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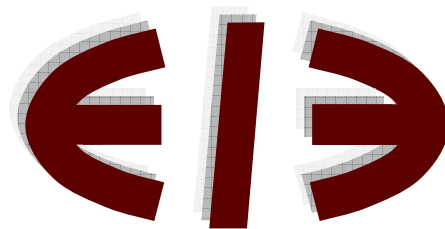## Economics and Econometrics Research Institute

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Nikolay Arefiev

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EERI
Economics and Econometrics Research Institute
Avenue de Beaulieu
1160 Brussels
Belgium

Tel: +322 299 3523
Fax: +322 299 3523
www.eeri.eu

THE INCONSISTENCY PUZZLE RESOLVED:
AN OMITTED VARIABLE

N. G. AREFIEV

MACROTEAM-HSE, MOSCOW

EUREQUA, PARIS-1, UNIVERSITÉ - SORBONNE

ABSTRACT. The contemporary version of the dynamic Ramsey problem omits expectations of a household's initial lump-sum wealth taxation due to policy revision; therefore, the attainable resource allocation set in this problem is ill-defined. This omission leads to misleading conclusions about the optimal policy in the short run and, in particular, that the Ramsey policy is dynamically inconsistent. The effect of introducing the expectations into the analysis of dynamic inconsistency is similar to that of introducing expected inflation into the Phillips curve: we show that only an unexpected policy surprise affects the attainable resource allocation set and the optimal policy. In contrast to Chamley (1986), we show that intensive capital income taxation at the beginning of an optimal policy does not imply a lump-sum taxation of household wealth and cannot reduce the excess tax burden. We also demonstrate that the Ramsey policy is dynamically consistent even without commitment. We resolve the Ramsey problem and compare our results to those of Chamley on optimal capital income taxation.

: *Key words*: Consistency, Equilibrium policy, Optimal taxation
: *JEL classification*: E61, E62, H21

The Ramsey approach to optimal policy remains the simplest way to analyze the consequences of distortionary taxation in representative agent models, such as endogenous growth or RBC-style models. Even if the objective of the analysis is not the Ramsey policy itself, but, for example, the outcome of a game between government and households (Chari, Kehoe, 1990) or the interactions between fiscal and monetary authorities (Dixit, Lambertini, 2003), a natural way to analyze the efficiency of the policy is to compare it with the policy that solves the Ramsey problem.

However, there is no satisfactory solution to the dynamic Ramsey problem in contemporary literature. Fisher (1980) notes that the Ramsey policy is dynamically inconsistent; consequently, it is not implementable. Chamley's (1986) recommendation to tax capital income at 100% at the beginning of the optimal plan and then switch to 0% resembles a mathematical artefact rather than a prudent recommendation to policymakers. A benevolent policymaker has no clear answer to the question "What policy should I implement in order to maximize household welfare?" The objective of this paper is to fill in this gap in the literature.

We show that the inconsistency result, as well as the Chamley "bang-bang" solution to the capital income taxation problem, stems from improper analysis of the implicit expropriation effect. Policy revisions induce jumps of the shadow price of household wealth, and, consequently, jumps of the household wealth measured in the units of the utility function. The latter are what we call the "implicit expropriation effect", or simply "expropriation". Previous researchers omit household expectations of such expropriations, and we explicitly introduce expected expropriation into the analysis.

The consequences of the introduction of expected expropriation are similar to that of introducing expected inflation into the Phillips curve. Previous researchers implicitly assumed that the attainable resource allocation set and optimal policy depend only on expropriation at time 0. However, we show that the attainable resource allocation set and optimal policy depend on the difference between the actual value and the expected value of expropriation, which we call the expropriation surprise. The omission of the expected expropriation in previous papers led to incorrect conclusions about the inconsistency of optimal fiscal and monetary policies.

The central result of this paper is that under rational expectations there is no reason for the dynamic inconsistency of optimal policy. Indeed, if expected expropriation equals actual expropriation, then the expropriation surprise is zero. We show that only a policy surprise affects the attainable resource allocation set and the optimal policy; consequently, *if expropriation is perfectly expected, the optimal policy and the resource allocation under any value of expropriation will be the same as under no expropriation.* This means that the government has no stimulus for inconsistency. In the Kydland and Prescott (1977) framework, this means that the optimum coincides with the equilibrium.

The hypothesis of rational expectations is methodologically attractive, but it is not necessary for the consistency result. In fact, however expectations are formed, we can maximize the government's objective on the set of attainable expropriation surprises and find the optimal dynamics of expropriation surprise. In this paper we show that the optimal policy is dynamically consistent regardless of the dynamics of the expropriation surprise.

We consider only the case of the Ramsey policy, where the government maximizes the representative household's welfare. However, the consistency result holds regardless of the government's objective, on the condition that the objective itself is dynamically consistent. For example, the policy may be inconsistent if we consider the case of hyperbolic preferences.

Our result is far beyond the scope of the Ramsey problem and is in sharp contrast to a substantial part of the literature on consistency, reputation, commitment, and policy interactions. Acceptance of ideas presented in this paper requires a deep revision of these results. In this paper we focus our attention only on the Chamley (1986) - Judd (1985) result of short term intensive capital income taxation.

The Chamley-Judd result holds if the government can commit to future policy, which means in our framework that expected expropriation is zero. Under commitment, the Chamley-Judd policy may achieve a higher value of the objective function than a policy without commitment, as considered in this paper, because of a positive expropriation surprise at the beginning of the optimal policy.

However, a few arguments cast doubt on the Chamley-Judd result. The central argument is that the Chamley-Judd policy is not implementable in a real economy, where this policy implicitly assumes that the government can expropriate some part of household wealth without affecting expropriation expectations. In fact, if expected expropriation for some reason differs from zero (either before or after the date the optimal policy is first implemented and the actual expropriation takes place), then the attainable resource allocation set will differ from the one considered by Chamley; this renders impossible implementation of the Chamley policy. In our

framework, under rational expectations the expropriation surprise is zero. Consequently, the government cannot achieve a lower value of initial household wealth by means of intensive capital income taxation at the beginning of the optimal policy. The only effect of intensive capital income taxation is an unnecessary consumption distortion.

Another argument against the Chamley policy is that for any given value of expropriation surprise there exists a policy that is better than Chamley's. Consumption taxation instead of capital income taxation can produce the desired value of expropriation surprise without producing the side effect of the unnecessary consumption distortion implied by Chamley's policy; see Chari and Kehoe (1999) and annex A of this paper for a discussion.

Finally, the value of actual expropriation in Chamley's framework is given quasi-exogenously. Chamley assumes that the consumption tax is zero and the capital income tax is bounded at 100%. If we relax either of these hypotheses, we get a higher value of expropriation than under the Chamley policy. Thus, the value of expropriation in Chamley's framework is determined by some *ad hoc* hypothesis.

The same criticism is applicable to the Judd (1985) approach. These arguments make us doubt that the policies of Chamley and Judd are in fact the policies that we are looking for. It is not surprising that these policies have not yet been implemented in any country.

We find the equilibrium policy, which is defined as the optimal policy without expropriation surprise, and compare it with Chamley's. Under the equilibrium policy, the Chamley-Judd result of zero capital income taxation may hold not only in the long term, but indeed from the very beginning of the optimal policy. However, our

solution differs from the solution of Chamley and Judd in two respects. First, our solution is dynamically consistent. Second, our solution is not just an application of Chamley-Judd's long term recommendation to the short term: if a policy revision is announced, the consumption and labor taxes should be adjusted at some date in a special way in order to compensate for the redistribution of wealth resulting from the abolition of capital income taxation.

We show our key result using the primal approach to the optimal fiscal policy problem developed by Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stockey (1983), Chari and Kehoe (1999) and many others. Chari, Christiano and Kehoe (1996) apply the primal approach to the optimal monetary policy problem; their method directly extends our results to the issue of the inconsistency of optimal monetary policy.

In the context of the optimal monetary policy, our solution may justify Woodford's (2003) "timeless perspective" approach. Woodford considers a policy which allows for the expropriation effect only at time $t \to -\infty$. Under some hypothesis, our approach implies the same policy as the timeless perspective approach[1]. However, Woodford does not justify this approach and proposes to consider the timeless perspective as a special type of commitment, *iter alia*. In contrast to Woodford, we get our result without assuming any commitment and derive it from the problem of maximization of household welfare with respect to policy.

Section 1 presents the model and explicitly introduces expropriation and expected expropriation into the analysis. We consider a continuous-time version of the neoclassical growth model with endogenous labor similar to the one used by

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[1]We get another solution to the optimal policy problem than Woodford either if the policy revision is not the first revision in history or if expectations are not rational.

Chamley. In sections 2 and 3 we show that both the attainable allocation set (section 2) and the optimal policy (section 3) depend only on the expropriation surprise but not on the expropriation and the expected expropriation taken separately. In section 4 we compare the equilibrium policy with that of Chamley. Section 5 concludes.

1. MODEL

The representative household maximizes expected utility, which depends on consumption C and labor L.

$$(1) \qquad \max_{[C,L]} E_0 \int_0^\infty e^{-\rho t} U(C, L)\, dt$$

We take the producer price of the final good to be equal to one. The consumer price of the final good is equal to $(1 + \tau_C)$, where τ_C is the consumption tax. The household's real wealth A consists of capital K and holdings of government debt B. The budget constraint is given by

$$(2a) \qquad \dot{A} = rA + WL - (1 + \tau_C)\, C$$

$$(2b) \qquad \lim_{t \to \infty} A(t) e^{-\int_0^t r(z)dz} \geq 0$$

$$(2c) \qquad A_0 - given,$$

where r and W are the real after-tax equilibrium rate of return and the real wage.

Policy revisions affect the shadow price of the household wealth γ (the co-state variable for equation (2a)) and, consequently, the household wealth measured in units of the utility function, $a = \gamma A$. We call jumps of a, stemming from policy

revisions, the *implicit household wealth expropriation effect*; see Appendix A for better intuition of this idea.

We use the Dirac delta function and the Heaviside function to formalize the household wealth expropriation effect due to policy revision. Let $X(t)$ be the Heaviside function, which accounts for the accumulated wealth expropriation effect at date t. We assume that X is constant during the periods in which the policy is not revised, and if a revision takes place, it discontinuously jumps in order to account for the new wealth expropriation effect:

$$(3) \quad dX = \begin{cases} 0, & \text{if there is no policy revision at date } t \\ -\lim\limits_{dt \to 0+} \frac{a_{t+dt} - a_{t-dt}}{a_{t-dt}}, & \text{if there is a policy revision} \end{cases}$$

Let x be the derivative of X with respect to time:

$$(4) \qquad\qquad\qquad\qquad x = \dot{X}$$

By definition, $x(t)$ is the Dirac delta function, with $x = 0$ on the intervals where the policy is not revised, and at the dates of policy revision, the value of x tends to infinity. However, the integral of x is bounded on any time interval.

The household takes into account the fact that the policy may be revised. It expects that during dt there will be a revision of the policy with probability $p\,dt$. If a revision takes place, there is an implicit expropriation of $\phi \times a$ of the wealth, where ϕ is a random variable defined on $(-\infty, 1)$ with a distribution function $\xi(\phi)$. Let x^e be the expected expropriation rate per time:

$$(5) \qquad\qquad\qquad\qquad x^e = p \int\limits_{-\infty}^{1} \phi \xi(\phi) d\phi$$

Similarly to x, the variable x^e may tend to infinity at some particular points in time, but the integral of x^e on any time interval remains bounded. In contrast to x, x^e may be positive on some time intervals.

The accumulated expected wealth expropriation effect is $X^E(t)$:

$$(6) \qquad\qquad X^E = \int_{-\infty}^{t} x_\tau^e d\tau$$

Expropriation surprise x^s and accumulated expropriation surprise X^s are introduced as follows:

$$(7a) \qquad\qquad x^s = x - x^e$$

$$(7b) \qquad\qquad X^s = X - X^e$$

The first order conditions of the household problem are (see annex B for details):

$$(8a) \qquad\qquad u_C = (1 + \tau_C)\gamma$$

$$(8b) \qquad\qquad u_L = -W\gamma$$

$$(8c) \qquad\qquad \dot{\gamma} = (\rho - r + x^s)\gamma$$

Neither the density function $\xi(\phi)$ nor the probability of expropriation p are present in the first order conditions (8); only x^s is important. Note that γ jumps at the dates where x^s tends to infinity.

Production is not of any particular importance in problems of optimal taxation; see Judd (1999) for a discussion. We assume perfectly competitive markets and constant returns to scale, which implies that there is no profit. The production function depends on labor L and capital K, and is given by

(9) $$Y = F(K, L)$$

The rate of depreciation is δ.

The capital income and labor taxes are τ_K and τ_L. The before-tax interest rate and wage are $\hat{r}$ and $\hat{W}$: $r = (1 - \tau_K)\hat{r}$ and $W = (1 - \tau_L)\hat{W}$. The firms' first-order conditions are given by

(10a) $$\hat{r} = F_K - \delta$$

(10b) $$\hat{W} = F_L$$

The government collects taxes to supply an exogenous amount of public goods G. Its budget constraint can be written as

(11a) $$\dot{B} = rB + G - \tau_C C - \tau_K \hat{r} K - \tau_L \hat{W} L$$

(11b) $$\lim_{t \to \infty} B(t) e^{-\int_0^t r(z)dz} \leq 0$$

(11c) $$B_0 - given.$$

Market clearing requires

(12a) $$\dot{K} = Y - C - G - \delta K$$

(12b) $$K(t) \geq 0 \; \forall t$$

(12c) $$K_0 - given.$$

The representative household cannot solve its optimization problem while fiscal policy is unknown. We suppose that there is a fiscal policy $[\tilde{\tau}_C(t), \tilde{\tau}_K(t), \tilde{\tau}_L(t)]_{t \in [0, \infty)}$

which may be suboptimal, but which is given *ex ante*. The household solves its op-
timization problem assuming that this policy may be implemented. However, the
household takes into account that the government can revise the policy.

The government solves a modified Ramsey (1927) problem: it maximizes house-
hold utility (1) with respect to the fiscal policy $[\tau_C(t), \tau_K(t), \tau_L(t)]_{t \in [0,\infty)}$, taking
into consideration the wealth expropriation effects that occur if the optimal policy
diverges from the *ex ante* policy.

2. Expropriation and the attainable allocation set

The set of allocations that are attainable by the social planner (who finds the
first-best allocation) is given by the resource constraint. This constraint may be
found by substitution of the production function (9) into the market clearing con-
dition (12):

$$\text{(13a)} \qquad\qquad \dot{K} = F(K, L) - C - G - \delta K$$

$$\text{(13b)} \qquad\qquad K(t) \geq 0 \ \forall t$$

$$\text{(13c)} \qquad\qquad K_0 - given.$$

The implementability constraint ensures that the allocation that resolves the
Ramsey problem can be decentralized without lump-sum taxes. This constraint
requires that for a considered allocation $[C(t), L(t)]_{t \in [0,\infty)}$, there exists a vector of
consumer prices that simultaneously satisfies the household budget constraint and
its first-order conditions[2]. We can derive the resource constraint from equations
(2) and (8). The expropriation-augmented implementability constraint is given by

---

[2]In an economy with two goods, the implementability constraint coincides with the price-
consumption curve.

(14a) $\dot{a} = \rho a - U_C C - U_L L - x^s a$

(14b) $\lim_{t \to \infty} a(t) e^{-\rho t} = 0$

Condition (14b) is derived from inequalities (2b), (11b), (12b), and $\lim_{t \to \infty} K(t) e^{-\int_0^t r(z)dz} \leq$ 0, which complete each other and together with (8c) ensure that (14b) is satisfied with equality.

There are two differences between the conventional implementability constraint (see, for example, Chari and Kehoe (1999)) and the expropriation-augmented constraint (14). First, there is a new term $x^s a$ in (14a). Second, for a given value of X_0, the value of a_0 is also given, while in the conventional constraint, a_0 is not determined.

The government finds the equilibrium under ex-ante policy $[\widetilde{\tau}_C(t), \widetilde{\tau}_L(t), \widetilde{\tau}_K(t)]_{t \in [0, \infty)}$ and arrives at $\widetilde{a}_0$. The value of X_0 is historically given. Consequently, the initial conditions for (14a) are given by

(14c) $a_0 \quad = \quad \widetilde{a}_0$

(14d) $X_0 \quad - \quad given$

Note that a policy revision that produces a wealth expropriation effect at date 0 will change not only a_0 but also X_0.

The resource and implementability constraints with the initial and transversality conditions exactly describe the set of allocations that may be implemented in a decentralized economy without lump-sum taxes. The proof of this fact is well known in the literature; see annex C for details.

Proposition 1. *The attainable resource allocation set depends on the expropriation surprise x^s but not on x and x^e separately.*

Proof. The attainable resource allocation set is given by the resource constraint (13) and the expropriation-augmented implementability constraint (14). We see that only x^s enters into these constraints. $\square$

Proposition 1 reveals why our conclusions differ from those of Chamley and Judd. Chamley and Judd implicitly assume that a positive value of x is possible only at $t = 0$ and $x^e = 0 \ \forall t \geq 0$. This is why they arrive at the result that the more the government expropriates at the beginning, the better the policy outcome.

However, expected expropriation affects the attainable resource allocation set, and there is no reason to believe that x^e is always zero. If we assume rational expectations and $x^e = x$, then $x^s = 0$ and the expropriation x does not affect the attainable resource allocation set.

3. Optimal policy for a given expropriation surprise

3.1. **The modified Ramsey problem.** Assume that the vector $[x(t), x^e(t)]_{t \in [0,\infty)}$ is given exogenously. The optimal policy problem takes the form:

(15a)
$$\max_{[C(t),L(t)]} \int_0^\infty e^{-\rho t} U\left(C, L\right) dt$$

(15b)
$$\dot{K} = F\left(K, L\right) - C - G - \delta K$$

(15c)
$$\dot{a} = \rho a - U_C C - U_L L - x^s a$$

(15d)
$$\lim_{t \to \infty} a(t) e^{-\rho t} = 0$$

(15e)
$$a_0 = \widetilde{a}_0$$

(15f)
$$K_0 \quad - \quad given.$$

The co-state variable is λ (negative) for the implementability constraint and μ (positive) for the resource constraint. The first-order conditions are:

(16a)
$$U_C\left(1 - \lambda\left(1 + H_C\right)\right) = \mu$$

(16b)
$$U_L\left(1 - \lambda\left(1 + H_L\right)\right) = -\mu F_L$$

(16c)
$$\dot{\lambda} = x^s \lambda$$

(16d)
$$\dot{\mu} = \left(\rho - \left(F_K - \delta\right)\right)\mu,$$

where the terms H_C and H_L are given by

(17a)
$$H_C = \frac{U_{CC}}{U_C}C + \frac{U_{CL}}{U_C}L$$

(17b)
$$H_L = \frac{U_{CL}}{U_L}C + \frac{U_{LL}}{U_L}L$$

The term H_i is a measure of the excess tax burden related to a particular form of taxation. It plays the same role as the inverse elasticity of demand in microeconomic analysis of the deadweight loss of taxation; see Atkinson and Stiglitz (1980). A

possible interpretation of $(-\lambda)$ is the marginal excess burden of taxation measured in terms of utility.

3.2. **Optimal policy.** Equations (16) and the constraints to the Ramsey problem (15) give the resource allocation under the optimal policy. In order to determine the policy itself, it is necessary to combine the first order conditions of the household's problem (8) with the first order conditions of the optimal policy problem (16), taking into consideration the initial condition (15e). For convenience, we introduce here a determining cumulative tax set that uniquely determines all tax distortions (see annex A for details). This set consists of the following 3 cumulative taxes:

$$
(18a) \qquad 1 + T_{C/L} \;=\; \frac{1 + \tau_C}{1 - \tau_L}
$$

$$
(18b) \qquad 1 + T_{C(t+z)/C(t)} \;=\; \frac{1 + \tau_C(t+z)}{1 + \tau_C(t)} \exp\left(\int_t^{t+z} \tau_K r(s)ds \right)
$$

$$
(18c) \qquad 1 + T_{C(0)/A(0)} \;=\; (1 + \tau_C(0))
$$

The optimal policy is given by

$$
(19a) \qquad 1 + T_{C/L} \;=\; \frac{\Phi_C}{\Phi_L}
$$

$$
(19b) \qquad 1 + T_{C(t+z)/C(t)} \;=\; \frac{\Phi_{C,t+z}}{\Phi_{C,t}} \exp\left[X_{t+z}^s - X_t^s \right]
$$

$$
(19c) \qquad 1 + T_{C(0)/A(0)} \;=\; (1 + \widetilde{\tau}_C(0)) \frac{U_C(C(0), L(0))}{U_C(\widetilde{C}(0), \widetilde{L}(0))},
$$

where

$$
(20a) \qquad \Phi_{C,t} \;=\; (1 - \lambda_t (1 + H_{C,t}))^{-1}
$$

$$
(20b) \qquad \Phi_L \;=\; (1 - \lambda (1 + H_L))^{-1}
$$

Equation (19a) was found from (8a), (8b), (16a) and (16b), equation (19b) was found from (8a), (8c), (16a) and (16d), and equation (19c) was found from the definition $a = \gamma A$, the constraint (15e), and equation (8a).

There is an infinite number of policies that implement (19) and decentralize the optimal allocation. In order to get the only policy, we exogenously define the dynamics of one of the tax rates. Suppose that if $x^s = 0$, then the consumption tax is constant and its value is chosen to satisfy (19c):

$$(21a) \qquad\qquad 1 + \tau_C \;\; = \;\; 1 + T_{C(0)/A(0)}$$

In this case, the optimal capital and labor taxes are given by

$$(21b) \qquad\qquad 1 - \tau_L \;\; = \;\; \frac{1 + \tau_C}{1 + T_{C/L}}$$

$$(21c) \qquad\qquad \tau_K \hat{r} \;\; = \;\; \frac{\dot{T}_{C(t)/C(0)}}{1 + T_{C(t)/C(0)}}$$

We assume that expropriation surprises are absorbed by simultaneous jumps of τ_c and τ_L.

Proposition 2. *For any given dynamics of x^s, the solution to the optimal policy problem (15) is dynamically consistent.*

Proof. From (15) it can be immediately seen that if x and x^e are given, the solution to the problem is dynamically consistent: all state variables are in fact *state* variables, which do not include forward-looking terms. If a formal argument is required, consistency may be shown, for example, by comparing the solutions obtained by two alternative methods: the Pontriagin and Bellman principles. The Pontriagin

principle maximizes the discounted value of the objective function and may be dynamically inconsistent. The Bellman principle recognizes that, in the future, a plan will be chosen that is optimal for that period and that resolves the consistency problem. From the fact that these two solutions are equivalent, it follows that the solution to the optimal policy problem is dynamically consistent.   □

A special case is $x_t^s = 0$ $\forall t$. An application of proposition 4 to this case is that under $x^s = 0$, the optimal policy is also dynamically consistent.

Proposition 3. *Optimal policy depends on the expropriation surprise x^s, but not on x and x^e separately.*

Proof. See the equations that describe the optimal policy (19).   □

Proposition 3 supplements proposition 1 and says that not only the attainable resource allocation set but also the optimal policy depends only on the expropriation surprise.

Propositions 1 and 3 encourage us to analyze the equilibrium policy, defined as $x_t^s = 0$ $\forall t \geq 0$, instead of the policy of Chamley (1986) and Judd (1985).

4. EQUILIBRIUM POLICY

Let us consider the case of *equilibrium policy*, $x_t^s = 0$ $\forall t \geq 0$. A special case of the equilibrium policy is *policy without expropriation*, where $x_t^e = x_t = 0$ $\forall t$.

For the case of equilibrium policy, our conclusions are similar to the long term conclusions of Chamley and Judd. If H_C is constant, the optimal cumulative tax $T_{C(t)/C(0)}$ is zero (see (19b), (20a) and (16c)). Equation (21c) shows that the optimal capital income tax in this case is also zero. This is possible in the two

cases: if preferences are isoelastic (for example, $U(C, L) = \frac{C^{1-\theta}-1}{1-\theta} + V(L)$) or if the economy is on the balanced growth path[3].

There are two differences between the equilibrium policy and the policy of Chamley and Judd. First, the equilibrium policy is dynamically consistent. Second, the optimal capital income tax under the equilibrium policy may be zero not only in the long term, but also in the short term.

However, our solution does not replicate the Chamley and Judd long term recommendations into the short term: in the short term, the optimal consumption and labor taxes are adjusted in order to avoid any change in a_0. For example, a capital income tax reduction increases the shadow price of household wealth. In order to compensate for this effect, the government needs to increase the consumption tax and to decrease the labor tax.

If we neglect certain second-order effects that we discuss in the next paragraph, then the required changes in consumption and labor taxes may be approximately calculated in the following manner. Suppose that a decrease in the capital income tax increases the after-tax interest rate by 10%. Then the capitalists become 10 % richer, and to compensate this effect, the consumer price of the final good $(1 + \tau_C)$ should be increased by 10%. The new value of the labor tax should ensure that the infratemporal government budget constraint is satisfied.

This arithmetic works well in Y=AK - type models, where the decrease of the capital income tax creates a permanent effect on the real interest rate and when the excess tax burden of distortionary taxation is not too high. However, in exogenous growth models, the effect of a decrease in τ_K on a is temporary; consequently, it

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[3]These two cases are not too different: the balanced growth path is possible only if preferences are isoelastic in consumption for the *realized* allocation.

requires a smaller increase in the consumption tax. In addition, the capital income tax reduction produces another effect: this tax should be substituted by others. This will increase the cumulative tax $T_{C/L}$, decrease the labor supply, decrease the before tax interest rate, and reduce a_0. These effects require a decrease in the consumption tax. In the general case, it is not clear whether the consumption tax, as well as the labor tax, should be increased or decreased.

Finally, note that on the balanced growth path, all taxes are constant. Consequently, the optimal debt to GDP ratio is also constant.

5. Conclusions

Previous papers implicitly assumed that the attainable allocation set and the optimal policy depend on the expropriation of household wealth due to a policy revision at the beginning of the optimal policy. However, we show that only an expropriation surprise affects the attainable resource allocation set and the optimal policy.

If we knew exactly what affects expected expropriation, we could define the attainable set of expropriation surprises and maximize the government's objective on this set. However, expectations of expropriation depend on a large number of factors, such as credibility, commitment, history, economic and cultural development, government debt, sunspots in the sense of Azariadis, and so on. The exact relationships are unknown, so we cannot solve the maximization problem.

However, a long discussion in the 1970s and 80s on the ways inflationary expectations are formed induced researchers to use rational expectations by default. The reason why the rational expectations hypothesis prevails in contemporary research is the weakness of the alternatives: any other particular hypothesis is worse

than that of rational expectations. In our framework, under rational expectations, $x = x^e$, whereby $x^s = 0$, and the government cannot affect the attainable allocation set by means of an implicit expropriation of household wealth at the beginning of the optimal policy.

Under $x^s = 0$, intensive capital income taxation at the beginning of the optimal policy does not imply a lump-sum taxation of households' initial wealth and creates only an unnecessary consumption distortion. Thus, in contrast to the Chamley result, we show that intensive capital income taxation at the beginning of the optimal policy is suboptimal.

The only reason for the inconsistency of the Chamley policy is the desire to produce a positive expropriation surprise. Under rational expectations, $x^s = 0$ and therefore an expropriation surprise is impossible and the policy is dynamically consistent.

Appendix A. Cumulative taxation and the wealth expropriation effect

The central conclusions of this appendix are well-known in the literature (see, for example, Chari and Kehoe (1999)). We review these results in a proper manner in order to achieve a better intuition of the central conclusions of this paper.

A.1. **Some definitions.** The only reason why distortionary taxation affects resource allocation is because it distorts the ratios of relative consumer prices to relative producer prices[4]. Consequently, we can specify all tax distortions using *cumulative tax rates*, which determine these ratios:

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[4]In the general case, relative prices may differ not only between consumers and producers but also between different groups of consumers or different groups of producers. Note that this may imply a violation of the Diamond-Mirrlees (1971) principle of production efficiency.

$$(22) \qquad\qquad 1 + T_{i/j} = \frac{p_i/p_j}{\hat{p}_i/\hat{p}_j},$$

where p_i and p_j are the consumer prices of goods i and j, $\hat{p}_i$ and $\hat{p}_j$ are the respective producer prices, and $T_{i/j}$ is the cumulative tax rate of good i with respect to good j.

It is easy to verify that two fiscal policies implement different resource allocations if and only if they implement different cumulative taxes.

By definition,

$$(23a) \qquad\qquad \left(1 + T_{i/j}\right) = \left(1 + T_{j/i}\right)^{-1}$$

$$(23b) \qquad\qquad \left(1 + T_{i/k}\right) = \left(1 + T_{i/j}\right) \times \left(1 + T_{j/k}\right)$$

Consequently, *all* the cumulative tax rates are not needed in order to specify all tax distortions, since some cumulative taxes may be derived from others. A *determining cumulative tax set* is a set that uniquely determines all tax distortions.

A.2. **The determining cumulative tax set.** In order to derive a determining cumulative tax set in the economy, we first write the households' budget constraint and the firms' net present value in integral form:

$$(24a) \qquad (1 - \tau_{A0})\, A_0 = \int_0^\infty e^{-\int_0^t r(z)\,dz} \left[(1 + \tau_C)\, C - WL\right] dt$$

$$(24b) \qquad NPV_0 = \int_0^\infty e^{-\int_0^t \hat{r}(z)\,dz} \left[Y - \hat{W}L - \dot{K} - \delta K\right] dt,$$

where τ_{A0} is the fictitious initial wealth tax. We assume that $\tau_{A0} = 0$ and introduce it simply to show how it may be substituted by other taxes.

Second, from (24), we find the present values of consumer and producer prices. For example, the present value of the consumer price of the final good is equal to the term that is multiplied by consumption in the integrand of equation (24a). All present values of prices are given in Table 1.

Table 1. Present values of prices

	Consumer	Firms
Final good $Y(t)$, $C(t)$	$(1 + \tau_C(t))\, e^{-\int_0^t r(z)dz}$	$e^{-\int_0^t \hat{r}(z)dz}$
Labor $L(t)$	$W(t)e^{-\int_0^t r(z)dz}$	$\hat{W}(t)e^{-\int_0^t \hat{r}(z)dz}$
Initial wealth A_0, NPV_0	$(1 - \tau_{A0})$	1

Finally, we derive a determining cumulative tax set from Table 1 by definition (22):

$$(25a) \qquad 1 + T_{C(t)/L(t)} \;=\; \frac{(1 + \tau_C(t))}{(1 - \tau_L(t))}$$

$$(25b) \qquad 1 + T_{C(t)/A(0)} \;=\; \frac{(1 + \tau_C(t))}{(1 - \tau_{A0})} \exp\left(\int_0^t \tau_K \hat{r}(z)\, dz \right)$$

The determining cumulative tax set $\left[T_{C(t)/L(t)}, T_{C(t)/A_0} \right]_{t \in [0,\infty)}$ uniquely determines all distortions that arise from distortionary taxation. All cumulative taxes can be derived from the determining cumulative tax set. For example,

$$(26a) \qquad 1 + T_{C(t+s)/C(t)} \;=\; \frac{1 + T_{C(t+s)/A0}}{1 + T_{C(t)/A0}}$$

$$(26b) \qquad 1 + T_{C(t+s)/L(t)} \;=\; \frac{1 + T_{C(t+s)/A0}}{1 + T_{C(t)/A0}} \left(1 + T_{C(t)/L(t)} \right)$$

$$(26c) \qquad 1 + T_{L(t)/A0} \;=\; \frac{1 + T_{C(t)/A0}}{1 + T_{C(t)/L(t)}},$$

and so on.

In section 4 we compare our policy with the policy of Chamley and Judd. We can use equations (25) to clarify their famous result of zero long term capital income

taxation. For this purpose, we will demonstrate how consumption and labor taxes can substitute capital income taxation.

Assume that the *ex ante* capital income tax $\widetilde{\tau}_K$ is a positive constant. The government sets the capital income tax rate to zero, $\tau_K = 0$. If it desires to keep the determining cumulative tax set unchanged, it needs to revise the consumption and labor taxes, τ_C and τ_L, in the following manner:

$$(27a) \qquad (1 + \tau_C(t)) = (1 + \widetilde{\tau}_C(t)) \exp\left(\int_0^t \widetilde{\tau}_K \hat{r}(z)\, dz\right)$$

$$(27b) \qquad (1 - \tau_L(t)) = (1 - \widetilde{\tau}_L(t)) \exp\left(\int_0^t \widetilde{\tau}_K \hat{r}(z)\, dz\right)$$

If the revised fiscal policy satisfies (27), then the determining cumulative tax set remains unchanged, and the revision does not affect the resource allocation. According to (27), capital income taxation at a positive constant rate perfectly substitutes consumption tax, which tends to infinity as time tends to infinity, and labor tax, which tends to minus infinity as time tends to infinity. There are two consequences of this fact. The first consequence follows from the perfect substitutability between taxes. If the consumption and labor taxes can perfectly substitute the capital tax, then the capital income tax is unnecessary. For example, if the consumption and labor taxes are set to their optimal values at the micro level, then the capital income tax should be set to zero. The second consequence follows from the tax dynamics when time tends to infinity. A constant capital income tax perfectly substitutes taxes, which tend in absolute values to infinity. Any tax tending to infinity is suboptimal; consequently, the optimal capital income tax should be set to zero at least

in the long term, even if consumption and labor taxes are not set to their optimal

levels[5].

A.3. **The wealth expropriation effect.** Let us determine the instruments that

allow the initial wealth taxation to be substituted, i.e. the instruments that produce

the *wealth expropriation effect.*

Proposition 4. *The consumption and labor taxes can perfectly substitute* $\widetilde{\tau}_{A0}$. *If*

the capital income tax is bounded, it can imperfectly substitute $\widetilde{\tau}_{A0}$.

Proof. Assume that the government substitutes the initial wealth tax $\widetilde{\tau}_{A0}$, which is

positive ex ante, by other taxes and attempts to keep the determining cumulative

tax set unchanged. The new policy should satisfy the following conditions:

$$(28\text{a}) \ (1 + \tau_C(t)) \exp \left(\int_0^t \tau_K \hat{r}(z) \, dz \right) = \frac{(1 + \widetilde{\tau}_C(t))}{(1 - \widetilde{\tau}_{A0})} \exp \left(\int_0^t \widetilde{\tau}_K \hat{r}(z) \, dz \right)$$

$$(28\text{b}) \ (1 - \tau_L(t)) \exp \left(\int_0^t \tau_K \hat{r}(z) \, dz \right) = \frac{(1 - \widetilde{\tau}_L(t))}{(1 - \widetilde{\tau}_{A0})} \exp \left(\int_0^t \widetilde{\tau}_K \hat{r}(z) \, dz \right)$$

Equations (28) show that an increase in τ_C and a simultaneous decrease in τ_L

can perfectly substitute $\widetilde{\tau}_{A0}$.

If τ_K is bounded, then the term $\exp \left(\int_0^t \tau_K \hat{r}(z) \, dz \right)$ is too small to substitute

$\widetilde{\tau}_{A0}$ for small t. For large values of t, the term $\exp \left(\int_0^t \tau_K \hat{r}(z) \, dz \right)$ in (28) can

be sufficiently large to substitute $\widetilde{\tau}_{A0}$. Thereafter, substitution of $\widetilde{\tau}_{A0}$ by τ_K is

imperfect, because it requires a revision of $T_{C(t)/A0}$ for small t. $\square$

Proposition 4 clarifies the short term recommendations of Chamley (1986) and

Judd (1985). Chamley and Judd set the consumption tax and the initial wealth tax

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[5]Equations (27) state that capital income taxation does not violate Diamond-Mirrlees (1971) principle of production efficiency.

to zero, so that the initial wealth in their economies can be taxed only by means of the capital income tax. However, the capital income tax substitutes the initial wealth tax imperfectly and creates extra distortions. When the optimal policy is first implemented, the effect of initial wealth taxation dominates and the capital income is taxed at its top bound level. At some date the effect of extra distortions prevails, and the optimal capital income tax switches to another value in order to compensate for the non-optimality of the taxation at the micro level (if it exists). In the long term the optimal capital income tax converges to zero.

Thus, consumption and capital income taxes are indirectly levied on the initial wealth. Note that this appendix confines the analysis to fiscal reforms such that the cumulative tax rates and the resource allocation remain unchanged, and this is why we do not consider many other ways to expropriate the initial wealth in this appendix. For example, a fiscal reform may affect the before-tax interest rate dynamics and thus influence the market value of household wealth. The approach used in the body of the paper accounts for all possible ways of wealth expropriations.

APPENDIX B. FIRST-ORDER CONDITIONS TO THE WEALTH

EXPROPRIATION-AUGMENTED HOUSEHOLD PROBLEM

Let $V(A(t), X(t), t)$ be the value function

$$(29) \qquad V(A(t), X(t), t) = \max_{[C,L]} E_t \int\limits_t^\infty e^{-\rho\tau} U(C, L) \, d\tau$$

Taking into account that

(30)

$$
\begin{aligned}
E_t V(A(t+dt), X(t+dt), t+dt) \;=\;& (1-pdt)\, V(A(t+dt), X(t), t+dt) \\
&+pdt \int_{-\infty}^{1} V(A(t+dt), X(t)+\phi, t+dt)\xi(\phi)d\phi,
\end{aligned}
$$

the Bellman equation can be written as:

(31)

$$
\begin{aligned}
0 \;=\;& \max_{[C,L]} \Big(e^{-\rho t} U(C, L) + \\
& \frac{V(A(t+dt), X(t), t+dt) - V(A(t), X(t), t)}{dt} + \\
& p \int_{-\infty}^{1} \big(V(A(t+dt), X(t)+\phi, t+dt) - V(A(t+dt), X(t), t+dt) \big)\, \xi(\phi)d\phi \Big)
\end{aligned}
$$

We will use a Taylor decomposition for the second term and substitute $\dot{A}$ from (2). Taking the limit as $dt \to 0+$, this gives:

$$
\begin{aligned}
(32) \quad 0 \;=\;& \max_{[C,L]} \Big(e^{-\rho t} U(C, L) + \\
& V_A(A(t), X(t), t)(rA + WL - (1+\tau_C)C) + V_t(A(t), X(t), t) + \\
& p \int_{-\infty}^{1} \big(V(A(t), X(t)+\phi, t) - V(A(t), X(t), t) \big)\, \xi(\phi)d\phi \Big)
\end{aligned}
$$

Equation (32) is the Bellman equation for the problem. The first-order conditions are:

$$
(33a) \qquad\qquad e^{-\rho t} U_C \;=\; (1+\tau_C) V_A(A(t), s(t), t)
$$

$$
(33b) \qquad\qquad e^{-\rho t} U_L \;=\; -W V_A(A(t), s(t), t)
$$

Let γ be the shadow price of the household's wealth:

$$(34) \qquad\qquad \gamma = V_A e^{\rho t},$$

then equations (33) give (8a) and (8b).

Application of the envelope theorem gives:

(35)

$$
\begin{aligned}
0 \;=\; & V_{AA}(A(t), X(t), t)\dot{A} + rV_A(A(t), X(t), t) + V_{At}(A(t), X(t), t) + \\
& p \int_{-\infty}^{1} \left(V_A(A(t), X(t) + \phi, t) - V_A(A(t), X(t), t) \right) \xi(\phi) d\phi \Bigg)
\end{aligned}
$$

Differentiate (34) with respect to time:

$$(36) \qquad\qquad \dot{\gamma} = \left(V_{AA}\dot{A} + V_{At} \right) e^{\rho t} + \rho\gamma$$

From equations (34), (35) and (36), taking into account (3), (4), and (5) we arrive at the last first-order condition to the expropriation-augmented household problem (8c).

Appendix C. The attainable allocation set (comments to section 2)

The derivation of the attainable allocation set that we use in section 2 is well-known in the literature; see, for example, Lucas and Stockey (1983).

We obtain the resource (13) and implementability (14) constraints from conditions that are satisfied for any equilibrium allocation; consequently, they are also satisfied for any equilibrium allocation.

If an allocation $[C(t), L(t)]_{t\in[0,\infty)}$ satisfies equation (14), then for any given strictly positive dynamics of one of the consumer prices $[r(t), \tau_c(t), W(t)]_{t\in[0,\infty)}$, there exist dynamics of the other prices such that the household will choose the

given allocation. Indeed, the first-order conditions (8) and definitions $a = \gamma A$ and (3) give prices such that these conditions are satisfied, and substitution of these prices into the implementability constraint gives the household's budget constraint. Thus, the household's budget constraint is also satisfied.

If an allocation $[C(t), L(t)]_{t \in [0,\infty)}$ satisfies the resource constraint (13), then we can find the dynamics of the producer prices $(\hat{r}, \hat{W})$ under which firms will choose an input-output vector such that the equilibrium market condition is satisfied. Indeed, from equation (13) and the initial conditions, we can calculate the dynamics of K that give the dynamics of the output $Y = C + G + \dot{K} + \delta K$. Knowing the dynamics of Y, K, and L, we can use the firms' first-order conditions to find the prices $(\hat{r}, \hat{W})$ under which the firms choose the considered allocation, $\hat{r} = F_K$ and $\hat{W} = F_L$.

If both constraints are satisfied, the government budget constraint is also satisfied by Walras' law. Thus, these constraints guarantee that there exist vectors of consumer and producer prices such that all equilibrium conditions are satisfied. The cumulative tax rates (introduced in annex A) that decentralize the considered allocation may be found from the difference between the consumer and producer prices. For example,

$$1 + T_{C,L} = \frac{(1 + \tau_C)}{W/\hat{W}}.$$

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