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Ethics and economics in Karl Menger: how did social sciences cope with Hilbertism

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Abstract

This paper deals with the contributions made to the social sciences by the mathematician Karl Menger (1902-1985), the son of the more famous economist, Carl Menger. Mathematician and a logician, he focused on whether it was possible to explain the social order in formal terms.[1] He stressed the need to find the appropriate means with which to treat them, avoiding recourse to historical descriptions, which are unable to yield social laws. He applied Hilbertism to economics and ethics in order to build an axiomatic and formalized model of the individual behavior and the dynamics of social groups.

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Introduction

The paper follows Menger's contributions to the social sciences chronologically from his early interest in the role of uncertainty in economic behavior, through his acceptance of Hilbert's formalism, to his application of Hilbert's formalism to economics and ethics. Some final remarks conclude the paper[2].

The main purposes of this paper are: to show how the application of Hilbertism in ethics led Menger to a formalistic ethics he applied to the neo-classical model of rational choice in

[1] In recent years, several papers on Menger's thought have appeared. His works and his personal history have been studied not only by mathematicians (Sigmund 1998, 2006; Kass 1996) and logicians (Dawson 1998) but also by philosophers (Gilles 1981, Golland 1996) and economists (Leonard 1998; Punzo 1994; Weintraub 2002). The role of his Mathematical Colloquium (founded in 1928) in building the neoclassical paradigm of general economic equilibrium, through the contributions of Schlesinger, Wald and von Neumann, has long been recognized (Weintraub 1983, Punzo 1989, 1991; Debreu 1998), and it has gained him an important place in the history of mathematical economics after the Second World War.

[2] The reconstruction will include some biographical sketches and historical remarks on the cultural milieu of Vienna during the years that Menger spent in that city (1923-38), since these will aid understanding of how he was influenced by the Viennese philosophical framework.

opposition to the incorporation of another formalistic ethics (the Kant's duty-based) into the neo-classical paradigm; and to show how the application of Hilbertism in economics led Karl Menger to results which can be considered as precursors of inquiries of the theory of expected utility of von Neumann and Morgenstern (1944) and of the theory of fair games into social decision-making of Luce and Raiffa (1957), (Perlmann and McCann 1998).

Menger and the St. Petersburg Paradox

Menger studied mathematics and logic in the 1920s in Vienna under Hahn, and thereafter in Amsterdam under Brouwer, one of the leading exponents of intuitionism. Between 1921 and 1927 Menger worked on topology and geometry (above all set theory and curve theory: Golland and Sigmund 2000). But he also concerned himself with economic theory: an interest possibly explained by the fact that he was the son of one of the foremost economists of the time. After the death of his father, in 1921, he started to revise the second edition of the *Grundsätze*, which was published two years later. In the same year he wrote his first paper on economic theory. This dealt with "the role of uncertainty in economics" (as appeared in the title of the paper itself), and it urged economists and psychologists to find a general function able to describe the constant behaviour of agents and deviations from it. The paper was a reflection on the St. Petersburg paradox[3] formulated in the eighteenth century by Nicholas Bernoulli and which highlighted the conflict between rational expectations and common sense. Nicholas's cousin, Daniel Bernoulli, claimed that mathematical expectations should be replaced with "moral" expectations, and was thus the first to investigate the meaning of so-called **"**expected utility**"** for a gambler who persists in playing the same game. He defined the notion of expected utility by decomposing the valuation of a risky venture as the sum of utilities from outcomes weighted by the probabilities of outcomes. He illustrated his famous paradox with the following situation: a coin will be tossed until a head appears; if the first head appears on the n^{th} toss, then the payoff is 2^n and the game is over. The paradox,

[3] When Menger wrote "Unsicherheitsmoment in der Wertlehere. Betrachungern in Anchluss an das sogenannte Petersburger Spiel", (Menger, 1967). Hans Mayer, then the editor of *Zeitschrift für Nationalökonomie,* refused to publish the paper because it made excessive use of mathematical formulas. The paper was discussed in 1927 at a meeting of the Viennese Economic Society and provoked differing reactions. It was only published in 1934, when Oskar Morgenstern - according to Menger "one of the very few Austrian economists who were free from prejudices against mathematical methods in economics" (Menger 1967, 259) - was appointed editor of the *Zeitschrift* (Becchio 2008).

of course, is that the mathematical expectation in this game is infinite.[4] Yet, although the expected payoff is infinite, one would not suppose, at least intuitively, that a real-world person would be willing to pay an infinite amount of money to play the game. In fact, there is a quite high probability that he would collect only a few units of money. Daniel Bernoulli's solution involved two ideas: (i), that people's utility from wealth, u(w), is not linearly related to wealth (w) but rather increases at a decreasing rate - the famous idea of *diminishing marginal utility*; (ii) that a person's valuation of a risky venture is not based on the expected return from that venture, but rather the *expected utility* from that venture.[5] Consequently, people would only be willing to pay a *finite* amount of money to play the game, even though its expected return is infinite. On replacing the first factor with the subjective (or, as he put it, 'moral') value of the gain – which, Bernoulli emphasized, depends not only on the amount A of the gain but also on the wealth W of the evaluating person – he had assumed the subjective value to be proportional to A and inversely proportional to W. Hence the subjective value became

$$c \log(1 + A/W)$$

where c = number independent that may differ among persons but is constant for each person. By considering the subjective value, we can obtain a finite expectation, and then a solution of the paradox.

Menger disagreed with this view for various reasons. Also the subjective expectation is infinite. There are many cases where man's behaviour fails to conform to mathematical expectations: games in which a player can win only one very large amount with a very small probability or games offering a single moderate amount with a very high probability. Furthermore, we can always find a sequence of payoffs x_1, x_2, x_3, .., which yield infinite expected value, and then propose, say, that $u(x_n) = 2^n$, so that expected utility is also infinite. Menger therefore proposed that utility must also be bounded above for paradoxes of this type to be resolved.

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[4] 1st toss: payoff = 1; 2nd toss: payoff = 4 and so on and $E(w) = (1/2^n)\cdot 2^n = (1/2)\cdot 2 + (1/4)2^2 + (1/8)2^3 + ... = 1 + 1 + 1 + ... = \infty$

[5] In the St. Petersburg case, the *value* of the game to an agent (assuming initial wealth is zero) is: $E(u) = (1/2^n)\cdot u(2^n) = (1/2)\cdot u(2) + (1/4)\cdot u(2^2) + (1/8)\cdot u(2^3) + ... < \infty$, which Bernoulli conjectured is finite because of the principle of diminishing marginal utility.

Menger's explanation was that most people systematically underrate very small and very high probabilities in their economic actions, while they overrate medium probabilities. Moreover, Menger pointed out, besides a person's wealth W, one must also consider the amount U that he needs in order to continue his present standard of living, as well as the amount U' that he absolutely needs to go on living. And the difference $W - U$ plays a crucial role in a rational person's attitude toward risk. Finally, other "personal" parameters may modify the way a person gambles. Hence Menger stressed that deviations of behaviour are very complex: even in a game with a finite number of solutions, individual choices may disregard mathematical expectations because individual choices are conditioned by psychological motivations or by taste (Leonard 1995).

It is perhaps no coincidence that Menger's interest in uncertainty in economic theory arose in the same period when he was editing his father's treatise. It is well known that in Carl Menger's theory (as well as in the subsequent Austrian tradition) human knowledge is incomplete, rationality is not full, and uncertainty performs a major role in human actions. Moreover, chapter II of his father's *Grundsätze* contains a section on uncertainty in economics. Unlike his father (as well as the following Austrian tradition), Menger sought to formalize the economic issue in mathematical terms. The "uncertainty" to which Menger referred in the title of the paper is the randomness of choices when they are made in a context of knowable probabilities. This laid the basis for the subsequent development of inquiry by mathematical economics into the formalization of expected utility that could not be formalized without referring to the philosophical and methodological framework on which it was rooted. This was the axiomatic formalism of Hilbert, which was chosen by Menger and by members of his Mathematical Colloquium.[6]

Menger and the debate on formalism, intuitionism and logicism

During the 1920s the philosophical climate in Vienna was influenced by the fact – strongly stressed by Karl Menger in his *Reminiscences* – that "Austrians never contributed to the German type of metaphysics that culminated in Fichte, Schelling and Hegel" (Menger 1994, 18). Austrian philosophy was mainly oriented to discussions on epistemology and logics: "the great thinkers born in the Austrian empire, Bolzano and Mach, used to philosophize

[6] The debate around formalism and economic explanation also in relation with Karl Menger's Mathematical Colloquium started with Mirowski's history of mathematical physics (1986) and his idea of "a structural homeomorphism" between the formalist program and the metaphor of game, as a formal system (1992).

along scientific lines" (18). The rejection of both idealism and neo-Kantianism culminated in the advent of the new empiricism developed by the Vienna Circle (founded in 1922 by Moritz Schlick) where there was a certain consensus on the merits of the logical analysis of language (following Wittgenstein's *Tractatus*), on the need to develop a rigorous epistemology for a scientific vision of the world, and on the unity of scientific explanation and knowledge in general (Stadler 2001)[7]. New inquiries into epistemology and logic were expounded in a different way also in the Mathematical Colloquium (founded by Menger in 1928), as well as "new applications of the exact sciences to problems of a sociological character", as Menger himself put it in an Italian paper (Menger 1935; recently translated as Menger 1998). Official members of the Colloquium were Abraham Wald, Kurt Gödel, Franz Alt, Georg Nöbeling; many other guests, such as John von Neumann and Oskar Morgenstern, gave lectures.[8] There was general agreement within the Mathematical Colloquium that logical-deductive treatment should be given exact science (from geometry to arithmetic) and that ethics and economics should be given a new foundation based on logical coherence (Menger 1935). The philosophical paradigm shared by the members of Colloquium was the axiomatic formalism of Hilbert (Miroswi 1992; Punzo 1989, 1991). 'Formalism' signifies that the sciences say nothing about facts: instead, they elaborate symbols with no connection with experience. Applying Hilbertism in the description of human behaviour entailed the development of an axiomatic system that had to satisfy the following conditions: no contradiction within the axiomatic system itself; the independence of individual axioms from each other; the self-sufficiency of the set of axioms used to deduce every other necessary proposition (Golland 1996).

[7] These matters formed the core of the international movement in the 1930s, when the philosophical position of the Vienna Circle was most prominently represented by Carnap's analysis of language and by Neurath's physicalism and its program for a unified rational reconstruction of science (including the human sciences) (Stadler 2006).

[8] Morgenstern and von Neumann were not officially members of the Colloquium, but their role in it, their influence on the subsequent development of its inquiries, and their friendships with its members, were so deeply important and widely recognized that they may be included in the Colloquium (Weintraub 1983; Punzo 1989; Golland and Sigmund 2000). As the Karl Menger archive testifies, relations between Menger and the other members of the Mathematical Colloquium were very close during the 1930s and in the following decades, when most of them moved to the United States. Menger's friendship with Oskar Morgenstern was particularly long-lasting because of their shared interest in the relationship among mathematics, logic and economics (Leonard 1995; 1998). They also had amicable relations with Gödel, who was constantly in contact with both of them.

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In order to clarify Menger's adherence to formalism, it is necessary to explain his position in the contemporaneous debate on logic. In 1932 Menger delivered a lecture in Vienna on "the new logic" (Menger 1979) which was published a year later and then translated into English in 1937 as *Philosophy of Science*. The paper was a historical reconstruction of the crisis of logic in the nineteenth and twentieth centuries. It represented the first step in Menger's thought towards his subsequent application of the new logic then emerging in the social sciences, and it should be regarded as a "sequel" to Menger's rejection of Brouwer's intuitionism and his adherence to Hilbertism (Menger 1979; Leonard 1998).

Menger's paper described the evolution of certain exact sciences (like geometry and physics) over the past decades as a development toward more scientific results. Menger rhetorically claimed that "[just] one subject, however, is generally supposed to be unchanging and unshakable. That subject is logic" (Menger 1937, 300). Aristotelian logic founded on the three principles of identity, non-contradiction and excluded middle had first been criticized by Leibniz, who realized that the logic based on "subject-predicate propositions" should be replaced by a logic of relations. He consequently proposed his *lingua universalis*, "which should permit all scientific propositions to be stated in precise form…and treat mathematically all methods of inference" (301). Leibniz's endeavour was obscured by Kantian logic, which took his rightful place in the history of science at precisely the time when mathematicians, whose efforts were directed at enumerating all the axioms of geometry and arithmetic, were asserting the inadequacy of the old logic for that purpose.

According to Menger, the refoundation of logic had consisted of five developments: of a calculus of classes (Boole's, Pierce's and Schröder's algebras of logic); of a calculus of propositions (Frege; Whitehead-Russell; Hilbert-Ackermann; Carnap), which led to Wittgenstein's definition of tautologies as "propositions always true"; of a calculus of functions (Frege, Pierce); of a calculus of relations first formulated? by Peano; and finally an expanded calculus of functions developed by Russell. According to Menger, the first three developments "may be interpreted as mere refinements on the old logic"; the fourth and fifth ones were revolutionary and had provoked an outright crisis in the old logic.

Menger maintained that there were three possible solutions for that still ongoing crisis: Russell's logicism, Hilbert's formalism, and Brouwer's intuitionism.

Russell thought that language could be analyzed into a perfect logical structure, and also that mathematics could become a part of logic; his position was shared by the scientists and

philosophers of the *Wiener Kreis*, who were also deeply influenced by Wittgenstein, for whom logic was something that both the world and language must have in common (it is only because language has something in common with the world that it can be used to picture the world; so it is only because of logic that our sentences have meaning at all).

Hilbert's formalism was based on the axiomatization of logical and mathematical theory: every branch of mathematics starts with a number of axioms or statements that are assumed to be true and with which all other statements in that branch can be proven. And the system is consistent. Unlike logicism and Hilbertism, intuitionism (Kronecker; Brouwer; Heyting) was a non-classical logic that refused to reduce either mathematics to logic or logic to mathematics. This was because mathematical and logical proofs simply work differently: in particular, Brouwer, Menger's mentor in the mid-1920s, showed that in some cases the law of excluded middle does not hold in mathematics (it is impossible in infinite sets).

Menger chose Hilbertism, that became the official position of the *Mathematical Colloquium*. As shown, Hilbertism was the methodological paradigm that Menger applied to ethics and economics as well during the mid 1930s. Nevertheless, Brouwer's influence on Menger was kept alive, so to speak, by his interest in non-classical (or non-standard) logics. Many-valued logic as a separate subject was created by the Polish logician and philosopher Łukasiewicz (1920), and developed first in Poland. Łukasiewicz's prime intention was to use a third, additional truth value for "possible", and thereby model the modalities "it is necessary that" and "it is possible that", although its intended application to modal logic did not materialize. The outcome of these investigations, however, were the Łukasiewicz systems, and a series of theoretical results concerning these systems. Essentially in parallel to the Łukasiewicz approach, the American mathematician Post (1921) introduced the idea of additional truth degrees and applied it to problems of the representability of functions. Menger severely criticized the idea – embraced by members of the Vienna Circle in the mid-1930s – that logic was unique. Contrary to standard logic (ordinary two-valued logic) he strongly supported the n-valued logics of Post and Lukasievicz in which neither the law of excluded middle nor the law of non-contradiction operated (in particular, Menger regarded Lukasiewicz's three-valued logic as being able to include uncertainty: "the third value being the excluded middle of the traditional two-valued system" (Leonard 1998, 16)).

Menger shared with Post and Lukasievicz the idea that logic could take various forms. Menger gave the name of "logical tolerance" to this theoretical stance that there is not one

but many logics (any expression in language is acceptable as long as there are sufficient rules governing its logical application). Menger considered the so-called uniqueness of logic to be a dogma, and he was highly sceptical of the notion. On several occasions, he defended what Carnap later called the principle of "logical tolerance" (Carnap 1934) and objected to employment of the word "meaningless" to signify that, for example - in Wittgenstein's terms - all mathematical propositions are tautologies. Menger's acceptance of many-valued logics "drew him gently but inexorably away from Hanh on logic and Neurath on the unity of science" (Menger 1979 p. 16). As Menger himself stated later, his tolerant attitude towards the logical foundations of mathematics was very close to Popper's criticism of essentialism, i.e. "the futility of defining a science [and] the arbitrariness of precisely circumscribing its object" (16).

How Menger applied Hilbertism to ethics

Menger's first attempt to apply formalism to social science was his book *Moral, Wille und Weltgestaltung, Grundlegung zur Logic der Sitten* (Menger 1974). [9] It represented a form of meta-theory in ethics, or meta-ethics, according to the Hilbertian program: a formal approach to moral problems, based on an unambiguous definition of ethical language; a form of positive, or not normative, ethics.

When Menger's book appeared in Vienna, in 1934, it was welcomed by Oskar Morgenstern as "the only examination of a *strictly* formal nature about social groups" (Leonard 1995, 746) and by Karl Popper as "one of a few books in which the author attempts to depart from the stupid talk in ethics" (Popper 1944-45). Both Popper and Menger, in fact, totally disagreed with the neopositivists of the Vienna Circle, who – influenced by Wittgenstein – considered norms to be "meaningless" because they cannot be deduced from facts. According to Popper, Menger did not fall in this fallacy and tried to develop a positive logic of norms. Also Hayek paid attention to Menger's work on ethics. When discussing compatibility among the plans of different individuals (Hayek 1937), he explicitly cited Menger's investigations in social theory, and he hoped that he would be able develop his first insights into an "exact sociological theory" (Hayek 1937) as he had promised. In a footnote Hayek wrote: "It has long been a subject of wonder to me why there should have been no systematic attempts in sociology to analyse social relations in terms of correspondence and non-correspondence, or

[9] It was translated into English as *Morality, decision and social organization toward a logic of ethics,* only in 1974, and printed as the Vienna Circle Collection's sixth volume (Menger 1974).

compatibility and non-compatibility, of individual aims and desires. It seems that the mathematical technique of *analysis situs* (topology) and particularly such concepts developed by it as that of *homeomorphism* might prove very useful in this connection, although it may appear doubtful whether even this technique, at any rate in the present state of its development, is adequate to the complexity of the structures with which we have to deal. A first attempt made recently in this direction by an eminent mathematician (Karl Menger 1934) has so far not yet led to very illuminating results, but we may look forward with interest to the treatise on exact sociological theory Menger has promised"(Hayek 1937, 38).[10]

Menger was convinced that ethics as normative science was impossible because experience reveals only what 'is' , not what 'ought to be'. His book on ethics should – he declared – be considered an application of "exact thinking in the field of ethics" free from any influence "by subjective feelings" (Menger 1974, 1). He would "steer clear of the search for ultimate meanings or essences such as 'the concept of morality' or 'the principle of virtue'" (Leonard 1998, 20). Menger's aim was to identify "specific rules of conduct" in a strict logical sense. He maintained that it is necessary to free ethics from metaphysical inquiries into "the concept of morality" or "virtue" or "the essence of good", and he sought "to confine [these] cognitive studies to facts". Menger's investigations were concerned "with the application of mathematical modes of thinking and deal only with the formal side of questions". His ethics was a formalist model: not a scale of values but rather a set of rules, a sort of Hilbertian programme extended to the moral field.

The main reference in formalist ethics is of course to Kant and his "practical reason" founded on three categorical imperatives (based on the conformity of any action with universality, autonomy, and to notion of humans as ends in themselves) in opposition to hypothetical ones (religion, laws, hedonistic pleasure and personal ideals) which do not confer morality on an action. Menger stated that his morality was very far from Kantianism. He considered ethics in order to understand individual decisions, and subsequently social organizations, and he argued that the Kantian categorical imperative was neither a necessary nor a sufficient condition for constituting cohesive (or peaceful) groups. Hence ethical imperatives are always hypothetical, never categorical. The unresolved question in Kantianism was "what

[10] Hayek was referring to a paper on social relations and groups that Menger had presented in the same year at the Third Annual Conference of the Cowles Commission in Colorado: "An Exact Theory of Social Relations and Groups", in Report of Third Annual Research Conference on Economics and Statistics, Cowles Commission for Research in Economics, Colorado Springs, 1937: 71-73), later published (Menger 1938).

concrete precepts result from the categorical imperative in specific situations" (Menger 1974, 9). In order to answer this question (how to apply the categorical imperative to a decision), it is necessary to consider a decision that implies a cognitive activity. This is the point where, according to Menger, ethics and logic are strictly connected. The ethical problem in Menger's thought was understanding how social coexistence comes about in concrete situations where "there are several mutually incompatible decisions to consider" (10) and the categorical imperative needs supplementary stipulations and additional norms in order to generate the well-being of a group.

An epistemology of morality could be developed by founding it on a logical basis expressed by mathematical tools able to construct an "ethics without morality and immorality". Menger consequently focused his attention on a set of decisions by individuals whether or not to adhere to certain norms: the main point was understanding why, when, and how a person accepts a definite system of norms, basing his decisions on that system. According to Menger, rational foundations for decisions are possible. By way of example, he cited the idea of maximization in economic theory and stated that if economists wished to claim that "the optimal distribution of commodities and the greatest welfare of mankind could be achieved under certain system of organization which they describe", they "indeed must first take the trouble to study logic"(31), and he added ironically that such an exercise would "without doubt increase their self-criticism". Cognitive action is the starting point for the explanation of how individual decisions and a social changes arise, although volition has a central role as well. But the problem is that, if we can study only cognition (the logical side of decision-making), volition remains a matter of individual taste. According to Menger, it is possible to develop a logic of ethics which considers every single norm or system of norms within a social group in which people (in connection with any norm) can approve the norm, disapprove of it, or be indifferent to it. Thus, for every norm (n), people are always divided into three groups: those who always follow the rule; those who sometimes do and sometimes do not follow it; and those who never follow it. It may happen that one (or two) of these sets is empty; if everybody always (or never) adheres to the norm the other two sets are empty. If someone is indifferent, he may or may not follow the rule: he hence forms another group comprising indifferent people who sometimes follow the rule and sometimes does not, so that the groups become three in number.

> Then, in a society which comprises a system of n norms, there are 3^n possible groups. These groups are

1. jointly exhaustive = each member of the class belongs to one of them;

2. mutually disjoint = there are no members in common: "each member of the class belongs to one and only one of the total groups of consentience";

3. within a group, agreement relations are transitive: two individuals (P) who completely agree with a third also completely agree with each another (43-44).

In 1938, Menger published a paper in which he explained more clearly the subject of his previous book, i.e. how it can be possible to form a cohesive group (Menger 1938). If we consider a group of men G as a total group, it can be divided into two *fundamental groups* with no overlaps: G_1 and G_2 (for example, men and women). Each member of G has four possible attitudes toward the association ($G^{1,}$ or G^2); with everybody ($G^{1,2}$) or with nobody (G^0). Groups are thus represented as G_a^b, where *a* is membership of a group and *b* is *compatibility* or *mutual acceptance*. Thus we have eight *main classes:*

G_1^1; G_1^2 ; G_2^1; G_2^2 ; $G_1^{1,2}$; $G_2^{1,2}$; G_1^0 and G_2^0

Groups G_1^1; G_2^2 ; $G_1^{1,2}$; $G_2^{1,2}$ are consistent = any member of G is willing to associate with any member of his own group (the last two also with members of the other group).

Groups G_1^2 ; G_2^1; G_1^0 G_2^0 are inconsistent = no member of G is willing to associate with a member of his own group (the last two with nobody).

Groups G_1^1 G_2^2 are antipathetic each other = any member of G is willing to associate only with a member of his own group.

Groups G_1^2 ; $G_2^{1,2}$and G_2^1 $G_1^{1,2}$ are mixed.

Menger's aim is to understand when is it possible to unify all the members of G into a consistent group. The answer lies in the tolerance of the mixed groups $G_1^{1,2}$ and $G_2^{1,2}$ that makes possible the overlapping between G_1 and G_2. Kant was the one who gave a great importance to the formal aspects of ethics, but his categorical imperative is unable to guarantee general harmony because it operates only in groups G_1^1 and G_2^2, i.e. only in those groups in which there is no need to find a tool of cohesiveness or peace, because they are formed by member who shared the same set of rules.

In the postscript to the English edition of his book (Menger 1974), Menger summed up its purpose as an attempt to apply *logico-mathematical* thought to ethical matters. He wrote that

it was possible to construct an exact system of thought in regard not to dealing with personal dilemmas but to treating social problems in what today is called group decision theory.

On 27 December 1961, Oskar Morgenstern wrote to Menger: "I do not know whether you have followed the literature on the problem of "fair division". You will find an interesting discussion in the book by Luce and Raiffa, *Games and Decisions.* This whole area has a close connection with your work on the logic of ethics, which is unfortunately quite unknown to these authors because of the time of its publication and its being available only in German. (...) The relation of the explorations in your book to game theory and fair division certainly bears further study, and I would be happy if I could stimulate you to resume when they were begun".

Morgenstern was referring to the well-known book by Luce and Raiffa, published in 1957, and dedicated to the memory of John von Neumann. That book was about game theory and placed particular emphasis on the social science point of view. Chapter thirteen dealt with individual decision-making under uncertainty, and chapter fourteen with group decision making, devoting a section to "games of fair division".[11]

Menger returned to his paper many years later, in 1983, when he proposed a general criterion for explaining how cohesive social groups come into being. The model was the same as in 1938, but the paper comprised some additions on the ethics of the Vienna Circle and on game theory that are worth recalling. Menger criticized both Kant's formalistic morality and the ethics proposed by the Vienna Circle and founded on Wittgenstein's *Tractatus*. According to Menger, Kant's categorical imperative was unable to form a cohesive group because "in most specific situations it is impossible to deduce specific precepts for behaviour unless the imperative is supplemented by the value judgments", and it is very difficult to find a "maxim that can become a general law". Menger was also dissatisfied with the Vienna Circle's notion that, after the complete elimination of value judgments from ethics, only

[11] Luce and Raiffa maintained that a fair rule is a mode of conduct considered social desirable. The rules are so mixed that when players act rationally following their own selfish interest, the outcome is shared by all the participants and the outcome of a fair procedure is Pareto optimal. A group decision welfare function is built by passing from individual values to social preferences. The main difficulty is devising a system which is sufficiently egalitarian and flexible to cope with the dynamic vicissitudes of individual tastes. In Arrow's social welfare problem, there are preference rankings of alternatives by members of a society and a fair method for aggregating this set of individual rankings into a single ranking for the society (from a n-tuple of individual rankings to a single ranking for the society in order to construct a social welfare function). As well known, Arrow's model states five requirements of fairness, and its result is the famous impossibility theorem.

historical and ethnographical descriptions of moral beliefs and conditions were possible. The multiplicity of beliefs and evaluations seemed to recommend the formal study of inner judgments and attitudes among human groups with incompatible wishes, and of conflicting decisions between individuals and individuals, individuals and groups, groups and groups (Menger 1983). In this paper, Menger claimed that experiments should "test the presence and demonstrate the evolution of ethical norms" (Perlmann and McCann, 1998, 441)

How Menger applied formalism to economics

As said, Menger was a strong advocate of Hilbertism, which he sought to apply to ethics and economics. According to Hilbert's program, a proof (or a theorem) is explained by sentences; the meta-theory checks the logical coherence of the passages between a sentence to another one: if, and only if, these passages are logically valid, then it is possible to express proofs (or theorems) in a mathematical terms. Menger's Hilbertism is mostly evident in his paper on the "Law of Diminishing Returns. A study in meta-economics".[12] He was the first of using this term "meta-economics" in order to build a meta-theory of economics (Buracas 2004, 1). As Menger himself explained: "following a suggestion of Hilbert, modern logicians refer to the study of the logical relations between the statements of a theory as the corresponding *meta-theory*. In this terminology, the contents of the present paper can be described as a chapter in *meta-economics*" (Menger 1979, 280). When Menger used the expression "meta-economics", he was referring to the "logical relations between economic propositions as distinct from the study of economic material" (Conrides 1983, 4). Hence in this paper Menger focused on those aspects of economics to which mathematics could be applied according to a meta-mathematical model *à la Hilbert* and he defined it as : "the first instance in economics of a clear separation between the question of logical interrelations among various propositions and the question of empirical validity" (Menger 1979, 300). Today the meaning of meta-economics is more complex than the original source of the Hilbertian model: generally speaking it is regarded as "a special meta-theoretical system analyzing the taxonomical contents of economic methods and criteria, also the nature of economic concepts and judgements through the analysis of the logical and semantic aspects interconnected directly with methodologies of the economic sciences" (Buracas 2004, 2).

[12] In 1936 Menger published "Bemerkungen zu den Ertragsgesetzen", which was translated into English in 1954 as "Remarks on the Law of Diminishing Returns. A Study in Meta-Economics", and was, according to Schumpeter, "a shining example of the general tendency towards increased rigor that is an important characteristic of the economics of our own period" (Schumpeter 1954, 1037).

The occasion was given by a talk between Menger and Ludwig von Mises who had claimed that: "certain propositions of economics can be *proved* [and] as an example he mentioned the law of diminishing returns [referring him] to the literature for the proofs" (Menger 1979, p. 279). Menger wrote a very articulate paper in which he introduced the fact that a sequence of inferences becomes a proof when propositions are assumed to have the same meaning and when logical quantifiers ("many", "all", "some" and so on) are properly applied; but this methodological and logical operation is often make without any attention and rigor and often what is regarded as a proof is not "precise enough to be subject to deductive treatment" (ibid). Hence Menger analyzed the proof of the law of diminishing return on land according to the formulation of Wicksell (1909), Böhm-Bawerk (1912) and von Mises (1933):

additional applications of capital and labor on a piece of lnd increase the total product, but after a certain point this output increases relatively less than further costs (281).

The law of diminishing imply law of diminishing product increment and of diminishing average product. These two laws are not the same, they are different. The law of diminishing product increment claim:

an increase of cost outlay yields a smaller increase of the product when added to a larger outlay than when added to a smaller outlay, provided that both exceed a certain outlay level (which may depend on the amount of the land used (281).

The law of diminishing average product claim:

as the cost outlay rises, the average product of every piece of and falls after a certain point has been reached (281)

Menger showed how economists often claim that the law of diminishing product increment and of diminishing average product are the same law expressed in different words: this is a logical mistake. Moreover the law of diminishing returns could be divided into many other sub-propositions that differ from the law and each other, hence "the question arise, which of these two propositions do economists mean when they speak of the law of diminishing return" (283). Menger's conclusion is that there are many forms of the law of return; some formulation is "empirical valid", but the meta-problem is "how the various propositions are related, which ones are consequences of others – these and similar questions are purely logical and have nothing to do with experience" (300).

Some concluding remarks

Now that some features of Menger's thought on economics and ethics have been explained, some considerations can be ventured. The main characteristic of his reflections was the endeavour to find a logical pattern able to explain the mental mechanisms involved in decision-making by individuals and groups. As shown, Menger used an individual approach in his economic analyses (expected utility in his commentary on the St. Petersburg paradox) and a holistic approach when discussing the cohesiveness of groups.

Like the early Vienna Circle, he was firmly convinced that the social sciences could be treated from a scientific point of view, but with some differences. Firstly, he rejected the idea that social sciences such as ethics and economics were, following Wittgenstein's contention, to be considered "meaningless" because they were too deeply embedded in human actions. He shared this critique (albeit in very different ways that led to very different results) against the neopositivism of the Vienna Circle with Popper and Hayek. Influenced by the debate on the relationship between mathematics and logics of those years, he considered it urgent to re-build social science and he chose the Hilbertian model, axiomatic and deductive, applied to social contexts (to give coherence to the system).

Examination of Menger's specific contributions to economics and ethics shows that he succeeded in applying Hilbertism to ethics (1934) and economics (1936). Nevertheless Menger's *demystified ethics* and Menger's meta-economics present some problems. Menger tried to develop an "exact theory of social groups and relations" based on the idea that there exists a logic able to drive social behavior in a different way from the logic of an individual in order to determine when a group is consistent. As we have seen, Menger defined groups by starting from a division into two non-overlapping groups and then building subgroups until cohesive groups are found. This procedure has no faults from a formal point of view, but in realistic terms it seems rather problematic. Firstly, uncertainty emerges when defining the behavior of groups; even if we accept the definition of cohesiveness, the model remains static and it does not envisage the possible presence of groups with a separate identity within cohesive sets (this is not the case of women and men, but it could be the case of smokers and non-smokers: there could be a class of people who normally do not smoke but sometimes smoke: in this case what is their behaviour?).[13]. Moreover even if the formal structure of

[13] In truth, Menger was partly aware of this criticism, and in his last paper (Menger 1983) he proposed introducing social experiments under minimal assumptions, as well as arrangements made feasible by computers.

ethics in Menger's thought seems to work, it isn't clear what he meant for "norm"; Menger did not gave an exact definition of norm: his formal ethics is anti-Kantian (Menger rejected the idea according to which "ought" implies "can") and it is a sort of revalidation of Hume's law (you can't deduce an "ought" from an "is"), but, above all in his second paper on social groups (1938), even if ethics is logically structured on an axiomatic systems of groups, it seemed to be an empty box able to explain the characteristics of peaceful groups but unable to describe in which way social dynamics are formed. About Menger's application of Hilbertism in economics, he named meta-economics, there are some problems as well. Meta-economics checked the validity of the economic discourse in a logical perspective, but in which sense a valid proposition can be considered true? Menger considered valid every proposition that is deduced in a logical proper way from another one, but his focus on the logical coherence of the economic discourse led apart the question of the urgency of empirical validity of a preposition.

It was not by chance that Hayek was very disappointed by Menger's following inquiries into social group decision theory after the publication of his book on morality: when he reprinted *Economics and Knowledge* (1937) as a chapter in *Individualism and Economic Order* (1948), the footnote in which Hayek claimed that he was waiting for the following research of Menger on social groups had been deleted.[14] At that time, in fact, Menger was completely integrated in the urgency of formalizing social sciences: he was carrying out an approach he started on the occasion of the writing of his first paper on uncertainty in individual behaviour. As Morgenstern stated, Menger's first paper on uncertainty in economics was a forerunner of the axiomatization of expected utility: Menger never considered uncertainty to be a state of non-knowledge outside the domain of probability. It was not by chance that his Hilbertism was taken up by Morgenstern too in the same years, when he claimed that the axiomatic method must be applied to economics with no regard to the realism of hypotheses. Morgenstern wrote: "It is possible to reduce pure economics into axioms" by applying mathematics to economics and rejecting all the problems connected with so-called "realistic conditions" in economics. According to Morgenstern, the need for realism had led scientists astray from the crucial issue of that that time: transforming economics into a rigorous science: "if a problem cannot be solved by a simple, very abstract, hypothesis, moreover it cannot be solved by more complicated hypotheses such as those *eo ipso* realistic ones" (Morgenstern [1935] 1976, 175). Some years later, Morgenstern's attitude gave rise to his

[14] I am indebted to Professor Roger Koppl for pointing this out to me.

collaboration with von Neumann and their jointly-authored book of 1944. Again, it was not by chance that Morgenstern considered Menger's treatment of cohesive groups to be the forerunner of fairness in group decision theory as explained in Luce and Raiffa (1957).

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