

Product Quality Under Regulated Monopoly

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Abstract. A monopolist regulated via a price cap may well have an incentive to change other variables of interest to consumers, in an attempt to shift the cost and demand curves in his favour. This paper develops a model in which the monopolist can vary product quality and the terms of a warranty, in response to price regulation. The regulated and unregulated monopoly outcomes are compared with the Pareto-efficient outcome.

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1. Introduction

It is frequently argued that monopolies may safely be left in the private sector, or transferred to it, provided they are properly regulated. The well-known welfare losses due to monopoly, it is said, can easily be mitigated by effective regulation (see Sappington and Stiglitz, 1987 for a review of these arguments). Thus the dramatic outbreak of privatisation in the British economy during the 1980s and 1990s was accompanied by the emergence of a new breed of regulator: OFTEL, OFFER, OFGAS, OFWAT etc.. Regulation typically takes the form of price ceilings imposed by the regulator on the monopoly, a type of constraint which naturally generates a great deal of noisy complaint from the monopolist. However the regulator generally has sufficiently draconian penalties available to impose her will in the end. One would expect therefore that the rational, profit-maximising monopolist would respond to binding price control by changing other variables, in an attempt to shift the demand and cost curves in his favour. Obvious candidates for this role are (a) product quality and (b) the contractual terms on which the product is offered to the market; for example the terms of any warranty which the monopolist may provide along with the good.

In order to analyse this problem it is necessary to develop a model which allows the firm to determine product quality and the terms of any warranty, as part of its overall (expected) profit-maximising decision. Unfortunately the existing literature does not provide much of a guide on these matters (though see George, 1996; Mikami, 1991; Besanko et. al., 1988; Kihlstrom and Levhari, 1977; and Spence, 1975 for related discussions). Rob (1994) considers the supply of electricity by a private monopoly, showing that a pure price cap induces the firm to reduce reliability.

In the management literature "product quality" is defined extremely broadly. The term has been used to refer to safety, availability, maintainability, reliability and usability (see e.g. Besterfield, 1986). Generally speaking quality is best thought of as a

characteristic of the product with the property that all consumers prefer more of it to less, **at a given price**. Some such characteristics will be known to the consumer before purchase, while others will not. For personal computers, for example, the former type of characteristic might include the size of the processor chip, speed of the processor chip (200Mhz., 266Mhz., 300Mhz. etc.) or amount of RAM, while the latter type might include product lifetime, repair costs etc.. Most goods have characteristics of both types, though the second notion of quality raises more interesting questions for firms, consumers, regulators and for economic theory. It is the notion of quality dealt with in this paper.

Firms devote considerable resources to influencing the quality of their products. This influence operates at the level of product design, production process and post-production quality control. The unity of the quality management process is often stressed in the management literature. The distinction between production and quality control decisions is frequently blurred. For example, a firm may seek to raise its production quality by, in effect, demanding tighter quality control from its components suppliers. Thus a production decision in one firm is inseparable from a post-production quality control decision in another. The model developed in this paper incorporates production, quality control, warranty and pricing decisions into the firm's overall (expected) profit maximising behaviour. Product design decisions are not considered.

In the model presented here 'quality' will be defined as a kind of 'durability'; more precisely as the probability of the product not breaking down during a particular time period (the warranty period). This warranty period will, for the sake of tractability be treated as exogenous. Both firms and consumers will be assumed to be ignorant at the moment of purchase, as to the quality of any given product, though they will be assumed to know the **average** quality of output. Thus the model is one of imperfect but symmetric information. It will further be assumed that the firm, though just as

ignorant as consumers, is less risk-averse. This assumption is readily justified, for example, in the consumer durables market, where each consumer typically owns one example of the good and is thus extremely concerned at the prospect of its breaking down. The firm, by contrast, supplies many examples of the good, and may well find it profitable to operate a risk-pooling warranty scheme. Under these assumptions there arises a demand, on the part of consumers, for insurance. This might, as mentioned above, be provided in the form of a product warranty offered by the firm, or an insurance policy provided jointly with the product. In the case of intermediate goods "consumers" may be thought of as firms and "warranties" as compensation clauses built into standard supply contracts. The model is applicable to any situation where the firm can offer to compensate consumers in the event of a product failure or service failure, (e.g. train arriving late, telephone connection breaking down). It could readily be modified to cover partial as well as total failure, or to cover product hazard and safety issues. Obvious applications are to consumer durables, new vehicles, some used vehicles (e.g. supplied under dealer approved used car schemes), new houses, privatised rail companies, privatised utilities and a variety of intermediate goods such as silicon chips and other components whose quality can be assessed independently of the final product.

Heal (1977) develops a model, involving warranties, which adopts precisely the informational assumptions discussed above. He remarks:

"Typically the quality control is sufficiently imperfect that no one (i.e. neither seller nor buyer) will know in advance of (a product's) use what (its) quality will be, and consequently some form of guarantee will be offered." (Remarks in brackets added)

In Heal's model the firm is assumed to produce a probability distribution of qualities which is simply taken as given. He does not seek to model the process by which the firm attempts to alter that distribution. In this paper the firm will be assumed able to

influence the average quality of its output, for example by quality control decisions. It will also be assumed able to offer a product warranty to the market.

A standard problem, often assumed away, in the literature on quality, is that of moral hazard on the part of consumers. If consumers can themselves influence the probability or size of a claim under the warranty, for example by failing to take proper care of the good during consumption, then the economic role of warranties may be reduced. See, for example McKean (1970), Oi (1973) and Priest (1981). Goering (1997) discusses the problem of moral hazard facing a durable goods monopolist. For simplicity moral hazard will be assumed away in this paper.

Warranties, whether voluntary or legally compelled, have an important bearing on quality management decisions because the higher the quality of a firm's marketed output, the lower the likely warranty costs experienced by the firm. Thus warranties provide the firm with an incentive to market high quality products. This connection between warranties and quality management has been apparent to managers for some time. Wright (1980), for example, describes events at General Motors:

"I instituted a programme for testing and repairing faulty cars as they came off the assembly line - and the results were phenomenal. It cost about \$8 a car, which drove The Fourteenth Floor up the wall. But I figured one way or the other we would end up fixing the defects or paying to have them fixed through recall campaigns or dealer warranty bills..... The internal quality control audit revealed a 66% improvement in the quality of a Chevrolet coming off the assembly line between 1969 and 1973 models. And most important, warranty costs of our new cars were down substantially."

The existing economics literature deals with both the notions of quality discussed above. When quality is known to consumers before purchase, the focus of interest is

screening. The seller will be ignorant as to the preferences of any individual consumer though he may be assumed to know the distribution of preferences across the population. His problem then is to provide a price-quality schedule, perhaps along with a warranty arrangement, to the market with a view to screening consumers and thus extracting the maximum surplus from them. The firm deliberately differentiates his product by quality. An obvious example is personal computers: most manufacturers produce a range of products involving different processing speeds, amount of RAM, size of hard disc etc. This situation merely enlarges the regulator's problem: she is, in effect, dealing with a multi-product instead of a single product monopolist. Authors who develop screening models of quality include Mussa and Rosen (1978) and Matthews and Moore (1987).

Signalling models, by contrast deal with a different asymmetry of information, namely one concerning the product itself. Such models are driven by exogenous "type uncertainty". That is to say "Nature" dictates a firm's quality which is then known to that firm but not to consumers. The firm's problem then is to signal its quality to consumers using price, warranties and possibly advertising. In a repeat purchase framework the firm may be able to build up a "reputation" for quality. Authors who develop signalling models of quality include Grossman (1981), Milgrom and Roberts (1982, 1986), Kreps and Wilson (1982), Klein and Leffler (1981), Shapiro (1983) and McClure and Spector (1991). Landon and Smith (1997) develop an empirical model of quality and reputation indicators, and apply it to the market for Bordeaux wine.

This paper is concerned with product quality that is unknown to the consumer before purchase since it is product quality in this sense which raises the greater problem for regulators. Screening models are of no help because they do not deal with this notion of quality. Signalling models, by contrast, do analyse this concept of quality, but they do so in a way which treats quality as exogenous and not affected by the firm's decisions. Again this is of little help in dealing with the regulator's problem as set out

above. Both screening and signalling models place the emphasis of the analysis on an **asymmetry** of information. The approach of this paper is to allow the firm's quality control decisions to influence product (or service) quality under conditions of imperfect but symmetric information.

The bulk of the paper develops a simple model intended to capture the situation described above. In section 2 the demand side is developed, assuming risk averse consumers, with a reservation price varying across the market. Section 3 develops the supply side, incorporating quality management costs and expected warranty costs into the monopolist's expected profit function. In section 4 a Pareto-efficient allocation of resources is characterised, in order to provide a benchmark for the regulator. Sections 5 and 6 characterise monopoly and regulated monopoly equilibria. Comparing the two types of equilibrium with each other and with the efficient allocation allows an analysis of the effects of regulation. The mathematical details are relegated to an Appendix. Section 7 concludes.

2. The Demand Side

The demand side of the market will be assumed to consist of z consumers, each consuming a single unit of the good. Each consumer has a different reservation price, and hence the market demand curve is downward sloping. For simplicity we take z to be a strictly positive real variable. Each consumer has a money budget M available and pays a price p for the good. As discussed in section 1, the good either breaks down or does not break down within the warranty period. In the latter case it generates a stream of services worth $f(z)$ to the z 'th consumer (note that $z > 0$ and $f'(z) < 0$). In the former case the product generates no services at all for the consumer, but the firm makes a warranty payment of b to her. Costs of writing and enforcing the warranty contract are ignored. Thus the z 'th consumer receives income stream:

$$x = M - p + f(z)$$

(1) if the product does not break down within the warranty period, and

$$y = M - p + b$$

(2)

if the product does break down within the warranty period.

The quality of a product will be defined as in section 1, as the probability that it does not break down within the warranty period. As discussed in section 1, neither firm nor consumer knows the quality of an individual product, only the average quality (Q) of the firm's output (i.e. the probability that a product picked at random from the firm's output will break down within the warranty period). Note that Q and b are determined by the decisions of the monopolist and that, by definition $0 \leq Q \leq 1$.

Consumers are assumed to be risk-averse maximisers of expected utility. Throughout the paper it will be assumed that consumers' subjective probability that the product does not break down during the warranty period is equal to the objective probability (Q). Of course Q is determined by the quality management decisions of the monopolist, and consumers cannot observe these decision directly. Moreover, the model set up here does not admit repeat purchasing, so the consumer cannot learn about average quality over time. Nonetheless there is no asymmetry of information, so consumers could in principle deduce the monopolist's quality management decision and thus deduce average quality.

The z 'th consumer maximises expected utility:

$$V = Q.U(M - p + f(z)) + (1 - Q).U(M - p + b) \quad (3)$$

Clearly $U'(\cdot) > 0$, and, to ensure risk aversion, it is assumed that $U''(\cdot) < 0$ (i.e. the function $U(\cdot)$ is assumed concave). Each consumer has a different reservation price, and thus the market demand curve slopes downwards. Note that the z 'th consumer is indifferent between consuming and not consuming when:

$$V = Q \cdot U(M - p + f(z)) + (1 - Q) \cdot U(M - p + b) = U(M) \quad (4)$$

since $U(M)$ is the utility she would get by not consuming the product (she will be referred to as the 'marginal consumer'). Equation (4) generates, for given values of Q and b , a relationship between p and z , namely the market demand curve. It is easy to establish that this demand curve slopes downwards (see figure 1). Note that Q and b are determined by the decisions of the monopolist, so that consumers can be thought of as consuming a 'bundle' consisting of the stream of services provided by the good, its expected quality and the warranty deal. They are not able to 'unbundle' these three things. If the monopolist raises Q or b the demand curve will shift upwards, except that, when a full 'money back' warranty is offered ($b = p$), the marginal consumer will be indifferent as to whether the product breaks down or not (since $x = y$ when $b = p$), and the demand curve will therefore rotate about the equilibrium, which will itself be immune to variations in Q . Note also that the $f(z)$ curve must be steeper than the demand curve (see figure 1) because it is the relationship between p and z which would hold **if b were continually kept equal to p** (this is clear from equation (4)). Of course the demand curve is defined *ceteris paribus* (i.e. holding everything constant except p and z).

[Figure 1 near here]

3. The Supply Side

Quality costs are discussed at some length in the management literature. Grocock (1986) points out:

"Because the products might be defective they must be inspected and tested. This results in appraisal costs.....Products may also fail a test or inspection, or may fail in the hands of customers. Failure costs are then incurred.....(since the firm) must rework or replace the failed product during manufacturing, or replace or repair the product for customers, for example, under warranty." (Grocock, 1986, p53)

The model developed here formalises these costs by assuming that production costs are increasing in the average quality (Q) of output, and by incorporating expected warranty costs into the firm's (expected) profit-maximising decision. Average and marginal production costs, at a given quality level, will be assumed constant. Note that z is the monopolist's output.

Adopting the assumptions set out above a suitable production cost function is:

$$z.C(Q), \quad \text{where } C'(Q) > 0 \text{ and } C''(Q) > 0 \text{ for } 0 < Q < 1. \quad (5)$$

The expected number of products breaking down within the warranty period is clearly $z.(1 - Q)$, and thus expected warranty costs are given by:

$$bz(1 - Q) \quad (6)$$

Thus the monopoly is a risk-neutral maximiser of expected profit:

$$F = pz - zC(Q) - bz(1 - Q) \quad (7)$$

4. Pareto-efficiency

We now characterise a Pareto-efficient allocation of resources to provide a benchmark for the regulator. Since there are no non-convexities, externalities or public goods in the model, a Pareto-efficient allocation of resources would be brought about by the operation of a perfectly competitive market. A Pareto-efficient allocation is defined as a 4-tuple (p, z, Q, b) which maximises each consumer's expected utility subject to the constraint that expected profits are non-negative. This problem is easily solved by taking a multiplier (λ) for the profit constraint and forming the Lagrangian:

$$L = Q.U(M - p + f(z)) + (1 - Q).U(M - p + b) + \lambda (pz - z.C(Q) - bz(1 - Q)) \quad (8)$$

By considering the first order conditions of this problem, two important results can be obtained. The proofs are relegated to the Appendix. Note that an asterisk denotes the efficient level of a variable.

Firstly the profit-constraint is binding: that is Pareto-efficiency requires zero (supernormal) profits and consumers extract all the surplus (Appendix, Proposition 1). In the absence of monopoly this could be achieved by free entry or by Bertrand competition. Note that price discrimination by the monopolist is assumed away.

Secondly, Pareto-efficiency requires that the marginal consumer is fully insured (Appendix, Proposition 2). Thus a full money back warranty must be offered (i.e. $b^* = p^*$, so that $x = y$, and the marginal consumer is indifferent as to whether the product breaks down during the warranty period or not).

5. Monopoly Equilibrium

In monopoly equilibrium there is a single supplier, maximising his expected profits, subject to the voluntary participation constraint. This is the constraint that each consumer obtains at least as much expected utility from purchasing the product as from not doing so. Mathematically it is simply:

$$Q.U(M - p + f(z)) + (1 - Q).U(M - p + b) \geq U(M)$$

(14)

In equilibrium z is determined at a level which makes this constraint bind (i.e. the z th. consumer is the marginal consumer, who is just on the point of leaving the market, and z is the monopolist's total output). A monopoly equilibrium is easily characterised by

taking a Lagrange multiplier (m) for the constraint (14) (noting equation (7), which specifies the monopolist's expected profits) and forming the Lagrangian:

$$M = pz - z \cdot C(Q) - bz(1 - Q) + m(QU(x) + (1 - Q)U(y) - U(M))$$

(15)

From the first order conditions for this problem the following results can be obtained (proofs in Appendix). Note that the superscript m denotes monopoly equilibrium values of the relevant variables.

Firstly the voluntary participation constraint is binding (Appendix, Proposition 3). This allows the monopoly output (z^m) to be determined.

Secondly, in (unregulated) monopoly equilibrium, risk is efficiently allocated (i.e. the marginal consumer is fully insured, so that $p^m = b^m$; Appendix, Proposition 4). This is a somewhat surprising result; since the firm has monopoly power, one would expect it to raise the price of the insurance services it provides above the efficient level, by lowering the warranty payment relative to the price of the good. It does not do so because of the bundling of quantity, quality and insurance which it offers to the market.

Note that the monopolist satisfies the familiar 'marginal cost = marginal revenue' condition. The monopoly equilibrium is illustrated in Figure 2.

[Figure 2 near here]

Manipulating the first order conditions allows a comparison between the (unregulated) monopoly levels of the relevant variables with their efficient levels. In particular:

$$Q^m > Q^*; b^m > b^*; p^m > p^*; z^m < z^* .$$

(16)

(Appendix, Proposition 5 and Corollaries 2,3,4)

The monopolist **raises** average quality above its efficient level but lowers output and raises price in the usual way. As remarked above it allocates risk efficiently. Thus the monopolised market could be invaded by competitive firms offering a greater quantity of lower quality, cheaper output. It could **not**, however be invaded by competitive insurers offering a better warranty deal.

6. Regulated Monopoly

The monopoly model of section 5 illustrates the standard sources of inefficiency under monopoly, namely a restricted output and raised price (relative to the efficient levels).

In addition to this, inefficiency arises because the monopolist supplies an average quality level **above** the efficient level. However, the monopolist does allocate risk efficiently by offering a full money-back warranty. In this section we consider the consequences of binding price regulation. Suppose the regulator imposes a binding price ceiling (p^r) on the monopolist such that: $p^m > p^r \geq p^*$. A reduction in average quality will clearly lower costs and may therefore increase the firm's expected profits.

With a money-back warranty in place, the marginal consumer is indifferent as to whether the product breaks down within the warranty period or not, and consequently the market equilibrium will be immune to variations in average quality. In fact, in the

model set out above, the effect of the price regulation is rather more complex than this, because it induces under-insurance and an inefficient allocation of risk. The complexity arise because of the bundling of quantity, quality and insurance. Because of the under-insurance, the market demand curve now **shifts** in response to changes in Q (rather than simply rotating about the equilibrium). In particular, lowering Q shifts the demand curve downwards (see figure 2). It is possible, for extreme forms of the functions $U(\cdot)$, $f(\cdot)$ and $C(\cdot)$, that this shift is so great as to force the regulated output level **below** the monopoly level. We rule out this extreme case and assume $z^r > z^m$ (as illustrated in figure 2).

The analysis of regulation is best approached by modifying the Lagrangian of section 5 (equation (15)), by adding a constraint on the price:

$$p \leq p^r \quad (17)$$

We take a multiplier η for this constraint, modifying equation (15) as follows:

$$M = pz - z.C(Q) - bz(1-Q) + m(QU(x) + (1-Q)U(y) - U(M)) + \eta(p^r - p) \quad (18)$$

The multiplier η represents the monopolist's marginal valuation of the price ceiling, i.e. the amount by which he could increase his expected profits if the price ceiling were relaxed by one marginal unit (or the maximum he would be willing to pay as a bribe in order to get a marginal, one unit, increase in the price ceiling). We are interested, in this paper, in **effective** price regulation and will therefore assume that, under regulation, $\eta > 0$. Note that equation (18) includes the unregulated case (i.e. when $\eta = 0$). In the Appendix the first order conditions of the Lagrangian (equation (18)) are derived, thus incorporating the regulated and unregulated cases into the same mathematical problem.

Using these first order conditions the regulated monopoly equilibrium ($n > 0$) can be characterised. Comparing this equilibrium with the unregulated case ($n = \infty$) the effects of regulation can be deduced. In particular, regulated monopoly output is below the efficient level (Appendix, Proposition 6). Moreover, regulation induces the monopolist to provide a less than money-back warranty (Appendix, Proposition 4 and Corollary 1). Thus the marginal consumer is not fully insured and risk is not efficiently allocated. The market demand curve now shifts in response to changes in average quality. The regulated monopolist also lowers average product quality (Appendix, Proposition 7). But it was inefficiently high to start with, so this may not concern the regulator. The cost savings associated with the quality reduction and the under-insurance are partly passed to consumers via the lower price, and partly taken as (supernormal) profits.

7. Conclusions

This paper has developed a simple model of monopoly in which the firm can vary average product quality and has an incentive to offer a warranty to the market. The model points to complications for the regulators of monopolies. Even if the imposition of an effective price ceiling on the monopolist has the standard effect of increasing quantity and decreasing price, it will also induce the monopolist to reduce product (or service) quality. This is consistent, for example, with the findings of Rob (1994), who shows that a pure price cap imposed on a private monopoly supplier of electricity induced the firm to reduce the reliability of supply. However, the model of this paper implies that average quality under monopoly will be inefficiently high, so that this quality effect may not concern the regulator. The model also implies that an unregulated monopolist will offer a full money-back warranty, thus allocating risk

efficiently. An effective price ceiling induces the monopolist to worsen the warranty terms, generating under-insurance and an inefficient allocation of risk. The cost savings arising from quality reduction and under-insurance are partly passed on to consumers via the lower price and partly taken by the monopolist as (supernormal) profits.

The model could be extended to analyse other problems, outside the area of regulation.

For example a firm which has joined a cartel may have to agree to restrict output. If such a firm can vary average product quality and the terms of a product warranty, it is likely to do so in pursuit of profits.

Appendix

This Appendix contains the proofs of the results discussed in the main text. Part A characterises a Pareto-efficient allocation of resources. Part B characterises monopoly equilibrium and regulated monopoly equilibrium. The approach is to derive and utilise the first-order conditions of the Lagrangians given in the main text.

A. Pareto-Efficient Allocation

Consider first the Lagrangian of equation (8). Its first order conditions characterise a Pareto-efficient allocation.

$$L = Q.U(M - p + f(z)) + (1 - Q).U(M - p + b) + \lambda (pz - z.C(Q) - bz(1 - Q))$$

(A1)

It is easy to prove:

Proposition 1. Pareto-efficiency requires that the expected profit constraint is binding (i.e. $F = 0$).

Proof. Differentiating equation (A1) w.r.t. p , and using equations (1) and (2), yields one of first-order conditions for an interior maximum:

$$\begin{aligned} L_p &= -QU'(x) - (1 - Q)U'(y) + \lambda z = 0 \\ \Rightarrow \lambda z &= QU'(x) + (1 - Q)U'(y) > 0 \end{aligned}$$

(A2)

But $z > 0$, hence $l > 0$, and it follows by complementary slackness that the expected profit constraint is binding.

We now establish that, Pareto-efficiency requires that a full "money back" warranty is offered. Thus a risk-neutral firm fully insures risk-averse consumers and the allocation of risk is efficient. First it is necessary to establish three useful Lemmas.

Lemma 1. Pareto-efficiency requires that $b = f(z)$.

Proof. Differentiating equation (A1) w.r.t. b and using equations (1) and (2) yields another first-order condition for an interior solution:

$$L_b = (1-Q)U'(y) - l z(1-Q) = 0$$

$$\Rightarrow (1-Q)U'(y) = (1-Q)(QU'(x) + (1-Q)U'(y)) \quad (\text{using equation (A2)})$$

$$\Rightarrow U'(y) = QU'(x) + (1-Q)U'(y) \quad (Q \neq 1 \text{ because we are seeking an interior solution}).$$

$$\Rightarrow QU'(y) = QU'(x)$$

$$\Rightarrow x = y \quad (Q \neq 0 \text{ because we are seeking an interior solution; } U'(\cdot) \text{ is invertible because } U''(\cdot) < 0).$$

Hence from equations (1) and (2) we have $b = f(z)$.

Lemma 2. Pareto-efficiency requires that $b = C'(Q)$.

Proof. Differentiating equation (A1) w.r.t. Q and using equations (1) and (2) yields another first-order condition for an interior solution:

$$L_Q = U(x) - U(y) + l(-z.C'(Q) + bz) = 0$$

(A3)

But $x = y$ (from Lemma 1), $l > 0$ (from Proposition 1) and $z > 0$ (by definition), hence equation (A3) implies that: $b = C'(Q)$, as required.

Lemma 3. Pareto-efficiency requires that: $p = f(z)$.

Proof. We have $x = y$ from Lemma 2. So equation (4) yields:

$$QU(x) + (1-Q)U(x) = U(M)$$

$$\Rightarrow U(x) = U(M)$$

$$\Rightarrow x = M \quad (U(\cdot) \text{ is invertible because } U'(\cdot) > 0)$$

Hence, from equation (1), $p = f(z)$, as required.

It is now straightforward to establish:

Proposition 2. Pareto-efficiency requires that a full "money back" warranty is offered (i.e. $p = b$). As mentioned above, this entails an efficient allocation of risk.

Proof. Combining Lemmas 1 and 3, we have $p = b$, as required.

We now establish the useful:

Lemma 4. Pareto-efficiency requires that: $C'(Q) \cdot Q = C(Q)$.

Proof. From Proposition 1, Pareto-efficiency requires that profits are zero. Thus, using Proposition 2 and equation (7) we obtain:

$$pz - z \cdot C(Q) - pz + pzQ = 0$$

$$\Rightarrow pQ = C(Q)$$

Combining Proposition 2 and Lemma 2, we have $p = b = C'(Q)$. Thus:

$$C'(Q) \cdot Q = C(Q) \text{ as required.}$$

B. Monopoly Equilibrium

In this section we characterise monopoly equilibrium, regulated and unregulated, by deriving the first order conditions of the Lagrangian specified in section 6 of the main text as equation (18)

$$M = pz - z \cdot C(Q) - bz(1-Q) + n(QU(x) + (1-Q)U(y) - U(M)) + n(p^r - p) \quad (\text{A4})$$

Note that $n > 0$ corresponds to the regulated case and $n = 0$ to the unregulated case.

Differentiating (A4) yields first-order conditions for an interior solution:

Differentiating wrt p :

$$z + m[-QU'(x) - (1-Q)U'(y)] - n = 0$$

(A5)

Differentiating wrt b :

$$-z(1-Q) + m(1-Q)U'(y) = 0$$

(A6)

Differentiating wrt Q :

$$-zC'(Q) + bz + m[U(x) - U(y)] = 0$$

(A7)

Differentiating wrt z :

$$p - C(Q) - b(1-Q) + mQU'(x)f'(z) = 0$$

(A8)

It is now straightforward to establish:

Proposition 3. In monopoly equilibrium (regulated and unregulated) the voluntary participation constraint binds. That is:

$$U(M) = QU(x) + (1-Q)U(y)$$

(A9)

Proof. (A6) implies that $m \neq 0$, since we are seeking an interior solution

($z > 0, I > Q > 0$). Hence, by complementary slackness, the voluntary participation constraint must bind.

We now establish:

Proposition 4. The unregulated monopolist provides the marginal consumer with full insurance, while the regulated monopolist provides under-insurance.

Proof. (A5) and (A6) together imply that: $Q(U'(y) - U'(x)) = \frac{n}{m}$. Thus, for the unregulated case ($n = 0$) we must have $x = y$. (Note that $U'(\cdot)$ is invertible because $U''(\cdot) < 0$). Hence, in the unregulated case, the marginal consumer is fully insured. For the regulated case we have $n > 0$, and hence, by a similar argument, $y < x$ and the marginal consumer is less than fully insured.

Corollary 1. For the unregulated case we have:

$$f(z) = p = b \quad (\text{A10})$$

and for the regulated case:

$$f(z) > p > b \quad (\text{A11})$$

Proof. Follows from (A9) and Proposition 4.

We now prove the inequalities (16) of the main text which express comparisons between (unregulated) monopoly levels and efficient levels of the relevant variables. We start by considering average quality Q .

Proposition 5. Average product quality in (unregulated) monopoly equilibrium is above the Pareto-efficient level.

Proof. (A6) and (A8) together imply that:

$$p + \frac{zQU'(x)}{U'(y)} f'(z) = C(Q) + b(1 - Q) \quad (\text{A12})$$

But in the unregulated case $x = y$, hence (A12) implies:

$$p + zQf'(z) = C(Q) + b(1 - Q) \quad (\text{A13})$$

Now note that, given $x = y$, (A7) implies that:

$$b = C'(Q) \tag{A14}$$

Combing (A13), (A14) and Corollary 1 yields:

$$C(Q) - QC'(Q) = zQf'(z) \tag{A15}$$

Now define the function $F(Q)$ as follows:

$$F(Q) = C(Q) - QC'(Q) \tag{A16}$$

Differentiating F gives:

$$F'(Q) = -QC''(Q) < 0 \tag{A17}$$

(noting that $C''(.) > 0$)

Using the function F we can now compare the monopoly level of Q with the efficient level. From Lemma 4:

$$F(Q^*) = 0 \tag{A18}$$

while (A15) gives:

$$F(Q^m) < 0 \tag{A19}$$

since $f'(z) < 0$. Hence (A17) implies that:

$$Q^m > Q^* \text{ as required.}$$

Corollary 2. $b^m > b^*$

Proof. Follows from Lemma 2, (A14) and Proposition 5, noting that $C'(Q) > 0$.

Corollary 3. $p^m > p^*$

Proof. Follows from Proposition 2, (A10) and Corollary 2.

Corollary 4. $z^m < z^*$

Proof. Follows from Lemma 3, (A11) and Corollary 3.

We now turn to the regulated monopolist. It is first useful to establish the following useful lemma:

Lemma 5. Under regulated monopoly: $C'(Q) < f(z)$

Proof. Using (A6) to eliminate m from (A7) yields:

$$C'(Q) = b + \frac{U(x) - U(y)}{U'(y)}$$

(A20)

The proof relies on the concavity of the utility function $U(\cdot)$ (see figure 3). Referring to figure 3, concavity implies that:

$$U(x) - U(y) < H < U'(y)(x - y) = U'(y)(f(z) - b)$$

$$\Rightarrow \frac{U(x) - U(y)}{U'(y)} < f(z) - b$$

(A21)

Combining (A20) and (A21) yields the result required.

[Figure 3 near here]

We can now establish:

Proposition 6. The regulated monopoly output is below the efficient level.

Proof. Combining Lemma 3 with (A11) yields:

$$f(z_r) > p^r \geq p^* = f(z^*)$$

(A22)

Hence, noting that $f'(\cdot) < 0$, it follows that:

$$z^r < z^* \text{ as required.}$$

Finally we establish:

Proposition 7. Average quality is lower under regulation than under unregulated monopoly.

Proof. Combining Lemma 5, (A10) and (A14) yields:

$$C'(Q^r) < f(z^r) < f(z^m) = C'(Q^m)$$

(A23)

(Noting that $f'(\cdot) < 0$ and $C'(\cdot) > 0$)

Hence: $Q^r < Q^m$ as required.

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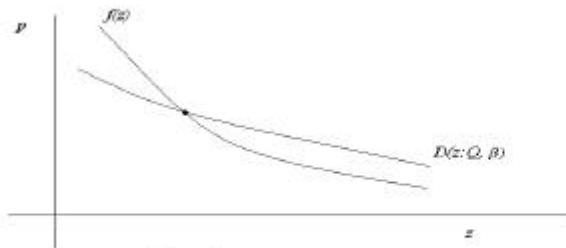


Figure 1

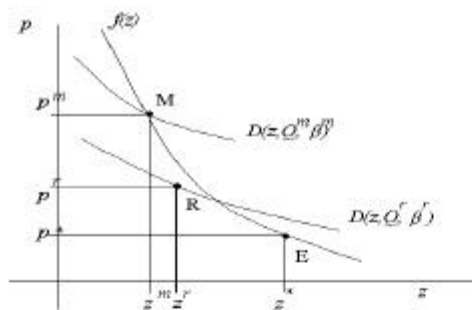


Figure 2. M is the unregulated monopoly equilibrium, R is the regulated monopoly equilibrium and E is the Pareto-efficient outcome

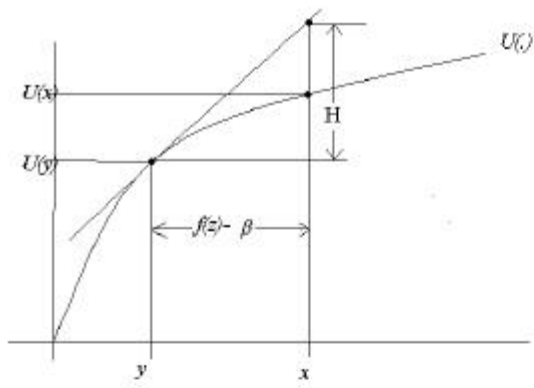


Figure 3.