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Multivariate Partial Distribution: A New Method of Pricing Group Assets and Analyzing the Risk for Hedging

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ABSTRACT Based on the Partial Distribution (Feng Dai, 2001), a new model to price an asset (MPA) is given. Going a step further, this paper puts forward the Multivariate Partial Distribution (MPD) for the first time. By use of MPD, we could gain a new kind of model for pricing the group assets (MPGA), in which the competition and cooperation are considered. Based on MPGA, the integrated risk of group assets can be divided to hedging risk and independent risk, and the corresponding models are given. So we could analyze the price risk of group assets in more particular way. The conclusions show that assets are hedged in simple way of one to one can not eliminates completely their market risk in many cases. So there should be an optimal ratio between underlying asset and its derivative in hedging. The approach to determine the optimal ratio in hedging is offered in this paper. By the MPA and MPGA, we also could interpret five of interesting economic propositions in analytic way.

KEYWORDS multivariate Partial Distribution; pricing assets; group assets; risk analysis; optimal hedging

1 Introduction

In the modern economical society, it is very important to price assets (capitals or commodities). Up to now, many outstanding studies and works have been done for the estimating and measuring of the price of capital asset, e.g. CAPM (capital asset pricing model, W.F. Sharpe, 1964, J. Lintner, 1965) and APT (arbitrage pricing theory, S.A. Ross, 1976), etc. Coming a further considering, we see, CAPM need a group of risk capitals. This is difficult to realize in reality, because it is not easy to make a whole samples indexes in a larger financial market. In general, CAPM is regarded as an example of APT, because CAPM is a method for a single asset, and APT is a method for multiple assets. In fact, CAPM can also be extended to multiple assets, for example, the Consumer Service Model (R.C. Merton, 1973). This model considered the risk premium of assets group, APT did not. Again, the consumption-based CAPM (T.Breeden, 1979) is more of imagination.

What is needed to point out, CAPM is based on the market equilibrium and is a result of investors behaving together. So CAPM must be under a series of assumptions, and some of assumptions are more rigorous in some time. APT is applicable to investment decision on group assets and emphasizes the rule of no-arbitrage. APT is based on the assets group, so it is not always right in pricing for single asset. In the other hand, CAPM and APT make the pricing on yield of asset or assets mainly. The prices of asset itself always change in a financial market. In many time, we need to know not only the yield of asset or assets, but also the current prices of asset itself, because both of them are influenced one another.

Based on Partial Distribution (Feng Dai, 2001) and multivariate Partial Distribution, this paper will give the new model of pricing asset. Comparing with CAPM and APT, the model given here can price the single asset, group assets, and other general commodities. This kind of model does not suppose the equilibrium and no-arbitrage. Of course, there must be some of general assumptions. By this kind of model, we also can analyze the hedging risk for group assets in market and give the optimal ratio between underlying asset and its derivative in hedging. it is worth to say that, based on Partial Distribution and multivariate Partial Distribution, the five of economic propositions can be interpreted in analytic way, like "the more the risk is, the larger the possible profit is", "the new asset must be developed continuously in order to acquire the higher sale profits", "the competition results in the raise in cost of asset, and the cooperation results in descending the cost of asset", and etc.

The models given in this paper can be applied to price the virtual products, invisible asset and etc., and to analyze the price risk of them.

2 The Definitions and Assumptions

2.1 The Definition of Partial Distribution

Definition 1 (Partial Distribution, PD for short) Let X be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

where $\mu\geq 0$ and $\sigma>0$. Then, X is called to follow a Partial Distribution, and note as $S\in P(\mu, \sigma^2)$.

The Partial Distribution is a kind of truncated Gaussian distribution. Partial Distribution is sharper than Gaussian and lognormal distribution as $x\rightarrow\mu$ and μ is less. If μ is big enough, the Partial distribution is approximately near to Gaussian distribution.

Partial Distribution has two basic characteristics. One characteristic is that the probability is equal to zero when the variable is less than zero, this is even corresponding to that the prices of any asset, like capitals, stocks, futures and commodity, is non-negative. Another characteristic is that the probability is not equal to zero when the variable is equal to zero. This is even corresponding that the prices of some asset may become zero in market, like the price of stock of a company closed down, the price of overdue food or medicine, etc. Both Gaussion and lognormal distribution do not have above two characteristics at the same time. Levy distribution is a better one for fitting the price behaviors in financial market now, but it can not be expressed as the elementary function except Gaussion and Cauchy distribution. Cauchy distribution has an infinite variance, so it is of inconvenience in application.

Moreover, we can get two important results in section 3 according to Partial Distribution. And these two important results can not be got by other probability distribution at the same time.

Definition 2 (Partial Process) Let $\{X(t), t\in[0,\infty)\}$ be a stochastic process. $\forall\, t\in[0,\infty)$, $X(t)$ follows the Partial Distribution $P(\mu(t), \sigma^2(\mathrm{t}))$, then the $\{X(t), t\in[0,\infty)\}$ is called a partial process.

Definition 3 (Multivariate Partial Distribution, MPD for short) if $X_1, \cdots, X_n$ ($n\geq 2$) are all the non-negative stochastic variables, and follow the multivariate distribution of density

$$f(x_1,\cdots,x_n)=\begin{cases} \dfrac{e^{-\frac{1}{2|M|}\left[\sum\limits_{i=1}^{n}|M_{ii}|(x_i-\mu_i)^2+\sum\limits_{i,j=1,i\neq j}^{n}|M_{ij}|\left(\sigma_i\left(x_j-\mu_j\right)\right)\right]}}{\int_0^\infty\cdots\int_0^\infty e^{-\frac{1}{2|M|}\left[\sum\limits_{i=1}^{n}|M_{ii}|(x_i-\mu_i)^2+\sum\limits_{\substack{i,j=1\\ i\neq j}}^{n}\sigma_i|M_{ij}|\left(x_j-\mu_j\right)\right]}dx_1\cdots dx_n} & 0\leq x_1,\cdots,x_n<\infty \\ 0 & \text{other cases} \end{cases} \tag{2}$$

where, $M=\begin{bmatrix}\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n}\\ \vdots & \vdots & \cdots & \vdots \\ \sigma_{n1} & \sigma_{n1} & \cdots & \sigma_{nn}\end{bmatrix}$, $\sigma_{ii}=\sigma_i^2, \sigma_{ij}=r_{ij}\sigma_i\sigma_i$ ($i\neq j$), $\sigma_i>0$, $|r_{ij}|\leq 1$, $i, j=1, \cdots, n$.

then we call $(X_1, \cdots, X_n)$ to follow n-dimensions Partial Distribution, and note as $\boldsymbol{X}\in P(\boldsymbol{\mu}, \boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma}, \boldsymbol{R})$. where, $\boldsymbol{X}=(X_1, \cdots, X_n)^{\mathrm{T}}$, $\boldsymbol{\mu}=(\mu_1, \cdots, \mu_n)^{\mathrm{T}}$, $\boldsymbol{\sigma}=(\sigma_1, \cdots, \sigma_n)^{\mathrm{T}}$, $\boldsymbol{R}=(r_{ij})_{n\times n}$, $\mu_1, \cdots, \mu_n\geq 0$, $\sigma_1, \cdots, \sigma_n>0$, r_{ij} is called the correlation coefficient between X_i and X_j, $r_{ii}=1$, $i, j=1, \cdots, n$.

As a special example of MPD, if the non-negative stochastic variables X and Y follow the multivariate distribution of density:

$$f(x,y)=\begin{cases} \dfrac{e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{x-\mu_1}{\sigma_1}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]}}{\int_0^\infty\int_0^\infty e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{x-\mu_1}{\sigma_1}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]}dxdy}, & 0\leq x,y<\infty \\ 0 & x<0 \text{ or } y<0 \end{cases} \tag{3}$$

then, (X, Y) is called to follow 2-dimensions Partial Distribution, and note as $(X, Y) \in P(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, r)$, where, the constants $\mu_1, \mu_2 \geq 0$, $\sigma_1, \sigma_2 > 0$, $-1 < r < 1$.

If $r=0$, thus we know, from expression (3) and (1), $f(x, y)=f_1(x)f_2(y)$, i.e. X is not correlating with Y. Where,

$$f_1(x)=\begin{cases} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} \Big/ \int_0^\infty e^{-\frac{(u-\mu_1)^2}{2\sigma_1^2}} du & x \geq 0 \\ 0 & x < 0 \end{cases}, \quad f_2(y)=\begin{cases} e^{-\frac{(y-\mu_1)^2}{2\sigma_2^2}} \Big/ \int_0^\infty e^{-\frac{(u-\mu_2)^2}{2\sigma_2^2}} du & y \geq 0 \\ 0 & y < 0 \end{cases}$$

When $r=1$, we know, according to reference [9], X is correlating with Y in linearity and on probability 1, i.e., the probability $P(Y=aX+b)=1$, where, $a>0$ if $r=1$, and $a<0$ if $r=-1$.

2.2 Estimating the parameters in MPD

Here we take the 2-dimension PD as an example. The samples series of stochastic variable1 and variable 2 are separately $x_{1,1}, x_{1,2}, \cdots, x_{1n}$ and $x_{21}, x_{22}, \cdots, x_{2n}$ ($x_{1i}, x_{2i} > 0$, $i=1,\cdots, n$).

According to the modified maximum likelihood estimation [10], we can obtain $\hat{\mu}_k$ (the estimated value of μ_k) and $\hat{\sigma}_k$ (the estimated value of σ_k), $k=1, 2$. Thus, the correlation coefficient can be estimated as expression (4):

$$\hat{r}_{1,2} = \frac{\sum_{i=1}^{n}(x_{1i} - \hat{\mu}_1)(x_{2i} - \hat{\mu}_2)}{\sqrt{\sum_{i=1}^{n}(x_{1i} - \hat{\mu}_1)^2 \cdot \sum_{j=1}^{n}(x_{2j} - \hat{\mu}_2)^2}} \tag{4}$$

2.3 The assumptions about the price of asset

As a theoretical basis of the discussions in this paper, we give some basic concepts and basic assumptions as follow:

Definition 4 The prices of an asset include the cost price and market price. The cost price of an asset is all the payment for holding the asset. The market price of an asset is its current exchange price in market.

As to the finance or capital asset, the cost price includes the paying for buying asset, ratepaying, interest paying and the other payment concerned. And as to the consumed asset, like the commodity, etc., the cost price includes the paying for buying raw and processed materials, production, transportation, advertisement, ratepaying and other payment concerned.

Assumption 1 For the price of asset, we suppose

1) The prices (cost price and market price) have been fluctuating with time. Any price and the fluctuation range (i.e., the variance or standard variance) of price are non-negative.
2) Both the cost price and the fluctuation of cost price of an asset are the basic elements to influence the market prices of the asset, and the market prices come into being on the market exchange.
3) The possibilities that the market price of asset is much lower than its cost price, or is much higher than its cost price, will be very small.

We will use the following basic notations:

μ—The cost price of asset (capital asset or commodity asset), $\mu \geq 0$.

σ—The fluctuation range of the cost price, i.e., the standard variance of the cost price, $\sigma > 0$.

X—The market price of an asset, $0 \leq X < \infty$.

r— The correlation coefficient.

Assumption 2 If the cost price and market price of an asset satisfy the assumption 1, than we suppose that the market price of the asset follows PD, i.e., $X \in P(\mu, \sigma^2)$.

If the price of asset changes along with time, then the market price of the asset $X(t) \in P(\mu(t), \sigma^2(t))$ (for any time $t \geq 0$), where $\mu(t)$ means the cost price of the asset, and $\sigma(t)$ means the standard variance of the cost price. If $\mu(t)$ and $\sigma(t)$ are continuous as to time t, the market price $X(t)$ will be continuous. If $\mu(t)$ or $\sigma(t)$ is dispersed as to time t, the market price $X(t)$ may be dispersed.

We shall regard the asset as a capital or a commodity, and $X(t) \in P(\mu(t), \sigma^2(t))$ as the asset or the market price of the asset, and we will mention no more in the following discussions.

3 The Basic Results about Partial Distribution

According to references [8], we have two basic results about Partial Distribution as follow:

Theorem 1 For any $x\in[0,\infty)$, the following formulas are correct approximately:

1) $\int_0^x e^{-\frac{t^2}{2}}dt=\sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$;

2) $\int_0^x e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx=\sqrt{\frac{\pi}{2}}\sigma\left((\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+\operatorname{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}}\right)$

where, $\operatorname{sgn}(x)=\begin{cases}1 & x>0\\ 0 & x=0\\ -1 & x<0\end{cases}$.

From theorem 1, we have

Theorem 2 Let X follow the PD, $X\in P(\mu,\sigma^2)$, thus

1) The expected value of X, $E(X)$, is as follows

$$E(X)=\int_0^\infty xe^{-\frac{(x-\mu)^2}{2\sigma^2}}dx\Big/\int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx,$$

i.e., $E(X)=\mu+\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1}$ (5)

2) The variance of X, $D(X)$, is as follows

$$D(X)=\int_0^\infty [x-E(X)]^2 f(x)dx,$$

i.e., $D(X)=\sigma^2+E(X)[\mu-E(X)]$ (6)

According to expression (1), theorem 1and theorem 2, Partial Distribution has also the following properties:

1) $f(0)=\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1}$, this means the probability is non-zero at x=0.

2) The expectation $E(X)=\mu+\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+1}$, this means $E(X)>\mu$.

3) The variance $D(X)=\sigma^2+E(X)(\mu-E(X))$, this means $D(X)<\sigma^2$.

4 The Pricing Models Based on DP

4.1 The pricing model for single asset

If the market price of an asset follows DP, i.e., $X\in P(\mu,\sigma^2)$, according to Theorem 2, we have

1) The average market price of the asset can be evaluated as

$$E(X)=\mu+\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)^2}}} \tag{7}$$

where, $R(X)=\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)^2}}}$ is the average sale profit of the asset.

Because of $E(X)>\mu$, this means the average market price should be higher than the cost price of asset.

2) The risk of market price of the asset can be evaluated as

$$D(X)=\sigma^2+E(X)[\mu-E(X)] \tag{8}$$

Because of $D(X)<\sigma^2$, this means the trading risk is less than the cost risk of asset.

According to theorem 2, we obtain two economic propositions as follow:

Proposition 1 The more the risk is, the larger the possible profit is.

From the expression (7), we have the average sale profit as following

$$R(X)=\sqrt{\frac{2}{\pi}}\frac{\sigma e^{-\frac{\mu^2}{2\sigma^2}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu}{\sigma}\right)^2}}}$$

We could see that, if the cost price μ is of fixedness, the more the cost price risk σ is, the larger the average sale profit $R(X)$ is.

Proposition 2 The new asset will bring the higher sale margin.

Suppose that X_i is the market price of an asset in ith trading and $X_i \in P[E(X_{i-1}), D(X_{i-1})]$, $E(X_0)=\mu$, $D(X_0)=\sigma^2$, $i=1, \cdots$. We have, from (7) and (8), $E(X_i)>E(X_{i-1})> \cdots >E(X_0)$, $R(X_i)<R(X_{i-1})< \cdots <R(X_0)$ and $D(X_i)<D(X_{i-1})< \cdots <D(X_0)$

These make clear that: if the economic environment and asset quality do not change, the average trading price of an asset would be higher and higher, the average sale profit will be lower and lower and the price risk of the asset trading will go down and down, but, the ranges which the average trading price, the average sale profit and the price risk change will be smaller and smaller. So, the new asset must be developed continuously in order to acquire the higher sale profits.

4.2 The pricing model for group assets

Similarly to theorem 1 and theorem 2, we obtain separately the theorem 3 and theorem 4 as follow:

Theorem 3 If both X_1 and X_2 are stochastic variables and follow 2-dimensions Partial Distribution, i.e., $(X_1, X_2) \in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, thus

1) $$\int_0^\infty\int_0^\infty e^{-\frac{1}{2(1-r^2)}\left[\left(\frac{u-\mu_1}{\sigma_1}\right)^2-2r\left(\left(\frac{u-\mu_1}{\sigma_1}\right)+\left(\frac{v-\mu_2}{\sigma_2}\right)\right)+\left(\frac{v-\mu_2}{\sigma_2}\right)^2\right]}dudv$$

$$=\frac{\pi}{2}\sigma_1\sigma_2(1-r^2)e^{\frac{r^2}{1-r^2}}\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}\right)\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}\right)$$

2) $$\int_0^{x_1}\int_0^{x_2} f(u,v)dudv=\frac{A_1(x_1)A_2(x_2)}{\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}\right)\left(1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}\right)}$$, $0 \le x_1, x_2 < \infty$

where, $A_i(t)=\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_i+r\sigma_i}{\sigma_i\sqrt{1-r^2}}\right)^2}}+\text{sgn}(t-\mu_i)\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{t-(\mu_i+r\sigma_i)}{\sigma_i\sqrt{(1-r^2)}}\right)^2}}$, the meaning of sgn($t$) is similar to theorem 1, i=1, 2.

If denoting:

$$f_{1r}(x)=\int_0^\infty f(x,y)dy=\begin{cases} e^{-\frac{[x-(\mu_1+r\sigma_1)]^2}{2\sigma_1^2(1-r^2)}} \Big/ \int_0^\infty e^{-\frac{[u-(\mu_1+r\sigma_1)]^2}{2\sigma_1^2(1-r^2)}}du & x\ge 0 \\ 0 & x<0 \end{cases}$$

and $$f_{2r}(y)=\int_0^\infty f(x,y)dx=\begin{cases} e^{-\frac{[y-(\mu_2+r\sigma_2)]^2}{2\sigma_2^2(1-r^2)}} \Big/ \int_0^\infty e^{-\frac{[u-(\mu_2+r\sigma_2)]^2}{2\sigma_2^2(1-r^2)}}du & y\ge 0 \\ 0 & y<0 \end{cases}$$

Theorem 4 If both X_1 and X_2 are stochastic variables and follow 2-dimensions Partial Distribution, i.e., $(X_1, X_2)\in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, thus

1) The expected values of each stochastic variable are

$$E_r(X_1)=\int_0^\infty xf_{1r}(x)dx=\int_0^\infty\int_0^\infty xf(x,y)dxdy$$

$$=\mu_1+r\sigma_1+\sqrt{\frac{2}{\pi}}\frac{\sigma_1\sqrt{1-r^2}e^{-\frac{1}{2}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}} \tag{9}$$

$$E_r(X_2)=\int_0^\infty yf_{2r}(y)dy=\int_0^\infty\int_0^\infty yf(x,y)dxdy$$

$$=\mu_2+r\sigma_2+\sqrt{\frac{2}{\pi}}\frac{\sigma_2\sqrt{1-r^2}e^{-\frac{1}{2}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}} \tag{10}$$

2) The variances of each stochastic variable are

$$D_r(X_1)=\int_0^\infty [x-E_r(X_1)]^2 f_{1r}(x)dx$$

$$=\sigma_1^2(1-r^2)+E_r(X_1)\left[\mu_1+r\sigma_1-E_r(X_1)\right] \tag{11}$$

$$D_r(X_2)=\int_0^\infty [y-E_r(X_2)]^2 f_{2r}(y)dy$$

$$=\sigma_2^2(1-r^2)+E_r(X_2)[\mu_2+r\sigma_2-E_r(X_2)] \tag{12}$$

We can validate that $D_r(X_1)=D(X_1)$ and $D_r(X_2)=D(X_2)$ if r=0.

If the X_1 and X_2 in theorem 3 and theorem 4 are two assets, then the expression (9) and (10) are separately the average market prices of X_1 and X_2 in group or correlation meaning, and the expression (11) and (12) are separately the risks of market price of X_1 and X_2 in group or correlation meaning. There are some important differences between the $E(X)$ in expression (7) and $E_r(X_1)$ in expression (9) or $E_r(X_2)$ in expression (10), and the $D(X)$ in expression (8) and $D_r(X_1)$ in expression (12) or $D_r(X_2)$ in expression (13).

In summing up, there are two differences:

Difference 1 The μ is replaced by the $\mu_1+r\sigma_1$ or $\mu_2+r\sigma_2$.

Difference 2 The σ is replaced by $\sigma_1\sqrt{1-r^2}$ or $\sigma_2\sqrt{1-r^2}$.

The correlation coefficient $r>0$ means that cost prices of two assets are positively correlated. If we regard $r>0$ as that two assets need the same cost resource, then the two assets will compete the same cost resource, so that the cost prices of two assets become higher according to difference 1. In contrary, $r<0$ means that cost prices of two assets are negatively correlated. if we regard $r<0$ as that two assets need the reverse cost resource, then the two assets will use the different cost resource, so that the cost prices of two assets become lower according to difference 1.

On the other hand, whether $r>0$ or $r<0$, the risk of cost prices of two assets will become lower according to difference 2. This means that both the competition and cooperation will reduce the price risk of assets.

So we have the following economic propositions:

Proposition 3 Resource competition results in the cost price getting higher and resource complementarity results in the cost price getting lower.

The cost prices of assets (especially to product and commodity) are closely related to the group of cost resources. The degree that the group of cost resources influences the cost prices of assets can mainly valuated by $r\sigma_1$ in (9) or $r\sigma_2$ in (10).

Proposition 4 The average profit of monopolized asset is higher than that of the correlation assets.

From (9) and (10), we have the expressions of average profit for two correlation assets:

$$R_r(X_1)=\sqrt{\frac{2}{\pi}}\frac{\sigma_1\sqrt{1-r^2}\,e^{-\frac{1}{2}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_1+r\sigma_1}{\sigma_1\sqrt{1-r^2}}\right)^2}}} \quad \text{and } R_r(X_2)=\sqrt{\frac{2}{\pi}}\frac{\sigma_2\sqrt{1-r^2}\,e^{-\frac{1}{2}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}{1+\sqrt{1-e^{-\frac{2}{\pi}\left(\frac{\mu_2+r\sigma_2}{\sigma_2\sqrt{1-r^2}}\right)^2}}}$$

If the value of $|r|$ is close to 0, the average profits $R_r(X_1)$ and $R_r(X_2)$ are higher. And the more the value of $|r|$ is close to 1, the lower the average profits $R_r(X_1)$ and $R_r(X_2)$ are. All of these indicate that the average profit of monopolized asset is higher than that of the correlation assets. If a company hold many of the correlation assets, although the average profit of any single asset is not higher, it can makes the sum of profits higher by reducing the competition resources and increasing the complementarity resources to reduce cost. This is the catchpenny basis in analytic way.

On the other hand, although the average profit of monopolized asset is higher, the market risk contained in monopolized asset is higher. Once the monopolized policies are abolished, the price system of monopolized asset may collapse.

We also see, from the two expressions above, that average profit of the positively correlated assets is generally lower than that of the negatively correlated assets.

Proposition 5 Comparing with single asset, the price risk of group assets will be lower correspondingly.

We know, from expression (11) and (12), if the market is large enough, the price risk of group assets will reduce because of the competition or cooperation. The higher the correlation degree of assets being combined is, whether competition or cooperation, the lower the price risk of assets is; whereas, The lower the correlation degree of assets being combined is, the higher the price risk of assets is. So the incorporate economy can reduce the market risk. Further more, the risk of market price of the positively correlated assets is generally larger than that of negatively correlated assets.

5 The Risk Analysis Model for Group Assets Based on MDP

5.1 The risk analysis model for group assets

If we have two assets X_1 and X_2, and $(X_1, X_2)\in P(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,r)$, the expression (11) and (12) are separately the computing formulas to evaluate the price risk of each asset in the meaning of correlation.

The computing formulas to evaluate the price risk of each asset on its independence are separately

$$\overline{D}(X_1)=\int_0^\infty [x-E(X_1)]^2 f_{1r}(x)dx = D_r(X_1)+\left[E_r(X_1)-E(X_1)\right]^2 \qquad (13)$$

$$\overline{D}(X_2) = \int_0^{\infty}[y - E(X_2)]^2 f_{2r}(y)dy = D_r(X_2) + \left[E_r(X_2) - E(X_2)\right]^2 \tag{14}$$

where, $E(X_i)$ comes from expression (7), $E_r(X_i)$ comes from expression (9) or (10), and $D_r(X_i)$ comes from expression (11) or (12), i=1,2.

The expression (13) and (14) mean the price risk of asset X_1 and X_2 relating to the average market price before they are combined. $\overline{D}(X_1)$ and $\overline{D}(X_2)$ are called separately the independent risk of X_1 and X_2.

We can validate that $\overline{D}(X_1) = D(X_1)$ and $\overline{D}(X_2) = D(X_2)$ if r=0.

If X is an underlying asset, Y is a derivative asset of X. Because there are always the differences in the price behaviors of underlying asset and its derivative asset, there should be an optimal ratio between underlying asset and its derivative in hedging. The optimal ratio is related to the correlation coefficient between underlying asset and its derivative asset. Thus, the holding group of X and Y should be $rX+\alpha Y$. Where, r is the correlation coefficient between the cost price of X and Y, α is the holding ratio of derivative asset Y in hedging. rX has a coequal measurement with Y in the meaning of price behavior. Also, α means the holding direction, α>0 means holding Y in the same direction with X, α<0 means holding Y in the reverse direction with X. Thus, the evaluating model for integrated risk on hedging is as follows:

$$Q^2 = \varphi[\overline{D}(X) + \overline{D}(Y)] + \psi\overline{D}(rX + \alpha Y)$$

or

$$Q = \varphi\sqrt{\overline{D}(X) + \overline{D}(Y)} + \psi\sqrt{\overline{D}(rX + \alpha Y)} \tag{15}$$

where, $\varphi+\psi=1$, φ, $\psi>0$. In general, $\psi=|r|^s$, $0<s<\infty$. Specially, we have s=1 or s=2.

In expression (15), $\overline{D}(X) + \overline{D}(Y)$ or $\sqrt{\overline{D}(X) + \overline{D}(Y)}$ can be applied to evaluate the price risk for X and Y in independence, $\overline{D}(rX + \alpha Y)$ or $\sqrt{\overline{D}(rX + \alpha Y)}$ can be applied to evaluate the price risk for hedging group of X and Y.

For convenience, we call Q in (15) the integrated risk, $\sqrt{\overline{D}(X) + \overline{D}(Y)}$ the independent risk, and $q=\sqrt{\overline{D}(rX + \alpha Y)}$ the hedging risk.

From the computing on the former analytic formulas, we could know:

When $|r|$=1, $Q=q$=0, the both independent risk and hedging risk reach their minimum, i.e. all the integrated risk and the hedging risk reach zero.

When r=0, i.e., X and Y are completely independent one with another, then α=0, so hedging risk is equal to zero. In this case, the assets hedged are not related one with another, so that hedging risk is zero, but the integrated risk reaches maximum. As a result, establishing a hedging group for assets can reduce the integrated risk effectively.

5.2 The hedging risk analysis about one derivative asset to one underlying asset.

Based on (15), and denoting $I=\overline{D}(rX + \alpha Y)$, then

$$I = E_r\{[r(X-E(X))]^2\}+2r\alpha E_r[(X-E(X))(Y-E(Y))]+ \alpha^2 E_r\{[Y-E(Y)]^2\}$$

$$=r^2\,\overline{D}(X)+2r\alpha[E_r(X)-E(X)][E_r(Y)-E(Y)] + \alpha^2\,\overline{D}(Y)$$

Let $\dfrac{dI}{d\alpha} = \dfrac{d\overline{D}(rX + \alpha Y)}{d\alpha} = 0$, we obtain

$$\alpha = -r\frac{[E_r(X) - E(X)][E_r(Y) - E(Y)]}{\overline{D}(Y)} \tag{16}$$

At this time, the hedging risk reaches minimum. It is

$$q=\sqrt{I_{\min}} = \sqrt{\frac{r^2}{\overline{D}(Y)}\left\{\overline{D}(X)\cdot\overline{D}(Y) - [E_r(X) - E(X)]^2[E_r(Y) - E(Y)]^2\right\}} \tag{17}$$

Because the price behavior of underlying asset does not accord completely with that of its derivative asset some time, the optimal ratio between underlying asset and its derivative in hedging should be of existence and the optimal ratio is given by the expression (16).

We know, from (16), when $r>0$, i.e. the underlying asset X is positively correlated with the derivative asset Y, the minimum risk group is holding in the reverse direction, means buying underlying asset and selling the derivative asset at the same time, or vice versa; when $r<0$, i.e. the underlying asset X is negatively correlated with the derivative asset Y, the minimum risk group is holding in the same direction, means buying underlying asset and buying the derivative asset at the same time, or vice versa.

If $r=0$, we see $\alpha=r=0$ from (16), i.e. $rX+\alpha Y=0$ and $D=0$. This means we need not make a hedging on these two assets.

if $|r|=1$, X is correlating with Y in linearity and on probability 1. Let $Y=cX+b$, the hedging risk is

$$\overline{D}(rX+\alpha Y)=\overline{D}[(r+\alpha c)X+\alpha b]$$

If $r+\alpha c=0$, $D=0$, hedging risk reaches minimum. At this time, $\alpha=-\dfrac{r}{c}$, i.e., $\alpha=-\dfrac{1}{c}$ when $r=1$; $\alpha=\dfrac{1}{c}$ when $r=-1$.

We see that the minimum risk group changes always according to r, so it is needed to make an adjustment on α in time along with the real price of asset.

5.3 The hedging risk analysis about one derivative asset to more underlying assets

Let X be an underlying asset, $Y_1, \cdots, Y_n$ be the derivative assets on X, and r_i be the correlation coefficient between X and Y_i, $i=1, \cdots, n$.

Model 1 The hedging risk about one derivative asset to multiple underlying assets which are not correlated.

If denoting: $r=\sum_{i=1}^{n} r_i$, $r\neq 0$. And we know

$$E_r\{[Y_i-E(Y_i)][Y_j-E(Y_j)]\}=\begin{cases}\overline{D}(Y_i) & i=j\\ [E_r(Y_i)-E(Y_i)][E_r(Y_j)-E(Y_j)] & i\neq j\end{cases}$$

Then, the holding group between X and $Y_1, \cdots, Y_n$ is $rX+\sum_{i=1}^{n}\alpha_i Y_i$, α_i is the holding ratio of derivative asset Y_i in hedging. If Y_i($i=1, \cdots, n$) are not correlated, have

$$I=\overline{D}\left(rX+\sum_{i=1}^{n}\alpha_i Y_i\right)$$

$$=r^2\overline{D}(X)+2r[E_r(X)-E(X)]\sum_{i=1}^{n}\alpha_i[E_r(Y_i)-E(Y_i)]+\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j E_r\{[Y_i-E(Y_i)][Y_j-E(Y_j)]\}$$

Let $\dfrac{\partial I}{\partial \alpha_k}=0$ ($k=1, \cdots, n$), we obtain

$$2\sum_{j=1}^{n}\alpha_j E_r\{[Y_k-E(Y_k)][Y_j-E(Y_j)]\}+2r[E_r(X)-E(X)][E_r(Y_k)-E(Y_k)]=0, \text{ i.e.}$$

$$B\alpha+rc=0 \tag{18}$$

where, $\alpha=(\alpha_1, \cdots,\alpha_n)^{\mathrm{T}}$, $c=(c_1, \cdots,c_n)^{\mathrm{T}}$, $c_i=[E_r(X)-E(X)][E_r(Y_i)-E(Y_i)]$, $B=(b_{ij})_{n\times n}$,

$b_{ij}=E_r\{[Y_i-E(Y_i)][Y_j-E(Y_j)]\}$, $i, j=1, \cdots, n$.

The solution of equations group (18) is $\alpha_1, \cdots, \alpha_n$. And α_i is the optimal holding ratio of Y_i, $i=1, \cdots, n$.

Model 2 The hedging risk about one derivative asset to multiple underlying assets which are correlated.

If $Y=(Y_1, \cdots, Y_n)^{\mathrm{T}}$ is the vector of derivative assets, $Q=(q_{ij})_{n\times n}$ is the matrix of correlation coefficients on

$Y \times Y$, where q_{ii}=1, q_{ij}=q_{ji} , i, j=1, ···, n. α=(α_1, ···, α_n)$^{\mathrm{T}}$ is the vector of holding ratios to Y=(Y_1, ···, Y_n)$^{\mathrm{T}}$. Thus, the hedging risk of assets group is

$$I=\overline{D}(rX + \alpha^{\mathrm{T}} QY)$$

i.e., $I = r^2 \overline{D}(X) + 2r[E_r(X) - E(X)] \sum_{i,j=1}^{n} \alpha_i q_{ij} [E_r(Y_j) - E(Y_j)] + \sum_{i,j=1}^{n} b_{ij} E_r\{[Y_i - E(Y_i)][Y_j - E(Y_j)]\}$,

where, $b_{ij} = \sum_{s=1}^{n} \sum_{t=1}^{n} \alpha_s \alpha_t q_{is} q_{tj}$. Let $\frac{\partial I}{\partial \alpha_k} = 0$ (k=1, ···, n), we obtain the equations group

$$B\alpha + rc = 0 \tag{19}$$

where, c=(c_1, ···,c_n)$^{\mathrm{T}}$, c_i=$[E_r(X) - E(X)] \sum_{j=1}^{n} q_{ij} [E_r(Y_j) - E(Y_j)]$,

$B = (b_{ij})_{n \times n}$, b_{ij}=$\sum_{s=1}^{n} \sum_{t=1}^{n} q_{is} q_{tj} E_r\{[Y_s - E(Y_s)][Y_t - E(Y_t)]\}$, i, j=1, ···, n.

The solution of equations group (19) is α_1, ···, α_n. And α_i is the optimal holding ratio of Y_i, i=1, ···, n.

5.4 The hedging risk analysis about group assets

If we have m underlying assets like X_1, X_2, ···, X_m and n derivative assets like Y_1, ···, Y_n. Denoting: $\boldsymbol{X}$ =(X_1, X_2,···,X_m)$^{\mathrm{T}}$, $\boldsymbol{Y}$ =(Y_1, Y_2,···,Y_n)$^{\mathrm{T}}$, and

$\boldsymbol{S}$=$(s_{ij})_{m\times m}$ is the matrix of correlation coefficients on $X \times X$, $\boldsymbol{L}$=$(l_{ij})_{n\times n}$, is the matrix of correlation coefficients on $Y \times Y$, $\boldsymbol{R}$=$(r_{ij})_{m\times n}$ is the matrix of correlation coefficients on $X \times Y$, where, s_{ii}=1, s_{ij}= s_{ji}, i, j=1, ···, m; l_{jj}=1, l_{ij}= l_{ji}, i, j=1, ···, n; 0≤|r_{ij}|≤1, i=1, ···, m, j=1, ···, n.

Suppose $\boldsymbol{X} \in P(\mu_X, \sigma_X^T \sigma_X, S)$, $\boldsymbol{Y} \in P(\mu_Y, \sigma_Y^T \sigma_Y, L)$, ($\boldsymbol{X}$, $\boldsymbol{Y}$)$\in P(\mu_X, \mu_Y, \sigma_X^T \sigma_X, \sigma_Y^T \sigma_Y, R)$, where, $\boldsymbol{\mu_X}$=(μ_{1X}, ··· , μ_{mX})$^{\mathrm{T}}$, $\boldsymbol{\mu_Y}$=(μ_{1Y}, ··· , μ_{nY})$^{\mathrm{T}}$, $\boldsymbol{\sigma_X}$=(σ_{1X}, ··· , σ_{mX})$^{\mathrm{T}}$, $\boldsymbol{\sigma_Y}$=(σ_{1Y}, ··· , σ_{nY})$^{\mathrm{T}}$, $\boldsymbol{R}$=$(r_{ij})_{n\times n}$. r_{ii}=1, i =1, ··· , n.

Denoting: $\boldsymbol{r}$=(r_1, ··· , r_m)$^{\mathrm{T}}$, where, $r_k = \frac{1}{n} \sum_{j=1}^{n} r_{kj}$. $\boldsymbol{\alpha}$ =(α_1,···, α_n)$^{\mathrm{T}}$ is the vector of holding ratios to Y=(Y_1, ···, Y_n)$^{\mathrm{T}}$. Thus, the hedging risk of assets group X =(X_1, ···, X_m)$^{\mathrm{T}}$ and Y=(Y_1,···,Y_n)$^{\mathrm{T}}$ is

$$I= D(r^{\mathrm{T}} X + \alpha^{\mathrm{T}} Y) \tag{20}$$

Let $\frac{\partial I}{\partial \alpha_k} = 0$ (k=1,···, n), and seeking the solution, we obtain

$$\boldsymbol{\alpha}=-\boldsymbol{C}^{-1}\boldsymbol{b} \tag{21}$$

where, $C = (c_{ij})_{n\times n}$, c_{ij}=$E_R\{[Y_i-E(Y_i)]\cdot[Y_j-E(Y_j)]\}$, $\boldsymbol{b}$=(b_1,···, b_n)$^{\mathrm{T}}$, b_i=$\sum_{t=1}^{m} r_t E_R\{[Y_i - E(Y_i)][X_t - E(X_t)]\}$, the vector $\boldsymbol{\alpha}$ in (21) is the optimal holding ratios of $\boldsymbol{Y}$.

Replacing $\boldsymbol{\alpha}$ in (20) by one in (21), we get the minimum hedging risk as follows

$$q=\sqrt{I_{\min}} = \sqrt{D(r^{\mathrm{T}} X - C^{-1} bY)} \tag{22}$$

And the integrated risk is:

$$Q = \varphi \sqrt{\sum_{i=1}^{m} \overline{D}(X_i) + \sum_{i=1}^{n} \overline{D}(Y_i) + \psi q} \tag{23}$$

where, $\varphi+\psi=1$, φ, $\psi>0$. In general, $\psi=\left(\dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n}|r_{ij}|}{m\times n}\right)^s$, $0<s<\infty$. Specially, $s=1$ or $s=2$.

6 The Empirical Researches

Here, we take the Stocks Index as an asset, and the real data of Stocks Index is gained from the Web : http://www.stockstar.com.cn. the parameter μ and σ in PD or MDP are all estimated by the modified maximum likelihood estimation[10], [11]. We adopt the significance level $\beta=0.001$, i.e. the fiducial level $1-\beta=0.999$ in the following statistic test.

6.1 The example of risk analysis about one derivative asset to one underlying asset

6.1.1 The risk analysis about the Integrated Index of Shanghai Stock Exchange in China (IISS)

Samples: the closed prices of IISS.

Time field of Samples: the trading days in the field of 2004.04.12-2005.06.17.

The estimated results of parameters are as follow:

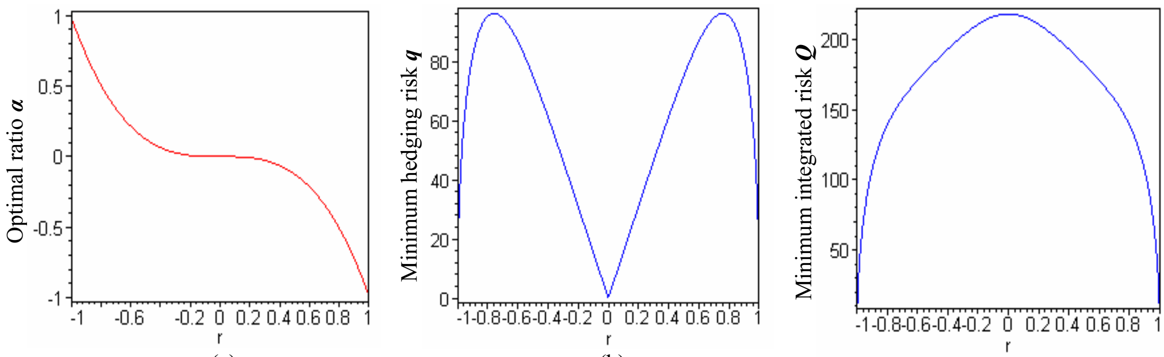
$\mu=1329.525414$, $\sigma=153.5404930$.

Because the number of divided fields on samples is $n=40$, and the fiducial test: $\chi^2=55.58249506<\chi^2(n-2-1)=\chi^2(37)=68.883$, we accept that IISS follows DP, i.e., $X\in P(1329.525414,\ 153.5404930^2)$ in the time field of 2004.04.12-2005.06.17.

From expression (7), $E(X)=1329.525414+R(X)$, where $R(X)=0.3201535603\times10^{-14}$. We see that the trading profit is very little in the average meaning.

Up to now, Shanghai Stock Exchange does not set up the futures of IISS, so we can not get the trading data of the futures of IISS. Here, we suppose that the Futures of IISS is F , and $F\in P(1329.525414e^{\delta(T-t)},\ [153.5404930e^{\delta(T-t)}]^2))$ according to reference [12], where , $\delta=\gamma-y$, γ is the risk-free rate, y is the convenience yields.

If $e^{\delta(T-t)}=e^{0.01}=1.010050167$, $F\in P(1342.887366,\ 155.083606^2)$. And if r is the correlation coefficient of X and F, according to formula (16), (17) and (15), we can compute separately corresponding to IISS: the optimal ratio of holding F corresponding to X , α, the minimum hedging risk q and the minimum integrated risk Q ($s=2$). When $r\in(-1,1)$, moving curves of α, q and Q are separately drawn in (a),(b) and (c) of figure 1.



(a) (b) (c)

Figure 1 The risk structure and moving curves about IISS

From (a) in figure 1, we see that α changes from 1 to -1 as the correlation coefficient r changes from -1 to 1. This indicates that holding the derivative asset should be in the reverse direction with its underlying asset in order to reach the minimum integrated risk, and the holding ratio to the derivative asset changes always as the correlation coefficient changes.

And we see, from (b) and (c) in figure 1, the risk of group assets in financial market includes two parts, one is the independent risk, another one is the hedging risk. Both of the two kinds of risks compose the integrated risk. When $|r|$ is near to zero, the hedging risk is smaller. Also see the (b) in figure 1. The reason is that the hedging effect is not very good owing to the bad relativity between the two hedging assets. At the same time, the independent risks of two assets are higher so that the integrated risk is higher. Also see the (c) in figure 1.

By (15) and other formula in section 5.1, we get $Q=q=0$ when $|r|=1$, this means both the hedging risk and the integrated risk reach their minimum. These results are not all shown in figure 1, the reason is that $|r|=1$ is a strange point in the formula (15) and other formula in section 5.1, but, $\lim_{|r|\to 0} Q = \lim_{|r|\to 0} q = 0$.

In addition, we can see, from (b) and (c) in figure 1, that the hedging risk reaches zero when $r=0$ i.e. $\alpha=0$, but the integrated risk reaches its maximum. So, it is necessary for us to establish the hedging group of assets to reduce the integrated risk.

Although we have no real data of futures here, the accuracy of empirical results is not influenced. It is the same to the following empirical analysis.

6.1.2 The risk analysis about the Components Index of Shenzhen Stock Exchange in China (CISZ)

Samples: the closed prices of CISZ.

Time field of Samples: the trading days in the field of 2004.04.16-2005.06.17.

The estimated results of parameters are as follow:

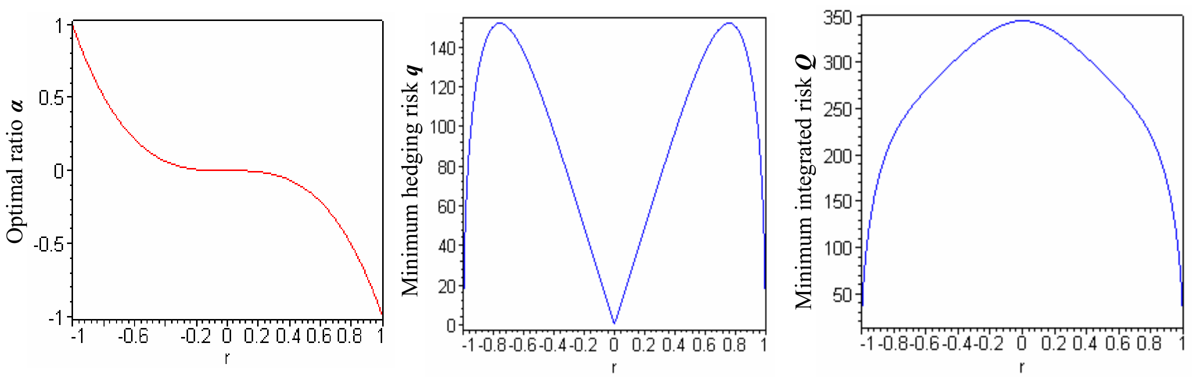
$\mu=3245.116068$, $\sigma=242.5451888$.

Because the number of divided fields on samples is $n=33$, and the fiducial test: $\chi^2=57.69646723<\chi^2(n\text{-}2\text{-}1)=\chi^2(30)=59.703$, we accept that CISZ follows DP, i.e., $X\in P(3245.116068,\ 242.5451888^2)$ in the time field of 2004.04.16-2005.06.17.

From expression (7), $E(X)=3245.116068+R(X)$, where $R(X)=0.5470348045\times 10^{-9427}$. So we see that there is almost no the trading profit in the average meaning.

Up to now, Shenzhen Stock Exchange does not set up the futures of CISZ also, we hve no the trading data of the futures of CISZ. So we suppose that the Futures of IISS is F, and $F\in P(3245.116068e^{\delta(T-t)}, [242.5451888e^{\delta(T-t)}]^2))$ according to reference [12], where , $\delta=\gamma\text{-}y$, γ is the risk-free rate, y is the convenience yields.

If $e^{\delta(T-t)}=e^{0.01}=1.010050167$, $F\in P(3277.730026,\ 244.9828085^2)$. And if r is the correlation coefficient of X and F, according to formula (16), (17) and (15), we can compute separately corresponding to CISZ: the optimal ratio of holding F corresponding to X, α, the minimum hedging risk q and the integrated risk Q ($s=2$). When $r\in(-1,1)$, moving curves of α, q and Q are separately drown in (a),(b) and (c) of figure 2.



(a) (b) (c)

Figure 2 The risk structure and moving curves about CISZ

We have the same discussions as in section 6.1.1.

Taking a sum of above two examples, we obtain the following results:

1) The integrated risk gets smaller and smaller as $|r|$ is near to 1, but the integrated risk never clear away if $|r|\neq 1$. So it is significant for us to know the optimal holding ratio.

2) The higher the degree of correlation between the underlying asset and its derivative asset is, the lower the integrated risk and hedging risk could be. This is tallies with the reality in financial market. So we should choose the derivative asset which correlate highly with the underlying asset if making a hedging business in financial market. If like that, we will control the market furthest.

The two results above are important especially for the large scale of hedging business across the finance markets.

6.2 The example of risk analysis about group derivative assets to group underlying assets

Here, we take IISS and CISZ as the underlying assets, and the futures of IISS and CISZ as the derivative assets. The notations are in accordance with those in section 5.4.

For the sake of convinience and elucidating problems, we suppose that the matrix of the correlation coefficient of the underlying assets is equal to zero, i.e., $\boldsymbol{S}$=0, the matrix of the correlation coefficient of the derivative assets is equal to zero, i.e., $\boldsymbol{L}$=0, and the matrix of the correlation coefficient of $\boldsymbol{X}\times\boldsymbol{Y}$ is

$$R= \begin{array}{cc} \begin{array}{cc} \boldsymbol{Y}_1 & \boldsymbol{Y}_2 \end{array} & \\ \begin{pmatrix} r_1 & r_1 \\ r_2 & r_2 \end{pmatrix} & \begin{array}{c} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{array} \end{array}$$

Thus, $(\boldsymbol{X}, \boldsymbol{Y})\in P(\mu_X, \mu_Y, \sigma_X^T\sigma_X, \sigma_Y^T\sigma_Y, R)$. According to expression (20), we have

$I= D(r^{\mathrm{T}}X + \alpha^{\mathrm{T}}Y)$, where, $\boldsymbol{r}=(r_1, r_2)^{\mathrm{T}}$, $\boldsymbol{\alpha}=(\alpha_1, \alpha_2)^{\mathrm{T}}$.

And from (21), have

$$\alpha_1 = -\frac{\begin{vmatrix} b_1 & c_{12} \\ b_2 & c_{22} \end{vmatrix}}{\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}} \quad , \quad \alpha_2 = -\frac{\begin{vmatrix} c_{11} & b_1 \\ c_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}}$$

where, $c_{ij}= E_R\{[Y_i-E(Y_i)]\cdot[Y_j-E(Y_j)]\}$, $b_i= E_R\left\{[Y_i - E(Y_i)]\sum_{t=1}^{m} r_t[X_t - E(X_t)]\right\}$,

$c_{ii} = \overline{D}_R(Y_i) = \nu_i^2(1-r_i^2) + E_R(Y_i)[\lambda_i + r_i\nu_i - E_R(Y_i)] + [E_R(Y_i) - E(Y_i)]^2$,

$c_{ij}=[E_R(Y_i)- E(Y_i)]\cdot[E_R(Y_j)- E(Y_j)](i\neq j)$, i, j=1,2.

The trading data of IISS and CISZ are the same as in section 6.1, then

$\boldsymbol{\mu_X}=(1329.525414,3245.116068)^{\mathrm{T}}$, $\boldsymbol{\sigma_X}=(153.5404930, 242.5451888)^{\mathrm{T}}$,

$\boldsymbol{\mu_Y}= (1329.525414e^{\delta(T-t)}, 3245.116068e^{\delta(T-t)})^{\mathrm{T}}= (1342.887366, 3277.730026)^{\mathrm{T}}$,

$\boldsymbol{\sigma_Y}=(153.5404930\, e^{\delta(T-t)}, 242.5451888\, e^{\delta(T-t)})^{\mathrm{T}}=(155.083606, 244.9828085)^{\mathrm{T}}$,

where, $e^{\delta(T-t)} = e^{0.01}$ =1.010050167.

When $-1<r_1, r_2<1$, the futures Y_1 and Y_2 should be held separately in the ratios α_1 and α_2. The changing characteristics of α_1 and α_2 are drawn separately in (a) and (b) of figure3.

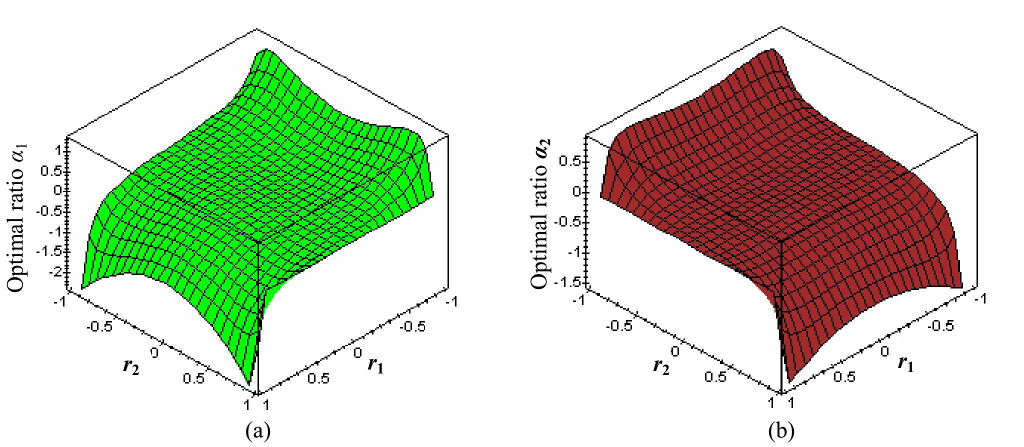
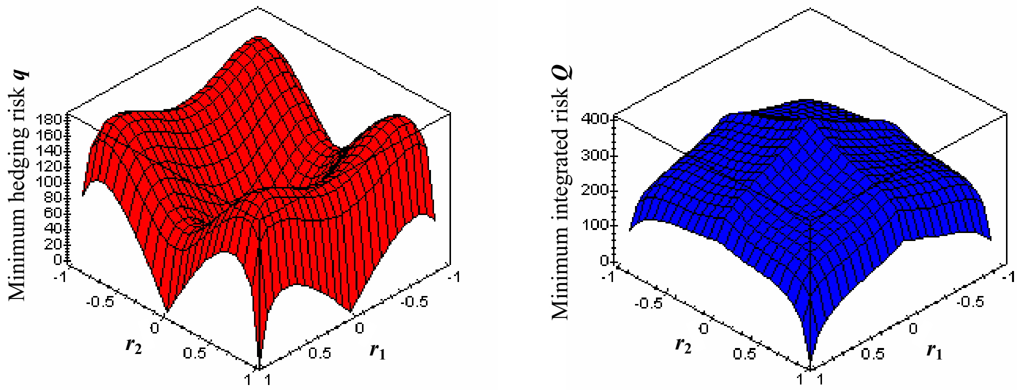


Figure 3 The optimal holding proportions for derivative assets. In order to make the integrated risk minimum, we should hold separately the futures Y_1 and Y_2 in the optimal ratios α_1 and α_2. The α_1 and α_2 change along with the correlation coefficients r_1 and r_2.

Moreover, when $-1<r_1$, $r_2<1$, q and Q, the minimum hedging risk and integrated risk of group assets, are drawn separately in (a) and (b) of figure 4. Where, the s in expression (23) is equal to 1, i.e., s=1.



(a) (b)

Figure 4 Risk characters of group assets. The hedging risk q has a different movement characteristic with that of integrated risk Q. When $r_1=r_2=0$ or $r_1=r_2=1$, the hedging risk q=0, reaches its minimum; and when $|r_1|=1$ or $|r_2|=1$, the hedging risk q is also lower. but, when $r_1=r_2=0$, the integrated risk Q reaches its maximum; when $r_1=r_2=1$, the integrated risk Q=0, reaches its minimum; when $|r_1|=1$ or $|r_2|=1$, the integrated risk Q is also lower. All of these mean that hedging group of assets is sure to reduce their market risk.

In many cases, like $r_1\neq1$ or $r_2\neq1$, the risk of group assets can not be eliminated completely by the way of hedging, but the risk will be higher if we do not make an assets hedging.

The effect of eliminating risk by the way of assets hedging is closely related to the correlation coefficients among the group assets, and the relations are very complex. In general, the higher the related degree among the group assets is, the lower the integrated risk may reduce to, until the risk is completely eliminated. In a word, we should eliminate risk in a various hedging way, not only the simple hedging way of one to one.

5 Conclusions and Remarks

In addition to the pricing method of single asset based on Partial Distribution (Feng Dai,2001), this paper gives a new pricing method for group assets based on the multivariate Partial Distribution (MDP). It is worth that this method considers the correlation coefficient among the assets grouped when make a pricing. This method has not many assumptions as a precondition, so it is different from other current method of asset pricing.

Also, we could evaluate the integrated risk on prices of group assets by the method on MDP. The integrated risk on prices of group assets includes two parts of price risks of assets, the hedging risk and independent risk. The integrated risk has the different movement character from the hedging risk.

By the idea of dividing the integrated risk into hedging risk and independent risk, we could analyze the price risk of assets in a deeper and more detailed way. The optimal ratio for hedging asset based on correlation coefficient should be important for actual financial business.

It is worth to say that five interesting economic propositions can be interprited in analytic way besed on DP or MDP. They are

1) The more the risk is, the larger the possible profit is.
2) The new asset will bring the higher sale margin.
3) Resource competition results in the cost price getting higher and resource complementarity results in the cost price getting lower.
4) The average profit of monopolized asset is higher than that of the correlation assets, but the price risk of monopolized asset is higher than that of the correlation assets.
5) Comparing with single asset, the price risk of group assets will be lower correspondingly.

In the empirical analysis, we have only discussed the cases of one derivative asset to one underlying asset and group derivative assets to group underlying assets. The other cases should be the similar results.

In section 6.2, if the matrix of the correlation coefficient of the underlying assets is not equal to zero, i.e., $\boldsymbol{S}\neq 0$, and the matrix of the correlation coefficient of the derivative assets is not equal to zero, i.e., $\boldsymbol{L}\neq 0$, the discussions will much more complex and authors will give a depiction in another paper.

Of course, the models and methods in this paper need to be demonstrated further. Otherwise, combining the conclusions in this paper with the model of option pricing in references [10]-[11], we could make the discussions on the group options pricing.

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