department of economics
"S. Cognetti de Martiis"

Via Po, 53 – 10124 Torino (Italy)
Tel. (+39) 011 6702704 - Fax (+39) 011 6702762
URL: http//www.de.unito.it

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Computer Simulations, Mathematics and Economics

Magda Fontana

Dipartimento di Economia "S. Cognetti de Martiis"

Centro di Studi sulla Storia e i Metodi dell'Economia Politica
"Claudio Napoleoni"
(CESMEP)

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Computer Simulations, Mathematics and Economics

Magda Fontana[*]
University of Turin

Abstract

Economists use different kinds of computer simulation. However, there is little attention on the theory of simulation, which is considered either a technology or an extension of mathematical theory or, else, a way of modelling that is alternative to verbal description and mathematical models. The paper suggests a systematisation of the relationship between simulations, mathematics and economics. In particular, it traces the evolution of simulation techniques, comments some of the contributions that deal with their nature, and, finally, illustrates with some examples their influence on economic theory.

Keywords: Computer simulation, economic methodology, multi-agent programming techniques.

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[*] Mail: Dipartimento di Economia, via Po 53, 10124 Torino. E-mail: magda.fontana@unito.it. The author gratefully acknowledges the financial support of MIUR (PRIN 2002: "*La matematica nella storia dell'economia. Studio di un tema controverso*"). I would like to thank an anonymous referee, R. Marchionatti, S. Fiori, and the participants to a seminar held at the University of Siena for comments and useful suggestions on an earlier version of this paper. The usual disclaimer applies.

1. Introduction

Computer has profoundly affected the way economists do and think about economics (Mirowski 2002, Duren 1988, Galison 1997). Not only it has relieved scholars from computational burden (originally, computers were persons hired to perform calculus), but it has also changed the mode in which economists approach their theories. By using simulations they are provided with metaphors of human thinking and problem solving, have the opportunity to subject social processes to laboratory experimentation and, finally, they are allowed to model economic agents in fashions that were precluded to mathematical and verbal modelling.

The paper deals with the relationships between computer simulations and economics. I investigate how the possibility of embedding economic theories within computer programs, that is running a simulation, has affected the process of theory-making and has accompanied some developments of economic thought. To this purpose, I will proceed along two lines. Firstly, I will try to characterise the concept of simulation by tracing its evolution and comparing it with other formal languages and modelling procedures. Secondly, I will try to identify those developments of simulations that have mostly affected economics. The analysis covers a period that starts from post World War II to the present days, while the places of action are mainly the faculties of engineering of the major universities of the United States. It will appear how simulation, that generated from very practical concerns, will subsequently circulate in different disciplines - such as economics- and will become a theoretical instrument. The central tenet of the paper is that simulation is a way of conducting research that is autonomous – i.e. it has distinctive properties and a different ability in capturing the phenomena under study - with respect to other modelling solutions.

2. The origin and evolution of computer simulation

The philosopher Peter Galison reconstructs the steps through which computer simulations come to stage:” *At first no more than a faster version of an electro-mechanical calculator, the computer became much more: a piece of the instrument, an instrument in its own right, and finally (through simulations) a stand in for nature itself. [...] In a non trivial sense, the computer began to blur the boundaries between the ‘self evident’ categories of experiment, instrument, and theory”* (1997, p. 44-45). The process, however, is far from being linear because the term “simulation” refers to a variety of techniques with different lineages and theoretical niches, and because simulation originates in hard sciences and ends up in social science. The paper will consider system dynamics, microsimulation, cellular

automata and agent-based models, which I consider the most significant for the theory of simulation and for economics. These simulations can be split in two groups. The first one is related to the development of system dynamics modelling under the influence of Norbert Wiener's[1] cybernetics. Computer simulations are representations of mathematical models within the computer aiming at extending tractability when a large amount of computation is needed. The debated point, when those simulations first appeared, was whether they could actually be considered as mathematics or, else, they were to be thought of as a mere a support to mathematical research. The second one, pioneered by John Von Neumann[2] and including cellular automata and agent-based computational models, tends to emancipate simulation from mathematical representations.
The next two sections will be devoted to the analysis of these ways of conceiving simulation. The leading theme will be the relationship between mathematics (read equation-based modelling) and simulation, an issue which is pervasive and permeates all related arguments. Originally, simulations were nothing but the numerical treatment of differential equations and, in addition, most of simulation techniques stemmed from extensions of mathematical analysis. Henceforth, it seems natural to treat the argument starting from the relationship between simulation and mathematics. Discussion will be completed by the description - in section 3. - of a further, more recent, approach to simulation (Ostrom 1988, Parisi 2001), which focuses on the features that derive from using the symbol system of programming language.

[1] (Cambridge Mass. 1894, Stockholm 1964). He started his studies in zoology at Harvard turning to philosophy at Cornell one year later. He received his Ph.D from Harvard at the age of 18 with a dissertation on mathematical logic supervised by Karl Schmidt. He then went to Cambridge (U.K.) to study under professors Russell and Hardy. In 1914 he was in Göttingen to study differential equations under Hilbert and also attended a course of group theory given by Edmund Landau. In 1920 Wiener joined the Massachusetts Institute of Technology, where he became (1932) professor of mathematics.

[2] (Budapest 1903 – Washington D.C. 1957). Doctorate in mathematics from the University of Budapest, undergraduate chemistry degree from the Eidgenossische Technische Hoschschule in Zurich. In 1930 he went to United States as a visiting lecturer at Princeton University where he was made full professor in 1931. In 1933 he joined the Institute for Advanced Study as a professor and retained that position for the rest of his life.

2.1 Simulation at MIT: mathematicians, engineers and physicians

As for the origin of computer simulation, the right place to look at is the MIT during and after WWII. In its laboratories, mathematicians, engineers and physicians were independently working on issues that later generated the first simulation techniques. As it will be shown, the origins and features of computer simulation are quite different the ones from the others, but are assimilated by one quality: their strong relation to mathematics. However, as simulation techniques grew more sophisticated, separation from mathematical endeavours started to be a widespread need among simulators. This separation, that in my opinion is now completed, took place through a progressive identification and conceptualisation of simulation's properties.
The attempt at tracing this process bring us to the discussion of different views ranging from those insisting that simulation is a pale imitation of mathematics, and concluding that it does not pertain to the realm of theorising but, rather, to the realm of measurement (Alker 1974), to the more recent ones that pinpoint the characterising elements of simulation and contrast them with mathematical and verbal modelling (Axelrod 1997a). Given that first applications openly intended to be mathematical, it took quite some times before the issue concerning the nature of simulation was ripe to discuss. Computer entered universities and research centres in the early 1960s and the debate started to spread about ten years later, however to grasp fully the terms of the problem, one has to go back to the very inception of simulation: WWII.

2.1.1 Cybernetics and System Dynamics

Our story begins with the American mathematician Norbert Wiener working on gunfire control at MIT in 1940. He devised a predictor of the behaviour of an aircraft trying to evade antiaircraft fire. He conceived the relation between man and machine system as essentialy similar to that of a servomechanism[3]. The pilot is considered as a part of the steering mechanism and thus it is possible to apply to the interaction between man and machine notions -such as feedback and stability- which were originally devised for mechanical systems and electrical circuits (Wiener 1961, p. 8; Mirowski 2002, chapter I). As time passed by, such flashes of insights were elaborated in a theory which Wiener named cybernetics (after the greek

[3] A servomechanism is an automatic control of a mechanical device; it regulates the mechanism in response to feedback.

word *kubernetes* which means steersman or governor) and passed over to meteorology, sociology and economics. While the cybernetics contributions to science are highly controversial (Heims 1980, 1991), this paper focuses on its influence on simulations. Briefly, Wiener's idea (1956, p. 251-252) was that, in order to obtain a complete mathematical treatment of a system, it was necessary to assimilate its parts to a single root either human or mechanical. Since understanding of mechanics appeared far ahead of psychological understanding, he chose to construct a mechanical model of the relation between human and mechanical: the feedback which, roughly, implies circular causation. This line of thought was applied to simulation by Jay Forrester[4] an engineer that, in the same period, was studying feedback control systems (control of radar antennas and gun mounts) at MIT's Servomechanism Lab. Wiener's influence on Forrester is strong, the concept of feedback and the theory of causes and effects in fact are central to his *system dynamics* simulation. Following cybernetics, Forrester moved away from looking at isolated events and their causes (usually assumed to be other events), and started to look at phenomena as systems made of interacting parts. He believed that the "events cause events" orientation was not very helpful in the understanding of a system and in altering its undesirable performances. This because it is always possible to find yet another event that caused the one that was thought to be the cause. This is almost a *regressio ad infinitum* and thus it is difficult to determine where to stop searching for causes and begin to act in order to improve performance. System dynamics takes the alternative viewpoint that the internal structure of the system (the way parts are interrelated) is often more important than the external events in generating the behaviour of the system. According to Forrester, a proper definition of such interrelation is the feedback: e.g. the situation of X affecting Y and Y in turn affecting X perhaps through a chain of causes and effects. The idea is that it is impossible to study the link between X and Y, independently, because it is precisely the link between Y and X that will generate system behaviour.
System dynamics, in Forrester's thought, is a way to investigate counterintuitive and surprising outcomes that can arise in systems of multiple non-linear equations (Gilbert and Troitzsch, 1999, chapter III). A system dynamics model describes the target system by means of large

[4] Forrester Jay (1918) Nebraska, Engineering College University of Nebraska, graduated MIT 1939. He then joined the High Voltage Lab and transferred a year later to the Servomechanism Lab.

systems of (discontinuous) differential equations from which the trajectories of variables over (discrete) time are plotted. The target system is an undifferentiated whole whose properties are described by means of levels (the state of the entire system) and rates (its changes). The model starts with individuating the pattern of behaviour exhibited by the variable of interest over time (e.g. exponential growth) and by describing the system structure in terms of feedback or causal loop. In terms of economic methodology, system dynamics modelling is a pattern modelling process. With appropriate refinements, a system dynamics model can be converted into a tipology called a 'generic structure' (i.e. a model that encapsulates the essential relationships that appear in a multiplicity of pattern models within a group). Such generic structure, when properly parameterised can reproduce any patterns within its group (Radzicki, 2003, p. 151).

The lesson that can be learnt from this kind of simulation is that, in practice, it was meant to be mathematical, its main modelling feature being the ability of extending analytical treatment to discrete time and non-differentiable differential equation. Another attribute that contributed to direct the following debate was that Forrester's vocation was practical: "*early system dynamics analyses were in the consultant mode in which the system dynamicist would study a corporation, go away, build a model, and come back with recommendations*" (Forrester 1989).

Early system dynamics was thus tuned on problem solving for corporations, bureaucrats and policy makers, and as such had a "business" flavour that did not facilitate its recognition in the academic field. Forrester retained the idea that research and theory must be related to the field application, and described his work as an attempt to run from mathematical theory to the operating field. In fact, the beginning of system dynamics was an inventory control system with pencil and paper simulation for General Electric. The actual simulation arrived a bit later, when he asked a computer programmer to write down the code for his 1958's article "Industrial Dynamics a Major breakthrough for Decision Maker"[5]. The programmer created a compiler that would automatically generate the computer code and called it "SIMPLE" the acronym for "Simulation of Industrial Management Problems with Lots of Equations". The presence of a compiler accelerated modelling to such an extent that it rapidly expanded and nowadays it is still widely used especially by an active group at the Sloan College at MIT. In 1969, Forrester applied system dynamics to the description of urban dynamics, a work that raised a

[5] The article later became chapter II of *Industrial Dynamics* (1961).

lot cricisms. Together with the mayor of Boston who he had met at MIT as a professor of Urban Affairs, he portrayed the city as a system of interacting industries, housing and people. His conclusions produced strong reactions. The model suggested that "*all the major urban policies that the United States was following lay somewhere between neutral and highly detrimental, from the view point of either of the city as an institution, or from the viewpoint of the low income, unemployed residents, and that the most damaging policy was to build low-cost housing [because] such housing used up space where job could be created, while drawing in people who needed jobs*" (Forrester, 1989).

In a time in which low cost housing policy where believed to be essential, the publication of *Urban Dynamics* (1969) did not contribute much to the fortune of simulation. A similar reaction was reserved to *Limits to Growth* (Meadows et al., 1972), a book that adopted Forrester's system dynamics to look at the scenarios for human population growth and industrial production in the world over the next century. A computer model was used to simulate resources production and food supply to keep up with the growing system. The authors concluded that the world could not support the present rates of economic and population growth much beyond the year 2100. Those models made a major impact but also diffused the feeling that simulation was somewhat non-scientific as it became clear that results heavily depended on the specific quantitative assumptions made about the model's parameters and that many of them were backed by rather little evidence (Gilbert and Troitzsch, 1999, p. 6). As it will be shown, the feeling among scientists was that system dynamics had made questionable policy recommendations, as in the case of Forrester (1969), as well as inaccurate predictions as in the case of Meadows (1972).

2.1.2 Microsimulations

In the same years at MIT, Guy Orcutt [6], a researcher who was trained in engineering, physics and economics developed *microsimulation* (1957;

[6] University of Michigan: B.S. Physics, Phd Economics. Soon after finishing his doctoral dissertation in 1944 he was appointed at MIT. Orcutt's works, alone and with Donald Cochrane, are part of every econometrician's tool kit.

Orcutt et al., 1986), a technique attempting at modelling social phenomena in highly disaggregated way[7].

Orcutt's interest in economics was motivated by the nation economic difficulties at the time (Orcutt 1990). From his studies in economics he become convinced that economic models were in urgent need for stronger empirical basis. Around 1950, by focusing on data aggregated at the national account level, he realised that they were not accurate enough to provide a useful guide for policy. Orcutt thought that economic models should have been built at the micro level and, therefore, that a firm understanding of the behaviour of micro-units was required. As for policy implications, models should have taken into account that the overall impact of such policies may depend on how their consequences are distributed over non-homogeneous individuals. Aggregate time series cannot capture those aspects, and, even if it is possible to establish robust behavioural relationships at the microunit level, there remained the problem of aggregating them in order to appreciate the macroeconomic consequences of policies or exogenous shocks (Watts 1991, p.173). Orcutt's answer to this problem was the conceptualisation and implementation of microsimulation. Microsimulation represented the convergence of the ideas he nurtured during his training (Watts 1991, p.174): the first was *Monte Carlo simulation* (see below) he had been using in the context of electrical analogue models to explore the consequences of autocorrelation in regression estimates; the second one was neoclassical economics imprinting that drew his attention on the market as a system in which many agents interact; the third one, attributable to his studies in physics, was that the world is recursive, ruled by the response-follow-to stimulus motto. A microsimulation is a computer code that applies to a dataset of micro units (e.g. households or firms). It starts from a representative sample of the population that contains all the information of interest (e.g. age, sex, marital status, participation to education, income) observed in given moment in time. The simulation consists in observing the state of the sample under different prospects. A microsimulation can be either static (in which case the sample does not change) or dynamic (in which case the units undergo some transformation in response to time or to behavioural pressure). The static microsimulation is best used to calculate the day after effect (before behavioural reactions) of a policy change.

[7] The structure of the microsimulation models was described in 1957 and the first application, regarding demographic processes, labour supply and education demand, began around that time and appeared in Orcutt et al. (1961).

Imagine we want to compute the effect on the government revenues of a reform of the income tax. For each unit the sample displays the (gross) income level and the other relevant variables such as appliable allowances and deductions (e.g. allowances for children, cost of education, social security contribution…). The computer code calculates individually the taxable income and, subsequently, the tax to be paid. The procedure can be repeated for different tax structures. Results can then be observed at the individual and aggregate level. It is worth noting that, apart from the tax formula, all the other features are kept constant (there is no aging process, no birth or change in occupational status). In dynamic simulation, on the contrary, the units of the sample can change as the simulation runs. Following life tables, at each time step individuals are aged, and – according to age – they can either give birth to a child or retire from work, die and so on. Consequently, changes are computed for all the related attributes such as income, participation in education, employment. Dynamic microsimulation is applied to long-run prediction of demographic change and to its effects on social expenditure, and to long term behaviour of labour supply, consumption and the like. For example, in order to know how many people of 60 years or older will have adult near relatives who could nurse them if they needed care, one cannot simply run a system dynamics demographic simulation which computes the future structure of the population as a whole. Rather, a model of the kinship networks within the sample and including their transformation in time is needed. Hence, individual data are used and birth and marriage probabilties are applied to update the sample year by year. After the desired number of runs the results are interpreted as the evolution of the initial sample[8].

As compared to system dynamics, the modelling approach is considerably different. While the former considers the target system as a whole which disregards the features of the units and therefore produces very aggregated information, microsimulation generates individual information that can be aggregated at the desired level (group, age range…). As for the more general features of this modelling approach, it is worth stressing that is has no pretension to explain the phenomenon under study: it simply aims at prediction. Secondly, there is no attempt at modelling interaction among units, rather each of them has a given trajectory which is independent from

[8] A further type of microsimulation called longitudinal applies to the entire life of an age cohort. For further discussion see Gilbert and Troitzsch (1999, pp. 7; 53-73) from which these examples are borrowed.

that of the others. Finally, when units change their state they do so exogenously (for instance according to life tables) and not in response to some behavioural rule.
Microsimulation found lasting employment and wide acknowledgment within economics (particularly in gender and population economics and in tax policy analysis)[9], but this did not help much in contributing to simulation's success. In fact, it played a role in circulating the idea that simulation is merely a measurement procedure, something that has to do mainly with statistics and perhaps with econometrics, but cannot stand independently of mathematical and verbal modelling.
On the other hand, scientists were bothered by the empirical failures of system dynamics and by the difficulty of framing computer simulation within the traditional categories of science[10]. A typical example of this kind of reasoning comes from Hayward Alker's paper of 1974[11]. Alker compares simulation (read system dynamics) with mathematical models and natural language descriptions. However, he feels none of them fit it completely. System dynamics are large system of equations that are written in a programming language and that include qualitative statements. Moreover, the procedure to obtain the output is different from analytical solution and this, he believes, changes the nature and reliability of results. They are not as reliable as those generated by elegant and soluble mathematical representation of social processes, the lack of rigour being due to the absence of a shared set of formal rules of representation and to the lack of received procedures for the solution of the models (Alker, 1974, p. 152). He therefore concludes that simulation is "*bad mathematics and poor social science*" (p. 140) and thus it is closer to verbal representations[12]. The use of simulation

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[9] Moreover, in many countries (Australia, Israel, United Kingdom, Germany, Sweden…) specialised institutes carry out this kind of simulation and publish their results on a regular basis. See for instance the publications of the National Centre for Social and Economic Modelling of the University of Canberra (www.natsem.canberra.edu.au).

[10] See for instance the debate appeared in different issues of *Scienc*e in 1973. Disputes surrounded especially the works of Forrester (1971) and Dreyfus (1972).

[11] Alker is a political scientist that uses simulation to model decision making in political processes. For an example see Alker and Christensen (1972).

[12] Recent studies (Edmonds 2003) echo Alker's opinions.

must be secondary with respect to the traditional mathematical and natural descriptions and must be used to explore hypotheses rather than to formalise them. The results of a simulation can be considered, at most, as hints for social scientists that must not be trusted since a lot of them "*are contradicted by available evidence; others have not even been carefully tested, a few fit the evidence within limited content areas reasonably well"* (Alker 1974, p. 152).

Alker's opinion reflects the difficulty in including simulation in the existing categories and the negative impression it has made on social science. It will take some more time before new categories are imported from computer science and new simulation techniques are perfectioned to clarify the status of simulation. Let us see how this happened.

2.2 Simulation gains autonomy: mathematicians, computer scientists and economists

Apart from microsimulation, little was heard about simulation in the 80'. However, there were forces at work. On the one hand, new techniques, such as *agent based models*, bring simulation farther from mathematics. On the other hand, in this decade simulation starts to be considered as an autonomous way of doing research. In fact, if for system dynamics and microsimulation computer was necessary to extend computational abilities, as simulation techniques evolve (embedding, for instance, spatial descriptions), computer is needed because the model can be "solved" only within the machine since it is not possible to write and solve an equivalent, equation based, mathematical representation. In addition, there is a progressive shift of focus from a macro approach (whose attention is devoted to the whole target system) to a modelling approach that stresses the relevance of decentralised interaction and learning models. As we have seen, in microsimulation agents do not actually interact, whereas in this stream of simulations the core of the analysis is the study of macro regularities that emerge out of proper local interaction. Let me give an example: if in a microsimulation the decision to give birth to a child depends on life tables, in the simulations I am about to describe the same decision would be made by taking into account the status of the neighbours (for instance, a female agent can generate an offspring only if she has a male neighbour) or the environmental conditions (for instance, reproduction will take place only if in a given location of space there are enough resources to sustain it). It follows that, contrarily to microsimulation, agents have explicit behavioural rules and, if learning algorithms are introduced, they can learn from experience and exhibit innovative behaviour. With respect to system dynamics these simulations can, at least in the most recent developments, quite easily manage the representation of physical space separately from agents[13].

2.2.1 Von Neumann and Cellular Automata

The impulse to such innovations, similarly to what happened for system dynamic and microsimulation came from physics and engineering. The incipit of this stream of simulations is to be found, yet again, in WWII and comes from the most eclectic member of the cybernetics group: John von Neumann. His writings on computer theory (1958; 1961-63; 1966)[14] anticipated many of the contemporary issues on computer simulation and inspired the simulation techniques discussed in this section.

From 1943 to 1955, von Neumann worked at Los Alamos National Laboratories as a consultant to the armed forces. He was collaborating to the making of the atomic and hydrogen bomb. In order to deal with complicated physical processes that he could not directly observe and experiment, such as the possibility that the test of the atomic bomb would ignite the atmosphere, he developed a method to simulate hydrodynamics, turbulence, and chain reaction in the computer that lately has come to be known as Monte Carlo simulations[15]. The method was born out of his dissatisfaction with mathematical knowledge of non linear partial differential equations. The procedure he pioneered was to employ computer to solve numerically cases and to use the results as heuristic guide to theorising (von Neumann 1966, p. 3). This heuristic use of computers consisted in discovering regularities by solving many (non – linear differential) equations and in generalising results. Solutions were not sought for their own sake, but as an aid to discover useful

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[13] For a more technical explanation see Epstein and Axtell (1996, p. 15-16).

[14] He agreed on writing a book on this topic in connection with some lectures given at University of Illinois. The outline of these lectures together with the recording and typescript have been reorganised by Arthur Burks and published in 1966. Among the previous contributions included in the Collected Works see also: "The General and Logical Theory of Automata" (1948,vol. 5, pp. 288-328); "Probabilistic Logics and the Synthesis of Reliable Organisms from Unreliable Components" (1952, vol. 5, pp. 329-378).

[15] A Monte Carlo simulation is a stochastic technique that samples a large system in a number of random configurations, so that data can be used to describe the system as a whole.

concepts and general theories: "*The heuristic use of computers is similar to and may be combined with the traditional hypothetical-deductive-experimental method of science. In that method one makes a hypothesis on the basis on the available information and derives consequences from it by means of mathematics, tests the consequences experimentally, and forms a new hypothesis on the basis of the findings. This sequence is iterated indefinitely. In using a computer heuristically one proceeds in the same way, with computation replacing or augmenting experimentation. One makes an hypothesis about the equation under investigation, attempts to pick up some crucial special cases, uses a computer to solve these cases, checks the hypothesis against the results, forms a new hypothesis, and iterates the cycle. The computations may also be compared with experimental data. When this is done the heuristic use of computer becomes simulation. Computation in itself can only provide answers to purely mathematical question, so when no comparison is made with empirical fact the heuristic use of computers contributes to pure mathematics*" (1966, p. 4).

With respect to previous positions, in this assessment we find novel elements. The difference between computation and simulations, and the contribution of the latter to theorising on a pair with mathematics are explicitly stated. With respect to Forrester and Orcutt, the approach here is very different. Forrester's system dynamics was driven by practical concerns, i.e. how to solve housing problems in a given city or how to improve the efficiency in a corporation, while Orcutt was trying to highlight the consequences of public policy across heterogeneous groups. According to von Neumann, simulation is on the same level as the deductive methods, has a general scope, and can be used in devising theories. In a close relation with his applied work with simulation, in the late '40 he started developing a theory of automata that, due to his premature death, was left incomplete. Being convinced of the existence of important similarities between computer and natural organisms and of the usefulness of comparing such related systems, he sought a theory that would cover them both. He called it the "Theory of cellular automata". It was concerned with the structure and organisation of both natural and artificial systems and the role of language and information, programming and control in such systems (von Neumann 1966, p.18). The blueprint of the theory was both mathematical and logical (von Neumann 1966, p. 25-28) while the study of actual automata provided its empirical core. Mathematician Stanislaw M. Ulam (1960, Chapter 8) liked to concoct games for the computer at Los Alamos: given certain fixed rules, the computer would produce changing patterns (to take an example, a square would evolve into a crystal-like growth). Ulam's games were cellular games played on limitless lattices: each pattern was composed of square cells that changed as simulated (discrete) time passed. At each step, the state of a given cell depended only on the states of its neighbouring cells. Ulam suggested to von Neumann to adopt the "cellular" framework for his analysis of machine reproduction. In doing so, he would have been able to exploit the cellular structure that reduces the otherwise infinitely many possible connections between machine components to a controllable plan: the model would have been complete enough to cover all the essentials of machine operation but, at the same time, as simple as possible. Von Neumann used an infinite chessboard as his universe. In the latter, each square cell could be in any of a number of states corresponding approximately to machine components (it follows that a "machine" was a pattern of such cells), and the rules governing the world would be those of a cut down physics. Cellular automata, with their ambition to embrace both natural and artificial system, would become in the following years an established way of modelling social systems in which local interaction takes place. However, from physics to social systems the route is not short. The first concerted effort to apply explicitly cellular automata to social science was accomplished, without the use of a computer, by an economist: Thomas Schelling. His Segregation Model (1969, 1971a, 1971b, 1978) consisted in placing pennies and dimes on a chessboard and moving them around according to given rules. He interpreted the board as a geographical space (e.g. a city), with each square of the board representing ,say, a house, pennies and dimes as agents, representing any two groups in a society. The neighbourhood of an agent occupying any location on the board consisted of the squares adjacent to this location. Rules specified whether a particular agent was satisfied with its current location: if she was dissatisfied with it, she would shift to another place on the board. Schelling found that the board quickly evolved into a strongly segregated configuration even if the agents' satisfaction rules expressed only a weak preference for having neighbours of their own type.

2.2.2 The Santa Fe Institute and Agent-based Simulations

In the 80', the use of cellular automata was made more practical by the development of general purpose cellular automata simulator programs to be applied to problems of adaptation and optimisation. In the same years, Schelling's simulations with its developments *in silico* were worked out along different lines, mainly at the Santa Fe Institute[16]. The outcomes of

[16] The Santa Fe Institute, founded in 1984, is devoted to the creation of a scientific research community that emphasises multidisciplinary collaboration in the pursuit of

these researches have found a systematisation and a number of seminal extensions by Joshua Epstein[17], an economist working at the Santa Fe Institute. Epstein's research was presented in the book written with Robert Axtell[18]: *Growing Artificial Society: Social Science from the Bottom Up* (1996). It proposes an original approach to economics (which will be discussed in section 5) that explicitly acknowledges its lineage with Schelling and von Neumann: agent based economics[19]. The book, which is a sort of manifesto for agent based computational economics, introduces Sugarscape, an artificial society in which demographic, environmental, and economic processes take place. This work is important to us, not only because it represents the most recent evolution of Von Neumann's simulation techniques, but also because it is the most organised attempt to characterise simulation as a research methodology. Agent-based economics aims at analysing "*fundamental social structures and group behaviours as emerging from the interaction of individuals operating in artificial environments under rules that place only bounded demands on each agent's information and computational capacity. We view artificial societies as laboratories where we attempt to grow certain social structures in the computer […] the aim being to discover fundamental local or micro mechanism that are sufficient to generate the macroscopic social structures and collective behaviour of interest*" (Epstein and Axtell 1996, p.4). The idea recalls closely that of the economic phenomena as the unintended result

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understanding the common themes that arise in natural, artificial, and social systems. Among the economists, in the Science Board seats Kenneth Arrow. Among the businness members there is the Los Alamos National Laboratory.

[17] PhD MIT 1981, B.A. Amherst College 1976. He is senior fellow in economic studies at the Brookings Institution, member of external faculty of Santa Fe Institute, member of the National Academy of Sciences. He previously worked for the Rand Corporation, the Council on Foreign Relations, the U.S. Department of State and the U.S. Senate Armed Services Committee.

[18] Ph.D., Carnegie Mellon University, 1992; B.S., University of Detroit, 1983. He is senior fellow in economic studies at the Brookings Institution.

[19] The publication of Growing Artificial Society is conventionally taken as the milestone of agent-based economics. However, such models and the label 'agent-based models' were already in use among researchers in the first years of 1990.

of decentralised interaction of individuals with the inclusion of the more recent ideas of Simon's bounded rationality and of Hayekian incomplete knowledge (Vriend 2002). Technically this is implemented by programming on three interacting levels. Firstly, the model needs agents (firms, consumers….) that can have the desired levels of heterogeneity (technology, endowments, gender, age...). Secondly, agents populate an environment that is separate from them and with which they interact. Finally, the model needs rules that govern agents' and environmental behaviour as well as the interaction between the two. For instance, imagine a population of agents living on a lattice gathering and exchanging a resource necessary for survival. There will be individual rules of movement, gathering and trade for the agents and rules of reproducibility of resources for the environment. The computer will place the agents on the grid and let them behave according to the rules without any intervention on the researcher side. The behaviour of the system will be observed at the micro level (the story of each agent can be tracked) and at the macro level as an aggregation of the behaviour of the individuals. With its emphasis on the effects of interaction, agent-based simulations are similar to cellular automata to an extent that the latter are often subsumed in the former category, as in the case of Axelrod (1997b). Technically, since a cellular automaton's decision rule makes reference to the states of other cells in the neighbourhood, cellular automata are best suited to model situations where interaction is local, whereas agent-based model can include many different kinds of relationships among agents such as global (i.e. the single unit interacts with all the other units in the population) or random (i.e. the single unit interacts with one or more units randomly picked from the population) interaction.
In agent based economics, social science is interpreted as an experimental science[20]: models are laboratories in which one can make different hypotheses on the phenomenon under study and observe the outputs: regularities emerging from micro rules and robustness of such regularities. As in biology, results are interpreted in terms of candidate explanation (sufficiency of rule to generate a given regularity) and not in terms of general laws[21].

[20] This is intended as laboratoy experimentation and not as meant by experimental economics, that aims at determining which rules are actually utilized by individuals.

[21] As a relatively new approach, agent based simulation still has to solve important issues such as the problems arising when there is more than one microspecification that generates the macrostructure of interest. Moreover, for what concerns the

Mostly important to us, contraposition to equation based modelling is sought and explicit (Epstein 1999). Thinking of such kind of simulation in terms of mathematical representation becomes more difficult than in the past, since there is not a readily available mathematical technique which is able to translate such models in analytical terms. In fact, even if in principle every computation has a corresponding and equivalent partial recursive function[22], in the case of agent based models, it is not clear how to write down the appropriate equations and how to solve them if formulated (Epstein 1999, p. 51) and actually, to my knowledge it has not been done yet. It follows that agent based simulation is different from its predecessors in that, so far, there is not a mathematical representation at hand. With respect to microsimulation, results derive from decentralised interaction and not from the aggregation of the separate history of the units. As compared to system dynamics, the system is not considered in its entirety. In addition, there is a space which is distinct from the agents' population, whereas in ordinary differential equations models there is no spatial component (agents interact only in time but not in space).

Differently from the positions reported by Alker, according to whom the recognition of simulation was subordinated to the possibility of obtaining the status of mathematical science, here the detachment from the spirit and method of mathematics is explicit: "*no one would fault a "theoremless" laboratory biologist for claiming to understand population dynamics in beetles when he reports a regularity observed over a large number of experiments. But when agent based modellers show such results […] there is a demand for equations and proofs. […] one can do perfectly legitimate science with computer, sweeping the parameter space of one's model, and conducting extensive sensitivity analysis, and claiming substantial understanding of the relationship between model inputs and model outputs, just as in any empirical science for which general laws are not yet in hand*" (Epstein 1999, p. 51). This is a concept of science which resembles the one promoted by von Neumann through the heuristic use of computer: explore

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robustness of results the literature on sensitivity analysis of agent-based models is quite limited and still under development. For some early reflections on this topic see Axtell and Epstein (1994), for an interesting exercise in the alignment of computationl models (docking) see Axtell et al. (1996).

[22] This statement is known as the Church-Turing thesis.

the phenomenon via simulation to find regularities and then extract general principles and theories.

Agent based economics is different from the mathematical approach in the kind of sought explanations, even when a mathematical soluble representation is available. A question that any simulator must be ready to answer (as I did myself in many circumstances) is "Why do we need simulations if we can get a given result from an equation based model?" As Epstein (1999) points out, the answer relies on one's criterion of explanation. For instance, an oscillatory time series can be described by a function of the kind $y=f(x)$. The behaviour of y, the left hand side variable, is accurately described in mathematical terms, but what happens inside the system, which "rule of behaviour" generates, on aggregate, the observed oscillation, remains unknown. Simulation is therefore useful when the emphasis is on the process that generates a given regularity, while mathematics is more concerned with the description of the system. With agent based simulation, the path opened by von Neumann and Wiener has brought to some definitive conclusions: simulation, at least for what concerns agent-based models, is different and autonomous from mathematics and, in spite of its being an instrument generated to investigate physical and mechanical processes, it is well suited to study social systems too.

3. Other contributions: symbolic system and cognitive attitude

This section is devoted to the analysis of some recent interpretations of simulation. These are kept separated from the previous ones since they come from very different traditions: psychology and computer sciences. They rely on features that have been neglected by the scientists so far encountered, namely, the programming languages and the cognitive relationship between researcher and theory. These assessments are reported to witness both the increasing interest in giving simulation an independent status and the attempt at isolating its properties that runs parallel in many different disciplines.

In 1988, social psychologist Thomas Ostrom studies simulation on the side of the used symbol system. He believes that the characteristic attribute of simulation lies in its being a symbol system distinct from the traditional ones. Programming languages, according to Ostrom, are different both from natural and mathematical symbols "*many…regard computer simulation as merely a method. […] All of this could lead the reader to assume that computer simulation is merely a technology [instead] computer simulation is a symbol system; it is a medium through which theoretical concepts can be represented and communicated. Rather than being a special purpose technology, it is regarded as offering theorists in all areas of social* *psychology an alternative way of expressing their ideas*" (1988, pp. 382-3). He therefore concludes that simulation has to be considered as autonomous from mathematics and has to be given a status on a pair with it.

More recently, cognitive psychologist Domenico Parisi (2001) extends Ostrom's position by qualifying the concept of simulation as the third symbol system and by adding a further specification. He maintains that, not only simulation is a way of expressing theory, which is as covering and acceptable as mathematics and natural language, but also – as a symbol system – it exhibits a unique property. While the symbol systems of mathematics and natural language theories are semantic, the symbol system of simulation is syntactic. A semantic symbol is an object of reality that, perceived by a mind, generates a meaning in it. In fact, it is by comprehending such symbols that researchers can derive implications from theoretical statements and try to find empirical validation. This is not quite the case for the symbol system of computer simulation: programming languages. Those symbols are not human oriented, they are computer oriented. A computer program is a list of instructions that the scientist gives to the computer to obtain the desired operations and manipulations. According to Parisi, it is not necessary that a human being understands what the computer is going to do with the program for the simulation to produce the output correctly. In other words, these are generated only by the computer. Syntactic symbols do not work by virtue of their meaning but only by virtue of their physical characteristics. One can object that the symbols must produce some meaning at least in the person writing the program and interpreting the results. The answer is obviously positive, but a caveat is necessary. Modelling a phenomenon by means of a mathematical model requires the knowledge of the rules of writing and manipulating, say, a system of equations. When writing a program, the researcher knows how to write a list of commands, this list will be then translated by the computer into a "machine language"[23] that will tell the computer what to do. This latter language needs not to be known by the programmer neither he needs to know how it works (say, with a decimal or binary system). This distinction implies a special cognitive interaction between mind and theory. In other words, premises are stated in the programming language and implications are derived by the machine operating with its own language, independently of any human cognitive act. Results are presented in an output form that has

[23] The distinction between machine language (primary language) and programming language (secondary language) is due to von Neumann (1966).

to be interpreted by the researcher, but the process through which the former are generated has taken place outside the human mind and within the machine.

Leaving aside details that are strictly linked to cognitive aspects, it seems to me that Parisi raises an interesting aspect which has a general flavour: simulations are automated mental experiments. That is to say that the theory written in the programming code produces implications and predictions by means of the computer. This is obviously an important difference with respect to theories expressed in mathematical and natural languages that, under given circumstances, would justify the use of simulation. In fact, simulation has the desirable property of reducing the incidence of errors in the process of drawing conclusions from premises. This is true in a twofold sense: the steps of the procedure are correctly performed (for instance, there are not calculus mistakes), and the researcher's convictions cannot affect the output (for instance, she believes that a given statement descends logically from premises while, after the simulation is ran it emerges that it does not).

4. Some conclusions on the nature of simulation

At the origin, simulation was used in the hard sciences as a continuation of the mathematical modelling tradition: it was nothing but the numerical treatment of difference or differential equations where computer replicates and manipulates mathematical language. In the following years computer becomes a tool to manage the symbol of programming language (Troitzsch 1997), and computer simulation gains independency from mathematics and is used to model those aspects of phenomena that the latter cannot encompass (e.g. physical space and qualitative rules, endogenous change in the model structure). However, as Parisi and Ostrom stress, it is not only a matter of field of application: simulation implies a different way of approaching scientific research. Differences reside in the symbol system of programming languages and in the way of drawing implications and predictions. Not only simulation is a way of automating mental experiments, but its application also implies an experimental interpretation of science.

The experimental approach together with the absence of theorems and proof made it particularly hard for simulation to enter the realm of economics. Economists have been prone to accept numerical and statistical simulation, while they have been far less receptive for what concerns the theory-as-simulation such as agent-based models. The reason for this attitude together with the role of simulation in economics will be the subject of the following section.

5. Simulation and economics

The history of simulation in economics is a characteristic one. Its inclusion in the economists' armoury involves a shift of methodology and competences and, in some cases, implies a detour from the received economic theories. The analysis of all these facets falls out the scope of this study. Nevertheless, in this section I will concentrate my attention on two points that I believe are particularly seminal. The first one regards communication between branches of science, namely physics and economics, the second one concerns the relationship between orthodox and heterodox economics and the value added of simulation to the understanding of economic phenomena.

Simulation has been developed by scientists who were in close relationship with economics but, within this discipline, it has: "*not usually* [been] *explicitly defended, and certainly not with the fervor mixed with confusion that existence theorems and significance have been"* (McCloskey 1998, p. 184-5). This is probably due to the events dating back to the 40': in the departments of engineering simulations were employed to run controlled experiment on physical phenomena, while economics was turning into a divergent road. In those years, according to the ongoing project of axiomatisation, economics got focused on theorems and proofs and started looking mainly at existence and stability of equilibria. It seems almost trivial to consider that, since economics transformed into a mathematical science, there was no much room left for simulation that, with its empirical vocation and without axioms, was not a very appealing methodology to adopt. However, one must not think that simulation has not been used in economics. In fact, from the 60' onwards it has been intensively introduced within economiscs but overwhelmingly to "measure" economic phenomena in the style of microsimulation. Economists' perception of their science as an axiomatic one was so rooted that even those who contributed to the birth of simulation – chiefly von Neumann - switched from physics (cellular automata) to mathematics (theory of games) when dealing with economics[24].

[24] Traditionally, Von Neumann's research is split in two opposed periods: the first one devoted to economics and the second one devoted to computer related issues. I am not convinced that, in this case, such distinction holds since, even if works on automata follow those pertaining to economics, von Neumann started to deal with computer programs and simulations when he was about to publish the Theory of Game and Economic Behaviour. Rather it seems to me that, under many aspects the projects have been carried out together. For instance, the relationship between mathematics and empirics is stated in 1944 (chapter 1, p. 5) and corroborated in The

So it happened that, while on the one side both Wiener (through Forrester's system dynamics) and von Neumann contributed to the advances in simulation, on the other side they never applied this methodology directly to their economic endeavours. The hiatus between physics and mathematics must have seemed so deep that von Neumann and Wiener had to change language to be understood by their fellows economists. Moreover, and strangely enough, modern neoclassical economists have received in their theories the cybernetics insights on information flows (Mirowski 2002, p. 6) but have not adopted Forrester's methodology to explore their implications preferring a traditional mathematical modelling.
The legacy of von Neumann was then picked up by heterodox economists such as agent-based economists and used to criticise the axiomatic approach. Agent-based economics, which explicitly acknowledges the cybernetics lineage (Epstein and Axtell 1996, p. 2-3), gathers different instances of dissatisfaction with theorising and modelling in economics and proposes simulation as the natural way to approach social sciences. From a theoretical point of view, agent based simulation is concerned with the relationship of individual behaviours to macroscopic regularities and with dynamics as opposed to general equilibrium theory and game theory. It insists on agents' heterogeneity, bounded rationality, and imperfect knowledge. Agent-based simulations are computer simulations built in such a way that assumptions such as representative agents, auctioneer or faultless rationality can be relaxed (Tesfatsion, 2005). Agent-based economics is nowadays encountering a growing favour witnessed by the publication of many contributions on mainstream journals (Arifovic, 1996; Holland and Miller, 1991; Tesfatsion, 2001) [25] and by the interest shown by economists that tend to be included into orthodoxy (Anderson, Arrow et al., 1988; Arrow, 1994). Its increasing relevance to economists is recorded by Colander (2003) that, in reporting the occurrences in two conferences held at the Santa Fe Institute nearly a decade apart, points out a dramatic change. The first conference, held in the mid 80s, featured a set of mainstream economists and defenders of general equilibrium orthodoxy and a set of physicists. "*At that first conference the economists mostly attempted to defend their axiomatic*

Theory of Self-Reproducing Automata (see further references in Burks' introduction to the book, 1966, p. 18).

[25] For some statistics on where economic simulation models have been published from 1969 to 2004 see Fontana (2005).

approach, facing sharp challenges and ridicule from the physicists for holding relatively simplistic views" (Colander 2003, p. 8). The second one, held in the mid 90s', faced a very different tone and result e than the first one: "*No longer were mainstream economists adhering to general equilibrium orthodoxy. Now they were using methods adopted from biologists and physicists, many suggested at the early conference, in innovative ways*"[26]. Another hint of the reception of simulation into economics comes, in a much less enthusiastic tone, from Frank Hahn (2001, p. 50): "*not only will our successors have to be far less concerned with general laws than we have been, they will have to bring to the particular problems they will study particular histories and methods capable of dealing with the complexity of particular, such as computer simulation. Not for them […] the pleasure of theorems and proofs. Instead, the uncertain embrace of history, sociology and biology*".

Let me now turn to my second point: What can simulation say that (neoclassical) mathematical models cannot? A first instance concerns the contrast between linear and non-linear modelling of phenomena. While it is widely conceded that, due to presence of many interacting agents, non-linearity is ubiquitous in economic phenomena, on the modelling side the latter are often reduced to linear system. The most important example of such procedure is the traditional Walrasian representation of the market mechanism: all agents are identical in means (i.e. perfect rationality, full information) and ends (maximisation of the same objective), and the behaviour of such market is simply the summation of the individuals' actions.

From the modelling point of view, the behaviour of linear systems is almost entirely understood. They can exhibit few typical independent motions whose stability is reasonably easy to appreciate. More interestingly to economics, they have the property that their behaviour in all regions of the state space is proportional to their behaviour in a small neighbourhood of the origin (Albin and Foley, 1998, p. 6). This leads to predictability and to a rather accessible mathematical treatment. When faced with the well known ciriticisms concerning the realism of such a market model (Kirman 1989, 1992), economists frequently answer that there is not a natural methodology for relaxing those assumptions about individuals. In fact, mathematical treatment of dispersed exchange among heterogeneous agents without the coordination of an auctioneer is definitely non-linear. Non-linear systems

[26] Studies presented at those conferences are collected in Anderson et al. (1988) and Brian et al. (1997) respectively.

exhibit a much wider range of behaviour than the linear ones. For example, the motions of a linear dynamical system can be combined, in which case the system could be unable to achieve a limit and wander indefinitely in a part of the state space (i.e. chaotic motion). With a reasoning which is typical of the analysis of financial markets, economists can, while incurring in some more trouble, partially predict the behaviour of a chaotic system by stating that it will not assume given configurations (Albin and Foley 1988, p. 9). However, when non-linearity results in a system displaying self- organisation (i.e. the system ability of displaying a regularity without any external constraint on agent's behaviour) mathematics is less useful in providing general laws (e.g. when available, knowledge of the underlying dynamics would not lead to predictability of the system behaviour). Hence, the system should not be embedded into a set of equations but, rather, it should be described through simulations (Foster, 1995). Agent-based simulations can model agents (the components) and their rules of interaction through space and time separately, without assuming any a priori overall dynamics rule which, in fact, is expected to be the output of the model.

In Sugarscape (Epstein - Axtell 1996), agents are non-neoclassical: they live finite lives and have different evaluations of the gain extracted from trade. New agents enter the market as offspring of the existing population and exchanges take place at non-equilibrium prices. This results in a greater variance of prices that, in turn, generates horizontal inequality and, above all, nothing that resembles equilibrium emerges. This result seems to show that alternative representations of agents are not trivial in determining the overall behaviour of system and also highlights how simulation can be used as a theoretical instrument to model those facets that are not included in traditional mathematical representations (Tesfatsion, 2005)[27].

A further example concerns endogenous change and evolution. In equation based models, the behaviour of the system is portrayed by the causal relationships that are present at a single moment in time and, if a change intervenes in the classification as initially defined, than the model is no longer suitable and needs to be changed. In order to overcome this limit, simulations can use adaptive agents. Adaptive agent's action can be assigned a value and the agent acts in order to increase this value over time. Adaptation may occur at the micro level (say, through learning algorithms) or at the macro level through differential survival and reproduction (Holland and Miller, 1991). Either way, consequences are very difficult to foresee when there are many autonomous interacting agents. Among these

[27] For a survey of the simulations of the market process see Mirowski (2004).

consequences, there may be a change in taxonomy caused by the relative success (selection) of some agents that are better adapted to the environment. Adaptation and selection lead to evolution and, yet again, to diverging results from traditional economics. While in economics the common thought (Alchian 1950, Williamson 1988, Friedman 1953) is that evolution should lead to ever improving forms of adaptation, simulation contributed to show that evolving system do not always reach optimality and often get lock onto inefficient equilibria (Arthur et al., 1987) and, more interestingly, helped in explore the "would be worlds" under different hypotheses of adaptation (Chattoe and Gilbert 1997).

Finally, simulation has shown that the inclusion of interaction in economic processes is not trivial, and, on the contrary, can give interesting insights in "sensitive" areas such as game theory (Axelrod 1997b). For example, Epstein (1998) has devised a version of the prisoner's dilemma in which cooperation emerges without assuming repeated play (e.g. tit-for-tat and the like) or features (tags) permitting defectors and cooperator to distinguish one another. In Epstein's simulation, agents with finite vision move to random sites on a lattice and play, without memory, a fixed inborn strategy of cooperate or defect against neighbours. Agents that accumulate certain amount of payoff can give birth to offspring of the same strategy while agent with negative payoffs 'die'. In contrast with the received view, he demonstrates that, for a wide range of initial configurations, cooperators do not disappear rather persist and prevail.

It seems to me that these examples can help in making my point: simulation can help in gaining a better understanding and explanation of economic facts. I also believe that inclusion of simulation in this science should not be realised by persevering in opposing simulation to mathematics but simply in recognising the different potentialities of these methodologies. Moreover, attention should be paid to the current contrast that, in my opinion, lies in the experimental versus axiomatic approach to economics (Mc Closkey 2005).

6. Concluding remarks

In this paper, I have reconstructed the lines of thought that have led to the view that simulation is an autonomous way of expressing theories. In economics, this recognition is of special interest because, as it has been shown, it involves the ways of thinking about economic phenomena and about economics as a science.

From the simulation perspective, economics is not separable from other neighbouring sciences (e.g. history or psychology), and while retaining a quantitative approach, there is no attempt of mimicking the hard sciences.

Possible explanations are opposed to general laws; detailed descriptions of runs and sensitivity analysis substitute theorems and statistical significance.
To conclude few words must be spent on the diffusion of simulations in economics. Starting from 1989, economists adopted simulation with a growing frequency[28]. However, this upsurge does not regard all the forms of simulations homogeneously, rather it is particularly intense for statistic and econometric techniques. The use of other forms of simulation, and especially of agent based simulations, is growing but still represents a minority.
This is quite intuitive for techniques that have been developed to model specific issues (such as microsimulation) whereas, for what concerns multi-purpose techniques (for instance, multi agent simulations) a reflection is necessary. The prevalence of (statistic and econometric) simulations confirms that modelling phenomena within computer is still perceived as being peculiar when applied to social systems. It seems to me that the fact that the evolution of simulation has taken place in proximity with economics has passed almost unnoticed, and therefore simulation is still considered as pertaining to hard science and not suitable for the social ones. A final remark has to be made. It is undeniable that its recent origin is responsible for its being a minority, but it also depends on the fact that the simulators scarcely communicate the ones with the others. In fact, in reading through articles that make use of simulation it is very rare to found a detailed description of the adopted technique. This diffuses the idea that simulation is not a crucial part of the work and allows for the conclusion that there is a substantial identity among techniques, that as it has been shown, is not true. A better communication and integration of simulators and a strong investment in discovering and circulating the links between simulation and economic theory would certainly help the diffusion and a better understanding of this methodology.

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