Communication for Public Goods

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Abstract

This paper studies information transmission between multiple agents with different preferences and a welfare maximizing decision maker who chooses the quality or quantity of a public good (e.g. provision of public health service; carbon emissions policy; pace of lectures in a classroom) that is consumed by all of them. Communication in such circumstances suffers from the agents' incentive to "exaggerate" their preferences relative to the average of the other agents, since the decision maker's reaction to each agent's message is weaker than in one-to-one communication. As the number of agents becomes larger the quality of information transmission diminishes. The use of binary messages (e.g. "yes" or "no") is shown to be a robust mode of communication when the main source of informational distortion is exaggeration.

Keywords: Communication, Public Good Provision, Cheap Talk, Committee, Non-binding Referendum

JEL Classifications: D71, D82, H41

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1 Introduction

In many social environments policy decisions are made after communication with interested parties. For example, a local authority may try to find the optimal public services policy for the community (e.g. how much to be spent on health care or education out of a given budget) by discussing with the residents, or a teacher may ask her students how fast or how difficult they would like her lectures to be. A government may ask environmental experts for advice on emissions policy. Non-binding referenda may be thought of as a communication device between a government and citizens. Suppose that a decision maker chooses the quality or quantity of a public good that is consumed by members of a group with different preferences but no monetary transfers are allowed. Before making her decision, the benevolent decision maker may communicate with the members to figure out the optimal provision of the public good. Are the agents willing to reveal their private information truthfully? How does the number of agents affect the nature of communication? Why is it very often the case that, in communication with many individuals, the decision maker asks them questions in a binary form (e.g. "yes or no", "agree or disagree" with a statement) even when policy choice, preferences and state (distribution of preferences) may not be binary?

This paper addresses these questions by modelling communication as "cheap talk", whereby each agent receives a private signal about his preference and sends a message to an uninformed decision maker who, on the basis of the information received from all agents, makes a decision that affects their utilities. We demonstrate that communication becomes distorted by agents' incentive to "exaggerate" their preferences, as the number of agents becomes larger and each agent has less influence on the decision.[1]

Specifically, in the presence of multiple agents, extreme messages are less informative, while moderate messages are more informative about agents' preferences. For instance, when a resident wants his local authority to increase spending on education (health care), he may stress his needs for educational (health care) support much more than his actual needs. It may thus be that his words should not be taken literally and the intensity of his preference should be adjusted to take his incentive to exaggerate into account. Similarly, when asked about the pace of lectures, a good (weak) student who finds it only *slightly* slow (fast) may nonetheless say the lecture is *very* slow (fast), in an attempt to influence the teacher more than if he answered completely truthfully. Extreme messages tend to be used by people with non-extreme as well as extreme preferences, and this reduces the informativeness of such messages.[2]

Another example where potentially valuable communication appears to suffer from exagger-

[1] By incentive to "exaggerate", we mean an agent's incentive to misreport in such a way that, if words are taken literally and believed by the decision maker, the agent whose type is high (low) "overstates" ("understates") his type by saying his type is even higher (lower). In other words, incentive to "exaggerate" includes both downward and upward biases simultaneously in a one dimensional policy/message space.

[2] In our formal analysis (and in many other cheap talk models) messages used are completely arbitrary and do not have to be taken literally. What matters for the equilibrium outcome is the correspondence between each agent's preference and the decision maker's induced action, so that what word (or language) is used to induce a particular action is irrelevant. However, throughout this paper we interpret our results by assuming that messages directly refer to agents' types.

ation (or the possibility of exaggeration) is policy debates on climate change. Although the vast majority of scientists and politicians seem to have acknowledged certain basic scientific findings, some media attention has been given to extreme claims from "global warming skeptics" such as "Global warming is a hoax."[3] Even among those who agree that carbon management is essential to prevent global warming, we observe a wide spectrum of opinions about how strict emissions regulations should be. Policy discussions and their informativeness often seem marred by exaggeration and allegedly extremist messages that are possibly not completely truthful but rather intended to increase public attention and influence on policy.[4]

Interestingly, however, we show that the incentive to exaggerate itself does not completely eliminate the possibility of mutually beneficial communication. Regardless of the number of agents there always exists an equilibrium where binary messages (e.g. "yes or no") are meaningfully communicated. Moreover, in certain settings as the number of agents becomes larger the most informative communication converges to binary communication. The intuition is very simple: when an agent has the choice between two messages and is unable to send any other messages credibly, he cannot exaggerate his preference in any way. Hence to the extent that the potential source of informational distortion is exaggeration, binary communication eliminates incentive to misreport. This might explain why the "choice between the two" ("yes or no", "agree or disagree", etc.) is a very common way of communicating when multiple interested parties are involved in a decision, even when neither the agents' preferences nor the decision made after communication is binary (e.g. quality of service, pace of lectures, or tightness of regulations). If an agent can only say whether he agrees or disagrees with a statement, he cannot misrepresent the intensity of (dis)agreement.

In policy debates, interested parties may have individual (partisan) bias as well as incentive to exaggerate. For instance, a scientist who receives donations from a company that produces low emission cars may be biased towards a stricter emissions policy, which might make his recommendation less credible even if he was the sole advisor to the government. In the environment policy committee, he would be one of several expert members and may have less influence on its decision. This give rises to incentive to exaggerate and thus another source of informational distortion, in addition to his individual bias. This paper also examines the interplay between these two different sources of informational distortion: individual bias, and incentive to exaggerate due to the presence of multiple agents.

The literature on public good provision has been concerned with mechanism design problems where agents reveal their preferences (partially or fully) by sending a message on or voting for the provision of a public good (Palfrey and Rosenthal, 1984; Bagnoli and Lipman, 1989; Ledyard, 1995). Typically monetary transfers are allowed and the decision maker is assumed to be a mechanism designer who is able to commit to a mechanism (i.e. a mapping from messages to

[3]For instance: "As evidence mounts that humans are causing dangerous changes in Earth's climate, a handful of skeptics are providing some serious blowback." (Washington Post, May 28, 2006, p. W08) and "Global Warming Skeptics Insist Humans Not at Fault" (Washington Post, March 4, 2008, p. A16).

[4]Some scientists and politicians have criticized former US vice president Al Gore's film *An Inconvenient Truth* for being "alarmist". See e.g., "Some Heated Words for Mr. Global Warming" (Washington Post, March 22, 2007, p. A02).

the decision including transfers). Having received the messages the decision maker implements the provision and compels transfers, according to a pre-specified rule. The main source of moral hazard is the free rider problem, where agents have incentive to "understate" true preferences for the public good, because given the amount of the public good everyone prefers to incur lower costs. Without a truthful revelation mechanism the agents are negatively biased in reporting their preferences.

The present paper sheds light on a different set of problems in public good provision. First, we focus on situations where no transfers among members (including the decision maker) are available and therefore each agent's costly contribution is not a concern. As we have suggested earlier, settings with no transfers characterize many important aspects of decision making in regulatory and political relationships and other organizations. In many of these circumstances monetary transfers are often infeasible or deemed inappropriate. Second, we assume that the decision maker cannot commit to a mechanism. In other words, the decision maker makes her decision strategically after hearing or reading the messages, which seems to be relevant to many practical situations, especially where legally binding contracts are unavailable or the decision maker does not have strong reputational concerns.

In this paper we extend the standard cheap talk model of Crawford and Sobel (1982) to a setting with multiple agents. If there is only one informed agent, our model collapses to that of Crawford and Sobel (1982). This enables us to contrast the effect of the presence of multiple agents, which is our primary focus, and the effect of individual bias in one-to-one communication, which is the focus of their analysis. Alonso, Dessein and Matouschek (2008) consider decision making on a single action that affects multiple agents with possibly different preferences that are private information. Like us, they identify the incentive to exaggerate, and derive equilibria with a similar structure to ours.[5] However, they focus on communication with two agents. Carrasco and Fuchs (2008) propose a simple dynamic allocation rule that implements the optimal outcome in a model similar to Alonso, Dessein and Matouschek (2008), again with two agents. Remarkably, Carrasco and Fuchs (2008) show that their allocation rule is implementable by a (utilitarian) decision maker who lacks commitment to a mechanism. In a related context but with a mechanism design approach Martimort and Semenov (2008) study whether the decision maker should allow a coalition of informed parties.[6] Unlike Alonso, Dessein and Matouschek (2008), Carrasco and Fuchs (2008) and Martimort and Semenov (2008), we focus on how the nature of communication changes according to the number of agents.

Morgan and Stocken (2008) study communication between a single decision maker and many agents in the context of polls.[7] They analyze a cheap talk model related to ours but assume that the message space is binary while preferences and policy space are continuous. We do not restrict the message space a priori, and find that finer communication is generally available. However, we also identify some desirable properties of binary communication.

[5] See also Melumad and Shibano (1991), Gordon (2007) and Blume, Board and Kawamura (2007).

[6] Bester and Strausz (2000) study the revelation principle in a related model but do not examine communication in equilibrium.

[7] Feddersen and Pesendorfer (1997) analyze an information aggregation problem with a voting model. They assume that the message (voting) space is binary.

Other papers that study communication with multiple informed parties include Krishna and Morgan (2001), Battaglini (2002, 2004), Ottaviani and Sørensen (2001) and Baliga, Corchon, and Sjöström (1997) where agents observe the same or correlated states of nature while each agent has a different bias or ability. Among models of communication with multiple senders, our model is closer to Austen-Smith (1993) and Wolinsky (2002) where agents observe independent signals (types). Austen-Smith (1993) focuses on the comparison between simultaneous and sequential reporting, and Wolinsky (2002) considers information sharing between senders.

This paper proceeds as follows. The following section describes the model and examines informative equilibria when the distribution of agents' types is known to the decision maker. Section 3 extends the analysis to the case where the type distribution is imperfectly known. Section 4 concludes.

2 Independent Preferences

Let us consider communication between a single decision maker and n agents labelled by $i \in \{1, 2, .., n\}$. Each agent may have a different preference for the decision maker's action, denoted by $y \in \mathbb{R}$, and the utility of agent i is given by

$$U^{Ai}(y, \theta_i) = -(y - \theta_i)^2 \text{ for all } i, \tag{1}$$

where θ_i represents the agent's preference for y and is private information to agent i. In the context of public services example, the local authority chooses the level of service y (e.g. how many staff to hire) provided to the residents who have different preferences. Since (1) is a quadratic loss function, each agent has an ideal policy $y = \theta_i$ that maximizes his utility. The decision maker maximizes the sum of all agents' utilities

$$U^{DM}(y, \boldsymbol{\theta}) = -\sum_{i=1}^{n}(y - \theta_i)^2, \tag{2}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_i, ..., \theta_n] \in [0, 1]^n$. The decision maker can be thought of as a utilitarian social welfare maximizer who nonetheless acts as a "player". She is a "player" in the sense that she determines her action strategically after receiving the messages. In other words, she does not commit to a pre-determined mechanism that automatically prescribes and implements y according to the reported messages. There is no conflict at the individual level. That is, if $n = 1$, both the decision maker and the agent share the same utility function.

The agent's type θ_i is independently and uniformly distributed on $[0, 1]$.[8] Let $m_i \in M$ be the message agent i reports to the decision maker, where the message space M has enough elements to cover all types, and is shared by all agents. This means that, unlike the voting literature where the message space is typically *assumed* to be binary, we do not impose a strong a priori restriction on the messages to be reported.

Each agent reports a costless message after learning his type but before the decision maker takes her action. Prior to choosing y the decision maker updates her belief on θ_i according to the

[8] We relax this independence assumption in Section 3.

message from agent i, since the decision maker cannot observe the agent's type directly. Except for a later subsection (Section 2.4), throughout this paper we assume that all agents adopt the same strategy, which means that any agent with an identical type induces the same (distribution of) action from his viewpoint.[9] The timing is restated as follows:

1. All agents privately learn their types;
2. The agents send messages to the decision maker;
3. The decision maker chooses her action y.

The decision maker's maximization problem can be written

$$\begin{aligned} \max_y \; E\left[-\sum_{i=1}^{n}(y-\theta_i)^2 \middle| m_1, m_2, ..., m_n \right] &= \sum_{i=1}^{n} -E\left[(y-\theta_i)^2 \middle| m_i \right] \\ &= \sum_{i=1}^{n} \left[-(y - E[\theta_i \mid m_i])^2 - var(\theta_i \mid m_i) \right]. \end{aligned}$$

The first equality follows because each agent's type is independently distributed and therefore the message from agent i is not informative about the other agents' types. In this maximization problem we can ignore the variance term $var(\theta_i \mid m_i)$, which is constant from the decision maker's viewpoint. Therefore the first order condition gives the decision maker's best response function

$$y(m_1, m_2, ..., m_n) \equiv \frac{1}{n}\sum_{i=1}^{n} E[\theta_i \mid m_i]. \tag{3}$$

Let us consider agent i's strategy. From his viewpoint, after sending his message the decision maker's action is still a random variable, because it depends on the other agents' messages too. In other words, a message induces a corresponding distribution of the decision maker's action y. However, since the utility functions are quadratic and the choice of message does not change the variance of the induced distribution, it suffices to consider the expected value of the decision maker's action. Since each agent does not observe the other agents' types or messages, (3) implies that the expected action from the agent's viewpoint conditional on his own message is given by

$$\begin{aligned} y_A(m_i) &= \frac{1}{n}E[\theta_i \mid m_i] + \frac{n-1}{n}E\left[E[\theta_{-i} \mid m_{-i}]\right] \\ &= \frac{1}{n}E[\theta_i \mid m_i] + \frac{1-n}{n}\frac{1}{2}, \end{aligned} \tag{4}$$

where θ_{-i} denotes the type of any agent other than i. The second equality follows from the fact that

$$E\left[E[\theta_{-i} \mid m_{-i}]\right] = E[\theta_{-i}] = \frac{1}{2}.$$

We call $y_A(\cdot)$ the *reaction* function, or the decision maker's expected *reaction* to the message from a particular agent. This is to be distinguished from the *best response* function $y(m_1, m_2, ..., m_n)$ in (3), which is a function of the messages from all agents.

Let us illustrate how $n \geq 2$ gives rise to incentive not to reveal truthfully. An agent's message has less influence on the decision maker's action as n becomes larger, since his conditional expected

[9] This assumption is necessary for the calculation of the decision maker's expected utility but not for the construction of equilibria in this section.

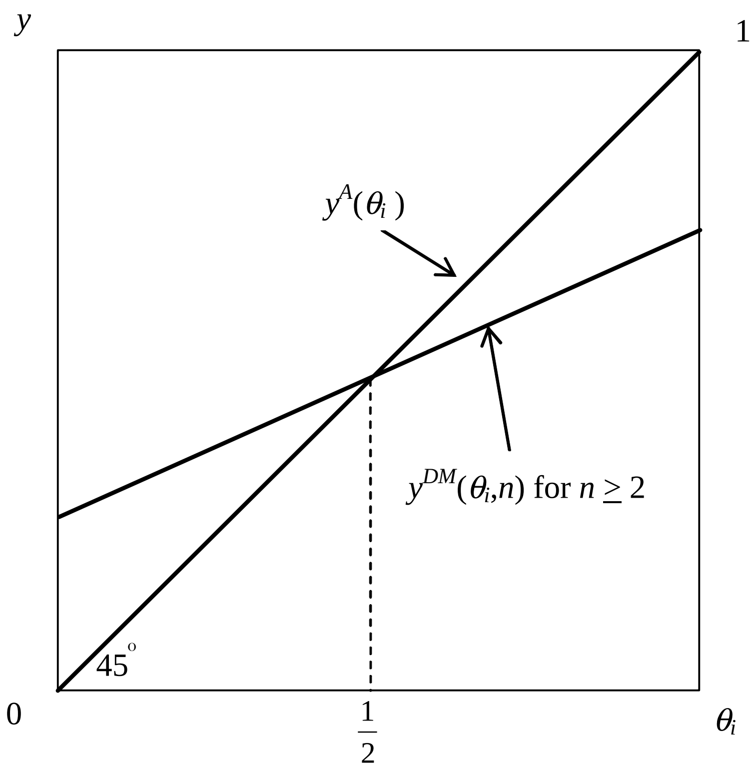

Figure 1: Agent's ideal action and decision maker's reaction

type is weighted at $1/n$. The expected reaction is weighted towards the expected type of any other agent, 1/2.

Define

$$\begin{aligned} y^A(\theta_i) &\equiv \theta_i \\ y^{DM}(\theta_i, n) &\equiv \frac{1}{n}\theta_i + \frac{n-1}{n}\frac{1}{2}, \end{aligned}$$

where $y^A(\theta_i)$ denotes the agent's ideal action given his type θ_i, and $y^{DM}(\theta_i, n)$ is the decision maker's expected reaction given that θ_i is perfectly revealed to her.[10] If the agent's type is $\theta_i = 1/2$ we have $y^A(\theta_i) = y^{DM}(\theta_i, n)$ for any n. Except for $\theta_i = 1/2$, $y^A(\theta_i)$ and $y^{DM}(\theta_i, n)$ do not coincide when there are two or more agents.

The agent's ideal action and the decision maker's reaction for given θ_i are depicted in Figure 1, where the horizontal axis represents the agent's type θ_i and the vertical axis represents the decision maker's action y. When the decision affects only a single agent ($n = 1$), the decision maker's reaction given that the agent reports his type θ_i truthfully is $y^{DM}(\theta_i, 1) = \theta_i$, the 45 degree line, which implies $y^{DM}(\theta_i, 1) = y^A(\theta_i) = \theta_i$. In this case, the agent can induce his ideal action simply by revealing truthfully, because both parties' interests are perfectly aligned for all θ_i. However, when $n \geq 2$ the agent's ideal action may be higher or lower than the decision

[10] Note that $y_A(m_i)$ denotes the decision maker's expected reaction as a function of the agent's message, while $y^A(\theta_i)$ is the agent's ideal action as a function of his type.

maker's reaction depending on his type. If $\theta_i < 1/2$, we have $y^A(\theta_i) < y^{DM}(\theta_i, n)$ so that the agent's ideal action is lower than the decision maker's expected reaction. On the other hand, if $\theta_i > 1/2$ then $y^A(\theta_i) > y^{DM}(\theta_i, n)$, which implies that the agent's ideal action is higher.

Figure 1 summarizes the nature of informational distortion we are considering. If the decision maker naively believes the agents, they have incentive to exaggerate their types. An agent whose type is low (i.e. below 1/2) would report an even lower type than his type, and an agent whose type is high (above 1/2) would report an even higher type. Hence the following proposition holds:

Proposition 1 For any $n \geq 2$ there does not exist a fully revealing equilibrium.

2.1 Equilibrium

Let us examine the informative equilibria that take into account the agents' incentive to exaggerate.[11] We first derive the agent's equilibrium strategy given the decision maker's reaction (4). Let us introduce an alternative representation of the decision maker's reaction. Let $\underline{a}$ and $\overline{a}$ be two points in $[0, 1]$ such that $\underline{a} < \overline{a}$. From (4) and the assumption that θ_i is uniformly distributed

$$E[\theta_i \mid \theta_i \in [\underline{a}, \overline{a})] = \frac{\underline{a} + \overline{a}}{2}.$$

Define

$$\bar{y}_A(\underline{a}, \overline{a}) \equiv \frac{1}{n}\frac{\underline{a} + \overline{a}}{2} + \frac{n-1}{n}\frac{1}{2}. \tag{5}$$

$\bar{y}_A(\underline{a}, \overline{a})$ is the expected reaction from the agent's viewpoint, conditional on the decision maker's belief that an agent's type is such that $\theta \in [\underline{a}, \overline{a})$.[12] If $\theta_i = a$ then we write $\bar{y}_A(a, a)$. While $y_A(m)$ is defined as a function of the agent's message, $\bar{y}_A(\underline{a}, \overline{a})$ is a function of an interval although they both denote the decision maker's reaction. Note that the decision maker's action is a random variable from the agent's viewpoint. However, the randomness is caused only by messages from the other agents. Hence, the variance of the decision maker's action is independent from the agent's strategy (message). The quadratic utility functions imply that we can focus our attention on the decision maker's expected reaction $\bar{y}_A$.

In any equilibrium partition each boundary type $a_j \in (0, 1)$ must satisfy the "arbitrage" condition which says that the agent with $\theta_i = a_j$ is indifferent between inducing $\bar{y}_A(a_{j-1}, a_j)$ and $\bar{y}_A(a_j, a_{j+1})$. Solving the condition

$$-(\bar{y}_A(a_{j-1}, a_j) - a_j)^2 = -(\bar{y}_A(a_j, a_{j+1}) - a_j)^2 \tag{6}$$

by using (5) we obtain a second-order difference equation

$$\frac{1}{n}a_{j+1} - \left(4 - \frac{2}{n}\right)a_j + \frac{1}{n}a_{j-1} = \frac{2}{n} - 2. \tag{7}$$

From (7) we can easily construct the following example of an informative equilibrium:[13]

[11] To be precise, by informative equilibrium we mean an equilibrium where with strictly positive probability the decision maker's action is different from the action she chooses based only on her prior belief. The uninformative equilibrium refers to the equilibrium where the receiver's action is based only on her prior belief with probability 1.

[12] We drop the subscript i for y_A and boundary types a_j to simplify notation.

[13] Solve (7) for $a_0 = 0$ and $a_2 = 1$.

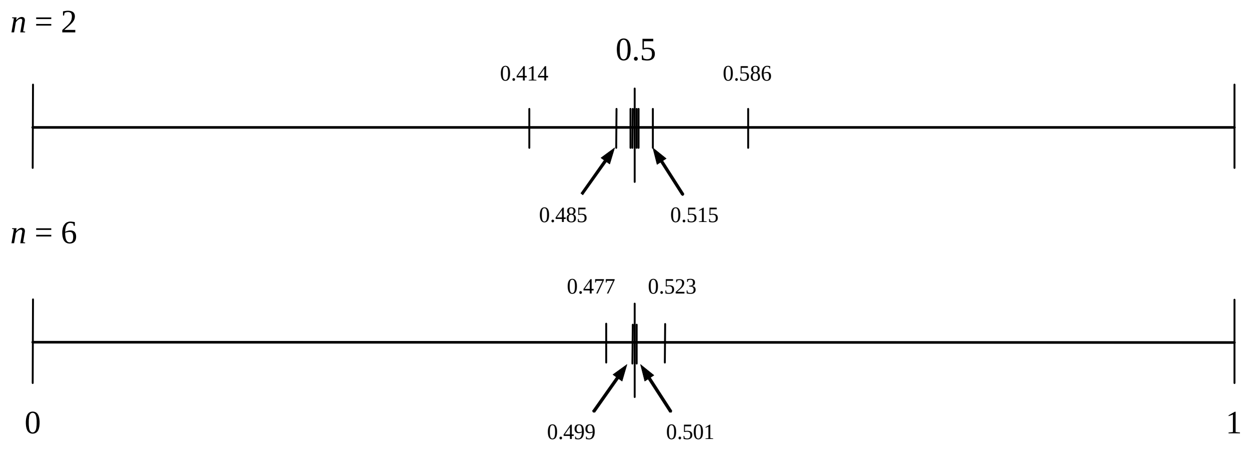

Figure 2: Equilibrium with infinite partition

Example 1 The partitional strategy $\{[0, 1/2), [1/2, 1]\}$ supports a perfect Bayesian equilibrium for any n.

This example points to the "robustness" of binary communication to exaggeration. As we have seen in Figure 1, the more agents there are, the stronger the incentive to exaggerate is. However, regardless of the number of agents, when an agent has the choice between two messages and is unable to send any other messages credibly, he has no room for exaggerating his preference. Since the only source of informational distortion is exaggeration in the present setting, binary messages completely eliminate the incentive to misrepresent.

However, binary communication is not the unique informative equilibrium. In fact, it is easy to see that (6) can generate an infinite number of informative equilibria. The following proposition states that, if we look for the equilibrium where the ex ante (i.e., before the agents learn their types) expected utilities of the decision maker and the agents are highest for given n, we should focus on the equilibrium that has the largest number of intervals.

Proposition 2 For any $n \geq 2$, both the decision maker and the agents are ex ante better off in an equilibrium with more intervals.

Proof. See Appendix I. ■

Let us consider the partition in the equilibrium with the largest number of intervals, which we also call the *most informative equilibrium*. Solving (7) with respect to a_j explicitly, we can let $J \to \infty$, which give us the following sequences

$$a_j = \frac{1}{2} - \frac{1}{2}\left(-1 + 2n - 2\sqrt{n(n-1)}\right)^j \tag{8}$$

and

$$a'_j = \frac{1}{2} + \frac{1}{2}\left(-1 + 2n - 2\sqrt{n(n-1)}\right)^j \tag{9}$$

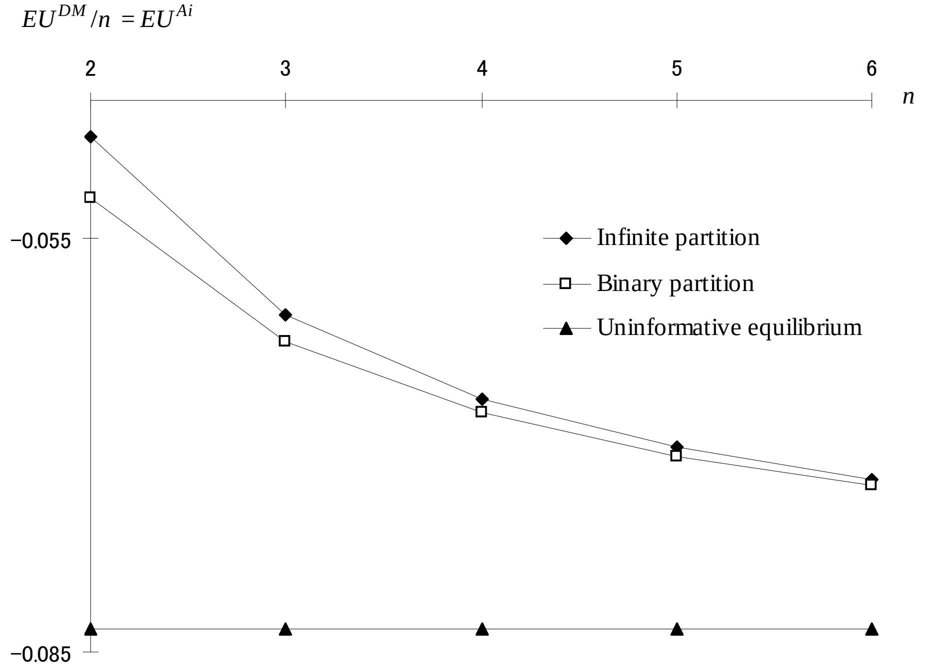

Figure 3: Decision maker's expected utility per agent (= agent's expected utility)

where $a_1 = 0$ and $a'_1 = 1$. Since

$$0 < -1 + 2n - 2\sqrt{n(n-1)} < 1 \text{ for } n \geq 2,$$

(8) and (9) give strictly increasing sequences both of which converge to the average type 1/2. In this equilibrium an infinite number of messages are sent with positive probability but the messages are not fully revealing. The equilibrium partition is illustrated in Figure 2, where the horizontal lines denote the type space of an agent $\theta_i \in [0, 1]$. We can see that in the most informative equilibrium there are an infinite number of intervals in the neighbourhood of 1/2. The length of intervals is longer as they are away from 1/2 and is narrower as they are closer to 1/2, which implies that agent types are more accurately inferred when they are closer to the average.

Figure 3 indicates that when the number of agents is larger, the loss from playing the binary partition equilibrium we have seen in Example 1 as opposed to the most informative one (with the infinite partition) can be very small. In Figure 3 the number of agents is -on the horizontal axis and an agent's expected utility is on the vertical axis. We can see that the difference between the expected utility in the most informative equilibrium and the expected utility in binary communication diminishes as n becomes larger.[14] The diminishing difference implies that messages in the most informative equilibrium become less precise due to stronger incentive to exaggerate.

[14] We can also do a similar calculation for EU^{DM} and confirm that the difference between EU^{DM} with the infinite partition and EU^{DM} with the binary partition diminishes as n becomes larger.

2.2 Application to Multiple Choice Questionnaires

When more than a few people are affected by a decision, multiple choice questions are commonly asked to find the preferences of the interested parties. Such questions can be simple "Yes or No", or a slightly elaborated one such as "Strongly Agree; Agree; Neutral; Disagree; Strongly Disagree" (five choices). The analysis so far indicates that i) an arbitrary number of informative messages can be used, but that ii) the benefit of having additional messages in equilibrium can be severely limited when the number of agents is large. The prevalent use of five or less choices may suggest that the informational benefit of allowing richer messages (e.g. free answer to a question) is indeed smaller as the number of people involved becomes larger.

Another observation from our analysis is that, when the question involves four or more choices the weight to be put on each choice must be adjusted to take into account the incentive to exaggerate. Let us consider the following example of informative equilibria with five intervals, which corresponds to the choice of "Strongly Agree; Agree; Neutral; Disagree; Strongly Disagree":

Example 2 The following partitional strategy[15] supports a perfect Bayesian equilibrium for any n:

$$\{[\underbrace{0}_{a_0},\underbrace{\frac{8n^2-8n+1}{16n^2-12n+1}}_{a_1}),[\underbrace{\frac{8n^2-8n+1}{16n^2-12n+1}}_{a_1},\underbrace{\frac{8n^2-6n}{16n^2-12n+1}}_{a_2}),[\underbrace{\frac{8n^2-6n}{16n^2-12n+1}}_{a_2},\underbrace{\frac{8n^2-6n+1}{16n^2-12n+1}}_{a_3}),$$
$$[\underbrace{\frac{8n^2-6n+1}{16n^2-12n+1}}_{a_3},\underbrace{\frac{8n^2-4n}{16n^2-12n+1}}_{a_4}),[\underbrace{\frac{8n^2-4n}{16n^2-12n+1}}_{a_4},\underbrace{1}_{a_5}]\}.$$

It is easy to see that the first boundary a_1 is increasing in n and the last boundary type a_4 is decreasing in n. In the case of a questionnaire with five choices, as the number of agents increases, a wider range of agent types report extreme messages, namely such as "Strongly Agree" and "Strongly Disagree". Moreover, the first (last) boundary type increases (decreases) in n at a faster rate than the second boundary type a_2 (third boundary type a_3). Therefore, their informativeness may be very limited and the expected types of the agents who sent those messages may be substantially closer to the average than the wording "Strongly" literally suggests. If n is very large, most types fall into the first or the last intervals (i.e. send extreme messages), although message from the types in the middle interval $[a_2, a_3)$ becomes more informative.

2.3 Approximation to Binary Partition

How do the characteristics of the most informative equilibrium change when the number of agents n increases? Since

$$-1+2n-2\sqrt{n(n-1)}$$

in (8) and (9) is decreasing in n for any $n \geq 2$, (8) and (9) imply that as n increases, every boundary type except for $a_0 = 0$ and $a'_0 = 1$ becomes closer to $1/2$. Intuitively, as the number of agents becomes larger the intervals in the most informative equilibrium are more concentrated around $1/2$, because the incentive to exaggerate is stronger and messages from agents whose types

[15] Solve (7) for $a_0 = 0$ and $a_5 = 1$.

are away from 1/2 become less and less informative. In particular, as $n \to \infty$, we have $a_1, a_1' \to 1/2$: as the number of agents goes to infinity the most informative equilibrium converges to the equilibrium with two intervals. Even if the decision maker and the agents play the equilibrium with an infinite number of intervals, the probability that each agent induces either $\bar{y}(0, a_0)$ or $\bar{y}(a_1', 1)$ may be close to 1, in which case communication may look as if each agent faces a binary choice of message.

Note however that the agents' types are assumed to be drawn independently from a known distribution. One important consequence of this assumption is that as the number of agents goes to infinity, by the law of large numbers the average type converges to 1/2 with probability 1. This means that even without any communication $y = 1/2$ is a near-optimal action when n is very large. Assuming independent draw from a known distribution seems reasonable when the number of agents is relatively small and the decision maker has some prior information about the group of the agents as a whole, as in expert committees, classrooms and small local communities. In Section 3 we consider the case in which the distribution of agent types is imperfectly known.

2.4 Individually Biased Agents

So far we have assumed that there is no intrinsic preference divergence between the decision maker and each agent. That is, if $n = 1$, both parties interests' perfectly coincide and perfect communication is possible. However, in policy debates or expert committees, often the participants are known to be biased towards a particular direction of policy. For example, a scientist who receives donations from a company that produces low emission cars may be biased towards a stricter emissions policy, though he may indeed possess valuable information for policy making. Here we examine the interaction between such individual bias and the incentive to exaggerate caused by the presence of multiple agents.

To formalize the idea, the utility of the decision maker is given by $-\sum_{i=1}^{n}(y-\theta_i)^2$ as above, but that of agent i is $-(y-\theta_i-b_i)^2$. We assume that $b_i \geq 0$ (e.g. the scientist's political standpoint) is common knowledge but θ_i is private information to agent i, and independently and uniformly distributed on $[0, 1]$. The agent's ideal action and the decision maker's expected reaction when the agent reveals truthfully are

$$y^A(\theta_i, b_i) \equiv \theta_i + b_i$$

and

$$y^{DM}(\theta_i, n) \equiv \frac{1}{n}\theta_i + \frac{n-1}{n}\frac{1}{2},$$

respectively. We have $y^A(\theta_i, b_i) = y^{DM}(\theta_i, n)$ for

$$\theta_i = \frac{1}{2} - \frac{bn}{n-1} \equiv \hat{\theta}.$$

We call $\hat{\theta}$ the *agreement type*, whose ideal action coincides with the decision maker's expected reaction under truthful revelation. Clearly when the agent is not individually biased, we have $\hat{\theta} = 1/2$ as $b_i = 0$. As we will see shortly, the incentive to exaggerate takes a different from the

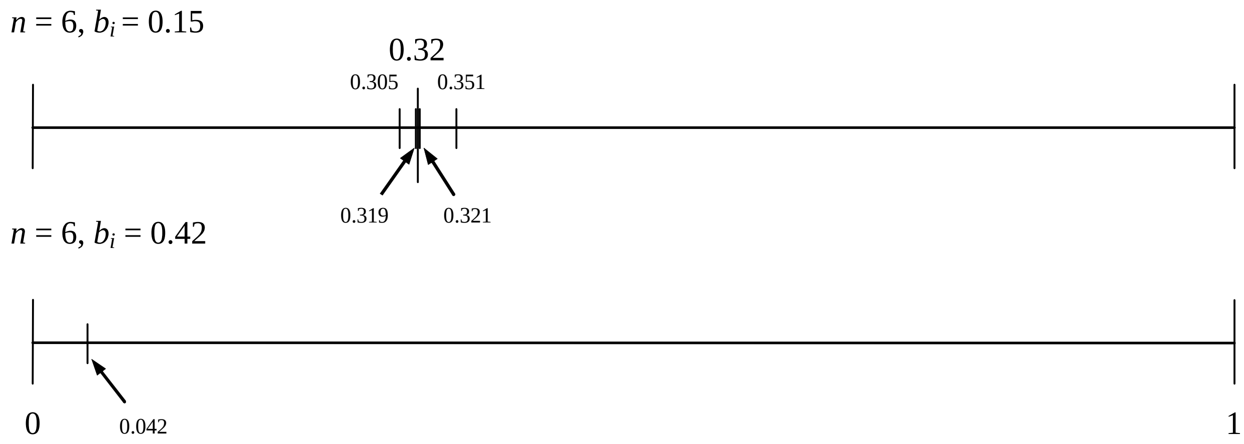

Figure 4: Communication with individually biased agent

one in the case where $b_i = 0$. Now the agent has the incentive to exaggerate his type relative to the agreement type $\hat{\theta}$, not necessarily $1/2$.

Let us derive equilibrium partitions. The arbitrage condition (6) can be rewritten:

$$-(\bar{y}_{Ai}(a_{j-1}, a_j) - a_j - b_i)^2 = -(\bar{y}_{Ai}(a_j, a_{j+1}) - a_j - b_i)^2. \tag{10}$$

From (10) we obtain the following second-order difference equation:

$$\frac{1}{n}a_{j+1} - \left(4 - \frac{2}{n}\right) a_j + \frac{1}{n}a_{j-1} = 4b_i + \frac{2}{n} - 2. \tag{11}$$

(11) determines the equilibrium partition for communication between the decision maker and agent i. Note that the equilibrium partitions obtained from (11) depend on each individual bias b_i and thus differ among agents with different biases. When $b_i = 0$ the structure of informative equilibria are the same as the ones we have seen earlier.

Example 3 For $b_i \leq \frac{1}{2} - \frac{1}{4n}$, the partition $\{[0, \frac{1}{2} - \frac{2b_i n}{2n-1}), [\frac{1}{2} - \frac{2b_i n}{2n-1}, 1]\}$ supports a perfect Bayesian equilibrium.[16]

Unlike the case without individual bias, the informative equilibrium with binary partition may not exist when b_i is very large, which is consistent with the model by Crawford and Sobel (1982) where $n = 1$ and $b_i > 0$. Figure 4 illustrates the equilibrium partitions with the largest number of intervals when $n = 6$. In the first case where $b_i = 0.15$, we have the agreement type $\hat{\theta} = 0.32$ and an infinite number of intervals in its neighbourhood.[17] Note that the length of an interval is longer as it is farther from 0.32, which implies that the incentive to exaggerate *relative to* 0.32 is taken into account in equilibrium. In other words, an agent who is biased in one direction at the individual level may still be biased towards both directions in the presence

[16] Solve (11) for $a_0 = 0$ and $a_2 = 1$.

[17] We can show that the ex ante expected utilities of both the decision maker and agents are higher in an equilibrium with more intervals. The proof is almost identical to that of Proposition 2.

of multiple counterparts. For example, in a policy committee that listens to several experts, a scientist with the known individual bias ($b_i \neq 0$) in a particular direction may still have both upward and downward biases, depending on the information (θ_i) he has.

However, if individual bias is very large, it dominates incentive to exaggerate. See the second partition in Figure 4 for $b_i = 0.42$, where at most two intervals can be supported in equilibrium. There the equilibrium partition accommodates only the upward bias due to a large bias $b_i = 0.42$. Messages from types in the upper interval $\theta_i \in [0.042, 1]$ are less informative than those from types the lower interval $\theta_i \in [0, 0.042)$.

3 Communication with a Large Population

When the number of agents is very large and the decision maker has little information about the nature of the population, she may need to infer the distribution itself. This assumption seems appropriate when analyzing large-scale polls or non-binding referenda. Let us consider the case where the distribution of preferences itself is uncertain. This implies that even if the number of agents is very large, the decision maker needs communication to infer the realized distribution. We obtain two main results related to the previous setting with a known iid distribution: namely i) fully revealing equilibrium does not exist, and ii) equilibrium with binary partition exists for any n.

Assume that the decision maker and the agents have the same quadratic utility functions without individual bias ($b_i = 0$ for all i) as given in (1) and (2). We also assume that θ_i is uniformly distributed on $[\alpha, \alpha + \beta]$, but that α and β are both uniformly distributed on $[1/2 - \beta, 1/2]$ and $(0, 1/2]$, respectively.[18] Note that now θ_i has a bell-shaped prior density on $[0, 1]$. Under this distributional assumption, ex ante neither the decision maker nor the agents know the location (α) or the length (β) of the realized distribution. We can consider α and β as variables that parametrize the "state" of nature. Generically, in order to infer the realized distribution (α and β) perfectly, the decision maker needs both fully revealing communication and an infinite number of agents. In any partitional equilibrium, the decision maker updates her belief on the distribution of the agents according to the received messages, and computes the expected type of the agents in each interval. Here we maintain the assumption that all agents adopt the same strategy, in the sense that any agent with an identical type induces the same distribution of action from his viewpoint.

Each agent updates his belief on the distribution of the other agents according to his own type θ_i. Suppose that $0 \leq \theta_i \leq 1/2$. Given β (, which is not observed) and θ_i, the posterior distribution of α is uniform on $[1/2 - \beta, \theta_i]$. Hence the expected type of the other agents is given by

$$E[\theta_{-i} \mid \theta_i, \beta] = \int_{1/2-\beta}^{\theta_i} \frac{\alpha + (\alpha + \beta)}{2} \frac{1}{\theta_i - (1/2 - \beta)} d\alpha = \frac{1}{4} + \frac{1}{2}\theta_i.$$

[18] A related formulation of type distribution is used by Alesina and Rosenthal (1996, 2000).

Similarly for $1/2 < \theta_i \leq 1$,

$$E[\theta_{-i} \mid \theta_i, \beta] = \int_{\theta_i-\beta}^{1/2-\beta} \frac{\alpha + (\alpha+\beta)}{2} \frac{1}{1/2 + (\theta_i - \beta)} d\alpha = \frac{1}{4} + \frac{1}{2}\theta_i.$$

Since the conditional expectation turns out to be independent of β, we can write

$$E[\theta_{-i} \mid \theta_i, \beta] = E[\theta_{-i} \mid \theta_i] = \frac{1}{4} + \frac{1}{2}\theta_i \text{ for } \theta_i \in [0,1]. \tag{12}$$

Contrary to the setting with a known distribution, from an agent's viewpoint the expected type of the other agents changes according to his own type. However, no agent knows the exact type distribution.

Proposition 3 For any $n \geq 2$ there does not exist a fully revealing equilibrium.

Proof. Suppose that the decision maker believes all agents reveal truthfully, and suppose also that all agents except agent i reveal truthfully. Without loss of generality consider direct revelation, where $m_i = \tilde{\theta}_i \in [0,1]$. Truthful revelation implies $\tilde{\theta}_i = \theta_i$. The expected action by the decision maker from the agent's viewpoint is given by

$$y(\tilde{\theta}_i) = \frac{1}{n}\tilde{\theta}_i + \frac{n-1}{n}\left(\frac{1}{4} + \frac{1}{2}\theta_i\right).$$

Since the agent's ideal action is θ_i, the message $\tilde{\theta}_i^*$ that maximizes his expected utility is given by

$$\tilde{\theta}_i^*(\theta_i) = \begin{cases} 0 & \text{if } \frac{\theta_i(n+1)}{2} - \frac{(n-1)}{4} \leq 0 \Rightarrow \theta \leq \frac{1}{2} - \frac{1}{1+n} \\ \frac{\theta_i(n+1)}{2} - \frac{(n-1)}{4} & \\ 1 & \text{if } \frac{\theta_i(n+1)}{2} - \frac{(n-1)}{4} \geq 1 \Rightarrow \theta \geq \frac{1}{2} + \frac{1}{1+n}. \end{cases}$$

Note that $\tilde{\theta}_i^*(\theta_i) = \theta_i$ only if $\theta_i = 1/2$. Otherwise $\tilde{\theta}_i^*(\theta_i) \neq \theta_i$, which contradicts full revelation. Therefore a fully revealing equilibrium does not exist. ■

The intuition for this result is straightforward and very similar to that in the setting where the realized distribution is known. Given that all the other agents reveal truthfully, an agent does not reveal truthfully because he has incentive to exaggerate his type relative to the conditional mean of the other agents' types $E[\theta_{-i} \mid \theta_i] = \frac{1}{4} + \frac{1}{2}\theta_i$. In particular, when n is large the message $\tilde{\theta}_i^*$ is likely to be the "extreme", either 0 or 1 in direct revelation. Note that, since there is no fully revealing equilibrium, the decision maker cannot infer the distribution perfectly even if $n \to \infty$. This is due to the fact that not only the location (mean) but also the length (variance) of the realized distribution is unknown.

In an informative equilibrium, the decision maker updates her belief on the entire distribution of the agents' preferences according to the received messages, and then computes the expected type of the agents who sent the same message (i.e. those in each particular interval). The updating process and corresponding strategies are intractable in our present setting,[19] but we are able to show that there exists an equilibrium with binary partition for any n.

[19] Note that estimation of a realized distribution is extremely complex when the sample size is finite and/or observed data are partitional (grouped) as in our model. Related problems arise even without strategic incentive to misrepresent information. See, e.g. Reddy and Minoiu (2007) and Minoiu and Reddy (2008).

Proposition 4 For any $n \geq 1$ there exists an equilibrium with two intervals, $\{[0, 1/2), [1/2, 1]\}$.

Proof. See Appendix I. ■

This confirms the intuition behind Proposition 3 and Example 1 we have seen earlier: binary communication is "robust" to exaggeration because it eliminates incentive to exaggerate. The existence of binary partition equilibrium holds for more general distributions, if sending a higher (lower) message induces higher action (higher expected type), given all the other messages.[20] The robustness of binary communication therefore might account for the extensive use of binary communication in polls and (non-binding) referenda, where richer communication potentially leads to a better estimation of population preferences but incentive to exaggerate may severely limit the informativeness.

4 Conclusion

Communication on a decision that affects multiple interested parties is subject to exaggeration, which becomes more severe as the number of agents increases. In principle, it is possible to transmit information with an arbitrary number of meaningful messages, but the value of adding more messages is decreasing in the number of agents. Our model can shed light on the nature of communication for public good provision, and how the nature of communication may change according to the number of agents affected by the decision. We have demonstrated that the concern for exaggeration may lead to binary communication, where reporting parties cannot possibly exaggerate their preferences. We have also examined how the incentive to exaggerate and individual bias interact in communication. This paper contributes to the literature on public good provision by offering an analysis of communication where the decision maker cannot commit to a mechanism and no transfers are available, which seems relevant to a lot of practical situations, including political or regulatory decision making as well as choice of action in classrooms or organizations.

[20] See Appendix II.

5 Appendix I: Proofs

5.1 Proposition 2

Before we prove the proposition, we provide some useful lemmas and outline how we construct the main proof. Let us call a sequence $(a_0, a_1, ..., a_J)$ that satisfies the arbitrage condition (6) a "*solution*" to (6). We make use of the monotonicity condition (M) in Crawford and Sobel (1982, p.1444) which requires that, for given n, if we have two solutions a^+ and a^{++} with $a_0^+ = a_0^{++}$ and $a_1^+ > a_1^{++}$, then $a_j^+ > a_j^{++}$ for all $j = 2, 3, ...$ In other words, (M) says that starting from a_0, all solutions to (6) must move up or down together. Solving (7) with respect to a_j explicitly with $a_0 = 0$, we obtain[21]

$$\begin{aligned} a_j &= \frac{1}{2} + \frac{a_1 - 1 + n - \sqrt{n(n-1)}}{4\sqrt{n(n-1)}} \left(-1 + 2n\left(1 + \sqrt{n(n-1)}\right)\right)^j \\ &\quad - \frac{a_1 - 1 + n + \sqrt{n(n-1)}}{4\sqrt{n(n-1)}} \left(-1 + 2n\left(1 - \sqrt{n(n-1)}\right)\right)^j \end{aligned} \tag{13}$$

Lemma 1 Any solution to (6) satisfies (M).

Proof. From (13) we have

$$\frac{da_j}{da_1} = \frac{1}{4\sqrt{n(n-1)}} \left[\left(-1 + 2n\left(1 + \sqrt{n(n-1)}\right)\right)^j - \left(-1 + 2n\left(1 - \sqrt{n(n-1)}\right)\right)^j\right] > 0,$$

which implies (M). ■

In order to show that the players' expected utility is higher in an equilibrium with more intervals, Crawford and Sobel (1982) deform the partition with J intervals to that with $J + 1$ intervals, by continuously increasing the player's expected utility throughout the deformation. We follow this method, but we need to proceed with a two-step deformation, rather than one, because the deformation takes place towards the opposite directions for the right-hand and left-hand sides of $1/2$ on $[0, 1]$. Intuitively, as the number of interval increases, the each boundary type on the left hand side of $1/2$ move to the left (except for $a_0 = 0$) while each boundary type of the right hand side of $1/2$ move to the right (except for $a_J = 1$). We need to perform a different comparative statics for each case.

Let $a(J)$ be the equilibrium partition of size J. We show that $a(J)$ can be deformed to $a(J + 1)$ by two steps, continuously increasing the players' expected utility in each step. Let the sub-partition of $a(J)$ equal or below $1/2$ be $\underline{a}(J) \equiv (a_0(J), a_2(J), ..., a_K(J))$ where $a_0(J) = 0$. In other words, K satisfies $a_k(J) \leq 1/2 < a_{k+1}(J)$. In the following we proceed in the following two steps:

1. We fix $a_K(J)$ and make the sub-partition $(a_K(J), a_{K+1}(J), ..., a_J(J))$ deform continuously to $(a_K(J), a_{K+1}(J + 1), a_{K+2}(J + 1), ..., a_{J+1}(J + 1))$, increasing the expected utility.

[21] (13) is used to obtain (8) and (9) in the main text.

2. We make the sub-partition $(a_0(J), a_1(J)..., a_K(J))$ deform continuously to $(a_0(J+1), a_2(J+1), ..., a_K(J+1))$, increasing the expected utility.

Lemma 2 If $a(J)$ and $a(J+1)$ are two equilibrium partitions for the same n, then $a_{j-1}(J) < a_j(J+1) < a_j(J)$.

Proof. See Lemma 3 in Crawford and Sobel (1982, p.1446). The proof follows directly from (M). ■

The first step of deformation is carried out as follows. Let $(a^x_K, a^x_{K+1}, ..., a^x_j, ..., a^x_{J+1})$ be the sub-partition that satisfies (6) for all $j = K+1, K+2, ..., J$ with $a^x_K = a_K(J)$, $a^x_J = x$ and $a^x_{J+1} = 1$. If $x = a_{J-1}(J)$ then $a^x_{K+1} = a^x_K = a_K(J)$. If $x = a_J(J+1)$ then we have $(a_K(J), a_{K+1}(J+1), ..., a_J(J+1))$, where (6) is satisfied for all $j = K+2, K+3, ..., J$. We are going show that, if $x \in [a_{J-1}(J), a_J(J+1)]$, which is again a non-degenerate interval by Lemma 2, then the agent's expected utility is strictly increasing in x.

In the second step, let $(a^z_0,\ a^z_1, ..., a^z_j, ..., a^z_K)$ be the sub-partition that satisfies (6) for $j = 1, 2, ..., K-1$, with $a^z_0 = 0$ and $a^z_K = z$. If $z = a_K(J)$ then $a^z_j = a_j(J)$ for all $j = 0, 1, ..., K$. If $z = a_K(J+1)$ then $a^z_j = a_j(J+1)$ for all $j = 0, 1, ..., K$.. We will show that when $z \in [a_K(J+1), a_K(J)]$, which is again a non-degenerate interval by Lemma 2, the agent's expected utility is strictly decreasing in z.

Lemma 3 Suppose that $(a_0, a_1, ..., a_j, ..., a_J)$ is a solution to (6). Then for all $j = 1, 2, ..., J-1$ if $a_j > (<) 1/2$ then $a_j - a_{j-1} < a_{j+1} - a_j$ $(a_j - a_{j-1} > a_{j+1} - a_j)$. If $a_j = 1/2$ then $a_j - a_{j-1} = a_{j+1} - a_j$.

Proof. The sequences that satisfy (6) are described by (7). Rearranging (7) we have

$$(a_{j+1} - a_j) - (a_j - a_{j-1}) = n\left(4a_j + \frac{2}{n} - 2\right) - 4a_j. \tag{14}$$

The left hand side $(a_{j+1} - a_j) - (a_j - a_{j-1}) = 0$ if

$$\begin{aligned} n\left(4a_j + \frac{2}{n} - 2\right) - 4a_j &= 0 \Rightarrow \\ a_j &= \frac{1}{2}. \end{aligned}$$

Since the right hand side of (14) is increasing in a_j, if $a_j > 1/2$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) > 0$, and if $a_j < 1/2$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) < 0$. ■

The above lemma says that an interval $[a_{j+1}, a_j)$ is longer (shorter) than the previous interval $[a_{j-1}, a_j)$ when $a_j > (<) 1/2$. The intuition is captured in Figure 2. The following lemma is similar but cannot be implied by Lemma 3. Since by definition a^x_K and a^z_{K+1} are fixed throughout the respective deformation, (6) is not satisfied at $a_j = a^x_{K+1}$ or $a_j = a^z_K$.

Lemma 4 $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$ and $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$.

Proof. From Lemma 3 we have $a^x_{K+1} - \tilde{a}_K < a^x_{K+2} - a^x_{K+1}$ where $\tilde{a}_K$ is defined such that $\{a_{j-1} = \tilde{a}_K,\ a_j = a^x_{K+1}, a_{j+1} = a^x_{K+2}\}$ satisfies (7). Since $a_K(J+1) < \tilde{a}_K < a_K(J) = a^x_K$ from Lemma 2, we have $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$. This proves the first part of the Lemma.

Similarly we have $a^z_K - a^z_{K-1} \geq \check{a}_{K+1} - a^z_K$ where $\check{a}_{K+1}$ is defined such that $\{a_{j-1} = a^z_{K-1}, a_j = a^z_K, a_{j+1} = \check{a}_{K+1}\}$ satisfies (7). Lemma 2 implies $a^z_{K+1} = a_{K+1}(J+1) < \check{a}_{K+1} < a_{K+1}(J)$. Hence we have $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$. ■

Proof of Proposition 2.

- Agent

The decision maker's action from an agent's viewpoint is a random variable, and since the utility functions are quadratic, we can separate the expected value terms and the variance terms. Let $y_i(m_i)$ be the decision maker's (random) action from the agent's viewpoint. The agent's utility in this separated form conditional of his report is given by

$$\begin{aligned} &E\left[-(y_i(m_i) - \theta_i)^2 \mid m_i\right] \\ &= -var(y_i(m_i)) - (Ey_i(m_i))^2 + 2\theta_i Ey_i(m_i) - {\theta_i}^2 \\ &= -var(y_i) - (Ey_i(m_i) - \theta_i)^2, \end{aligned} \tag{15}$$

where from (4)

$$Ey_i(m_i) \equiv y_A(m_i) = \frac{1}{n}E\left[\theta_i \mid m_i\right] + \frac{n-1}{n} \times \frac{1}{2}.$$

The variance term is independent of the agent's message since the randomness is caused by the other agents' messages unobservable to the agent. Let agent i's expected type given his message be $\hat{a}_i(a_j, a_{j+1})$. If a message is sent from $\theta_i \in [a_j, a_{j+1})$, then

$$\hat{a}_i = \frac{a_j + a_{j+1}}{2}.$$

From (3) the decision maker's action is the mean of all posterior expected types. Hence, from agent i's viewpoint

$$var(y_i) = var\left(\frac{1}{n}\left(\sum_{l\neq i}\hat{a}_l + \hat{a}_i\right)\right) = \frac{1}{n^2}var\left(\sum_{l\neq i}\hat{a}_l + \hat{a}_i\right) = \frac{n-1}{n^2}var(\hat{a}_i),$$

where $var(\hat{a}_i)$ is the variance of the expected type of an agent given his equilibrium strategy. The last equality follows from independent type distributions and symmetric strategies. In what follows we drop the subscript i.

The expected utility for the first part of deformation is given by

$$\begin{aligned} EU^A \equiv &-\sum_{j=1}^{K}\int_{a^x_{j-1}}^{a^x_j}\left(\frac{a_{j-1}+a_j}{2n} + \frac{n-1}{2n} - \theta\right)^2 d\theta - \sum_{j=K+1}^{J+1}\int_{a^x_{j-1}}^{a^x_j}\left(\frac{a^x_{j-1}+a^x_j}{2n} + \frac{n-1}{2n} - \theta\right)^2 d\theta \\ &-\frac{n-1}{n^2}\left[\sum_{j=1}^{K}(a_j - a_{j-1})\left(\frac{a_{j-1}+a_j}{2}\right)^2 + \sum_{j=K+1}^{J+1}(a^x_j - a^x_{j-1})\left(\frac{a^x_{j-1}+a^x_j}{2}\right)^2 - \frac{1}{4}\right]. \end{aligned}$$

It follows that

$$\begin{aligned}\frac{dEU^A}{dx} \equiv & \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \Bigg\{ -\left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 \\ & - \frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta \right] \\ & - \left[\frac{n-1}{2n^2} (a_{j+1}^x)^2 - (a_{j-1}^x)^2 + \frac{(a_{j-1}^x + a_j^x)^2 - (a_j^x + a_{j+1}^x)^2}{2} \right] \Bigg\}. \end{aligned} \tag{16}$$

For the first line we have[22]

$$\begin{aligned} & -\left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 \\ = \; & \frac{n-1}{2n^2} (a_{j+1}^x - a_{j-1}^x)(1 - 2a_j^x) + \frac{(a_{j+1}^x - a_{j-1}^x)(a_{j-1}^x - 2a_j^x + a_{j+1}^x)}{4n^2}. \end{aligned}$$

Also for the second line,

$$\begin{aligned} & -\frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta + \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta \right] \\ = \; & \frac{n-1}{2n^2} \left[(a_{j+1}^x)^2 - (a_{j-1}^x)^2 - (a_{j+1}^x - a_{j-1}^x) \right]. \end{aligned}$$

Hence, all terms in the curly brackets in (16) can be written

$$\begin{aligned} & -\left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 \\ & -\frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - \theta \right) d\theta \right] \\ & -\frac{n-1}{2n^2} \left[(a_{j+1}^x)^2 - (a_{j-1}^x)^2 + \frac{(a_{j-1}^x + a_j^x)^2 - (a_j^x + a_{j+1}^x)^2}{2} \right] \\ = \; & \frac{a_{j+1}^x - a_{j-1}^x}{2n} \left[\frac{a_{j-1}^x - 2a_j^x + a_{j+1}^x}{2} \right] > 0. \end{aligned}$$

The inequality follows because from Lemmas 3 and 4, we have $a_j - a_{j-1} < a_{j+1} - a_j \Rightarrow a_{j-1}^x - 2a_j^x + a_{j+1}^x > 0$ for all $j = K+1, K+2, ..., J$. We have $\frac{da_j^x}{dx} > 0$ by (M). It follows that

$$\frac{dEU^A}{dx} \equiv \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \left\{ \frac{a_{j+1}^x - a_{j-1}^x}{2n} \left[\frac{a_{j-1}^x - 2a_j^x + a_{j+1}^x}{2} \right] \right\} > 0.$$

[22] For $j = K+2, K+3, ..., J-1$ we can use the fact that a_j^x satisfies (6) or

$$-\left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - a_j^x \right)^2 = 0$$

to simplify the calculation, alhtough later exposition will become more complex because this does not apply to $j = K$.

We have the second part of deformation as follows:

$$\begin{aligned}\frac{dEU^A}{dz} &\equiv \sum_{j=1}^{K}\frac{da_j^z}{dz}\left\{-\left(\frac{a_{j-1}^z+a_j^z}{2n}+\frac{n-1}{2n}-a_j^z\right)^2+\left(\frac{a_j^z+a_{j+1}^z}{2n}+\frac{n-1}{2n}-a_j^z\right)^2\right.\\ &\quad-\frac{1}{n}\left[\int_{a_{j-1}^z}^{a_j^z}\left(\frac{a_{j-1}^z+a_j^z}{2n}+\frac{n-1}{2n}-\theta\right)d\theta+\int_{a_j^z}^{a_{j+1}^z}\left(\frac{a_j^z+a_{j+1}^z}{2n}+\frac{n-1}{2n}-\theta\right)d\theta\right]\\ &\quad\left.-\frac{n-1}{2n^2}\left[(a_{j+1}^z)^2-(a_{j-1}^z)^2-\frac{(a_{j-1}^z+a_j^z)^2-(a_j^z+a_{j+1}^z)^2}{2}\right]\right\}\\ &=\sum_{j=1}^{K}\frac{da_j^z}{dz}\left\{\frac{a_{j+1}^z-a_{j-1}^z}{2n}\left[\frac{a_{j-1}^z-2a_j^z+a_{j+1}^z}{2}\right]\right\}<0.\end{aligned}$$

The inequality follows because $\frac{da_j^z}{dz}>0$ by (M), and from $a_0, a_1, ..., a_K \leq 1/2$ and Lemmas 3 and 4 we have $a_j - a_{j-1} > a_{j+1} - a_j \Rightarrow a_{j-1}^z - 2a_j^z + a_{j+1}^z < 0$ for all $j = 1, 2, ..., K$.

Since we have completed the deformation from $a(J)$ to $a(J+1)$ in two steps while increasing the expected utility, we conclude that the agent's expected utility is higher in an equilibrium with more intervals.

- Decision Maker

Since the decision maker's utility is the sum of the agents' utilities, we can apply the above result for an agent's expected utility directly to show that the decision maker's expected utility is higher with an equilibrium with more intervals. ■

5.2 Proposition 4

Here we present the proof of Proposition 4, but we first develop a lemma for the proposition with a slightly more general distributional assumption than in the main text, which we will use in Appendix II. However, throughout the appendices we keep the assumption in Section 3 that all agents' types are drawn from the same realized distribution.

Let $G_i(y \mid \theta_i \in [\underline{a}, \overline{a}))$ be the distribution function of the decision maker's action from agent i's viewpoint, conditional on the decision maker's belief that $\theta_i \in [\underline{a}, \overline{a})$. In the equilibrium with two intervals $\{[0, a), [a, 1]\}$, the agent chooses whether to induce $G_i(y \mid \theta_i \in [0, a))$ or $G_i(y \mid \theta_i \in [a, 1])$ by his message. Let k be the number of agents whose types belong to the lower interval $[0, a)$.

Assumption 1 The expected type of agents in each interval conditional on k is non-increasing in k.

In the Proof of Proposition 4 we show that the distribution we consider in Section 3 satisfies this assumption. However, Assumption 1 should hold for a wider range of distributions. The assumption is satisfied for the distribution we have seen in Section 2 and *any* iid prior distribution.

Let $\tilde{y}(k \mid a)$ denote the decision maker's best response given that k agents' types are in the lower interval $[0, a)$.

$$\tilde{y}(k \mid a) = \frac{k}{n} \frac{\int_0^a \theta_i f(\theta_i \mid k) d\theta}{\int_0^a f(\theta_i \mid k) d\theta} + \frac{n-k}{n} \frac{\int_a^1 \theta_i f(\theta_i \mid k) d\theta}{\int_a^1 f(\theta_i \mid k) d\theta}, \tag{17}$$

where $f(\theta_i \mid k)$ is the posterior density of θ_i given the messages from all agents. Note that if the distribution is not iid (as in Section 3), the decision maker updates her belief on the distribution and infers the expected type of the agents in each interval. Assumption 1 guarantees that $\tilde{y}(k \mid a)$ is strictly decreasing in k.

Define

$$V(0, a, \theta_i) \equiv -\sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} (F(a \mid \theta_i))^k (1 - F(a \mid \theta_i))^{n-1-k} \times (\tilde{y}(k+1 \mid a) - \theta_i)^2 \tag{18}$$

and

$$V(a, 1, \theta_i) \equiv -\sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-1-k)!} (F(a \mid \theta_i))^k (1 - F(a \mid \theta_i))^{n-1-k} \times (\tilde{y}(k \mid a) - \theta_i)^2. \tag{19}$$

$V(0, a, \theta_i)$ is the expected utility conditional on his type θ_i and $G_i(y \mid \theta_i \in [0, a))$. $V(a, 1, \theta_i)$ is the expected utility conditional on his type θ_i and $G_i(y \mid \theta_i \in [a, 1])$. The cumulative conditional distribution of θ_{-i} from agent i's viewpoint is given by $F(a \mid \theta_i)$, i.e. the probability that $\theta_{-i} \leq a$ conditional on θ_i.

Lemma 5 Suppose that Assumption 1 is satisfied. If $a^* \in [0, 1]$ satisfies $V(0, a^*, a^*) = V(a^*, 1, a^*)$, the partition $\{[0, a^*), [a^*, 1]\}$ supports a perfect Bayesian equilibrium.

Proof. Assumption 1 implies $\tilde{y}(k+1 \mid a^*) < \tilde{y}(k \mid a^*)$ for any k. From (18) and (19)

$$\frac{\partial}{\partial \theta_i} V(0, a^*, \theta_i) < \frac{\partial}{\partial \theta_i} V(a^*, 1, \theta_i).$$

Therefore if $\theta_i < a^*$ the agent strictly prefers $G_i(y \mid \theta_i \in [0, a^*))$, and if $\theta_i > a^*$ the agent strictly prefers $G_i(y \mid \theta_i \in [a^*, 1])$. If $\theta_i = a^*$ then the agent is indifferent. Hence we conclude that the partition $\{[0, a^*), [a^*, 1]\}$ supports a perfect Bayesian equilibrium. ■

Lemma 5 establishes that the informative equilibrium with two intervals is characterized by the indifference condition, as in the known iid setting we have studied in Section 2.

Proof of Proposition 4. We consider the decision maker's inference problem as the estimation of a binomial distribution. According to the messages the decision maker computes the posterior distribution of $p \equiv \frac{1/2 - \alpha}{\beta}$, the proportion of the realized distribution that belongs to the lower interval $[0, 1/2)$. Recall that k is the number of agents whose types belong to $[0, 1/2)$. We can think of p as the success probability of the Bernoulli distribution, for which k is a sufficient statistic. To obtain the posterior of θ_i, the decision maker combines the posterior distribution of p with the expected value of θ_i conditional on p. Since ex ante p is uniformly distributed on $[0, 1]$, the density of p conditional on k is given by[23]

$$f(p \mid k) = \frac{p^k (1-p)^{n-k}}{B(k+1, n-k+1)}, \tag{20}$$

[23] See DeGroot (1970, p.165), for example.

where $B(\cdot,\cdot)$ is the beta function. For a given p, α is uniformly distributed on $[\frac{1-p}{2},\frac{1}{2})$, where $\frac{1-p}{2}=\alpha$ when $\beta=1/2$ and $\frac{1}{2}=\alpha$ when $\beta=0$. Hence the conditional expectation of θ_i is given by

$$E[\theta_i \mid p, \theta_i \in [0,1/2)] = \int_{\frac{1-p}{2}}^{\frac{1}{2}} \frac{\alpha+1/2}{2} \frac{1}{\frac{1}{2}-\frac{1-p}{2}} d\alpha = \frac{4-p}{8}, \tag{21}$$

where α is the lower bound of the realized distribution for given α. Similarly,

$$E[\theta_i \mid p, \theta_i \in [1/2,1]] = \int_{\frac{1-p}{2}}^{\frac{1}{2}} \frac{1/2+\frac{1/2-\alpha+\alpha p}{p}}{2} \frac{1}{\frac{1}{2}-\frac{1-p}{2}} d\alpha = \frac{5-p}{8}, \tag{22}$$

where $\frac{1/2-\alpha+\alpha p}{p}=\alpha+\beta$ is the upper bound of the realized distribution for α. From (20) and (21) we obtain

$$E[\theta_i \mid k, \theta_i \in [0,1/2)] = \int_0^1 \frac{4-p}{8} \frac{p^k(1-p)^{n-k}}{B(k+1,n-k+1)} dp = \frac{4n-k+7}{8n+16}. \tag{23}$$

Likewise, from (20) and (22)

$$E[\theta_i \mid k, \theta_i \in [1/2,1]] = \int_0^1 \frac{5-p}{8} \frac{p^k(1-p)^{n-k}}{B(k+1,n-k+1)} dp = \frac{5n-k+9}{8n+16}. \tag{24}$$

Clearly both $E[\theta_i \mid k, \theta_i \in [0,1/2)]$ and $E[\theta_i \mid k, \theta_i \in [1/2,1]]$ are decreasing in k, and therefore the distribution of types in Section 3 satisfies Assumption 1. Using (23) and (24), the decision maker's action given k is written

$$\tilde{y}(k \mid a=1/2) = \frac{1}{n}\left[k\frac{4n-k+7}{8n+16} + (n-k)\frac{5n-k+9}{8n+16}\right]. \tag{25}$$

Now let us consider an agent's binary strategy. Suppose $\theta_i = 1/2$. Then the conditional density of θ_{-i} from agent i's viewpoint is symmetric with respect to $1/2$. Thus apart from i, the expected number of the agents whose types belong to $[0,1/2)$ and that of the agents whose types belong to $(1/2,1]$ are the same, $\frac{n-1}{2}$. By substituting $k=\frac{n-1}{2}+1$ into (25) the expected reaction from agent i's viewpoint for $G_i(y \mid \theta_i \in [0,1/2))$ is

$$\bar{y}_A(0,1/2) = \frac{4n^2+7n-1}{8n^2+16n}.$$

Also by substituting $k=\frac{n-1}{2}$ into (25) we have the expected reaction from agent i's viewpoint for $G_i(y \mid \theta_i \in [1/2,1])$

$$\bar{y}_A(1/2,1) = \frac{4n^2+9n+1}{8n^2+16n}.$$

It is easy to check that $|\bar{y}_A(0,1/2)-1/2| = |\bar{y}_A(1/2,1)-1/2| = \frac{n-1}{16n^2+8n}$. That is, the expected distance from the ideal action ($\theta_i = y = 1/2$) and the induced action is the same for $G_i(y \mid \theta_i \in [0,1/2))$ and $G_i(y \mid \theta_i \in [1/2,1])$. Symmetry implies that both distributions have the same variance. Hence the agent is indifferent between $G_i(y \mid \theta_i \in [0,1/2))$ and $G_i(y \mid \theta_i \in [1/2,1])$. From this and Lemma 5 we conclude that the partition $\{[0,1/2),[1/2,1]\}$ supports a perfect Bayesian equilibrium. ■

6 Appendix II: General Distributions

In this Appendix we consider the existence of binary communication equilibrium with more general settings. The utility functions are given by (1) and (2), where $b_i = 0$. All agents adopt the same strategy. In addition to Assumption 1 in Appendix I we introduce the following distributional assumption:

Assumption 2 The prior density of θ_i is positive and continuous on $[0, 1]$. Any posterior density of θ_i is also positive and continuous on its support.

Clearly both assumptions are satisfied for the models we have seen in Sections 2 and 3. The following proposition extends the observations we have made in Example 1 and Proposition 4.

Proposition 5 Suppose Assumptions 1 and 2 hold. Then there exists an equilibrium with two intervals.

Proof. Recall the definition of $V(\cdot,\cdot,\cdot)$ in (18) and (19). For notational convenience let us define

$$D(a) \equiv V(0, a, a) - V(a, 1, a).$$

$D(a)$ is the difference between the expected utilities of the agent with the boundary type a ($\theta_i = a$) when he induces $G_i(y \mid \theta_i \in [0, a))$ and $G_i(y \mid \theta_i \in [a, 1])$, respectively.

The rest of the proof proceeds as follows. We first show that $D(0) > 0$ and then $D(1) < 0$. Since $D(a)$ is continuous on $[0, 1]$, by the intermediate value theorem there exists $a^* \in (0, 1)$ such that $D(a^*) = 0$. Then from Lemma 5, a^* supports an equilibrium.

Suppose that $a = 0$ and $\theta_i = a = 0$. Recall that $\tilde{y}(k \mid a = 0)$ is the decision maker's best response when k agents' type is $\theta_i = 0$. Since the agents' types are drawn from a continuous density function, almost surely all the other agents types are in $(0, 1]$, or $k = 0$. That is, if the agent induces $G_i(y \mid \theta_i = 0)$, the decision maker's action is, almost surely, $\tilde{y}(1 \mid a = 0)$; and if he induces $G_i(y \mid \theta_i \in (0, 1])$ the decision maker's action is, almost surely, $\tilde{y}(0 \mid a = 0)$. Hence

$$V(0, 0, 0) = -(\tilde{y}(1 \mid a = 0) - 0)^2$$

and

$$V(0, 1, 0) = -(\tilde{y}(0 \mid a = 0) - 0)^2.$$

Since $\tilde{y}(k \mid a)$ is decreasing in k,

$$\tilde{y}(1 \mid a = 0) < \tilde{y}(0 \mid a = 0). \tag{26}$$

By strict concavity of the utility function, the agent is strictly prefers $\tilde{y}(1 \mid a = 0)$ to $\tilde{y}(0 \mid a = 0)$. Therefore, $D(0) = V(0, 0, 0) - V(0, 1, 0) > 0$.

Suppose on the contrary that $\theta_i = a = 1$. If the agent induces $G_i(y \mid \theta_i = 1)$, then the decision maker's action is, almost surely, $\tilde{y}(n-1 \mid a = 1)$; if he induces $G_i(y \mid \theta_i \in [0, 1))$ then the decision maker's action is, almost surely, $\tilde{y}(n \mid a = 1)$. Hence we have

$$V(0, 1, 1) = -(\tilde{y}(n \mid a = 1) - 1)^2 < -(\tilde{y}(n - 1 \mid a = 1) - 1)^2 = V(1, 1, 1).$$

Therefore $D(1) = V(0,1,1) - V(1,1,1) < 0$.

Since the utility functions and the prior and posterior densities of θ_i are assumed to be continuous, $\tilde{y}(a \mid k)$ and consequently $D(a)$ are continuous on $a \in [0,1]$. Therefore, by the intermediate value theorem there exists $a^* \in (0,1)$ such that $D(a^*) = 0$. From Lemma 5 in Appendix I, $\{[0,a^*),[a^*,1]\}$ supports a perfect Bayesian equilibrium. ■

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