EERI

Economics and Econometrics Research Institute

A constant-utility criterion linked to an imperfect economy affected by irreversible global warming

Andrei V. Bazhanov

EERI Research Paper Series No 03/2009

ISSN: 2031-4892

EERI
Economics and Econometrics Research Institute
Avenue de Beaulieu
1160 Brussels
Belgium

Tel: +322 299 3523
Fax: +322 299 3523
www.eeri.eu

A constant-utility criterion linked to an imperfect economy affected by irreversible global warming

Andrei V. Bazhanov[a,b*]

[a]Department of Economics, Queen's University, Kingston, ON, K7L 3N6, Canada

[b]Far Eastern National University, 8 Ulitsa Sukhanova, Vladivostok, 690600, Russia

January 02, 2009

Abstract

The question of formulation of a social planner criterion for an imperfect economy is examined using an example of a polluting economy negatively affected by growing temperature. Imperfection of the economy is expressed here in deviations from the optimal initial state. It is shown that a criterion not linked to a specific initial state almost always implies either unsustainable or inefficient paths in the economy. In this paper, I link the constant-utility criterion to the initial amount of the resource reserve. This criterion implies efficient resource use and the paths of utility asymptotically approaching some constants, which depend on the parameters of the temperature function. The criterion can be formulated for the cases when the reserve estimate changes over time and when the high level of temperature can cause extinction.

JEL Classification: O13; Q32; Q38

Keywords: essential nonrenewable resource, imperfect polluting economy, economy-linked criterion, semisustainable development, semiefficient extraction

*Tel.: 1-(613)-533-6000 ext. 75468; fax: 1-(613)-533-6668.
E-mail: bazhanov@econ.queensu.ca

1 Introduction

Stollery (1998) examined a problem where utility and/or production were negatively affected by global warming resulting from oil use. He showed that the standard Hartwick saving rule (Hartwick, 1977) is still optimal in this framework under the constant-utility criterion. Stollery obtained the closed form solutions for the Dasgupta-Heal-Solow-Stiglitz (DHSS) model (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974), considering the case when the temperature affects only production, but he did not consider the case when the temperature affects utility, noting that "exactly the same energy path results from temperature effects in a standard constant elasticity utility function" (Stollery, 1998, p. 734).

However, the case when utility is affected by global warming raises some very interesting and important questions if one applies Stollery's model to a specific imperfect[1] economy with initial conditions that are close to the behavior of the real economy. Stollery used a conventional approach for defining a closed form solution. The approach implies that the owner of the known resource stock (or a social planner) defines the equilibrium (the optimal) path of the resource depletion, including the initial value of the rate of extraction. In this sense, the initial conditions are treated as "the future" because the problem is supposed to be solved before starting the extraction. In this situation, the solution is optimal and sustainable in the sense implied, for example, by the constant-utility criterion. Assume now that one applies this result to a specific real economy that has been extracting the resource for some period of time. It is known that government policies change with time, for example, due to changes in knowledge and in institutions. Then, accord-

[1] Arrow, Dasgupta and Mäler (2003) define imperfect economies as the "economies suffering from weak, or even bad, governance" (p. 648).

ing to the Bellman's Principle of Optimality, each time the social planner makes a decision about intertemporal resource allocation, the new solution "must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman 1957, p. 83). The Principle means, first, that the initial state is treated as "the past," and, second, that the optimal solution must be consistent with the initial state regardless of the values of this state and regardless of the reasons that have caused this state. This approach implies that the use of a theoretical result for a specific real economy will, as a rule, imply imperfection of the economy in the sense that the initial conditions will not be optimal with respect to the criterion. Inconsistencies in decisions of social planners are not the only reason for the imperfection of an economy. Another reason is, for example, a well-known uncertainty in the estimates of the resource reserves. It is known that the world's oil reserves estimated by Cambridge Energy Research Associates (CERA, 2006) are about three times more (3.74 trillion barrels) than the conventional estimate being published in the December issues of *Oil&Gas Journal*. This uncertainty alone implies that neither the initial state nor any consecutive state of a resource-extracting economy can be exactly optimal. The optimality can be thought of only in terms of probability.

This situation raises the following questions:

(1) is a qualitative theoretical result (sustainable development[2]) stable[3]

[2] I use here the notion of the weak form of sustainable development (nondecreasing per capita consumption) as a synonym for growth. Saving rule in an aggregate model implies the growth in man-made capital including the substitute technologies, which do not use oil. Therefore, the growth of the economy includes qualitative changes in technologies. Then the notions of growth and development can be used as synonyms if consumption includes all the benefits associated with development. I think that the question of accuracy of an aggregate model in this case deserves separate attention.

[3] I reinterpret here the notion of stability of an equilibrium (Leonard and Long, 1992, p. 90), namely, the result (sustainable development of an economy) is stable if the economy remains sustainable regardless of any small changes in the initial conditions. Formal

with respect to deviations from the optimal initial conditions?

(2) if the result is not stable, then how should the formulation of the problem be modified in order to avoid unacceptable consequences?

In Section 2 of this paper, I show that Stollery's problem is not globally stable (there exists an initial state that implies unsustainability of the economy). I consider an example where a social planner applies the constant-utility criterion to an economy with constant extraction during an initial period. This situation implies a conflict between the criterion and the combination of the production function and the temperature function considered by Stollery. I obtained some paths numerically for this problem in Bazhanov (2008a). The paths were sustainable only for initial states that did not reflect the behavior of the real economy. Plausible initial states in this framework led to unsustainable extraction, rapid growth of temperature, and collapse of the economy.

These results, of course, can imply inadequacy of either the temperature function or the production function in Stollery's model.[4] I consider these questions in Section 3. However, there is one more important question. Assume that there is an economy with the production and the temperature functions identical to the ones considered by Stollery but with the constant resource extraction in the initial period. This economy is not consistent with the constant-utility criterion because the technological opportunities cannot provide the rate of growth of consumption, which could compensate for disutility caused by the economy's pattern of extraction. The example raises again the question of relationship between optimality and sustainability of economic growth considered, for example, by Baranzini and Bourguignon

definition needs an indicator that equals, say, zero when the economy is sustainable.

[4]For example, Stollery's model does not explicitly include technical change.

(1995).[5] Koopmans called this situation "preferences not adjusted to opportunities" (Kopmans, 1965).[6] This situation originates from the lack of connection between a criterion and the opportunities of a specific imperfect economy to maintain the optimal path in the long run.

The case, when the path of extraction must be linked to the initial rate, implies that this path cannot already be linked to the initial amount of the resource reserve. In this situation, the constant-utility criterion almost always implies either unsustainable or inefficient paths in the economy. Section 4 considers these unacceptable consequences in an example of an imperfect economy with utility affected by temperature (hazard). Section 5 offers a modification of the constant-utility criterion that is linked to an initial reserve and that results in utility paths gradually declining to acceptable levels instead of collapsing to zero in finite time. Section 6 provides the analysis of the modified constant-utility criterion when the initial reserve estimate is being reappraised over time. Section 7 considers the case when a high level of hazard can cause the extinction of an economy regardless of the level of consumption. Section 8 offers conclusions.

[5] Baranzini and Bourguignon introduced a probability of an economy extinction in a standard optimal-growth model. The probability positively depends on the growth of consumption related to the stock of resources. As a result, the optimal growth path does not exist for some values of the initial conditions and for some kinds of preferences expressed in the discount rate and in the parameters of the utility function.

[6] Koopmans showed using a simple model that the optimal path does not exist for some values of the discount rate.

2 Technological opportunity vs. criterion

Stollery considered an example of the oil-burning economy with the Cobb-Douglas production function negatively affected by growing temperature

$$q(t) = c + \dot{k} = q\left[k(t), r(t), T(s_0 - s)\right] = k^{\alpha}(t) r^{\beta}(t) T^{-\gamma}(s_0 - s), \qquad (1)$$

where the lower-case variables are in per capita units: q – output, k – reproducible capital, r – current resource use, c – consumption, $\dot{k} = i$ – investments ($\dot{k} = dk/dt$); α, $\beta, \gamma \in (0, 1)$ are constants ($\alpha + \beta < 1, \alpha > \beta$). The average global temperature T is growing with the accumulated[7] extraction $s_0 - s$, where s_0 is the initial oil stock, $s = s(t)$ - current oil stock following $r = -\dot{s}$. Stollery assumed that the effect of growing population is compensated by technological progress and so there are no technical advances in the model and population is constant.[8]

Stollery also assumed that the rate of increase in concentration of the greenhouse gases in the atmosphere and the corresponding growth of temperature are proportional to the rate of oil extraction that implied the temperature function in the form of $T(t) = T\left[s_0 - s(t)\right] = T_0 \exp\left\{\Theta \int_0^t r(\xi) d\xi\right\} = T_0 \exp\left\{\Theta(s_0 - s)\right\}$,[9] where T_0 (the initial temperature) and the parameter Θ are positive.

In this framework a social planner maximizes a constant level of utility using a carbon tax.[10] Stollery considered the example where temperature

[7] Stollery did not consider the process of natural stabilization of temperature, explaining this assumption by referring to evidence that global warming can reduce natural regenerative capabilities. As a result, his temperature function, as a worst case scenario, depends on the extracted stock rather than on the current rate of extraction.

[8] I think that a more plausible alternative to this assumption would be a TFP compensating for the capital decay. I consider this assumption in Bazhanov (2008b & 2008c).

[9] Stollery's (1998, p. 735) formula reads $T[s(t)] = T_0 \exp\{-\Theta s(t)\}$, where T_0 is interpreted as $T_0 = T(0)\exp\{\Theta s_0\}$ - the maximum value of T.

[10] The tax biuniquely corresponds to the path of extraction; therefore, Stollery considered

affects only production function and claimed that "exactly the same energy path results from temperature effects in a standard constant elasticity utility function $\widetilde{u}(c,T) = -c^{-(\eta-1)}T^{-\omega}$." Here ω is, presumably, a constant parameter specifying the effect of temperature on utility.[11] The utility $\widetilde{u}(c,T)$ is measured here in negative numbers since $T_0 > 0$ and $c \geqslant 0$. Then ω must be negative in order that the temperature be a bad good. Consumption c is a normal good here only for $\eta > 1$. Using the assumption that c is always a normal good and temperature in the problems of global warming is always a bad good, the utility function can be formulated as follows: $u(c,T) = (cT^{-1})^{(1-\eta)}/(1-\eta)$. In this formulation, temperature is measured in the units consistent with the units of consumption and so the parameter Θ absorbs the parameter ω. A survey of literature on various forms of damage from pollution is offered in Kolstad and Krautkraemer (1993). More recent contributions can be found, for example, in Schou (2002) or in Grimaud and Rouge (2005).

The social planner determines the conditions for maximum constant per capita utility $\overline{u}$. In other words, the expression

$$u(c,T) = \left(cT^{-1}\right)^{(1-\eta)}/(1-\eta) = \overline{u} = const \tag{2}$$

is considered as a simple criterion for sustainable development.

The path of extraction obtained by Stollery (1998, formula (11), p. 735) uniquely defines the initial extraction that "starts at a lower level" and has the "less rapid" decline depending on the parameters of the hazard factor. Assume now that the social planner chooses the constant-utility criterion in

the extraction and consumption as control variables.

[11] Stollery used the letter ω before this example in his paper to denote a carbon tax (Stollery, 1998, p. 733). Apparently, here the sense of ω is different; otherwise, it would have meant that the carbon tax by itself influences our perception of changes in temperature.

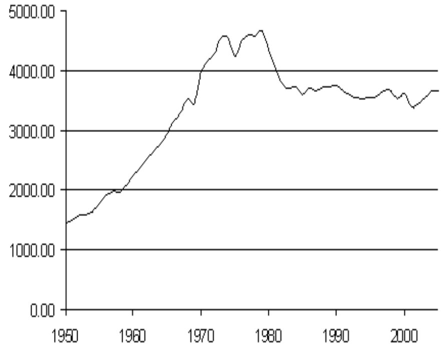


Figure 1: Per capita world's oil extraction [mln t/ year].

an economy with a specific pattern of extraction in the initial period that deviates from the optimal one. One need not apply stabilty theory here in order to see that, in this case, the qualitative result (sustainability of the economy) is not globally stable (depends on the initial extraction). Indeed, for simplicity, assume that the growing temperature affects only utility and that the initial extraction is constant (see Fig. 1 after 1980)[12]. Then $q(k,r) = k^{\alpha}r^{\beta}$, the economy follows the Hartwick saving rule $\dot{k} = rq_r$ ($q_r = \partial q/\partial r$), and $r(t) \equiv \overline{r} > 0$ for any $t \in [0, \overline{t}]$.

In this case $T(t) = T_0 \exp\{\Theta\overline{r}t\}$. Then the criterion (2) implies that $c(t) = c_0 \exp\{\Theta\overline{r}t\}$ and $q(t) = q_0 \exp\{\Theta\overline{r}t\} = c_0 \exp\{\Theta\overline{r}t\}/(1-\beta)$ since $\dot{k} = \beta q$. The saving rule gives $k^{-\alpha}\dot{k} = \beta\overline{r}^{\beta}$ or $k(t) = k_0\left[k_1 t + 1\right]^{1/(1-\alpha)}$ with the constants $k_0 = k(0)$ and $k_1 = \beta(1-\alpha)\overline{r}^{\beta}/k_0^{1-\alpha}$. The production function gives the expression for extraction: $r(t) = q^{1/\beta}k^{-\alpha/\beta} = \widehat{r}\exp\{\Theta\overline{r}t/\beta\}\left[k_1 t + 1\right]^{-\alpha/[\beta(1-\alpha)]}$, where $\widehat{r} = (q_0 k_0^{-\alpha})^{1/\beta}$. This expression implies that $r(t)$ is not a constant for

[12]I took the data for the world's oil extraction from December issues of *Oil & Gas Journal* and the data for the world population (population in 2006 equals to unity) from http://www.census.gov/ipc/www/worldpop.html (December, 2008).

$\overline{r} > 0$, contradicting the initial assumption $r(t) \equiv \overline{r}$, regardless of the parameters of the temperature function.

Another way to arrive at the contradiction in this problem with $r(t) \equiv \overline{r} > 0$ is to obtain the patterns of capital (a) from the criterion and the production function (capital, exponential in time) and (b) from the saving rule (capital, quasiarithmetic in time). In the same way, one can obtain this type of conflict for the case when the temperature affects both utility and production. The temperature weakens the productive capability in this case, making the conflict between the exponential growth, required by the criterion, and the maximum possible growth, restricted by the existing technology, even stronger.

Numerical simulations for this model gave sustainable paths only for the patterns of extraction with implausibly low initial rates that declined right from the initial moment (Bazhanov, 2008a). The case with the constant extraction at the initial moment implied the immediate sharp increase in extraction required by the necessity to maintain exponential growth of consumption. The growth in extraction was followed by the corresponding growth in temperature, more increase in extraction, and a very fast collapse of the economy. This pessimistic scenario does not reflect real life, where the per capita world's oil extraction has been fluctuating around a constant for more than 20 years (Fig. 1), while the average air temperature and per capita consumption are slowly growing. The following section provides some possible modifications of this problem that can reconcile it with existing patterns in the real economy.

3 Quasiarithmetic temperature

Nordhaus and Boyer (2000, p.22) use the model where the global average temperature linearly depends on the warming factor $F(t)$, which in turn logarithmically depends on the concentration of CO_2 : $F(t) = \overline{F}\left\{\log\left[M_{AT}(t)/M_{AT}^{PI}\right]/\log(2)\right\}+ O(t)$, where $M_{AT}(t)$ is the atmospheric concentration of CO_2, M_{AT}^{PI} is the preindustrial level of M_{AT}, and $O(t)$ – the total warming effect of other greenhouse gases.[13] In comparison with this model, Stollery's temperature posits an unrealistically "bad" scenario that gives no chance to the world's sustainability under the constant-utility criterion.

As an "average" simple temperature function, consider the following modification of Stollery's warming: $T(t) = T\left[r(t)\right] = T_0\left[\Theta\int_0^t r(\xi)d\xi + 1\right]^{\varphi} = T_0[\Theta(s_0 - s(t)) + 1]^{\varphi}$. This function can vary from constant to polynomial depending on the value of $\varphi \geqslant 0$. Then the criterion (2) with $r(t) \equiv \overline{r}$ implies the following optimal path of consumption: $c(t) = c_0\left[\Theta\overline{r}t + 1\right]^{\varphi}$, where $c_0 = \left[\overline{u}(1-\gamma)\right]^{1/(1-\gamma)} T_0$. Then the saving rule $\dot{k} = \beta q = \beta(c + \dot{k})$ gives $q(t) = q_0\left[\Theta\overline{r}t + 1\right]^{\varphi}$, where $q_0 = c_0/(1-\beta)$ and the specification of q gives $k(t) = k_0\left[\Theta\overline{r}t + 1\right]^{\varphi/\alpha}$ with $k_0 = q_0\overline{r}^{-\beta}$.

On the other hand, as was shown above, the saving rule gives a differential equation in capital with the solution $k(t) = k_0\left[k_1 t + 1\right]^{1/(1-\alpha)}$ that coincides with the expression implied by the criterion when $\varphi = \alpha/(1-\alpha)$ and $\Theta = k_1/\ \overline{r}$. Since $\overline{r} = r(0) = r_0$, the last condition can be rewritten as $\Theta = \beta(1-\alpha)q_0/(k_0 r_0)$. In other words, the constant-utility criterion can be

[13] Climate sensitivity is defined as the change in global mean surface temperature following a doubling of the atmospheric (equivalent) CO_2 concentration. A conventional estimation of this value is around 3°C (see, e.g., http://en.wikipedia.org/wiki/Climate_sensitivity (January, 2009)). This evidence implies that the dependence of global temperature on emissions is rather logarithmic than exponential.

consistent with a specific imperfect economy and with the quasiarithmetic temperature function only if the parameters of the latter are linked to the parameters of the former in a way defined by the criterion. Otherwise, the optimal path prescribed by the criterion can be unsustainable. It would be, of course, unrealistic to expect that the laws of physics depend on the economic technology or, say, on the current amount of capital in a way defined by human preferences. Not less implausible would be to assume that the aggregate production function depends on the specific warming properties of the air.

A number of approaches could reconcile Stollery's problem with real life. For example, an "additional" technical change could fill the gap between the requirements of the criterion and the technological capabilities. However, this case would also give the unnatural result that the technical progress should depend on some peculiarities of the atmosphere.

Another approach to eliminating this conflict is to assume that the economy should immediately switch from its current state to the optimal state required by the constant-utility criterion. In particular, these conditions could require that, for the given amount of capital, the rate of extraction must be substantially contracted at $t = 0$. Putting aside here the question of feasibility of this discontinuous shift, note that the production function implies in this case the corresponding discontinuous drop in consumption, while the temperature will be still growing with the lower rate. In other words, the immediate fall down in extraction immediately contradicts the constant-utility criterion, not to mention that it contradicts the Bellman's Principle of Optimality and the Hadamard's (Hadamard 1902) principle requiring the continuity of a solution with respect to initial conditions for a correctly (or well-) posed mathematical problem.

Henceforth, in order to avoid these contradictions, I will consider the paths uniquely defined by the given initial state. Then, following Koopmans (1965) or Baranzini and Bourguignon (1995), one can ask the question: when is the optimal path sustainable? The next section provides an analysis of this question using a simple example with production not affected by temperature.

4 Temperature in the utility function alone

The constant-utility criterion (2) implies that $cT^{-1} = \widetilde{u} = const$, where $\widetilde{u} = [\overline{u}(1-\eta)]^{1/(1-\eta)}$. Then, since $q = c + \beta q$, the optimal path of output must be $q(t) = q_0 \left[\Theta \int_0^t r(\xi)d\xi + 1\right]^{\varphi}$, where $q_0 = c_0/(1-\beta) = \widetilde{u}T_0$. Raising to the power $1/\varphi$ follows $q^{1/\varphi} = q_0^{1/\varphi} \left[\Theta \int_0^t r(\xi)d\xi + 1\right]$ (restriction $\varphi \neq 0$ will be lifted below). Time derivative, applying $r = q^{1/\beta}k^{-\alpha/\beta}$, is $q^{1/\varphi-1}\dot{q}/\varphi = q_0^{1/\varphi}\Theta r = q_0^{1/\varphi}\Theta q^{1/\beta}k^{-\alpha/\beta}$. This equation with the saving rule gives a system of two differential equations in q and k :

$$\begin{aligned} q^{1/\varphi-1-1/\beta}dq/dt &= \varphi q_0^{1/\varphi}\Theta k^{-\alpha/\beta}, \\ dk/dt &= \beta q. \end{aligned}$$

Following Schubert and d'Autume (2008), the system can be solved by eliminating time ($dt = dk/(\beta q)$): $q^{1/\varphi-1/\beta}dq = A_1 k^{-\alpha/\beta}dk$, where $A_1 = \varphi q_0^{1/\varphi}\Theta/\beta$. Integration gives $q^{1+1/\varphi-1/\beta}/(1+1/\varphi-1/\beta) = A_1 k^{1-\alpha/\beta}/(1-\alpha/\beta) + C_1$ or $q^a = A_2 k^{1-\alpha/\beta} + C_2$, where $a = 1 + 1/\varphi - 1/\beta = [\varphi(\beta-1)+\beta]/(\varphi\beta)$, $A_2 = aA_1/(1-\alpha/\beta)$. Calibration at $t = 0$ gives $C_2 = q_0^a(1 - B_1 k_0^{1-\alpha/\beta})$, where $B_1 = A_2 q_0^{-a} = q_0^{1/\beta-1}\Theta\left[\varphi(1/\beta-1)-1\right]/(\alpha-\beta)$. Then

$$q = q_0\left(B_1 k^{1-\alpha/\beta} + C_3\right)^b,$$

where $C_3 = 1 - B_1 k_0^{1-\alpha/\beta}$ and $b = 1/a = \varphi\beta/[\varphi(\beta - 1) + \beta]$. Henceforth, the restriction $\varphi \neq 0$ will be not relevant. The obtained expression for q combined with the saving rule gives a differential equation in capital and then the dynamics of the economy is defined by the following system:

$$\dot{k} = \dot{k}_0 \left(B_1 k^{1-\alpha/\beta} + C_3\right)^b, \tag{3}$$

$$r(t) = q(t)^{1/\beta} k(t)^{-\alpha/\beta}, \tag{4}$$

where $\dot{k}_0 = \dot{k}(0) = \beta q_0$.

The specific case with $\varphi = 0$ implies $b = 0$ that gives linear capital $k(t) = \beta q_0 t + k_0$, which coincides with Stollery's solution for $\gamma = 0$, and extraction in the form of $r(t) = r_0 \left(r_1 t + 1\right)^{-\alpha/\beta}$, where $k_0 = k(0), r_0 = r(0)$ and $r_1 = \beta q_0/k_0$.

In the general case, I will consider the positive values of φ, which are less than $\overline{\varphi} = \beta/(1-\beta)$. These values imply growing capital because in this case $b > 0$ and $B_1 k^{1-\alpha/\beta} + C_3 > 0$. It is enough for the objectives of the paper to study the behavior of the economy in the range $0 < \varphi < \overline{\varphi}$ since, even in this restricted case, one can see the main qualitative consequences of using the constant-utility criterion in a specific imperfect economy.

Following Nordhaus and Boyer (2000), consider $\alpha = 0.3$ and $\beta = 0.25$. The β implies that the interest rate ($f_k(0) = \alpha q_0/k_0$) equals 0.07 for the economy, growing with the rate $\dot{q}_0/q_0 = 0.02$ and following the world's pattern of extraction at the initial moment: $r_0 = 3.618$, the initial reserve $s_0 = 2 \cdot 182.42 = 364.85$ [bln t],[14] and the rate of extraction is growing

[14]Extraction: $r_0 = 72,361.1$ [1,000 bbl/day] $\times 365 = 26,411,765$ [1,000 bbl/year] (or 3.618 bln t/year); reserve: $S_0 = 2 \times 1,331,698,077$ [1,000 bbl] (or 2×182.42 bln t) (World[a], 2007). Here one ton of crude oil equals 7.3 barrel. The initial value of $\dot{r}$ is $\dot{r}_0 = 0.04$ that is close to the average $\dot{r}$ since 1984. Methodology of estimation of historical values for $\dot{r}$ is described in Bazhanov (2006). I use here the "average" of the estimates for S_0 provided in World[a] (2007) and in CERA (2006).

with the acceleration $\dot{r}_0 = 0.04$ (for simplicity, the population is constant here and so the extraction is growing, unlike in the Fig. 1, where the extraction is related to the growing population). The expression for $\dot{q}/q$ and the saving rule imply the initial value for capital expressed in terms of extraction data:

$$k_0 = \left\{ \left[\left(\frac{\dot{q}}{q} \right)_0 \frac{1}{\beta} - \frac{\dot{r}_0}{r_0} \right] / \left(\alpha r_0^\beta \right) \right\}^{\frac{1}{\alpha-1}} = 12.935.$$

That value gives the corresponding value of the initial consumption $c_0 = (1-\beta)q_0 = (1-\beta)k_0^\alpha r_0^\beta = 2.23$. In order to make the plots more visible, assume that the temperature function $T(t) = T_0 \left[\Theta \int_0^t r(\xi) d\xi + 1 \right]^\varphi$ has the following parameters: $T_0 = 1$, $\Theta = 1$, and the parameter φ changes between 0 and $\overline{\varphi}$. For this economy the constant-utility criterion (2) with $\eta = 0.5$ implies the paths of extraction and consumption depicted in Fig. 2 and 3 for the various values of φ. The exact solution for $\varphi = 0$ is in crosses.

It is known (Bazhanov, 2008c) that externalities in an economy can lead to growing patterns of extraction. One can see this effect in the economy with the Cobb-Douglas production function from the following equation: $\dot{r}/r = -[(1-w)f_k + \tau]/(1-\beta)$ (Bazhanov 2008c, p. 13), where $w = w(t) \in (0,1)$ is a saving rate and τ is the Hotelling Rule modifier ($\tau = \dot{f}_r/f_r - f_k$). The Hartwick rule implies $w \equiv \beta$, and then the equation takes the form:

$$\dot{r}/r = -[f_k + \tau/(1-\beta)]. \tag{5}$$

The modifier τ can deviate from zero due to influence of various phenomena including externalities.[15] Equation (5) shows that even small negative values of τ ($\tau < -f_k[1-\beta]$) can imply growing paths of extraction. The

[15] The most recent analysis of the reasons of distortions in the Hotelling Rule and the variants of the modified formulations were provided by Gaudet (2007).

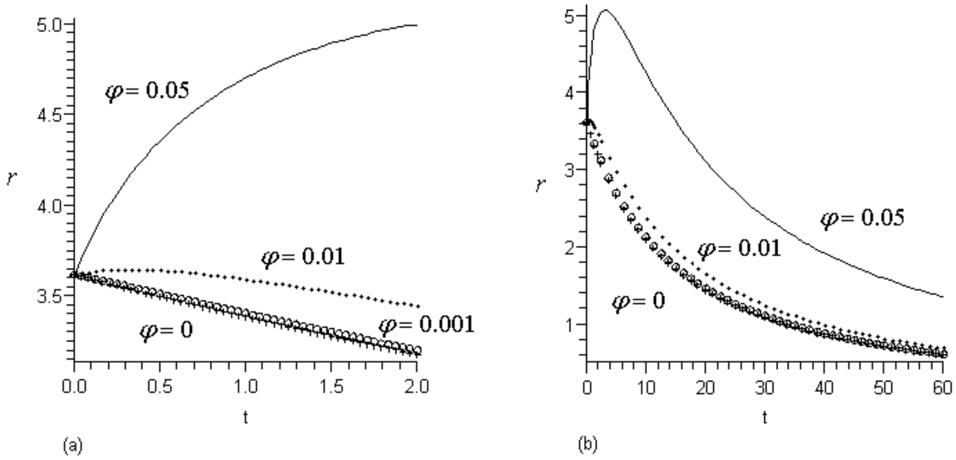


Figure 2: Extraction under the constant-utility criterion: (a) - short run, (b) - long run; closed form solution for $\varphi = 0$ is in crosses, solution for $\varphi = 0.001$ is in circles, for $\varphi = 0.01$ - as a dot line, for $\varphi = 0.05$ - as a solid line.

modified Hotelling Rule in the Stollery's framework is $\dot{f}_r/f_r = f_k + (f_T + u_T/u_c)T_{s_0-s(t)}/f_r$ (Hartwick 2008). In the case with the temperature in the utility alone, the modifier is $\tau(t) = u_T T_{s_0-s(t)}/(u_c f_r) = -\varphi\Theta c/(f_r[\Theta(s_0 - s) + 1])$ that is non-positive for $\varphi, \Theta \geqslant 0$. Note also that negative values of τ (for $\varphi, \Theta > 0$) follow the growth of consumption in this economy (Bazhanov 2008c, Proposition 1, p. 15). These properties of the problem are illustrated in Figures 2 and 3. The path of extraction is growing in the initial period for $\varphi = 0.01$ and $\varphi = 0.05$ but declining for $\varphi = 0.001$ and $\varphi = 0$ (Fig. 2a). It is easy to show that $\dot{r}(0) = 0$ when φ equals $\varphi^0 = f_k(0)f_r(0)(1-\beta)/(\Theta c_0) = 0.0047639$.

Fig. 3 shows that consumption is growing in the initial period for all positive φ and it is constant for $\varphi = 0$. The qualitative difference in these scenarios is that the constant-utility criterion requires different resource poli-

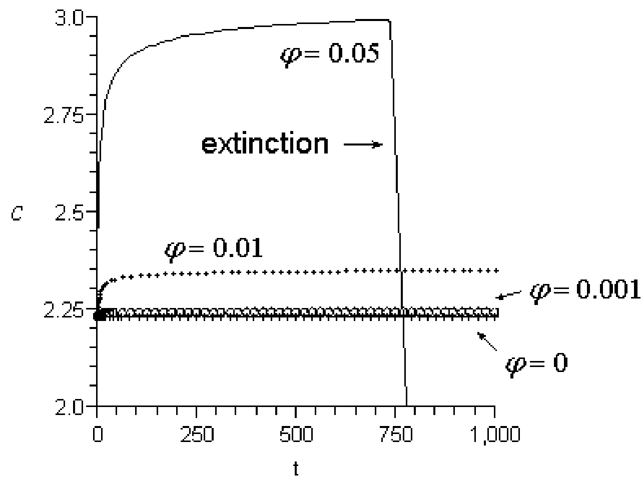


Figure 3: Consumption under the constant-utility criterion: closed form solution for $\varphi = 0$ is in crosses, solution for $\varphi = 0.001$ is in circles, for $\varphi = 0.01$ - as a dot line, for $\varphi = 0.05$ - as a solid line.

cies depending on the parameters of the temperature function. For $\varphi = 0.01$ the initial reserve s_0 is completely extracted during the infinite period of time, satisfying the efficiency condition, while for $\varphi = 0.001$ the part of the reserve is left in the ground (about 44 bln. t.) and for $\varphi = 0.05$ the resource is exhausted during 777 years following the collapse of the economy.[16]

This example specifies a mechanism of "economy extinction." This mechanism was considered in a general model by Baranzini and Bourguignon (1995) with the hazard function that did not depend on the resource stock

[16]The solution, of course, could be calibrated at $t \to \infty$, but then it will follow the inconsistency of the paths with the given initial conditions (incorrectly posed problem). This means that different values of φ will imply different initial states making the paths in the different cases incomparable. Here, different φ imply only different initial accelerations $\dot{r}(0)$. For the sake of simplicity, I assume here that the pattern of extraction can be changed in a non-smooth way (discontinuously in $\dot{r}$) depending on the parameters of the temperature function. The question of possibility of this non-smooth shift raises some technical and normative problems that deserve separate attention.

nor on consumption. The result, illustrated in Fig. 3, shows that the preferences expressed in the parameters of a criterion and/or a utility function can imply that the optimal path is nonexistent or unsustainable. This situation could be described, for example, as the impossibility of finding the optimal policy in some cases. In addition, one can claim that the notions of sustainable and optimal growth are different and they must not be confused. However, how then should be qualified the optimal paths for $\varphi < 0.01$? These paths are optimal and sustainable, and they even are locally stable in the sense that the economy is still sustainable despite some small deviations in the resource reserve. At the same time, they are Pareto-inferior to the path for $\varphi = 0.01$ due to inefficiency implied by the criterion.[17]

Hence, the example shows that the preferences linked to the properties of the air (instead of linking to the economic opportunities) almost always imply either unsustainable or inefficient paths in a specific imperfect economy. "Almost always" means here *except the unique case when the parameters of the temperature function imply the sustainable and efficient optimal path by chance*, as in the case with $\varphi = 0.01$ in the example above. The case when an economy is artificially sentenced to follow a Pareto-inferior path can be considered the opposite to unsustainability, but also as an unacceptable consequence. Koopmans argued that preferences should be adjusted to the economic opportunities, "viewing physical assets as opportunities," (Koopmans 1964, p. 253). This approach implies the necessity of linking a criterion to the technological properties and the initial state of the economy. Otherwise, "ignoring realities in adopting 'principles' may lead one to search for a nonexistent optimum, or to adopt an 'optimum' that is open

[17] I use a standard definition of efficiency that states that a path is efficient if the corresponding path of consumption c^E is such that there is no feasible path of consumption that is Pareto-superior to c^E (e.g., Dasgupta and Heal 1979, p. 213).

to unanticipated objections" (Koopmans 1965, p. 229). The following section describes a simple approach to avoid unacceptable consequences while applying the constant-utility criterion to a specific oil-burning economy.

5 Semisustainable variant of the constant-utility criterion

A straightforward way to reconcile the constant-utility criterion with sustainability of an imperfect economy is based on the properties of the sustainable and efficient solution for $\varphi = 0.01$ (Fig. 3). The general idea is that the preferences should be used to formulate a criterion only in a general form with some parameters that should be specified for a given economy. This specification must be done in such a way that the indicator of sustainability formalized in the criterion would asymptotically approach an "affordable" constant that positively depends on the opportunities of the economy and negatively on the potency of the hazard factors. Here, a simple example of this indicator is a utility function $u(c, T)$ that aggregates the benefits (c) and the disadvantages (T) of economic growth.[18] It is commonly accepted that, whatever criterion is used, it must select the optimum among the efficient paths. Then a combination of notions of efficiency and sustainability would define sustainable development as an efficient program with non-decreasing u over time. This means that the requirement of sustainability restricts the set of feasible paths. This restriction can be reflected in formulation of the criterion.

For example, in the constant-utility case, the criterion can be specified

[18] For the sake of argument, I assume here that it is possible to aggregate all the benefits and disadvantages in one indicator. In the general case, this indicator is just a component of a vector of such indicators.

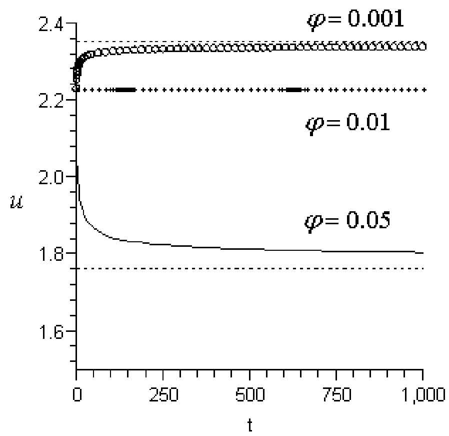


Figure 4: Semisustainable utility paths for different "real" values of the warming parameter φ: utility for $\varphi = 0.001$ is in circles, for $\varphi = 0.01$ - as a dot line, for $\varphi = 0.05$ - as a solid line.

for the temperature function with the parameters that imply the efficient program ($\varphi = 0.01$ in the example above); in other words, the general form of the criterion is defined by the preferences ($u = u(cT^{-1})$), where the actual temperature function is substituted by the "instrumental" function, which parameter(s) is(are) calibrated on the initial state of the economy instead of the ones that reflect real properties of the air. For the example above, it means that[19] $\varphi = \varphi(s_0) = 0.01$. Then, regardless of the specific parameters of the hazard function T, the economy will generate an efficient and sustainable flow of benefits c (Fig. 3, in dots), and the utility will asymptotically approach a constant that corresponds to the combination of the economy's

[19] Actually, the parameter Θ of the temperature function could also be calibrated on the initial value of $\dot{r}_0$ in order to lift the question of possibility of non-smooth switch in extraction, but for simplicity, I will not consider it here.

initial conditions and the intensity of the damage (Fig. 4),[20] expressed here in the parameters of the temperature function T. This approach implies that the utility can be growing if the damage factors are "weaker" than the economic potential, and it can be asymptotically declining to a constant if the economy is not able to maintain the current level of utility forever due to the strong influence of hazards. In the latter case, following Baranzini and Bourguignon (1995), the feasible value for the asymptote can be restricted from below by a minimum survival level.

The offered variant of the economy-linked criterion would be inapplicable if the efficient path had implied that the growing benefits c could not compensate for the damage T, causing extinction, as happened with Stollery's exponential temperature. The case with the probability of such irretrievable losses is considered in Baranzini and Bourguignon (1995), in a general model. However, this scenario is not the case for the efficient path with the quasi-arithmetic temperature, where the modified constant-utility criterion seems to imply more acceptable consequences than the one offered by Stollery. Another reason for applicability of this modification is that Stollery's constant-utility criterion in combination with the Cobb-Douglas production function does not require less extraction in an imperfect economy when the temperature grows faster with the extraction (greater φ). Just the opposite – it requires more extraction in order to compensate for growing damage by increasing consumption and to increase faster capital, giving the opportunity to reduce the extraction afterwards. Therefore, the economy-linked variant works as a stabilizer, preventing overheating when the damage factors are strong and avoiding recession when they are weak.

The paths generated by the economy-linked criterion raise also an in-

[20] The technique of constructing plots in Fig. 4 is described in the Appendix.

teresting terminological question, namely, can these paths be classified as sustainable? In the general case, no, if the utility is considered as a simple indicator of sustainability, because the utility in the example above is decreasing for $\varphi > 0.01$ (Fig. 4) while the conventional definition of (weak) sustainability requires non-declining values of an indicator. However, the paths asymptotically declining to an acceptable level of utility with ever-growing consumption seem "more sustainable" than the paths with the utility and consumption dropping to zero at some finite moment of time. Therefore, as the second best scenario, the paths that can guarantee an acceptable level of an indicator should be included in a more general notion of sustainability. The path with this property could be constructed, for example, in an overheated economy as the only possible way to avoid extinction. Hence, these paths could be treated as "almost-" or "second-best-" sustainable.

A kind of "almost-sustainable" path is called in sustainability literature "quasisustainable." This term denotes a more flexible form of strong sustainability,[21] which recognizes the necessity of extraction of nonrenewable resources for economic development and recommends "to exploit nonrenewables in a quasi-sustainable manner by limiting their rate of depletion to the rate of creation of renewable substitutes" (Daly 1990, p. 4). This approach is also called quasi-sustainable, for example, in Bretschger and Egli (2001, p. 186). This way of extraction relates in some sense to the problem considered in the current paper if the Hartwick rule is interpreted as investment of the resource rent into renewable substitutes, and if it is assumed that the extraction is not restricted by the rate of development of the substitutes. However, here this process is not the main reason of necessity in a more general concept

[21] Strong sustainability implies that both natural and man-made forms of capital must be maintained intact.

of sustainability.

The similar-sounding term "quasistability" is used in analysis of dynamic systems with multiple equilibria (optima). It means that the system may converge not to a given equilibrium but to any equilibrium depending on the initial conditions (for example, Uzawa 1961, p. 618). Quasistability can be compared to global stability, which implies that the system converges to the unique equilibrium regardless of the initial conditions (Leonard and Long, 1992, p. 90).[22] This sense of "almost-sustainability" could be relevant to the current problem if the effect of the changing in the initial conditions on sustainability had been studied; however, this interpretation of the problem is also not exactly the case, since here, non-optimal initial conditions are treated as given, and an indicator of the economy's development converges to a not desirable but to some acceptable level depending on the influence of hazard.

A similar term was introduced by Campbell and Rose (1979). They called a matrix A stable if some indicator $I(A, t)$ (they used $I(A, t) = \exp[At]$) converges to a given limit (they used zero) with $t \to \infty$. However, they needed a more general concept when $I(A, t)$ converges to any (finite) limit and they called this matrix *semistable*. Using this analogy, the economy's development can be called *semisustainable* if a sustainability indicator converges to some (acceptable) limit and its value is bounded by an acceptable constant for any $t \geqslant 0$.[23] This notion includes the conventional concept of sustainability, introduced in the Brundtland Report (World[b] 1987), as the first best solution, and it implies more optimistic scenarios than extinction for the cases

[22]The term quasistability has the same meaning, e.g., in quantum mechanics: http://en.wikipedia.org/wiki/Quasistable (November, 2008).

[23]McKibben (2005) used this term for the agriculture in Cuba, which experienced substantial reforms and eventually managed to converge to an acceptable sustainable level after the collapse of the Soviet Union.

when some of the goals of sustainability cannot be achieved due to either natural disasters or unpredictable consequences of human activity.

6 Variable reserves

Uncertainty in reserve estimate implies the corresponding uncertainty in formulation of any criterion that is linked to the initial resource stock. It is known that the values of the proven recoverable oil reserves are being updated annually. This value decreases because of the extraction and it can increase due to discoveries of new oil fields and due to changes in oil prices and in extracting technologies. The conventional estimate of the world's oil reserve s_0 has been slowly growing (World[a], 2007) during the last 150 years. This reappraisal implies the corresponding updates in the information about the reserves that were available for the future production at any moment of time in the past. The requirement of sustainability implies that the parameter of the "instrumental" temperature function in the modified constant-utility criterion should be linked to a reliable initial value of a variable reserve $s_0(t)$. This initial value can be defined, for example, by an estimate that provides a sustainable optimal path at a sufficiently high level of confidence. Then, due to the changes in the reserve estimates, the "instrumental" parameter φ, which is linked here to the initial reserve, should change according to the updates in s_0 over time. In other words, the preferences expressed in the economy-linked criterion are, in this case, being dynamically adjusted in response to new information.

Assume, for the sake of argument, that $s_0(t)$ grows with time and asymptotically approaches a constant $\widehat{s}_0$,[24] for example, in the following way (Fig.

[24]By the variable initial reserve $s_0(t)$ I mean here the amount of recoverable reserve at $t = 0$, which is reestimated at all moments of time $t \geqslant 0$.

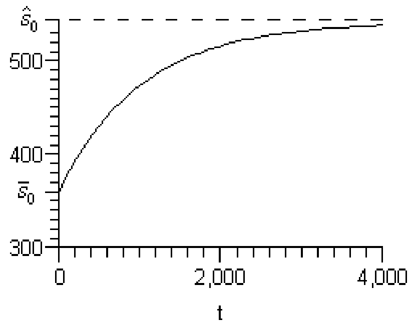


Figure 5: Updates in reserve estimate.

5):

$$s_0(t) = \widehat{s}_0 - e^{-wt}(\widehat{s}_0 - \overline{s}_0). \quad (6)$$

I will consider here $s_0(0) = \overline{s}_0 = 2 \cdot 182.42 = 364.84$ [bln t] and $\widehat{s}_0 = \lim_{t\to\infty} s_0(t) = 3 \cdot 182.42 = 547.26$ [bln t] (CERA's reserve estimate). The parameter w equals 0.001.

The amount of initial reserve $s_0 \geqslant s_{0\,\min}$ biuniquely defines the value of $\varphi \geqslant 0$, which implies the efficient and sustainable path of extraction under the constant-utility criterion (see Appendix). Biuniqueness of this dependence provides the link between updates in s_0 and the semisustainable paths in the economy via the path of the "instrumental" value of $\varphi\,[s_0(t)]$. In this case, the coefficients of equation (3) are variable: $B_1 = B_1(\varphi\,[s_0(t)]), C_3 = C_3(\varphi\,[s_0(t)])$, and $b = b(\varphi\,[s_0(t)])$.

In the numerical example, I consider the semisustainable path of utility $u\,[s_0(t)]$ dynamically consistent with the reserve updates (Fig. 6, in circles) in comparison with the two semisustainable precommitment paths, one of which, $u(\overline{s}_0)$, is constructed for the initial reserve estimate $s_0 = \overline{s}_0$ at $t = 0$, and the other one, $u(\widehat{s}_0)$, is obtained under the assumption of full knowledge

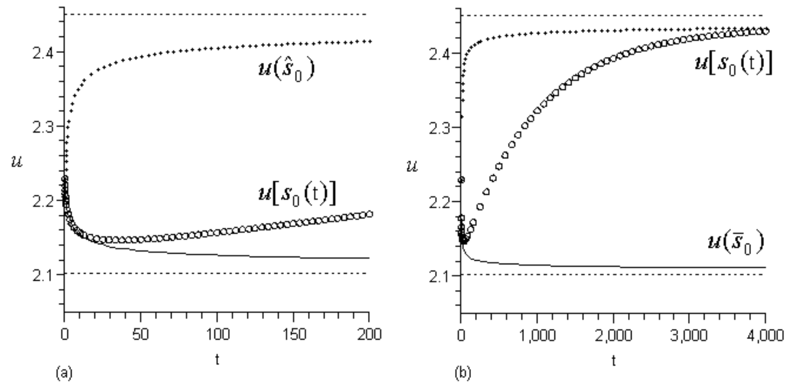


Figure 6: Semisustainable utility paths for the "real" warming parameter $\varphi = 0.02$ and different reserve estimates: $\overline{s}_0 = 364.84$ [bln t] (solid line), $\widehat{s}_0 = 547.26$ [bln t] (in crosses), and variable reserve (6) (in circles); (a) in the short run; (b) in the long run.

about all existing reserves at $t = 0$. In order to make the comparison interesting, I assumed that the parameter φ of the "real" temperature equals 0.02. For this value of φ, the reserve $\overline{s}_0$ is not enough to maintain constant utility forever; therefore, the utility declines to the asymptote $\overline{u} = 2.102$; the reserve $\widehat{s}_0$ is more than necessary for sustainability, and it implies monotonic growth of utility to the asymptote $\widehat{u} = 2.45$. In this situation, the path $u\,[s_0(t)]$ is nonmonotonic (Fig. 6a, in circles) because the economy relies only on the available information that is not favorable in the short run. However, the updates in reserves and/or technologies make it possible to reformulate the preferences with time and to increase the sustainable level of utility.

Note that, besides the uncertainties in reserve estimates, which can affect sustainability, there are uncertainties in the patterns of technical change and in estimates of the elasticity of substitution between natural resources and man-made capital.[25] The example with linking a criterion to the estimate of the natural resource reserve shows that reliance on too optimistic models or estimates of some values can cause unsustainability of the economy including its extinction. At the same time, if the influence of "positive" factors is underestimated due to the lack of reliable knowledge, then the criterion linked to these underestimated benefits will lead to inefficient paths that will asymptotically approach the efficient ones with updates in knowledge.

The following section considers the case when the initial reserve cannot be completely used in production because of the threat of extinction due to the high level of hazard.

7 Temperature as a restriction for growth

I have assumed so far, following Stollery, that irreversible global warming alone does not cause pathological changes that can follow extinction. The only reason of extinction was an unsustainable pattern of extraction in an attempt to compensate for damages in utility resulting from the growing temperature. Assume now, following Baranzini and Bourguignon (1995) that the economic growth can cause such a high level of hazard that extinction will follow, regardless of the economic "opportunities." Then the optimal sustainable program can be defined in the Stollery's framework by requiring the maximum asymptote for utility restricted by the "safe level" $T_{\max}$ for the hazard function:

$$\overline{u}(r) = \lim_{t\to\infty} u(r,t) \longrightarrow \max_{r}, \qquad (7)$$

$$T(r,t) \leqslant T_{\max} \quad \text{for any } t \geqslant 0. \qquad (8)$$

[25] See, e.g., Neumayer, (2000).

This criterion looks, at first glance, like dictatorship of the future (Chichilnisky 1996) because, in the general case, the level of asymptote $\overline{u}$ is not sensitive to any changes in the present utility. However, this is not the case here, since the higher level of asymptote implies in this framework the Pareto superior path of utility (Fig. 4). This follows from the given resource rent investing rule and from the monotone dependencies between extraction, temperature, and consumption.

The interesting case is when the restriction (8) is active, namely, when $T_{\max} < T_0 \left[\Theta s_0 + 1\right]^{\varphi}$. In this case, $T_{\max}$ uniquely defines the amount of the resource s_∞ that must be left in the ground forever in Stollery's framework:[26] $s_\infty = s_0 - \left[(T_{\max}/T_0)^{1/\varphi} - 1\right]/\Theta > 0$, where φ and Θ are the parameters of the "real" temperature function. Then the semisustainable variant of the constant-utility criterion implies that the parameter φ of the "instrumental" temperature function should be calibrated on the reduced amount of the initial reserve $\widehat{s}_0 = s_0 - s_\infty$. Namely, for the quasiarithmetic temperature this parameter equals $\varphi_{\max} = \ln T_{\max}/\ln\left(T_0\left[\Theta\widehat{s}_0 + 1\right]\right)$, implying that problem (7), (8) is equivalent to the following: to choose the path of extraction $r(t)$ that gives

$$u\left(c, \widehat{T}\right) \equiv \left[c(t)/\widehat{T}(t)\right]^{(1-\eta)}/(1-\eta) = const, \tag{9}$$

where $\widehat{T}(t) = T_0\left[\Theta\left(\widehat{s}_0 - s(t)\right) + 1\right]^{\varphi_{\max}}$. Then the utility paths for different values of φ will resemble the ones depicted in Fig. 4 with two differences: (1) the path for $\varphi = 0.01 > \varphi_{\max}$ will be declining (the path for $\varphi = \varphi_{\max}$

[26] More precisely, taking into account uncertainty of the reserve, the value of $T_{\max}$ defines the maximum amount of the resource that can be burned; the rest of the resource s_∞ should not be used in technological processes causing exhaustion of greenhouse gases. Therefore, the resource policy implied in this framework, in the general case, does not coincide with the one resulted from the assumption that the resource has an amenity value (Krautkraemer, 1985; Schubert and d'Autume, 2008).

will be constant); (2) the level of utility along the paths will be lower.

Note that formulation of problem (7), (8) with active $T_{\max}$ implies that the optimal program is inefficient because, in this case, all the feasible programs use only a part of the reserve, violating a necessary condition of efficiency (Dasgupta and Heal, 1979, p. 165). In this situation, when the efficiency of the resource extraction contradicts the very existence of the economy, the programs that satisfy the condition of a "complete reduced" extraction, $\widehat{s}_0 = \int_0^\infty r(t)dt$, can be called the *second-best efficient* or *semiefficient.* Alternatively, the amount s_∞ could be thought of as not accessible for extraction and then the efficiency condition could be reformulated for the "new" initial reserve $\widehat{s}_0$.

8 Concluding remarks

This paper has shown that, if a social planner applies a growth criterion not linked to the economic opportunities in an imperfect economy, then it almost always implies either unsustainable or inefficient paths. I associated the imperfect economy here with the initial state deviating from the optimal one. The result is shown using an example of Stollery's (1998) model of irreversible global warming in an oil-burning economy with the Cobb-Douglas technology (Section 4). I considered constant utility as a criterion of the social planner. Stollery showed that the Hartwick rule is still optimal in this framework despite the externality in the form of growing temperature. Therefore, I used here the Hartwick saving rule as a given.

In order to construct a variant of the economy-linked criterion, I "adjusted preferences" here by using an "instrumental" temperature function with a parameter linked to the initial reserve estimate. This modification implied an efficient extraction and the paths of utility asymptotically approaching some constants (Section 5). The “adjusted” utility is growing (instead of generating Pareto-inferior consumption in the case with non-linked criterion) when the economy is “stronger” than temperature, and it is declining to an acceptable level (instead of collapsing to zero) when the consequences of hazard cannot be adequately compensated by growing consumption.

The criterion was specified for the cases when the initial reserve estimate was updated over time (Section 6) and when the high level of hazard, caused by oil use, followed extinction (Section 7). The former case showed that the semisustainable path of utility, consistent with the updates in the reserves, can be nonmonotonic depending on the parameters of the hazard and on the values of changes in the reserves. In the latter case, the solution implied a natural result: a corresponding part of the resource reserve must not be used in burning technologies.

9 Acknowledgments

I am grateful to John M. Hartwick and Cees Withagen for very useful comments and advice.

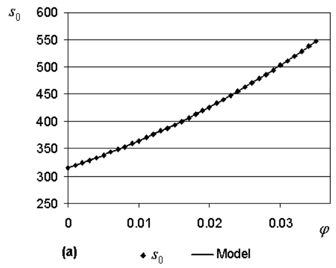
10 Appendix (construction of semisustainable utility paths for different parameters of real temperature)

The semisustainable variant of the constant-utility criterion implies that the paths of capital and extraction are to be obtained from system (3), (4) with coefficients B_1, C_3, and b calculated with the “instrumental” value of the warming parameter φ. This value of φ can be uniquely defined numerically from the one-to-one correspondence (due to smoothness and strict monotonicity) between φ and the amount of the resource that is extracted during the infinite period under the constant-utility criterion. The total extracted amount grows strictly monotonically with φ because the increase in φ leads to the higher level of temperature for the same path of extraction. Therefore, the criterion requires more extraction for higher levels of φ in order to compensate for growing hazards by growth in consumption. The increase in extraction leads to more growth in temperature (secondary effect), etc. Hence, the higher value of φ always requires the higher value of the initial reserve s_0 in order to maintain constant utility over time when the initial rate of extraction is given. Using the example provided in Section 4, this monotonic dependence, estimated numerically, is depicted in Fig. 7a (in dots). Note that the conventional DHSS model, used in this example, and the conventional estimate of the world's oil reserve, provided by *Oil&Gas Journal (*$s_0 = 182.42$ bln t*),* imply that the currently available amount of oil reserve is not enough to maintain constant per capita consumption even in the case when there are no hazards from growing temperature ($\varphi = 0$). Minimum reserve, required in this case, can be easily estimated from the exact solution of system (3), (4). The solution gives $\int_0^\infty r(t)dt = r_0 k_0 / \left[q_0(\alpha - \beta)\right] = s_{0\,\min} = 314.867$ bln t.

I use the following empirical function as the relation between s_0 and φ :

$$s_0 = d_1 \exp\left(d_2 \left\{\exp\left[\exp(d_3 \varphi) - 1\right] - 1\right\}\right), \qquad (10)$$

where $\ln d_1 = 5.7522, d_2 = 5.2008$, and $d_3 = 2.75$. The plot in Fig. 7b shows the residuals of model (10) in logarithms. Biuniqueness of the correspondence between s_0 and φ implies that for any $s_0 > s_{0\,\min}$ there exists a unique value of φ that can be used as an "instrumental" parameter in the semisustainable



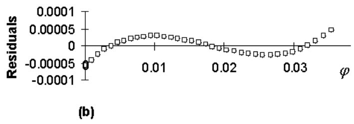


Figure 7: The amount of resource s_0 extracted during the infinite period under the constant-utility criterion for different values of the warming parameter φ: (a) the values of s_0 estimated numerically (dots) and empirical model (10) (solid line); (b) residuals of the model (10) in logarithms.

variant of the constant-utility criterion. Model (10) provides this value in the following form: $\varphi^i = \varphi[s_0] = \ln\{\ln[(\ln s_0 - \ln d_1)/d_2 + 1] + 1\}/d_3$. For example, the "average" estimate of the world's oil reserve, which I use here in the numerical example (Section 4), implies that the "instrumental" value of φ equals $\varphi^i = \varphi[364.84] = 0.010015$.

The "instrumental" value φ^i specifies the constants in equation (3) and uniquely defines the efficient path of extraction and the path of current reserve $s(t)$. The path $s(t)$ can be obtained, for example, from the "instrumental" temperature defined from equation (3). The saving rule and the criterion imply that $\dot{k} = \beta q = \beta\widetilde{u}T^i/(1-\beta)$, which, after solving (3), gives the expression for the "instrumental" temperature: $T^i(t) = \dot{k}_0(1-\beta)\left(B_1 k^{1-\alpha/\beta} + C_3\right)^b/(\beta\widetilde{u})$. On the other hand, $T^i(t) = T_0\left[\Theta\left(s_0 - s(t)\right) + 1\right]^{\varphi^i}$. Then $s(t) = s_0 - \left[\left(T^i(t)/T_0\right)^{1/\varphi^i} - 1\right]/\Theta$. This expression defines the path of “real” temperature for any parameter φ: $T(t) = T_0 \left[\Theta\left(s_0 - s(t)\right) + 1\right]^{\varphi}$, and this “real” temperature defines a semisustainable path of “real” utility $u(c, T)$.

References

[1] Arrow K.J., Dasgupta P., Mäler K.G., 2003. Evaluating projects and assessing sustainable development in imperfect economies. Environ. Resource. Econ. 26: 647-685.

[2] Baranzini A., Bourguignon F., 1995. Is sustainable growth optimal? International Tax and Public Finance. 2: 341-356.

[3] Bazhanov A.V., 2006. Variatsionnye printsipy modelirovaniya v resursnoj economike. Vestnik DVO RAN, 6: 5-13. Available at http://mpra.ub.uni-muenchen.de/1309/ (in Russian).

[4] Bazhanov A.V., 2007. The transition to an oil contraction economy. Ecological Economics, 64(1): 186-193.

[5] Bazhanov A.V., 2008a. On Stollery's case of temperature in the utility function alone. In: Aronsson T., Löfgren K.G. (Eds.), Handbook of Environmental Accounting. To be published by Edward Elgar Publishing Limited.

[6] Bazhanov A.V., 2008b. Maximin-optimal sustainable growth with nonrenewable resource and externalities. EERI Research Paper Series No 11/2008.

[7] Bazhanov A.V., 2008c. Sustainable growth in a resource-based economy: the extraction-saving relationship. MPRA Paper No. 12350, December 24, 2008.

[8] Bellman R., 1957. Dynamic programming. Princeton, New Jersey: Princeton University Press.

[9] Bretschger L. and Egli H., 2001. Sustainable growth in open economies. In: International Environmental Economics: A Survey of the Issues. Oxford University Press, Oxford, pp. 183-208.

[10] Campbell S.L., Rose N.J., 1979. Singular perturbation of autonomous linear systems. SIAM J. Math. Anal., 10(3): 542-551.

[11] CERA, 2006. Why the Peak Oil Theory Falls Down: Myths, legends, and the future of oil resources. (November 10, 2006). Available at http://www.cera.com/aspx/cda/client/report/reportpreview.aspx? CID=8437&KID= .

[12] Chichilnisky G., 1996. An axiomatic approach to sustainable development. Social Choice and Welfare, 13: 231-257.

[13] Daly H.E., 1990. Toward some operational principles of sustainable development. Ecological Economics, 2: 1-6.

[14] Dasgupta P. and Heal G., 1974. The optimal depletion of exhaustible resources. Rev Econ Stud, 41: 3-28.

[15] Dasgupta P. and Heal G., 1979. Economic Theory and Exhaustible Resources. Cambridge University Press, Cambridge, England, 501 pp.

[16] Gaudet G., 2007. Natural resource economics under the rule of Hotelling. Can. J. Econ., 40: 1033-1059.

[17] Grimaud A., Rouge L., 2005. Polluting non-renewable resources, innovation, and growth: welfare and environmental policy. Resource Energy Econ 27: 109-129.

[18] Hadamard J., 1902. Sur les problemes aux derivees partielles et leur signification physique. Princeton University Bulletin, 13: 49-52.

[19] Hartwick J.M., 1977. Intergenerational equity and the investing of rents from exhaustible resources. Amer. Econ. Rev., 67: 972-974.

[20] Hartwick J.M., 2008. Sustainable consumption programs. In: Aronsson T., Löfgren K.G. (Eds.), Handbook of Environmental Accounting. To be published by Edward Elgar Publishing Limited.

[21] Kolstad C.D., Krautkraemer J.A., 1993. Natural resource use and the environment. In: Kneese, A.V., Sweeney, J.L. (Eds.), Handbook of Natural Resources and Energy Economics, vol. III. Elsevier Science Publishers, 1219-1265.

[22] Koopmans T.C., 1964. On flexibility of future preference. In G. L. Bryan and M. W. Shelly, II, (Eds), Human judgments and optimality (pp. 243-254), New York, John Wiley.

[23] Koopmans T.C., 1965. On the concept of optimal economic growth. In Pontificiae Academiae Scientiarum scripta varia, 28: 225-300.

[24] Krautkraemer J., 1985. Optimal growth, resource amenities and the preservation of natural environments. Rev. Econ. Stud., 52(1): 153-170.

[25] Leonard D., Long N.V., 1992. Optimal control theory and static optimization in economics. Cambridge University Press, Cambridge, Mass. 353 pp.

[26] McKibben B., 2005. The Cuba Diet: what will you be eating when the revolution comes? Harper's Magazine, April, 61-69.

[27] Neumayer E., 2000. Scarce or abundant? The economics of natural resource availability. Journal of Economic Surveys, 14(3): 307-335.

[28] Nordhaus W.D., Boyer J., 2000. Warming the world: Economic models of global warming. Cambridge, Mass. : MIT Press.

[29] Schou P., 2002. When environmental policy is superfluous: growth and polluting resources. Scandinavian Journal of Economics, 104: 605-620.

[30] Schubert K., d'Autume A., 2008. Hartwick's rule and maximin paths when the exhaustible resource has an amenity value. Paper presented at the Thematic Meeting of the French Economic Association, Toulouse, France, June 9-11.

[31] Solow R.M., 1974. Intergenerational equity and exhaustible resources. Rev. Econ. Stud., 41: 29-45.

[32] Stiglitz J., 1974. Growth with exhaustible natural resources: Efficient and optimal growth paths. Rev. Econ. Stud., 41: 123-137.

[33] Stollery K.R., 1998. Constant utility paths and irreversible global warming. The Canadian Journal of Economics, 31(3): 730-742.

[34] Uzawa H., 1961. The stability of dynamic processes. Econometrica, 29(3): 617-631.

[35] World[a], 2007. Worldwide look at reserves and production. Oil and Gas Journal 105(48): 24-25.

[36] World[b], 1987. World Commission on environment and development. Our common future. Oxford/New York: Oxford University Press.