 1986, p. 1265). It follows that the adoption of different mathematical views (such as the ones sketched in the paragraph devoted to dynamic complexity) does not only imply a change in the modelling tools, it also involves a (further) change of method of reasoning.

Attempts to explore complex adaptive systems bring about a necessary shift towards nonlinear mathematics. On the analytical stance, non-linear mathematic is less elegant and rigorous than the linear one. In fact, one reason for the widespread use of linear differential and difference equations in economics is that such equations are always solvable, whereas nonlinear models offer no such guarantee. Moreover, whereas linearity is unique, there exist countless possible nonlinear forms. In situations when we only know that a generic nonlinear functional relation exists with given qualitative properties (for example, conditions on the first order partial derivatives, such as their sign and given bounds), the feasible analysis is purely qualitative[14]. The choice of a specific nonlinear functional relationship, unless there are compelling theoretical or empirical reasons in favour of a certain form, can in fact be as arbitrary as linearity. In some cases, specific nonlinear functional forms may however serve the important purpose of showing that a certain dynamical behaviour is possible. Their role may be simply to highlight that a certain motion cannot be ruled out, with no claim that the model constructed is more general or more accurate than the corresponding linear model.

If we keep thinking - in Debreu's vein- that mathematical forms dictate the rules of economic thinking, then non-linear mathematics involves a completely different view and, therefore, a different method, in which generality and abstraction cannot be the hallmarks and in which traditional techniques do not apply.

For instance, let us think in terms of connective complexity. In the traditional view, the net of links that shapes the economy is kept very simple due the hypotheses of complete information and perfect knowledge. These allow to assume that each element of the economy can 'contact' and 'evaluate' all the others elements at no cost, so that the network of connections is irrelevant to the functioning of the system. This is functional to the possibility of conducting an equilibrium analysis in mathematical forms: '[interconnections] are akin to mathematical operators which must stay fixed if logical deductions concerning equilibrium outcomes are sought' (Foster 2005, p. 884). This condition does not apply to complex systems which are subject to changes that alter the very structure of the system; the existence, the position, the nature of the connections; and, thus, impair the possibility of focusing on equilibria.

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[14] Indeed, in economic theory purely qualitative nonlinearity is often regarded as the true generalization of linear dynamics (Gandolfo, 1997).

Moreover, the continuous variations taking place in connections due to adaptation and non-simultaneity of actions - together with the dynamic unfolding through historical time often impede the use of optimisation techniques. Optimisation is practicable under the hypothesis of knowing all possible outcomes of the process under analysis and the probability associated with each of them. These conditions are not met in complex systems. Within complex systems, uncertainty is inescapable. Therefore, optimisation cannot be taken as a metaphor for individual or organisational decision-making, and there are not the conditions to apply it as mere technique.

In the light of these considerations the claims I made in the introduction seems strengthened: the theoretical and methodological apparatus of the complexity approach is irreconcilable with the Samuelsonian view of economic science and economic facts. As long as there is a substantial agreement on the nature of complex systems, it seems naive not to accept its consequences on economic methods. I am keen to conclude that most of the quotes reported in the introduction that do not support this idea depend on a tendency to neglect methodological issues (Vriend 1999). If we accept the provisional conclusions that the complexity approach is in fact a methodological revolution, what would the economics of complexity be?

IV. THE ECONOMICS OF COMPLEXITY: THE ELEMENTS OF A NEW PARADIGM

1. Premises

The complexity view of economic phenomena pivots on uncertainty, limited cognitive and computational skills on the side of decision makers, stresses heterogeneity and focuses on processes instead of equilibrium. In the following sections the implications of these underpinnings on the notion of economic science and on theory making and modelling will be drawn.

1.1 On the role of mathematics

Drawing the characteristics of the economic of complexity requires some more reflections on the role of mathematics in economics. I have already shown that the mathematics adopted by the mainstream is unable to describe complex systems. However, it must be said that taking the complexity perspective to its extreme consequences leads to criticisms to mathematics *tout court* – i.e. including chaos and bifurcations.

The high number of elements of a complex system and their heterogeneity do not allow grouping those elements in a few broad categories (e.g. the consumer, the firm, the representative agent) and therefore would require mathematical systems made of a great number of equations, increasing the computational load and the difficulty in analytical treatment.

Under connective complexity, a complex system assumes a given configuration according to the properties of its elements (heterogeneity) and to the nature of the connections between them. The

state of the system is subject to evolution and selection and therefore there are variations in topology and dynamics. In other words, it is important to consider not only the system's dynamics, but also how the dynamics themselves change over time. It follows that it is often impossible to freeze the behaviour of the system in a model made of equations (no matter whether they are linear or not) since its dynamic is not fixed but evolutionary.

When it comes to capturing innovation, any kind of model expressed in the form of equations, being inherently deterministic, cannot generate 'new behaviour' of the components of the system or new elements: in order to account for similar phenomena the equations must be reformulated. The evolutionary process is made of adaptation and selection: agents change their rules of behaviour according to some index of performance conveyed by the environment, while differential reproduction of the fittest individual and mutation trough combinatorial reproduction imply changes in the population ecology.

These phenomena escape mathematical treatment by systems of simultaneous equations: the mathematics of complexity depicts non-linear systems that are non-adaptive. It seems that the understanding of complex economic phenomena involves taking further distance from mathematical modelling.

1.2 The loss of certainty

The project of building a unified economic theory, which has fascinated economists in the last two decades, has failed. Its accomplishment would have guaranteed certainty – machine-like precision, objectivity - to economic science. Its failure implies the abandonment of a neat distinction between subject (*homo oeconomicus*) and object (well-defined problems), of the possibility of grounding on few propositions (e.g. rational choice) the entire micro-macroeconomic theoretical apparatus and the fading of its predictive power (Arthur 1994a).

Contemporary economists are left with a research agenda which is completely different from that characterising the beginning of the last century. The complexity approach - inherently distant from positivist thinking - does not seem to suffer much from the loss. As it will appear in the course of this section, the nature of a complex system is *per se* irreconcilable with a science that seeks certainty. It is not only a matter of non-linearity and unpredictability: agents acting in complex environments have no clear image of the problem to be solved nor are they separated from it. The process of problem representation in a context of heterogeneity in environments, in the way agents frame problems and in the way they devise solutions makes the set of possible outcomes explode. Theoretically, it is the recognition of heterogeneity that makes the project of building economic theories as if they were Chinese boxes inconceivable. The complexity approach to economics is less mechanistic and more organic than the traditional one.

1.3 The loss of generality

Continuous endogenous change together with the relevance of heterogeneity, connections and historical time to the overall configuration of a complex system imposes a shift from the search for general laws to a search for general patterns at best, and an increased importance of contingent information on the specific system being studied. Even where the mathematical derivation of results is possible and takes – as it often does – the standard form of theorems and proofs, their purport is limited to the system being studied or to a strict range of its instances.

Loss of certainty is a logical consequence of the premises presented above and is indeed acknowledged by some prominent economists: Frank Hahn writes: 'not only will our successors have to be far less concerned with general laws than we have been, they will have to bring to the particular problems they will study particular histories and methods capable of dealing with the complexity of particular, such as computer simulation. Not for them […] the pleasure of theorems and proofs. Instead, the uncertain embrace of history, sociology and biology' (2001, p. 50).

Complexity resort requires resorting to observation of a large number of instances in order to gain understanding of the phenomenon under study 'as a 'theoremless' laboratory biologist […]' just as in any empirical science for which general laws are not yet in hand'(Epstein 1999, p. 51).

1.4 An organic approach

The account provided by Colander (2003) of two conferences held at the Santa Fe Institute nearly a decade apart, highlights a dramatic change. The first conference, held in the mid 80s, featured a set of mainstream economists and defenders of general equilibrium orthodoxy and a set of physicists. At this first conference, Colander remembers, the economists mostly attempted to defend their axiomatic approach, 'facing sharp challenges and ridicule from the physicists for holding relatively simplistic views' (Colander 2003, p. 8). The second one, held in the mid 90s, was characterised by a very different atmosphere and result: 'No longer were mainstream economists adhering to general equilibrium orthodoxy. Now they were using methods adopted from biologists and physicists, many suggested at the earlier conference, in innovative ways' (ibidem)[15].

Complexity theory compels economics to leave behind nineteenth century physics and to move towards a more organic approach. The loss of certainty and generality, the role of the researcher as an observer of particular regularities, the importance of history and time in determining the behaviour and the performance of economies impair the use of abstract and general explanations. The features of complex systems recall the idea of economies as organisms that adapt, innovate,

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[15] Studies presented at those conferences are collected in Anderson et al. (1988) and Brian et al. (1997) respectively.

develop and eventually decline. This idea was at the very inception of complexity, as conjectured by N. Wiener, J. von Neumann and the cybernetic group. The object of cybernetics was to understand (mainly mathematically at that time) the functioning of highly organised but decentralised systems composed of very large numbers of individual components such as life, biological evolution, economies, and machines. The endeavour was founded on the assumption that these diverse systems have features in common – in spite of evident differences in scale, elements and rules - that permit a unified approach.

Complexity takes a more organic stance not just as a vague pretension of taking biology as a reference point - it is rather a research agenda proper. Instruments used to emulate problem solving, adaptation and innovation are explicitly grounded on biological models and metaphors. For instance, genetic algorithms (Holland 1975), used to simulate learning and adaptation, are modelled on the processes of biological evolution. A genetic algorithm manipulates a set of structures called population. Each structure is assigned a value (fitness) based on the result of its interaction with the environment. Genetic algorithms operate on the population by replicating making copies of individuals in proportion to the observed fitness, i.e. the fittest ones have a higher chance to be reproduced. The outcome is a population of individuals that adapt increasingly well to the environment[16]. It is worth recalling that all these operations take place out of the researcher's control and independently of her degree of knowledge of the system under study. Economists engaged in this line of thought observe human living beings, their organisations and their institutions with the aim of trying to uncover the rules guiding their behaviour and generating the phenomena of interest.

Arthur (1994a) uses the term 'organic' also to highlight that economics deals with living beings, with people having emotions and weaknesses, dealing with an ever-changing environment made of other living beings that likewise try to cope with complexity. In this context, another element of the complexity paradigm emerges: in a world of mutual adaptation and limited information, it is impossible to use deduction in order to formulate satisfactory decisions. In the El Farol bar problem illustrated by Arthur (1994b), agents must foresee attendance to the bar by observing attendance in the previous weeks. The underlying idea is that if the bar is very crowded no one wishes to go and vice versa, a typical situation in which the payoffs of an action are higher the fewer the people undertaking it. A corresponding economic example is that of buying when everybody is selling and the price is falling. Agents form their expectation self-referentially: if they expect the bar to be crowded they will not show up, therefore invalidating the forecast, and vice versa. In self-referential situations, decision makers rely on induction, trying different routines and choosing the best one in

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[16] Other learning algorithms such as classifier systems and Neural Networks work in a similar way.

terms of performance. Since each choice alters the performance of the others, the ecology of routines changes over time. Since self-reference is typical of complex systems, we can conclude that the scope for deduction in the complexity approach is greatly reduced, and the distance from the mainstream point of view increased.

2. Microfoundations
2.1 Procedural rationality

In complex environments, decision makers have to face diversity, unpredictability, self-reference, change and Knightian uncertainty. This consideration forces us to shift from Olympic rationality, as assumed by mainstream economics, to procedural rationality. In the former, knowing the objectives of the decision maker and the objective data of the problem to solve is sufficient in order to label behaviour as rational. In the latter, it is recognised that individuals and organisations, being endowed with limited cognitive and computational skills, cannot cope with the huge amount of information embedded in a complex system and cannot process the fluxes of information flowing in it. Mainstream economics defines rationality as a relation of means to ends: a decision maker has the entire set of relevant information and can work out the outcome of different courses of action; among them, he can choose the best action in order to pursue its ends. This view is not applicable to complexity. H. Simon (1962, 1969) pointed out that the agents have bounded rationality, i.e. are not able to perform the above computation even in rather simple settings. Bounded rationality aroused much interest among economists captured the interest of the economists, was rapidly adopted by heterodox economists and was eventually integrated in the mainstream. Actually, bounded rationality does not apply to indeterminate situations: rather than an alternative concept it is a weaker formulation of the rationality postulate.

However, complex systems pose a further challenge to the analysis of decision-making. In an ever-changing environment, it is almost impossible to prefigure the outcome of decisions with a satisfactory degree of precision. This impairs the whole concept of rationality in terms of adequacy of actions to achieve given goals. In order to define the decision making process under such occurrences Simon introduces procedural rationality. Decision making under uncertainty in complex environments takes into account the agent's conceptualisation of the problem to be solved and her ability in drawing inference based on available information (Simon 1969). The focus is on the skill in building an adequate representation of the problem and to adapt it in reaction to environmental response with the aim of improving the performance of actions. Behaviour can be dubbed as rational if it derives from an appropriate deliberation. The decision maker concocts alternatives on the ground of her own (partial) information and adopts the first strategy that is expected to satisfactorily (not optimally) solve her problem. In this decision-making process,

learning plays a crucial role. By learning, the information set is updated and new options are generated. The agents adapt to the environment and, by innovating, contribute to its change.

The mainstream view agrees with Lucas who states 'in cases of uncertainty, economic reasoning will be of little value' (1981, p.224). On the contrary, procedural rationality offers a theoretical tool allowing to extend economic reasoning in the domain of uncertainty.

At the operational level, abandoning the rationality postulate amounts to leave behind the representation of decision making as an optimisation procedure and the definition of its outcome as equilibrium. Firstly, optimisation applies only when the set of future outcomes and their associate probability are known. Secondly, in order to consider the output of such a procedure as an equilibrium, it is necessary that all the rest of system remains unchanged. None of these conditions is satisfied in a complex environment. Procedural rationality can be modelled by using learning algorithms (neural networks, genetic algorithms and the like) that respect the assumptions of limited information, limited computational skills, adaptation and induction.

2.2 Explanation, Solution and Prediction

The notions of explanation, solution and prediction that are relevant for the economics of complexity derive from Joshua Epstein concept of generative science (1999)[17]. In generative science, explaining a phenomenon amounts to finding the micro rules and the configuration of links that are sufficient to make it emerge from decentralised autonomous interaction (Epstein 1999, 42). That is to say that explaining a phenomenon amounts to generating it from the bottom up – a neat difference with respect to the mathematical treatment of mainstream economics. Mainstream economics, in fact, holds to have explained a phenomenon when a mathematical expression that can replicate the dynamics of a system is found: explanation coincides with description[18]. Faced with such mathematical expression, a complexity theorist would speak of description, since there is no knowledge of the underlying mechanisms that generate it. If a given set of rules generates the macro regularity of interest then it is a candidate explanation. If there is more than one candidate explanation then further investigation is required. In the 90s, techniques apt to explore the output of agent-based models were at their dawning. The problem of distinguishing between specifications, comparing models and testing their sensitivity was a prevalent one (Axelrod 1997, Axtell and Epstein 1994, Holland and Miller 1991) since the relatively scarce experience in this field.

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[17] See also Epstein (2006), where the same concepts are enriched with interesting applications.

[18] Epstein (1999, p. 51) puts it in a slightly different way, contrasting explanation with description.

Recently, the state of things much improved (Phan and Amblard 2007, Windrum, Fagiolo and Moneta 2007) and this concept of 'explanation' is strengthened by the possibility of choosing the best candidate explanation among alternative ones. This change in the notion of explanation is ripe with implications. Firstly, in line with the Santa Fe program (Anderson and Arrow 1988; Arthur, Durlauf and Lane 1997), economies are seen as sets of processes. This is a further shift from the pivotal importance of equilibrium in mainstream economics. The existence (and the persistence) of equilibrium is of secondary interest, while the main work to be done is to uncover the process that generates it –from the bottom up.

It has already been said that equation-based models cannot reproduce nor explain adaptive processes. Agent-based models overcome the problems of equation-based models since they allow for the representation of numerous instances of heterogeneous agents that interact autonomously. In addition, by using learning algorithms (genetic algorithms, neural networks and classifier systems) they also introduce the adaptation-selection mechanism necessary to foster evolution and free the researcher from the necessity to assume maximising behaviour[19].

One could object that a simulation is indeed a computation, and as - according to the Church-Turing thesis - for every computation there is an equivalent representation in terms of equations, simulation are nothing but mathematics (Fontana 2006). In principle, this is undeniably true. However, when it comes to writing down and then solving such a model (possibly comprising hundreds of instances of different individuals and evolutionary algorithms) computational complexity arises: as discussed above, the choice of the appropriate functional form is not trivial and the same must be said about its solution. The concept of solution becomes weaker than in traditional mathematical modelling. Instead of a 'specific element of a well designed function space' (Epstein 1999, p. 52) it represents an interval of elements or, in the case of computational models, it is a 'sample path of a stochastic process' (ibidem). The aim for generality is greatly reduced.

A further point is the role of prediction in economic science, which was crucial in the influential work of Milton Friedman. He thought (1953, p. 7) that the 'ultimate goal of a positive science is the development of 'theory' or 'hypothesis' that yields valid and meaningful (i.e., not truistic) predictions about the phenomena not yet observed'. Given that complex systems are ontologically unpredictable (see section 2.1) a science dealing with them cannot take prediction as its aim: the attempt at predicting well the behaviour of the system cannot constitute the benchmark against to which evaluate the goodness of a theory. The concept of explanation as above illustrated founds the

entire building of generative science. Unpredictability is not an obstacle to explanation: 'electrostatics explains lightning but does not predict their occurrence' (Epstein 1999, p. 55).

3. A paradigm shift

Various approaches critical towards mainstream economics have developed since the 60s. The term heterodox economics includes different streams of thought that show more or less pronounced difference with the received theories. Most of them, however, were isolating violations of the axioms of the received theory while keeping its framework almost intact. A leading example is experimental economics, which has shown violations of the expected utility theory axioms. As far as complexity theory is concerned, on the contrary, *all* of its manifestations are in contrast with the mainstream approach marking a methodological revolution meeting all the conditions set by Kuhn (1962). A similar operation cannot be conducted in the context of complexity theory: all its manifestations are in contrast with the mainstream approach. Economics is facing a methodological revolution which meets all the conditions set by Kuhn (1962). According to Kuhn, a scientific revolution takes place when scholars find anomalies that cannot be reconciled with the commonly acknowledged paradigm within which research has until then been conducted. Kuhn's definition of a paradigm is based on four elements: what is to be examined, what kind of questions are supposed to be asked and how answers have to be found and how these questions have to be structured, and how the results of investigations should be interpreted. Roughly, economics in all its declinations observes the behaviour of individual and organisations, and its results in terms of production and of distribution – among people and among uses - of resources in time and space. In doing so, mainstream economics takes as subject the *homo oeconomicus* endowed with a high degree of information and with the ability of processing it, and, as object, well-defined problems with likewise well-defined (probable) outcomes. Complexity economics places limited demand on the agent's cognitive skill and endows her with limited information (self-reference, adaptation and innovation do not allow for different assumptions). Problems are not well-defined: firstly, decision makers have to build their own representation of the problem on the basis of limited knowledge of the surrounding environment; secondly, agents often are not able to assign probabilities to the outcomes of their action or simply cannot figure out what these will be. They act under genuine uncertainty. Mainstream economics sees agents and their aggregations such as markets or economies as relentlessly engaged in the search for equilibria, and proceeds by comparing them those equilibria to find out about their properties. Complexity economics observes the economies and the decision-making as processes that never end. When equilibrium is taken into account, the relevant question that is posed is how it happened that a given system reached such configuration, whose existence is only one of the many possible interesting epiphanies of economic behaviour.

Economists working within one paradigm or the other pose different questions. Whether equilibrium exists and persists or not, and whether agents have maximised their objective function in that state of the system or not, are questions that would not bother much a complexity theorist who investigates instead which micro rules are able to generate equilibrium or any other macro pattern. Questions posed under the two paradigms also have different structures. Mainstream economics proceeds by deductive formal proofs of theorems whose power is positively related to the range of the applicability of solutions. Complex system are increasingly explored by using computer simulations that are able to master heterogeneity of agents, physical space, historical time, learning and autonomous decentralised interaction. Computer simulations are a sort of mental experiment conducted *in silico*: scientists wonder which micro rules can generate a macro configuration, write them down in an algorithm and let the computer unfold their implications. Computers greatly extend the computational skills of researchers and make the need for simplifying assumptions less stringent. Results of the research, in complexity, are interpreted as candidate explanations whose goodness needs to be tested through sensitivity analysis and validation. Good solutions are regarded as particular explanations whose scope is limited in space and time. Whereas mainstream economists consider an explanation good if it is valid (correctly deducted from its logical premises) and general. Considering complexity economics as an exception, a violation of the mainstream view would be bold: complexity economics is in all respects a paradigm competing with the dissolving mainstream theory and with the other heterodox approaches. In some of the complexity approach scholars the sense that this competition has already been who by complexity is particularly vivid. For instance, Barkley Rosser (2003, p. IX), in describing the growingly widespread disaffection from theorems, uses the verbs in their past form as if economics were already subsumed under the complexity paradigm. Whereas scholars outside of the complexity school see this transformation as a still ongoing process. The paper in which Frank Hahn, in the form of a regretful prophecy, depicts the future of economics as the domain of particular solutions sought by means of computer simulations is significantly entitled *The Next Hundred Years* (2001) -.

V. COMPLEXITY THEORY: WHAT DOES IT TELL ABOUT ECONOMICS THAT WE DID NOT ALREADY KNOW?

My argument has been developed by showing the fundamental differences between mainstream economics and the complexity approach and by stressing that the science of complexity is an internally coherent paradigm that offers modelling tools consistent with the assumed microfoundation. A further question consists in appreciating whether this different route to explanation and understanding has led to genuinely new findings. The bringing of complexity theory to economics is controversial. Criticisms range from questioning its applicability to social

phenomena *tout court* to the charge of having developed metaphors that are powerful (the butterfly effect, fractals, self-organised criticality, the edge of chaos, and so on) but that do not explain anything new (Horgan 1997). Some weaknesses are acknowledged even by theorists within the complexity approach: 'Studies which use a complexity approach often end up justifying themselves by how they correspond with already observed facts, rather than with the new insights they provide' (Rosser 1999, p. 184).

The contribution of this field of enquiry to economics, however, does not appear to behave like typical academic fads, characterized by a fast increase in the number of published papers followed by a collapse within few years. It is difficult to set a starting point or a clear divide between complexity theory and earlier fields such as cybernetics, catastrophe theory and chaos theory, but one can safely argue that economic analyses based on a complex systems approach began to appear in leading economic journals in the 1980s and were increasingly present through the 1990s and 2000s: too long a time span to dismiss them as a mere intellectual bubble.

A detailed survey of the contributions of complexity to economics falls beyond the scope of this work[20]. Here I only highlight those areas in which they are more numerous or in which discussion has been more lively and significant.

Probably the most uncontroversial result is Brian Arthur's analysis of increasing returns, in which the economy is seen – in opposition with the static neoclassical Walrasian conception – as stochastic dynamic process governed by positive feedbacks. His analysis has given a major impulse to the issue of equilibrium selection by showing how an arbitrary small historical event could be amplified so as to drive the economy towards a given equilibrium[21]. The relevance of his treatment of increasing returns goes beyond the importance of the analytical achievement per se. Until then, increasing returns had been treated as anomalies, dangerous for the local stability of the equilibrium and for the presence of competition between firms. On the contrary, looking at the dynamics of increasing returns reveals that, while positive feed-backs disrupt the traditional competition leading to the equality of marginal values, there are other forces – such as the life-cycle of the firm and

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[20] For a comprehensive survey see Markose (2005)

[21] While these ideas were initially appreciated by economic historians, they found opposition within economics. The attitude changed when Arthur went to Santa Fe Institute in 1987: 'At a conference there the physicists turned around and explained to the economists, including Kenneth Arrow, that my approach was absolutely standard in physics […] after that I could see the economists in the room relax. I started to have a lot of support from first rate economists' (in Delorme and Hodgson 2005, p. 18). Arthur's idea was initially applied to the adoption of technology and then was applied to economic geography, the evolution of patterns of poverty and segregation, and institutional analysis (Arthur 1999, p.108) It has also affected important jurisprudential decisions such as the one involving Microsoft's antitrust issues.

innovation – preventing firms from growing endlessly. Therefore, Arthur's analysis provides an important example of how complexity economics can deal with problems that were intractable in the previous framework.

Another influential contribution concerns the emphasis on the relevance of micro interaction to the configuration of the macrostructures. Thomas Schelling[22] has shown that, under local interaction, agents with a slight preference for having neighbours of the same type generate a segregated world. The hiatus between micro behaviour and macro consequences emphasises the need for testing the robustness of economic theories to the (often tacit) assumptions concerning interactions: if these are non-linear, the behaviour of the economy can be very sophisticated even in the presence of very simple individual behaviours. Moreover, Föllmer (1974) showed that under stochastic interaction – even in the case of a large number of agents – the use of an average agent is not accurate since the effects of random interactions do not offset each other.

In the wake of Simon's and Hayek's works, many studies aimed at questioning the conviction that the invisible hand should have 'rational fingers' (Epstein 1999). There have been interesting works that decouple rationality from equilibrium. For instance, Axtell and Epstein (1999) showed that equilibrium (although not necessarily the one that an agent endowed with perfect rationality would have chosen) could be reached by non-rational agents. In a previous work (Epstein and Axtell, 1996), they also demonstrate that Olympically rational agents were unable to reach any equilibrium. This stream of studies shows that assuming Olympic rationality is not only unrealistic but also unnecessary and not sufficient to obtain meaningful models.

A further relevant implication, grounded on the above sketched arguments, results in a criticism to the rational expectation hypothesis. In particular, the assumption that all agents use the same (true) model to assess the consequences of their actions and that such a model is common knowledge. Complexity theory explores the hypothesis that the model governing the environment in which the decision making takes place and the decision maker expectations are not given, rather they have to be built (rectius: induced) by her (Arthur 1994a, Holland et al. 1997)[23]. The emphasis is on the learning process that generates hypotheses concerning the functioning of the system that are confirmed or rejected according to their performance. An interesting foray into this domain in which there is not a 'true' model of the system nor a priori correct expectations is the above

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[22] While Schelling works on segregation are acknowledged as pioneering generative science (see Epstein and Axtell 1996) and he is often included in the economists who 'dissent' from mainstream economics, in an interview (2005, p.38) he declared 'I consider myself in the rational-choice school, absolutely. But I am more interested in exceptions than many other economists tend to be'.

[23] In this sense, complexity stems from the 'multitude of elements in the form of beliefs models that adapt to the aggregate environment they jointly create' (Arthur 1994b, p. 410).

mentioned 'El Farol Bar Problem' (Arthur 1994b). Arthur's work maintains that in many contexts (e.g. stock market, oligopoly pricing) the most appropriate model of rationality is the inductive one, consistently with procedural rationality.

VI. NEW PARADIGM, OLD ROOTS

The science of complexity proper has its roots in cybernetics, but as far as economics is concerned, the attempt at capturing the way in which regularities emerge from decentralised interaction of autonomous and heterogeneous agents has a longer tradition the permeates the entire history of the discipline, since Adam Smith's Invisible Hand metaphor. Some scholars, however, have been closer to the complexity view in its current meaning.

The most lucid reflections on the complex nature of economics are to be found in Marshall, Keynes and Hayek as discussed at length in Marchionatti (2002) and (Vriend 1999), on which I rely.

These scholars singled out many of the aspects and implications of the complexity of contemporary science. For instance, Marshall thought that biology – and not mathematical physics – was the science closest in spirit to economics and that there was no such thing as a general economic law which could be as precise as the laws of physics (Marshall 1961, p. 14)[24]. Moreover, in his view exactness could not be attained because of the 'variety and uncertainty of human action' (ibid., p. 781). Deductive reasoning applies only in very simplified contexts because as we proceed in the speculation, the number of circumstances and their reciprocal influences increase as to impair any useful conclusion. Consequently, Marshall disapproved the extensive use of mathematics – in its typical deductive form.

Keynes was concerned with the nature of economic material which is '[…] in too many respects, not homogeneous through time" (Keynes 1973: 297). In fact, economics has to cope with "motives, expectations, psychological uncertainties" that change in time and make the analysis less precise[25]. That is, in a context of limited knowledge and structural uncertainty the object of analysis becomes complex. Non-homogeneity through time requires inductive analysis and attention to the particular characteristics of the historical world. In the light of this consideration, it is clear that for Keynes generalisation is not possible.

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[24] Marshall is quoted in the most disparate situations. Milton Friedman (1953, p. 7) quoting 'The Present Position of Economics' (1885) to candidate formal logic and factual observation as 'systematic and organized methods of reasoning' is paradigmatic.

[25] The similitude with Arthur thought in this case is striking: 'economy relies on human being and not on orderly machine components. Human beings with all their caprices, emotions, and foibles' (1994a, p.1).

Hayek, in writing about sensory and spontaneous orders, describes their nature of self-organising structures characterised by self co-ordination and self-control. Order is inter-connective: 'its actions are determined by the relation and mutual adjustment to each other of the elements of which it consists' (Hayek 1967b, p. 73) and the many connections, 'proceeding at any moment, can mutually influence each other' (Hayek 1952, p. 112). For what concerns social spontaneous orders, they are based exclusively on the attempts of individuals to reach self-set goals (Hayek 1978, chap XII).

While it would be bold to argue of a continuous line linking these authors to the present endeavour, I will however put forward a conjecture. In the absence of methods able to encompass heterogeneity, uncertainty and connectivity, the speculations of these scholars on the complex nature of economics and economic phenomena have been interpreted as abstract considerations: Marshall's evolutionary metaphor of the trees in the forest, Keynes's animal spirits, and Hayek's spontaneous order were often regarded as non susceptible of formalisation. The mathematisation and axiomatisation of economics – which gave scholars a language which was not only descriptive but also analytic – seemed a more viable route to economic analysis. The computational methods developed by the science of complexity bridge this gap between ideas and analyses, offering a convenient way to actually apply the 'abstract' insights reviewed above (Vriend 1999). Progresses in mathematical techniques and advances in computational power have made old propositions and intuitions tractable. The application of the categories of the complexity paradigm is likely to reveal hidden insights in the theories of those economists who in the past have caught the complexity of their discipline (see for instance, Fontana and Marchionatti 2007).

VII. CONCLUDING REMARKS

Complexity is still a young and evolving branch of science. In economics its boundaries are growing more and more precise, the number of researchers defining themselves as 'complexity scholars' is rising and specialised journals devoted to their works have been founded. At the same time, their presence on established journals is increasing. Here I have tried to draw the implications of these facts. A comparison between the mainstream and the complexity view has shown that the latter is incompatible with the former. That is, adhering to the complexity perspective implies a rejection of the received conceptual categories and tools. I have focused on the role of mathematics stressing the importance it has had in economic science so far. Its relevance is necessarily is reduced in complexity since endogenous changes and heterogeneity make complex systems intractable by means of system of equations, while self-reference impairs deductive reasoning.

Having appreciated that complexity theory cannot be included in the mainstream approach to economics, the paper has dealt with its internal consistency: I have shown that complexity theory can be considered a paradigm, according to Kuhn's categories, and its tenets have been described. A

further level of the analysis has regarded the results of the line of research developed within the complexity view. These works explore new ambits of economic phenomena: increasing returns cease to be considered anomalies, the effects of interaction and the emergence of macro pattern are encompassed, inductive reasoning is applied to solve paradoxes generated by self-reference. A final reflection has been devoted to those scholars that, in the history of economics, have caught the complex nature of their discipline. The short foray in the past of economic theory is not only an exercise in the history of economic thought, it also suggests that the effects of many interesting theoretical propositions have not been investigated due to the lack of available methods apt to cope with complexity.

In spite of some ambiguities in contemporary scholars' statements, we can state - echoing Brian Arthur - that 'the economics of complexity is not an adjunct to standard economic theory but theory at a more general, out-of equilibrium, level' (Arthur 1999, p. 108).

REFERENCES

Albin, Peter (1998) *Barriers and Bounds to Rationality: Essays on Economic complexity and Dynamics in Interactive Systems* Princeton: Princeton University Press.

Anderson, Philip W. and Kenneth J. Arrow and David Pines (eds.) (1988). *The Economy as an Evolving Complex System.* Redwood: Addison Wesley.

Arthur, W. Brian (1994b), Inductive Reasoning and Bounded Rationality. *American Economic Review.* 84: 406-411.

Arthur, W. Brian and Steven N. Durlauf and David Lane (1997). *The Economy as an Evolving Complex System II.,* Redwood: Addison Wesley.

Arthur, W. Brian (1994a), The End of Certainty in Economics. Talk delivered at the Conference *Einstein Meets Magritte*, Free University of Brussels, 1994. Appeared in *Einstein Meets Magritte*, D. Aerts, J. Broekaert, E. Mathijs, eds. 1999, Kluwer Academic Publishers, Holland. Reprinted in *The Biology of Business*, J.H. Clippinger, ed., 1999, Jossey-Bass Publishers.

Arthur, W. Brian (1999). Complexity and the Economy, *Science.* 284: 107-109.

Axelrod, Robert (1997). *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration.* Princeton, NJ: Princeton University.

Axtell Robert and Joshua M. Epstein (1994). Agent-based Modelling: Understanding Our Creations. *The Bulletin of the Santa Fe Institute*: 28-32.

Axtell Robert and Joshua M. Epstein (1999). Coordination in Transient Social Networks: An Agent-Based Model of the Timing of Retirement, In Henry Aaron (Ed.). *Behavioral Dimension of Retirement Economics.* New York: Russel Sage.

Bak, Peer (1997). *How Nature Works.* Oxford: Oxford University Press.

Barkley Rosser, John Jr. (1999), On the Complexities of Complex Economic Dynamics. *Journal of Economic Perspectives.* 13(4): 169-192.

Barkley Rosser, John Jr. (2004), Introduction , in: Barkley Rosser, John Jr. (2004) (ed.), *Complexity in Economics, Methodology, Interacting Agents, and Microeconomic Models.* Cheltenham: Edward Elgar.

Barnett, William A. and Mark Salmon and Alan P. Kirman} (Eds.) (1996). *Nonlinear Dynamics and Economics.* Cambridge, UK: Cambridge University Press.

Benhabib, Jess (ed.) (1992). *Cycles and Chaos in Economic Equilibrium..* Princeton. NJ :Princeton University Press.

Binmore, Kenneth (1987), Modeling Rational Players I. *Economics and Philosophy.* 3: 9-55.

Blume, Lawrence and Steven Durlauf (2001). The Interaction-Based approach to Socioeconomic Behavior, in: Steven Durlauf and Peyton Young (eds.). *Social Dynamics.* Cambridge (Mass.): MIT Press.

Blume, Lawrence E. and Steven N. Durlauf (2005). *The Economy as an Evolving Complex System III.* Oxford: Oxford University Press.

Brock, William A. and David A. Hsieh and Blake Le Baron (Eds.) (1991), *Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence.* Cambridge, MA: MIT Press.

Brown, James H.(1994). Complex ecological systems, in: George Cowan and David Pines and

David Meltzer (eds.) *Complexity: Metaphors, Models, and Reality.* Reading (MA): Addison Wesley:419-43.

Chiarella, Carl (1990). *The Elements of a Nonlinear Theory of Economic Dynamics.* Berlin and New York, NY: Springer-Verlag.

Colander D. (2003), The Complexity Revolution and The Future of Economics, *Middlebury College Discussion Paper* No. 03/19.

Cournot, Augustin [1838] (1960). *Recherces sur les Principes Mathématiques de la Théorie des Richesses.* New York : Augustus M. Kelley.

Creedy, John and Vance Lindsay Martin (Eds.) (1994). *Chaos and Non-linear Models in Economics: Theory and Applications.* Aldershot, UK: Edward Elgar.

Day, Richard H. (1994). *Complex Economic Dynamics*- Cambridge, MA and London, UK: MIT

Debreu, Gerard (1986) . Theoretical Models: Mathematical Forms and Economic Content. *Econometrica.* 54:1259-1270.

Dechert, W. Davis (ed.) (1996). *Chaos Theory in Economics: Methods, Models and Evidence.* Cheltenham, UK: Edward Elgar.

Delorme, Robert and Geoffrey M. Hodgson (2005).Complexity and the Economy: An Interview with W. Brian Arthur, in Finch John and Magali Orillard (Eds.). *Complexity and the Economy:*

Implications for Economic Policy. Cheltenham, U.K. and Northampton, MA.: Elgar in association with the European Association for Evolutionary Political Economy: 17-32.

Devaney, Robert L. (1989). *An Introduction to Chaotic Dynamical Systems*. Reading (MA): Addison Wesley.

Devaney, Robert L. (1992). *A First Course in Chaotic Dynamical Systems: Theory and Experiment*. Reading (MA): Addison Wesley.

Epstein Joshua M. (1999). Agent-Based Computational Models and Generative Social Science. *Complexity*. 4(5): 41-60.

Epstein Joshua M. (2006). *Generative Social Science. Studies in Agent-Based Computational Modeling*. Princeton-New York: Princeton University Press.

Epstein, Joshua M. and Robert Axtell (1996). *Growing Artificial Societies - Social Science from the Bottom Up*. Cambridge MA: MIT Press.

Follmer, Hans (1974), Random Economies with Many Interacting Agents. *Journal of Mathematical Economics*. 1: 51-62.

Fontana, Magda (2006). Computer Simulations, Mathematics, and Economics. *Rivista Internazionale di Scienze Economiche e Commerciali*. 53(1): 96-124.

Fontana, Magda and Roberto Marchionatti (2007). Endogenous Animal Spirits and Investment An Agent-based Model. *CESMEP - Centro di Studi sulla Storia e i Metodi dell'Economia "Claudio Napoleoni", working paper 9/2007*.

Foster John (2005). From Simplistic to Complex Systems in Economics. *Cambridge Journal of Economic*. 29: 873-892.

Foster John and Stan Metcalfe (2001). *Frontiers of Evolutionary Economics: Competition, Self-Organization and Innovation Policy*. Cheltenham UK: Edward Elgar.

Friedman, Milton (1953). *Essays in Positive Economics*. Chicago: University of Chicago Press.

Gaffeo, Edoardo (2005). Commento alla relazione di Kirman, in: Delli Gatti Domenico and Mauro Gallegati (eds.). *Eterogeneità degli Agenti Economici e Interazione Sociale: Teorie e Verifiche Empiriche*. Bologna: Il Mulino: 29-34.

Gandolfo, Giancarlo (1997). *Economic Dynamics*. Berlin: Springer-Verlag.

Gell-Mann, Murray (1995). Complex Adaptive Systems, in: Harold Morowitz and Jerome L. Singer *The Mind, the Brain, and Complex Adaptive Systems*. Reading, MA: Addison-Wesley: 11-23

Glass, Leon and Michael C. Mackey (1988). *From Clocks to Chaos. The Rhythms of Life*. Princeton, NJ.: Princeton University Press.

Goodwin, Richard M. (1991). Non-linear dynamics and economic evolution, in Thygesen Niels and Kumaraswamy Velupillai and Stefano Zambelli.(eds.), *Business Cycles: Theories, Evidence and Analysis*. London, UK: Macmillan :424-442.

Hahn , Frank (2001). The Next Hundred Years. *The Economic Journal.* 101: 47-50.

Hamouda, Omar F. and J. C. R. Rowley (Eds.) (1999). *Discrete and Continuous Systems, Cointegration and Chaos*. Cheltenham, UK : Edward Elgar.

Hanappi Hardy (2007) review of Blume, Lawrence E. and Durlauf, Steven N. (2005). *The Economy as an Evolving Complex System: III*. Oxford: Oxford University Press, in *Journal of Artificial Societies and Social Simulation* Volume 10(2).

Hayek F. A. (1952), *The Sensory Order. An Inquiry into the Foundations of Theoretical Psychology*, Routledge & Kegan Paul, London.

Hayek F. A. (1967), The Theory of Complex Phenomena, in *Studies in Philosophy, Politics and Economics.* London: Routledge & Kegan Paul: 71-104.

Hayek F. A. (1967b), The Results of Human Action but not of Human Design, in *Studies in Philosophy, Politics and Economics.* London: Routledge & Kegan Paul: 189-204

Hayek F. A. (1978), *New Studies in Philosophy, Politics and the History of Ideas.* London: Routledge & Kegan Paul.

Holland, John H. – John Miller (1991). Artificial Adaptive Agents in Economic Theory. *American Economic review.* 81(2): 365-370.

Holland, John H. (1995). *Hidden Order. How Adaptation Builds Complexity*. Reading, MA: Addison-Wesley.

Horgan John (1997). *The End of Science: Facing the Limits of Knowledge in the Twilight of the Scientific Age.* New York: Broadway Books.
http://jasss.soc.surrey.ac.uk/10/2/reviews/hanappi.html

Kauffman Stuart A. (1993). *The Origins of Order. Self-Organization and Selection in Evolution*. Kelsey D. (1988). *The economics of chaos and the chaos of economics*. Oxford Economic Papers. 40: 663-693.

Keynes, John .M. (1973 [1937]), *The General Theory of Employment, Money and Interest*, in *The Collected Writings of John Maynard Keynes*, vol. VII, (D. Moggridge ed.). London: Macmillan for the Royal Economic Society.

Kirman, Alan (2005). Interazione e Mercati, in: Delli Gatti Domenico and Mauro Gallegati (eds.). *Eterogeneità degli Agenti Economici e Interazione Sociale: Teorie e Verifiche Empiriche.* Bologna: Il Mulino: 17-28.

Koppl, Roger and John Barkley Rosser (2002). All That I Have to Say Has Already Crossed Your Mind . *Metroeconomica.* 53: 339-360.

Kuhn Thomas S. (1962), *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press

Lesourne, Jacques (2002). *The Economics of Order and Disorder.* Oxford: Clarendon Press.

Li Tzong-Yau and James A. Yorke (1975). Period three implies chaos. *American Mathematical Monthly.* 82: 985-992.

Lorenz, Edward N.(1963), Deterministic nonperiodic flow. *Journal of Atmospheric Sciences*. 20: 130-41.

Lucas, Robert E. (1981). *Studies in Business‑Cycles Theory*. Cambridge: MIT Press.

Marchionatti Roberto (2002). Dealing with Complexity. Marshall and Keynes and the Nature of Economic Thinking in *The Economics of Alfred Marshall* (Richard Aréna & Michel Quéré eds.), London: Palgrave, 2002, pp.32-52.

Markose S. (2005). Computability and Evolutionary Complexity: Markets as Complex Adaptive Systems (CAS). *The Economic Journal.* 115: 159-192.

Marshall Alfred (1961) [first edition 1890, eight edition 1920] *Principles of Economics*, 2 vols. (vol. I *Text*, vol. II *Notes*), edited by C. W. Guillebaud, London: Macmillan.

Marshall, Alfred (1885) *The Present Position of Economics*, London: Macmillan.
May, Robert M. (1974). *Stability and Complexity in Model Ecosystems*. Princeton, NJ: Princeton University Press.

McIntire, Lee (1998). Complexity: A Philosopher's Reflections. *Complexity*. 6: 26-32.

Medio, Alfredo and Giampaolo Gallo (1992). Chaotic Dynamics : Theory and Applications to Economics. Cambridge, UK: Cambridge University Press.

Morgenstern Oskar [1935] (1976). Perfect Foresight and Economic Equilibrium, in Schotter Andrew (ed.) *The Selected Economic Writings of Oskar Morgenstern*. New York: New York University Press.
New York: Oxford University Press.

Morgenstern, Oskar (1928) *Wirtschaftsprognose, eine Untersuchung ihrer Voraussetzungen und Möglichkeiten*. Vienna: Springer Verlag.

Peak, David and Michael Frame (1994). *Chaos Under Control. The Art and Science of Complexity.* New York, NY: W. H. Freeman.

Ruelle, David (1991). *Chance and Chaos.* Princeton, NJ: Princeton University Press.

Samuelson, Paul A. (1947). *Foundations of Economic Analysis*. Cambridge, MA: Harvard University Press [revised edition 1983].

Schelling, Thomas. C. (1960). *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

Schumpeter Joseph A. (1954). *The History of Economic Analyses*. New York: Oxford University Press.

Simon Herbert A. (1969). *The Science of Artificial.* Cambridge (Mass.): MIT Press.

Simon Herbert A. (1972). The Architecture of Complexity. *Proceedings of the American Philosphical Society.* 106: 467-482.

Sugden Robert (1996). *Spontaneous Order.* Manuscript.

Vriend, Nicolaas J. (1999). Was Hayek an Ace?. *Queen Mary and Westfield College Working Paper* No. 403.

Windrum, Paul and Giorgio Fagiolo and Alessio Moneta (2007). Empirical Validation of Agent-Based Models: Alternatives and Prospects. *Journal of Artificial Societies and Social Simulation* 10(2)8 <http://jasss.soc.surrey.ac.uk/10/2/8.html>.