Forecast Uncertainties in Macroeconometric Modelling: An Application to the UK Economy*

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Abstract

This paper argues that probability forecasts convey information on the uncertainties that surround macro-economic forecasts in a straightforward manner which is preferable to other alternatives, including the use of confidence intervals. Probability forecasts obtained using a small benchmark macroeconometric model as well as a number of other alternatives are presented and evaluated using recursive forecasts generated over the period 1999q1-2001q1. Out of sample probability forecasts of inflation and output growth are also provided over the period 2001q2-2003q1, and their implications discussed in relation to the Bank of England's inflation target and the need to avoid recessions, both as separate events and jointly. The robustness of the results to parameter and model uncertainties is also investigated by a pragmatic implementation of the Bayesian model averaging approach.

Keywords: Probability Forecasting, Bayesian Model Averaging, Long Run Structural VARs, Forecast Evaluation, Probability Forecasts of Inflation and Output Growth
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1 Introduction

With few exceptions, macroeconomic forecasts are presented in the form of point forecasts and their uncertainty is characterized (if at all) by forecast confidence intervals. Focusing on point forecasts is justified when the underlying decision problems faced by agents and the government are linear in constraints and quadratic in the loss function; the so-called LQ problem. But for most decision problems, reliance on point forecasts will not be sufficient and probability forecasts will be needed (see, for example, Granger and Pesaran, 2000a,b). It is also important that statements about economic policy are made in probabilistic terms, since the public's perception of the credibility of the policy has important implications for its success or failure, irrespective of whether the underlying decision problem is of the LQ type or not. A prominent example, discussed in Peel and Nobay (2000), is the choice of an optimal monetary policy in an economy where the government loss function is asymmetric around the inflation target. In this context, a stochastic approach to the credibility of the monetary policy will be required, and policy announcements should be made with reference to probabilistic statements, such as "the probability that inflation will fall in the range (π_L, π_U) is at least α per cent". Policy targets expressed in terms of a fixed range only partially account for the uncertainty that surrounds policy making. (See, for example, Yates (1995)).

One of the main advantages of the use of probability forecasts as a means of conveying the uncertainties surrounding forecasts is their straightforward use in decision theoretic contexts. In a macroeconomic context, the motivation for the current monetary policy arrangements in the UK is that it provides for transparency in policy-making and an economic environment in which firms and individuals are better able to make investment and consumption decisions. The range of possible decisions that a firm can make regarding an investment plan, for example, represents the firm's action space. The 'states of nature' in this case are defined by all of the possible future out-turns for the macro-economy. For example, the investment decision might rely on output growth in the next period, or the average output growth over some longer period, remaining positive; or interest might focus on the future path of inflation and output growth considered together. In making a decision, the firm should define a loss function which evaluates the profits or losses associated with each point in the action space and given any 'state of nature'. Except for LQ decision problems, decisions rules by individual households and firms will generally require probability forecasts with respect to different threshold values reflecting their specific cost-benefit ratios. For this purpose, we need to provide estimates of the whole probability distribution function of the events of interest, rather than point forecasts or particular forecast intervals which are likely to be relevant only to the decision problem of a few.

The need for probability forecasts is acknowledged by a variety of researchers and institutions. In the statistics literature, for example, Dawid (1984) has been advocating the use of probability forecasting in a sequential approach to the statistical analysis of data; the so-called "prequential approach". In the macroeconometric modelling literature, Fair (1980) was one of the first to compute probability forecasts using a macroeconometric model of the US economy. The Bank of England routinely publishes a range of outcomes for its inflation and output growth forecasts (see Britton, Fisher and Whitley, 1998, or Wallis, 1999); the National Institute use their model to produce probability statements alongside their central forecasts (their methods are described in Blake, 1996, and Poulizac et al., 1996); and in the financial sector, J.P. Morgan presents 'Event Risk Indicators' in its analysis of foreign exchange markets. However, it remains rare for forecasters to provide probability forecasts in a systematic manner. One explanation might be due to the difficulty in measuring the uncertainties associated with forecasts in the large-scale macroeconometric models typically employed. Another explanation relates to the various types of uncertainty that are involved in forecasting. For example, probability forecasts typically provided in the literature deal with future uncertainty only, assuming that the model and its parameters are known with certainty. This is true of the probability forecasts published by the National Institute, for example.

This paper considers probability forecasting in the context of a small long-run structural vector error correcting autoregressive model (VECM) of the UK economy. Particular events of interest include inflation falling within a pre-specified target range and/or output growth remaining positive over two subsequent quarters. For this purpose, we provide a pragmatic implementation of the Bayesian Model Averaging (BMA) approach that allows for parameter as well as model uncertainties. The 'benchmark' model used for computation of probability forecasts is based on a revised and updated version of the model in Garratt *et al.* (2001, forthcoming) and contains five long-run relations subject to 23 over-identifying restrictions predicted by economic theory. This version, specifically updated for forecasting purposes, employs the long-run relations estimated over a long sample period starting from 1965q1, but bases the estimation of the short-run coefficients on a shorter sample period starting from 1985q1. In addition we consider thirteen further models that focus on alternative assumptions regarding the number of long-run relations and the specification of an oil price equation, assumed as weakly exogenous with respect to the UK model. These 14 models are used in a probability forecast evaluation exercise over the period 1999q1-2001q1, as well as for generating out-of-sample point and probability forecasts of inflation and output growth over the period 2001q2-2003q1. The forecast evaluation exercise is carried out recursively and provides statistically significant evidence of forecasting performance both for the theory-based model and for the 'average' model using Akaike or equal weights. The average model based on the Schwarz weights does not perform as well.

In generating out-of-sample probability forecasts, amongst the many possible macroeconomic events of interest, we focus on the possibility of a "recession" and the likelihood of the inflation rate falling within the range 1.5%-3.5%, the target range currently considered by the Monetary Policy Committee (MPC) of the Bank of England. We consider these and a number of related events both singly and jointly. In particular, based on information available at the end of 2001q1 and using the benchmark model, we estimate the probability of inflation falling within the Bank of England's target range to be relatively high, with only a small probability of a recession. These results seem to be robust to model uncertainty of the type considered in this paper.

The lay-out of the rest of the paper is as follows. Section 2 considers different sources of forecast uncertainties and discusses alternative approaches used to deal with them. This Section also gives a brief review of the computational issues involved in estimation of probability forecasts in the presence parameter and model uncertainties. Sections 3 and 4 provide an application of the probability forecasting approach to the UK economy. Section 3 presents the model, its parameter estimates, and the results of a probability forecast evaluation exercise. Section 4 provides a brief account of inflation targeting in the UK, presents single and joint event probability forecasts involving output growth and inflation objectives at different forecast horizons both using the benchmark model and alternative model averaging procedures. Section 5 offers some concluding remarks. Details of how probability forecasts are computed are provided in an Appendix.

2 Alternative Approaches to Characterizing Forecast Uncertainties

Generally speaking model-based forecasts are subject to five different types of uncertainties: future, parameter, model, policy and measurement uncertainties. This paper focusses on the first three and considers how to allow for them in the computation of probability forecasts using an error correcting vector autoregressive model of the UK economy. Policy and measurement uncertainties pose special problems of their own and will not be addressed in this paper. Future uncertainty refers to the effects of unobserved future shocks on forecasts, while parameter and model uncertainties are concerned with the robustness of forecasts to the choice of parameter values (for a given model) and available alternative models more generally.

The standard textbook approach to taking account of future and parameter uncertainties is through the use of forecast intervals around point forecasts. Although such forecast intervals may contain important information about probability forecasts of interest to a particular decision maker, they do not allow for a full recovery of the forecast probability distribution function which is needed in decision making contexts where the decision problem is not of the LQ type. The relationships between forecast intervals and probability forecasts become even more tenuous when forecasts of *joint* events or forecasts from multiple models are considered. For example, it would be impossible to infer the probability of the joint event of a positive output growth and an inflation rate falling within a pre-specified range from given variable-specific forecast intervals. Many different such intervals will be needed for this purpose. In fact, even if the primary object of interest is a point forecast, as we shall see below, consideration of probability forecasts can help clarify how best to pool point forecasts in the presence of model uncertainty.

Suppose we are interested in a decision problem that requires probability forecasts of an event defined in terms of one or more elements of $\mathbf{z}_t$, for $t = T+1, T+2, ..., T+h$, where $\mathbf{z}_t = (z_{1t}, z_{2t}, ..., z_{nt})'$ is an $n \times 1$ vector of the variables of interest and h is the forecast (decision) horizon. Assume also that the data generating process (DGP) is unknown and the forecasts are made considering m different models indexed by i (that could be nested or non-nested). Each model, M_i, $i = 1, 2, ..., m$, is characterized by a probability density function of $\mathbf{z}_t$ defined over the estimation period $t = 1, 2, ..., T$, as well as the forecast period $t = T+1, T+2, ..., T+h$, in terms of a $k_i \times 1$ vector of unknown parameters, $\boldsymbol{\theta}_i$, assumed to lie in the compact parameter space, Θ_i. Model M_i is then defined by

$$M_i : \quad \{f_i(\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_T, \mathbf{z}_{T+1}, \mathbf{z}_{T+2}, ..., \mathbf{z}_{T+h}; \boldsymbol{\theta}_i), \quad \boldsymbol{\theta}_i \in \Theta_i\}, \tag{1}$$

where $f_i(.)$ is the joint probability density function of past and future values of $\mathbf{z}_t$. Conditional on each model, M_i, being true we shall assume that the true value of $\boldsymbol{\theta}_i$, which we denote by $\boldsymbol{\theta}_{i0}$, is fixed and remains constant across the estimation and the prediction periods and lies in the interior of Θ_i. We denote the maximum likelihood estimate of $\boldsymbol{\theta}_{i0}$ by $\widehat{\boldsymbol{\theta}}_{iT}$, and assume that it satisfies the usual regularity conditions so that

$$\sqrt{T}\left(\widehat{\boldsymbol{\theta}}_{iT} - \boldsymbol{\theta}_{i0}\right) |M_i \overset{a}{\backsim} N(\mathbf{0}, \mathbf{V}_{\theta_i}),$$

where $\overset{a}{\backsim}$ stands for "asymptotically distributed as", and $T^{-1}\mathbf{V}_{\theta_i}$ is the asymptotic covariance matrix of $\widehat{\boldsymbol{\theta}}_{iT}$ conditional on M_i.[1] Under these assumptions, parameter uncertainty only arises when T is finite. The case where $\boldsymbol{\theta}_{i0}$ could differ across the estimation and forecast periods poses new difficulties and can be resolved in a satisfactory manner if one is prepared to formalize how $\boldsymbol{\theta}_{i0}$ changes over time.

The object of interest is the probability density function of $\mathbf{Z}_{T+1,h} = (\mathbf{z}_{T+1}, \mathbf{z}_{T+2}, ..., \mathbf{z}_{T+h})$ conditional on the available observations at the end of period T, $\mathbf{Z}_T = (\mathbf{z}_1, \mathbf{z}_2, ..., \mathbf{z}_T)$. This will be

[1] In the case of cointegrating vector autoregressive models analysed in the next section, a more general version of this result is needed. This is because the cointegrating coefficients converge to their asymptotic distribution at a faster rate than the other parameters in the model. However, the general results of this section are not affected by this complication.

denoted by $\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T)$. For this purpose, models and their parameters serve as intermediate inputs in the process of characterization and estimation of $\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T)$. The Bayesian approach provides an elegant and logically coherent solution to this problem, with a full solution given by the so-called "Bayesian model averaging" formula (see, for example, Draper (1995), Hoeting et al. (1999)):

$$\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T) = \sum_{i=1}^{m} \Pr(M_i|\mathbf{Z}_T)\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i), \tag{2}$$

where $\Pr(M_i|\mathbf{Z}_T)$ is the posterior probability of model M_i,

$$\Pr(M_i|\mathbf{Z}_T) = \frac{\Pr(M_i)\Pr(\mathbf{Z}_T|M_i)}{\sum_{j=1}^{m}\Pr(M_j)\Pr(\mathbf{Z}_T|M_j)}, \tag{3}$$

$\Pr(M_i)$ is the prior probability of model M_i, $\Pr(\mathbf{Z}_T|M_i)$ is the integrated likelihood

$$\Pr(\mathbf{Z}_T|M_i) = \int_{\boldsymbol{\theta}_i} \Pr(\boldsymbol{\theta}_i|M_i)\Pr(\mathbf{Z}_T|M_i, \boldsymbol{\theta}_i)d\boldsymbol{\theta}_i, \tag{4}$$

$\Pr(\boldsymbol{\theta}_i|M_i)$ is the prior on $\boldsymbol{\theta}_i$ conditional on M_i, $\Pr(\mathbf{Z}_T|M_i, \boldsymbol{\theta}_i)$ is the likelihood function of model M_i, and $\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i)$ is the posterior predictive density of model M_i defined by

$$\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i) = \int_{\boldsymbol{\theta}_i} \Pr(\boldsymbol{\theta}_i|\mathbf{Z}_T, M_i)\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i, \boldsymbol{\theta}_i)d\boldsymbol{\theta}_i, \tag{5}$$

in which $\Pr(\boldsymbol{\theta}_i|\mathbf{Z}_T, M_i)$ is the posterior probability of $\boldsymbol{\theta}_i$ given model M_i:

$$\Pr(\boldsymbol{\theta}_i|\mathbf{Z}_T, M_i) = \frac{\Pr(\boldsymbol{\theta}_i|M_i)\Pr(\mathbf{Z}_T|M_i, \boldsymbol{\theta}_i)}{\sum_{j=1}^{m}\Pr(M_j)\Pr(\mathbf{Z}_T|M_j)}. \tag{6}$$

The Bayesian approach requires *a priori* specifications of $\Pr(M_i)$ and $\Pr(\boldsymbol{\theta}_i|M_i)$ for $i = 1, 2, ..., m$, and further assumes that one of the m models being considered is the DGP so that $\Pr(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T)$ defined by (2) is proper.

The Bayesian model averaging formula also provides a simple "optimal" solution to the problem of pooling of the point forecasts, $E(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i)$, studied extensively in the literature (see Clemen (1989) and Diebold and Lopez (1996) for reviews), namely

$$E(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T) = \sum_{i=1}^{m} \Pr(M_i|\mathbf{Z}_T)E(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i),$$

with the variance given by (see, for example, Draper (1995))

$$\begin{aligned} V(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T) &= \sum_{i=1}^{m} \Pr(M_i|\mathbf{Z}_T)V(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i) \\ &\quad + \sum_{i=1}^{m} \Pr(M_i|\mathbf{Z}_T)\left[E(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T, M_i) - E(\mathbf{Z}_{T+1,h}|\mathbf{Z}_T)\right]^2, \end{aligned}$$

where the first term accounts for within model variability and the second term for between model variability.

There is no doubt that the Bayesian model averaging (BMA) provides an attractive solution to the problem of accounting for model uncertainty. But its strict application can be problematic particularly in the case of high-dimensional models such as the vector error correcting model of the U.K. economy which we shall be considering in the next Section. The major difficulties lie in the choice of the space of models to be considered, the model priors $\Pr(M_i)$, and the specification of meaningful priors for the unknown parameters, $\Pr(\boldsymbol{\theta}_i | M_i)$. The computational issues, while still considerable, are partly overcome by Monte Carlo integration techniques. For an excellent over-view of these issues, see Hoeting et al. (1999). Also see Fernandez et al. (2001a,b) for specific applications.

Putting the problem of model specification to one side, the two important components of BMA formula are the posterior probability of the models, $\Pr(M_i | \mathbf{Z}_T)$, and the posterior density functions of the parameters, $\Pr(\boldsymbol{\theta}_i | \mathbf{Z}_T, M_i)$, for $i = 1, ..., m$. In what follows we shall consider probability forecasts of certain events of interest by considering different approximations of $\Pr(M_i | \mathbf{Z}_T)$ and $\Pr(\boldsymbol{\theta}_i | \mathbf{Z}_T, M_i)$ assuming that T is sufficiently large such that the sample observations dominate the choice of the priors; in essence adopting a classical stance within an otherwise Bayesian framework.

2.1 Computation of Probability Forecasts

Suppose the *joint event* of interest is defined by $\boldsymbol{\varphi}(\mathbf{Z}_{T+1,h}) < \mathbf{a}$, where $\boldsymbol{\varphi}(.)$ and $\mathbf{a}$ are the $L \times 1$ vectors $\boldsymbol{\varphi}(.) = (\varphi_1(.), \varphi_2(.), ..., \varphi_L(.))'$, $\mathbf{a} = (a_1, a_2, ..., a_L)'$, $\varphi_l(\mathbf{Z}_{T+1,h})$ is a scalar function of the variables over the forecast horizon $T+1, ..., T+h$, and a_j is the "threshold" value associated with $\varphi_j(.)$. To simplify the exposition, we denote this joint event by $\mathfrak{A}_{\varphi}$. The (conditional) probability forecast associated with this event based on model M_i is given by

$$\pi_i(\mathbf{a}, h; \boldsymbol{\varphi}(.), \boldsymbol{\theta}_i) = \Pr\left[\boldsymbol{\varphi}(\mathbf{Z}_{T+1,h}) < \mathbf{a} \ | \mathbf{Z}_T, M_i, \boldsymbol{\theta}_i\right]. \tag{7}$$

In practice, we might also be interested in computing probability forecasts for a number of alternative threshold values over the range $a_j \in [a_{\min}, a_{\max}]$.

If the model is known to be M_i defined by (1) but the value of $\boldsymbol{\theta}_i$ is not known, a *point estimate* of $\pi_i(\mathbf{a}, h; \boldsymbol{\varphi}(.), \boldsymbol{\theta}_i)$ can be obtained by

$$\pi_i\left(\mathbf{a}, h; \boldsymbol{\varphi}(.), \widehat{\boldsymbol{\theta}}_{iT}\right) = \int_{\mathfrak{A}_{\varphi}} f_i(\mathbf{Z}_{T+1,h} \left| \mathbf{Z}_T, M_i, \widehat{\boldsymbol{\theta}}_{iT} \right.) d\mathbf{Z}_{T+1,h}. \tag{8}$$

This probability distribution function only takes account of future uncertainties that arise from the model's stochastic structure, as it is computed for a given density function, M_i, and for a given value of $\boldsymbol{\theta}_i$, namely $\widehat{\boldsymbol{\theta}}_{iT}$. It is also known as the "profile predictive likelihood". See, for example, Bjørnstad (1990).

To allow for parameter uncertainty, we assume that conditional on $\mathbf{Z}_T$, the probability distribution function of $\boldsymbol{\theta}_i$ is given by $g(\boldsymbol{\theta}_i | \mathbf{Z}_T, M_i)$. Then

$$\tilde{\pi}_i(\mathbf{a}, h; \boldsymbol{\varphi}(.)) = \int_{\boldsymbol{\theta}_i \in \Theta_i} \pi_i(\mathbf{a}, h; \boldsymbol{\varphi}(.), \boldsymbol{\theta}_i) \ g(\boldsymbol{\theta}_i | \mathbf{Z}_T, M_i) \ d\boldsymbol{\theta}_i, \tag{9}$$

or, equivalently,

$$\tilde{\pi}_i(\mathbf{a}, h; \boldsymbol{\varphi}(.)) = \int_{\boldsymbol{\theta}_i \in \Theta_i} \int_{\mathfrak{A}_{\varphi}} f_i(\mathbf{Z}_{T+1,h} | \mathbf{Z}_T, M_i, \boldsymbol{\theta}_i) \ g(\boldsymbol{\theta}_i | \mathbf{Z}_T, M_i) \, d\mathbf{Z}_{T+1,h} d\boldsymbol{\theta}_i. \tag{10}$$

In practice, computations of $\pi_i\left(\mathbf{a}, h; \boldsymbol{\varphi}(.), \widehat{\boldsymbol{\theta}}_{iT}\right)$ and $\tilde{\pi}_i(\mathbf{a}, h; \boldsymbol{\varphi}(.))$ are typically carried out by stochastic simulations. For further details, see Section 4 and the Appendix.

In a Bayesian context, $g(\boldsymbol{\theta}_i |\mathbf{Z}_T, M_i)$ is given by (6). Alternatively, in the case where the asymptotic normal theory applies to $\widehat{\boldsymbol{\theta}}_{iT}$, it may be reasonable to compute the probability density function assuming

$$\boldsymbol{\theta}_i |\mathbf{Z}_T, M_i \overset{a}{\backsim} N\left(\widehat{\theta}_{iT}, T^{-1}\widehat{\mathbf{V}}_{\theta_i}\right). \tag{11}$$

In this case, the point estimate of the probability forecast, $\pi_i\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_{iT}\right)$, and the alternative estimate, $\tilde{\pi}_i(\mathbf{a},h;\boldsymbol{\varphi}(.))$, that allows for parameter uncertainty are asymptotically equivalent as $T \to \infty$. The latter is the "bootstrap predictive density" described in Harris (1989) who demonstrates that it performs well in a number of important cases. Also, both of these estimates under M_i tend to $\pi_i(\mathbf{a},h;\boldsymbol{\varphi}(.),\boldsymbol{\theta}_{i0})$, which is the profile predictive likelihood evaluated at the true value $\boldsymbol{\theta}_{i0}$. But for a fixed T, the two estimates could differ, as the applications in Section 4 demonstrate. See Bjørnstad (1990, 1998) for reviews of the literature on predictive likelihood analysis.

The probability estimates that allow for model uncertainty can now be obtained using the Bayesian averaging procedure. Abstracting from parameter uncertainty we have

$$\pi\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_T\right) = \sum_{i=1}^{m} w_{iT}\ \pi_i\left(\mathbf{a},h;\boldsymbol{\varphi}(.),\widehat{\boldsymbol{\theta}}_{iT}\right), \tag{12}$$

where $\widehat{\boldsymbol{\theta}}_T = \left(\widehat{\boldsymbol{\theta}}'_{1T},...,\widehat{\boldsymbol{\theta}}'_{mT}\right)'$, and the weights, $w_{iT} \geq 0$ can be derived by approximating the posterior probability of model M_i, by (see, for example, Draper (1995))

$$\ln \Pr(M_i |\mathbf{Z}_T) = LL_{iT} - \left(\frac{k_i}{2}\right)\ln(T) + O(1), \tag{13}$$

where LL_{iT} is the maximized value of the log-likelihood function for model M_i, which is the familiar Schwarz (1978) Bayesian information criterion for model selection. The use of this approximation leads to the following choice for w_{iT}

$$w_{iT} = \frac{\exp(\Delta_{iT})}{\sum_{j=1}^{m}\exp(\Delta_{jT})}, \tag{14}$$

where $\Delta_{iT} = SBC_{iT} - Max_j(SBC_{jT})$ and $SBC_{iT} = LL_{iT} - \left(\frac{k_i}{2}\right)\ln(T)$. Alternatively, following Burnham and Anderson (1998), one could use Akiake weights defined by $\Delta_{iT} = AIC_{iT} - Max_j(AIC_{jT})$, $AIC_{iT} = LL_{iT} - k_i$. While the Schwartz weights are asymptotically optimal if the DGP lies in the set of models under consideration, the Akiake weights are likely to perform better when the models under consideration represent mere approximations to a complex and (possibly) unknowable DGP.

When parameter uncertainty is also taken into account, we have

$$\tilde{\pi}(\mathbf{a},h;\boldsymbol{\varphi}(.)) = \sum_{i=1}^{m} w_{iT}\tilde{\pi}_i(\mathbf{a},h;\boldsymbol{\varphi}(.)), \tag{15}$$

where $\tilde{\pi}_i(\mathbf{a},h;\boldsymbol{\varphi}(.))$ is the bootstrap predictive density defined by (10) that makes use of the normal approximation given by (11). For details of computations when $\mathbf{z}_t$ follows linear vector error correcting models see the Appendix.

2.2 Estimation and Forecasting with Conditional Models

The density function $f_i(.)$ can be decomposed in two ways. First, a sequential conditioning decomposition can be employed to write $f_i(.)$ as the product of the conditional distributions on successive observations on the $\mathbf{z}_t$,

$$f_i(\mathbf{Z}_t; \mathbf{z}_0, \boldsymbol{\theta}) = \prod_{s=2}^{t} f_i(\mathbf{z}_s \mid \mathbf{Z}_{s-1}; \mathbf{z}_0, \boldsymbol{\theta}_i),$$

for given initial values $\mathbf{z}_0$. And second, since we frequently wish to distinguish between variables which are endogenous, denoted by $\mathbf{y}_t$, and those which are exogenous, denoted by $\mathbf{x}_t$, we can write $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{x}_t')'$ and use the factorization:

$$f_i(\mathbf{z}_t \mid \mathbf{Z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}) = f_{iy}(\mathbf{y}_t \mid \mathbf{x}_t, \mathbf{Z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_{iy}) \times f_{ix}(\mathbf{x}_t \mid \mathbf{Z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_{ix}), \tag{16}$$

where $f_{iy}(\mathbf{y}_t \mid \mathbf{x}_t, \mathbf{Z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_y)$ is the conditional distribution of $\mathbf{y}_t$ given $\mathbf{x}_t$ under model M_i and the information available at time $t-1$, $\mathbf{Z}_{t-1}$, and $f_{ix}(\mathbf{x}_t \mid \mathbf{Z}_{t-1}; \mathbf{z}_0, \boldsymbol{\theta}_{ix})$ is the marginal density of $\mathbf{x}_t$ conditional on $\mathbf{Z}_{t-1}$. Note that the unknown parameters $\boldsymbol{\theta}_i$ are decomposed into the parameters of interest, $\boldsymbol{\theta}_{iy}$, and the parameters of the marginal density of the exogenous variables, $\boldsymbol{\theta}_{ix}$. In the case where $\mathbf{x}_t$ is strictly exogenous, knowledge of the marginal distribution of $\mathbf{x}_t$ does not help with the estimation of $\boldsymbol{\theta}_{iy}$, and estimation of these parameters can therefore be based entirely on the conditional distribution, $f_{iy}(\mathbf{y}_t \mid \mathbf{x}_t, \mathbf{Z}_{t-1}; \boldsymbol{\theta}_y)$.

Despite this, parameter uncertainty relating to $\boldsymbol{\theta}_{ix}$ can continue to be relevant for probability forecasts of the endogenous variables, $\mathbf{y}_t$, and forecast uncertainty surrounding the endogenous variables is affected by the way the uncertainty associated with the future path of the exogenous variables is resolved. In practice, the future values of $\mathbf{x}_t$ are often treated as known and fixed at pre-specified values. The resultant forecasts for $\mathbf{y}_t$ are then referred to as *scenario (or conditional) forecasts*, with each scenario representing a different set of assumed future values of the exogenous variables. This approach under-estimates the degree of forecast uncertainties. A more plausible approach would be to treat $\mathbf{x}_t$ as strongly (strictly) exogenous at the estimation stage, but to allow for the forecast uncertainties of the endogenous and the exogenous variables jointly. The exogeneity assumption will simplify the estimation process but does not eliminate the need for a joint treatment of future and model uncertainties associated with the exogenous variables and the endogenous variables.

3 An Application to the UK Economy

3.1 A Cointegrating VAR Model of the UK Economy

In principle, probability forecasts can be computed using any macroeconometric model, although the necessary computations would become prohibitive in the case of most large scale macroeconometric models, particularly if the objective of the exercise is to compute the probabilities of joint events at different horizons. At the other extreme, the use of small unrestricted VAR models, while computationally feasible, may not be satisfactory for the analysis of forecast probabilities over the medium term. An intermediate alternative that we shall follow here is to use a cointegrating VAR model that takes account of the long-run relationships that are likely to exist in a macro-economy. A model of this type has been developed for the UK by Garratt *et al.* (2000, 2001, 2003). This model is based on a number of long-run relations derived from arbitrage conditions in goods and capital markets, solvency and portfolio balance conditions. The model comprises six domestic variables whose developments are widely regarded as essential to a basic understanding of the U.K.

economy; namely, output, inflation, the exchange rate, the domestic relative to the foreign price level, the nominal interest rate and real money balances. It also contains three foreign variables: foreign output, foreign interest rate and oil prices.

The five long-run equilibrium relationships of the model outlined in Garratt *et al.* (2001) are given by:

$$p_t - p_t^* - e_t = b_{10} + b_{11}t + \xi_{1,t+1}, \tag{17}$$
$$r_t - r_t^* = b_{20} + \xi_{2,t+1}, \tag{18}$$
$$y_t - y_t^* = b_{30} + \xi_{3,t+1}, \tag{19}$$
$$h_t - y_t = b_{40} + b_{41}t + \beta_{42}r_t + \beta_{43}y_t + \xi_{4,t+1}, \tag{20}$$
$$r_t - \Delta p_t = b_{50} + \xi_{5,t+1}, \tag{21}$$

where p_t is the logarithm of domestic prices, p_t^* is the logarithm of foreign prices, e_t is the logarithm of nominal exchange rate (defined as the domestic price of a unit of the foreign currency), y_t is the logarithm of real per capita domestic output, y_t^* is the logarithm of real per capita foreign output, r_t is the domestic nominal interest rate variable, r_t^* is the foreign nominal interest rate variable, h_t is the logarithm of the real per capita money stock, we also use the variable p_t^o which is the logarithm of oil prices and $\xi_{i,t+1}$, $i = 1, 2, .., 5$, are stationary reduced form errors.

A detailed account of the framework for long run macro-modelling, describing the economic theory that underlies the relationships in (17) - (21), is provided in Garratt *et al.* (2001). In brief, we note here that (17) is the Purchasing Power Parity (PPP) relationship which assumes that, due to international trade in goods, domestic and foreign prices measured in a common currency equilibrate in the long-run. The inclusion of a linear trend in the PPP relation is intended to capture the possible persistent effects of productivity differentials on the real exchange rate known as Harrod-Balassa-Samuelson effect. Equation (18) is an Interest Rate Parity (IRP) relationship, which assumes that, under conditions of free capital flows, arbitrage between domestic and foreign bond holdings will, equilibrate domestic and foreign interest rates in the long-run. Equation (19) is an "output gap" (OG) relationship implied by a stochastic version of the Solow growth model with a common technological progress variable in production at home and abroad; (20) is a real money balance (RMB) relationship, based on the condition that the economy must remain financially solvent in the long run; and (21) is the Fisher Interest Parity (FIP) relationship which assumes that, due to inter-temporal exchange of domestic goods and bonds, the nominal rate of interest should in the long-run equate to the real rate of return plus the (expected) rate of inflation .

The five long-run relations of the model, (17) - (21), can be written compactly as:

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1 (t-1) - \mathbf{b}_0, \tag{22}$$

where $\mathbf{z}_t = (p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$, $\mathbf{b}_0 = (b_{01}, b_{02}, b_{03,} b_{04}, b_{05})'$, $\mathbf{b}_1 = (b_{11}, 0, 0, b_{41}, 0)$, $\boldsymbol{\xi}_t = (\xi_{1t}, \xi_{2t}, \xi_{3t}, \xi_{4t}, \xi_{5t})'$, and

$$\boldsymbol{\beta}^{'} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\beta_{42} & 0 & -\beta_{43} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{23}$$

Under the assumption that oil prices are "long-run forcing", efficient estimation of the parameters can be based on the following *conditional* error correction model:

$$\Delta \mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y \left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \Gamma_{yi} \Delta \mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo} \Delta p_t^o + \mathbf{u}_{yt}, \tag{24}$$

where $\mathbf{y}_t = (e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$, $\mathbf{a}_y$ is an 8×1 vector of fixed intercepts, $\boldsymbol{\alpha}_y$ is a 8×5 matrix of error-correction coefficients, $\{\Gamma_{yi}, i = 1, 2, ..., p-1\}$ are 8×9 matrices of short-run coefficients, $\boldsymbol{\psi}_{yo}$ is an 8×1 vector representing the impact effects of changes in oil prices on $\Delta \mathbf{y}_t$, and $\mathbf{u}_{yt}$ is an 8×1 vector of disturbances assumed to be $IID(0, \Sigma_y)$, with Σ_y being a positive definite matrix. This specification embodies the economic theory's long-run predictions, defined by (22), by construction.

For oil prices to be long-run forcing for $\mathbf{y}_t$, it is required that the error correction terms, $\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)$, are not statistically significant in the equation for oil prices, although lagged changes in $\mathbf{z}_t$ could be statistically significant. See Pesaran et al. (2000) for details. Harbo, Johansen, Nielson and Rahbek (1998) also provide an alternative analysis and use the concept of "weak exogeneity" instead of long-run forcing. A general specification that satisfies this condition is given by

$$\Delta p_t^o = a_o + \sum_{i=1}^{p-1} \Gamma_{oi} \Delta \mathbf{z}_{t-i} + u_{ot}, \tag{25}$$

where Γ_{oi} is a 1×9 vector of fixed coefficients and u_{ot} is a serially uncorrelated error term distributed independently of $\mathbf{u}_{yt}$. This specification encompasses the familiar random walk model as a special case and seems quite general for our purposes.

Combining (24) and (25), and solving for $\Delta \mathbf{z}_t$ yields the following reduced form equation

$$\Delta \mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha} \left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{v}_t, \tag{26}$$

where $\mathbf{a} = \left(a_o,\ \mathbf{a}_y' - a_o \boldsymbol{\psi}_{yo}'\right)'$, $\boldsymbol{\alpha} = \left(0,\ \boldsymbol{\alpha}_y'\right)'$, $\Gamma_i = \left(\Gamma_{oi}',\ \Gamma_{yi}' - \Gamma_{oi}' \boldsymbol{\psi}_{yo}'\right)'$ and $\mathbf{v}_t = \left(u_{ot},\ \mathbf{u}_{yt}' - u_{ot} \boldsymbol{\psi}_{yo}'\right)'$ is the vector of reduced form errors assumed to be $iid(0, \Sigma)$, where Σ is a positive definite matrix.

3.2 Estimation Results and In-sample Diagnostics

Estimation of the parameters of the conditional model, (24), can be carried out using the long-run structural modelling approach described in Pesaran and Shin (2002) and Pesaran et al. (2000). With this approach, having selected the order of the underlying VAR model (using model selection criteria such as the *AIC* or the *SBC*), we test for the number of cointegrating relations using the conditional model, (24), with unrestricted intercepts and restricted trend coefficients. As shown in Pesaran et al. (2000), these restrictions ensure that the solution of the model in levels of $\mathbf{z}_t$ will not contain quadratic trends. We then compute Maximum Likelihood (*ML*) estimates of the model's parameters subject to exact and over-identifying restrictions on the long-run coefficients.[2] If there is empirical support for the existence of five long-run relationships, as suggested by theory, exact identification in our model requires five restrictions on each of the five cointegrating vectors (each row of $\boldsymbol{\beta}$), or a total of twenty-five restrictions on $\boldsymbol{\beta}$. These represent only a subset of the

[2]The computations were carried out using Pesaran and Pesaran's (1997) Microfit 4.1.

restrictions suggested by economic theory as characterized in (23), however. Estimation of the model subject to all the (exact- and over-identifying) restrictions given in (23) enables a test of the validity of the over-identifying restrictions, and hence the underlying long-run economic theory, to be carried out.

Such an empirical exercise is conducted by Garratt *et al.* (2001) using UK data over the period 1965q1-1999q4. Their results showed that: (i) a VAR(2) model can adequately capture the dynamic properties of the data; (ii) there are five cointegrating relationships amongst the nine macroeconomic variables; and that (iii) the over-identifying restrictions suggested by economic theory, and described in (17) - (21) above, cannot be rejected. For the present exercise, we re-estimated the model on the more up-to-date sample, 1965q1-2001q1. The results continue to support the existence of 5 cointegrating relations, and are qualitatively very similar to those described in Garratt *et al.* (2001). For example, the interest rate coefficient in the real money balance equation, β_{42}, was estimated to be 75.68 (standard error 35.34), compared to 56.10 (22.28) in the original work, while the coefficient on the time trend, b_{41}, was estimated to be 0.0068 (0.0010), compared to 0.0073 (0.0012).

Since the modelling exercise here is primarily for the purpose of forecasting, we next re-estimated the model over the shorter period of 1985q1-2001q1, taking the long-run relations as given. The inclusion of the long-run relations estimated over the period 1965q1-2001q1 in a cointegrating VAR model estimated over the shorter sample period 1985q1-2001q1, is justified on two grounds: (i) as argued by Barassi et al. (2001) and Clements and Hendry (2002), the short-run coefficients are more likely to be subject to structural change as compared to the long-run coefficients; and (ii) the application of Johansen's cointegration tests are likely to be unreliable in small samples. Following this procedure, we are able to base the forecasts on a model with well-specified long-run relations, but which is also data-consistent, capturing the complex dynamic relationships that hold across the macroeconomic variables over recent years.

Table 1 gives the estimates of the individual error correcting relations of the benchmark model estimated over the 1985q1-2001q1 period. These estimates show that the error correction terms are important in most equations and provide for a complex and statistically significant set of interactions and feedbacks across commodity, money and foreign exchange markets. The estimated error correction equations pass most of the diagnostic tests and compared to standard benchmarks, fit the historical observations relatively well. In particular, the $\overline{R}^2$ of the domestic output and inflation equations, computed at 0.549 and 0.603 respectively, are quite high. The diagnostic statistics for tests of residual serial correlation, functional form and heteroskedasticity are well within the 90 per cent critical values, although there is evidence of non-normal errors in the case of some of the error correcting equations. Non-normal errors is not a serious problem at the estimation and inference stage, but can be important in Value-at-Risk analysis, for example, where tail probabilities are the main objects of interest. In such cases non-parametric techniques for computation of forecast probabilities might be used. See the Appendix for further details.

3.3 Model Uncertainty

The theory-based cointegrating model is clearly one amongst many possible models that could be used to provide probability forecasts of the main UK macroeconomic variables. Even if we confine our analysis to the class of VAR(p) models, important sources of uncertainties are the order of the VAR, p, the number of the long-run (or cointegrating) relations, r, the validity of the over-identifying restrictions imposed on the long-run coefficients, and the specification of the oil price equation. Given the limited time series data available, consideration of models with $p = 3$ or more did not seem advisable. We also thought it would not be worthwhile to consider $p = 1$ on the grounds that the resultant equations would most likely suffer from residual serial correlation. Therefore, we confined the choice of the models to be considered in the BMA procedure to exactly identified VAR(2) models with $r = 0, 1, .., 5$ and two alternative specifications of the oil price equation, namely (25), and its random walk counterpart, $\Delta p_t^o = a_o + u_{ot}$. Naturally, we also included our benchmark model in the set (for both specifications of the oil price equation), thus yielding a total of 14 models to be considered. We shall use these models in the forecast evaluation exercise below and in Section 4 to investigate the robustness of probability forecasts from the benchmark model to model uncertainty.

3.4 Evaluation and Comparisons of Probability Forecasts

In the evaluation exercise, each of the fourteen alternative models was used to generate probability forecasts for a number of simple events over the period 1999q1-2001q1. This was undertaken in a recursive manner, whereby we first estimated all the 14 models over the period 1985q1-1998q4 and computed one-step-ahead probability forecasts for 1999q1, then repeated the process moving forward one quarter at a time, ending with forecasts for 2001q1 based on models estimated over the period 1985q1-2000q4. The probability forecasts were computed for directional events of interest. In the case of $p_t - p_t^*$, e_t, r_t, r_t^* and $\Delta\widetilde{p}_t$, we computed the probability that these variables rise next period, namely $\Pr\left[\Delta(p_t - p_t^*) > 0 \mid \Omega_{t-1}\right]$, $\Pr\left[\Delta e_t > 0 \mid \Omega_{t-1}\right]$), and so on, where $\Omega_{t-1} = (\mathbf{Z}_{t-1}, \mathbf{z}_0, \mathbf{z}_{-1})$. For the remaining trended variables, ($y_t, y_t^*, h_t - y_t$ and p_t^o), we considered the event that the rate of change of these variables rise from one period to the next, namely $\Pr\left[\Delta^2 y_t > 0 \mid \Omega_{t-1}\right]$, $\Pr\left[\Delta^2 y_t^* > 0 \mid \Omega_{t-1}\right]$), and so on. The probability forecasts are computed recursively using the parametric stochastic simulation technique which allow for future uncertainty and the nonparametric bootstrap technique which allow for parameter uncertainty, as detailed in the Appendix. To allow for the effect of model uncertainty we employed the BMA formulae, (12) and (15), with the weights, w_{iT}, set according to the following three schemes: Akiake, Schwarz and equal weights ($w_{iT} = 1/14$). The first two are computed using (14). The probability forecasts were then evaluated using a number of different statistical techniques.

A general approach to evaluation of probability forecasts would be to use the probability integral transforms

$$u_i(\mathbf{z}_t) = \int_{-\infty}^{\mathbf{z}_t} p_{it}(\mathbf{x})\, d\mathbf{x}, \qquad t = T+1, T+2, ..., T+n,$$

where $p_{it}(\mathbf{x})$ is the forecast probability density function for model i, and $\mathbf{z}_t$, $t = T+1, T+2, ..., T+n$, the associated realizations. Under the null hypothesis that $p_{it}(\mathbf{x})$ coincides with the true density function of the underlying process, the probability integral transforms will be distributed as iid $U[0,1]$. This result, originally due to Rosenblatt (1952), has been used by Dawid (1984) and more recently by Diebold, Gunther and Tay (1998) in evaluation of probability forecasts. In our application, we first computed a sequence of one step ahead probability forecasts (with and without allowing for parameter uncertainty) for the nine simple events set out above over the nine quarters $1999q1, 1999q2, ..., 2001q1$, and hence the associated probability integral transforms, $u_i(\mathbf{z}_t)$, for the benchmark models (with the two specifications of oil price equation), and the three 'average' models (using Akaike, Schwarz and equal weights). To test the hypothesis that these probability integral transforms are random draws from $U[0,1]$, we calculated the Kolmogorov-Smirnov statistic, $KS_{in} = \sup_x |F_{in}(x) - U(x)|$, where $F_{in}(x)$ is the empirical cumulative distribution function (CDF) of the probability integral transforms, and $U(x) = x$, is the CDF of iid $U[0,1]$. Large values of the Kolmogorov-Smirnov statistics, KS_{in}, is indicative of significant departures of the sample CDF from the hypothesized uniform distribution.[3] The test results are summarized in Table 2.

For the over-identified benchmark specification, we obtained the value of 0.111 for the Kolmogorov-Smirnov statistic when only future uncertainty was allowed for, and the larger value of 0.136 when both future and parameter uncertainties were taken into account. The corresponding statistics for the benchmark model with the alternative oil price assumption were 0.123 and 0.136, respectively. All these statistics are well below the 5% critical value of Kolmogorov-Smirnov statistic (which for $n = 81$ is equal to 0.149), and the hypothesis that the forecast probability density functions coincide with the true ones cannot be rejected. The KS statistics for the probability forecasts based on the BMA procedure are also well below the 5% critical value with the notable exception of the forecasts based on the Akaike weights in the absence of parameter uncertainty.

Alternative measures of the accuracy of probability forecasts can be obtained by converting the probability forecasts into event forecasts by means of probability thresholds. (See, for example, the discussion in Pesaran and Granger (2000a)). For example, occurrence of an event can be forecast if its probability forecast exceeds a given threshold value, say 0.5. Applying this procedure to the events identified above we have 81 event forecasts and their associated realizations. The proportion of events predicted correctly by the various models are summarized in Table 2. They are all above 60%, with the probability forecasts that allow for parameter uncertainty performing slightly worse, except for the ones based on the Schwarz weights. It is also interesting to note that the benchmark model that does not allow for parameter uncertainty produces the best result. To check the statistical significance of these estimated proportions, we also computed the PT statistic proposed in Pesaran and Timmermann (1992) which is defined by $PT_n = \left(\widehat{P}_n - \widehat{P}_n^*\right) / \left\{\widehat{V}(\widehat{P}_n) - \widehat{V}(\widehat{P}_n^*)\right\}^{\frac{1}{2}}$, where n is the number of events considered, $\widehat{P}_n$ is the proportions of correctly predicted events, $\widehat{P}_n^*$ is the estimate of this proportion under the null hypothesis that forecasts and realizations are independently distributed, and $\widehat{V}(\widehat{P}_n)$ and $\widehat{V}(\widehat{P}_n^*)$ are the consistent estimates of the variances of $\widehat{P}_n$ and $\widehat{P}_n^*$, respectively. Under the null hypothesis, the PT statistic has a standard normal distribution. For the forecasts based on the benchmark model, we obtained $PT = 3.36$ when only future uncertainty was allowed for, and $PT = 2.35$ when both future and parameter uncertainties were taken into account. Both of these statistics are statistically significant. The random walk specification for the oil price equation resulted in PT values of 2.70 and 2.09 in the absence and presence of parameter uncertainty, respectively. Similar results were also obtained when we allowed for model uncertainty. Focussing on the average models, Akaike weights performed best followed by the probability forecasts based on equal weights, with the Schwarz weights coming last. It is, however, important to note that the PT test turned out to be statistically significant in the case of all the forecasts, suggesting that forecasting skill identified for the benchmark model is likely to be robust to parameter and model uncertainties. The results also provide some support in favour of imposing the theory-based long-run restrictions, although the strength of the evidence seem to depend on the choice of the oil price equation and whether parameter uncertainty is taken into account.

4 Probability Forecasts of Inflation and Output Growth

Having shown the viability of the cointegrating VAR model in forecasting, we shall now present out-of-sample probability forecasts of events relating to inflation targeting and output growth which are of particular interest for the analysis of macro-economic policy in the UK. Inflation targets

[3]For details of the Kolmogorov-Smirnov test and its critical values see, for example, Neave and Worthington (1992, pp.89-93).

have been set explicitly in the UK since October 1992, following the UK's exit from the European Exchange Rate Mechanism (ERM). The Chancellor's stated objective at the time was to achieve an average annual rate of inflation of 2%, while keeping the underlying rate of inflation within the 1%-4% range. In May 1997, the policy of targeting inflation was formalized further by the setting up of the Monetary Policy Committee (MPC), whose main objective is to meet inflation targets primarily by influencing the market interest rate through fixing the base rate at regular intervals. Its current remit, as set annually by the Chancellor, is to achieve an average annual inflation rate of 2.5%, with the rate falling in the target range 1.5%-3.5%.

The measure of inflation used by the MPC is the Retail Price Index, excluding mortgage interest payments, (RPI-x), and the time horizon over which the inflation objective is to be achieved is not stated. Inflation rates outside the target range act as a trigger, requiring the Governor of the Bank of England to write an open letter to the Chancellor explaining why inflation had deviated from the target, the policies being undertaken to correct the deviation, and how long it is expected before inflation is back on target. The Bank is also expected to conduct monetary policy so as to support the general economic policies of the government, so far as this does not compromise its commitment to its inflation target.

Since October 1992, the Bank of England has produced a quarterly *Inflation Report* which describes the Bank's assessment of likely inflation outcomes over a two-year forecast horizon. In addition to reviewing the various economic indicators necessary to place the inflation assessment into context, the *Report* provides forecasts of inflation over two year horizons, with bands presented around the central forecast to illustrate the range of inflation outcomes that are considered possible (the so-called fan charts). The forecasts are based on the assumption that the base rate is left unchanged. Since November 1997, a similar forecast of output growth has also been provided in the *Report*, providing insights on the Bank's perception of the likely outcome for the government's general economic policies beyond the maintenance of price stability. For a critical assessment of the Bank's approach to allowing for model and parameter uncertainties, see Wallis (1999).

The fan charts produced by the Bank of England are an important step towards acknowledging the significance of forecast uncertainties in the decision making process and it is clearly a welcome innovation. However, the approach suffers from two major shortcomings. First, it seems unlikely that the fan charts can be replicated by independent researchers. This is largely due to the subjective manner in which uncertainty is taken into account by the Bank, which may be justified from a real time decision-making perspective but does not readily lend itself to independent analysis. Second, the use of fan charts is limited for the analysis of uncertainty associated with joint events. Currently, the Bank provides separate fan charts for inflation and output growth forecasts, but in reality one may also be interested in joint events involving both inflation and output growth, and it is not clear how the two separate fan charts could be used for such a purpose. Here, we address both of these issues using the benchmark long-run structural model and the various alternative models discussed in the previous section.

In what follows, we present plots of estimated predictive distribution functions for inflation and output growth at a number of selected forecast horizons. These plots provide us with the necessary information with which to compute probabilities of a variety of events, and demonstrate the usefulness of probability forecasts in conveying the future and parameter uncertainties that surround the point forecasts. But our substantive discussion of the probability forecasts focuses on two central events of interest; namely, keeping the rate of inflation within the announced target range of 1.5 to 3.5 per cent and avoiding a recession. Following the literature, we define a recession as the occurrence of two successive negative quarterly growth rates. See, for example, Harding and Pagan (2000).

4.1 Predictive Distribution Functions

In the case of single events, probability forecasts are best represented by means of probability distribution functions. Figures 1 and 2 give the estimates of these functions for the four-quarter moving averages of inflation and output growth for the 1-, 4- and 8-quarters ahead forecast horizons based on the benchmark model (i.e. the over-identified version of the cointegrating model, (24), augmented with the oil price equation, (25)). These estimates are computed using the simulation techniques described in detail in the Appendix and take account of both future and parameter uncertainties. As before the probability estimates that allow for parameter uncertainty will be denote by $\widetilde{\pi}$, to distinguish them from probability estimates that do not, which we denote by π.

Figure 1 presents the estimated predictive distribution function for inflation for the threshold values ranging from 0% to 5% per annum at the three selected forecast horizons. Perhaps not surprisingly, the function for the one-quarter ahead forecast horizon is quite steep, but it becomes flatter as the forecast horizon is increased. Above the threshold value of 2.0%, the estimated probability distribution functions shift to the right as longer forecast horizons are considered, showing that the probability of inflation falling below thresholds greater than 2.0% declines with the forecast horizon. For example, the forecast probability that inflation lies below 3.5% becomes smaller at longer forecast horizons, falling from close to 100% one quarter ahead (2001q2) to 70% eight quarters ahead (2003q1). These forecast probabilities are in line with the recent historical experience: over the period 1985q1-2001q1, the average annual rate of inflation fell below 3.5% for 53.9 per cent of the quarters, but were below this threshold value throughout the last two years of the sample, 1999q1-2001q1.

Figure 2 plots the estimated predictive distribution functions for output growth. These functions also become flatter as the forecast horizon is increased, reflecting the greater uncertainty associated with growth outcomes at longer forecast horizons. These plots also suggest a weakening of the growth prospects in 2001 before recovering a little at longer horizons. For example, the probability of a negative output growth one quarter ahead (2001q2) is estimated to be almost zero, but rises to 14% four quarters ahead (2002q1) before falling back to 12% after eight quarters (2003q1). Therefore, a rise in the probability of a recession is predicted, but the estimate is not sufficiently high for it to be much of a policy concern (at least viewed from the end of our sample period 2001q1).

4.2 Event Probability Forecasts

Here we consider three single events of particular interest:

A : Achievement of inflation target, defined as the four-quarterly moving average rate of inflation falling within the range 1.5%-3.5%,

B : Recession, defined as the occurrence of two consecutive quarters of negative output growth,

C : Poor growth prospects, defined to mean that the four-quarterly moving average of output growth is less than 1%,

and the joint events $A \cap \overline{B}$ (Inflation target is met *and* recession is avoided), and $A \cap \overline{C}$ (Inflation target is met *combined* with reasonable growth prospects), where $\overline{B}$ and $\overline{C}$ are complements of B and C.

4.2.1 Inflation and the Target Range

Two sets of estimates of $\Pr(A_{T+h} \mid \Omega_T)$ are provided in Table 3 (for $h = 1, 2, ..., 8$) and depicted in Figure 3 over the longer forecast horizons $h = 1, 2, ..., 24$. The first set relates to π, which only take account of future uncertainty, and the second set relates to $\widetilde{\pi}$ which allow for both future and parameter uncertainties. Both π and $\widetilde{\pi}$ convey a similar message, but there are nevertheless some differences between them, at least at some forecast horizons, so that it is important that both estimates are considered in practice.

Based on these estimates, and conditional on the information available at the end of 2001q1, the probability that the Bank of England will be able to achieve the government inflation target is estimated to be high in the short-run but falls in the longer run, reflecting the considerable uncertainty surrounding the inflation forecasts at longer horizons. Specifically, the probability estimate is high in 2001q2, at 0.87 (0.80) for $\widetilde{\pi}$ (π), but it falls rapidly to nearer 0.45 by the end of 2001/early 2002. This fall in the first quarters of the forecast reflect the increasing likelihood of inflation falling below the 1.5% lower threshold (since the probability of observing inflation above the 3.5% upper threshold is close to zero through this period). Ultimately, though, the estimated probability of achieving inflation within the target range settles to 0.38 (0.35) for $\widetilde{\pi}$ (π) in 2003q1. At this longer forecast horizon, the probabilities of inflation falling below and above the target range are 0.32 and 0.30, respectively, using $\widetilde{\pi}$ (or 0.42 and 0.23 using π), so these figures reflect the relatively high degree of uncertainty associated with inflation forecasts even at moderate forecast horizons. Hence, while the likely inflation outcomes are low by historical standards and there is a reasonable probability of hitting the target range, there are also comparable likelihoods of undershooting and overshooting the inflation target range at longer horizons.

4.2.2 Recession and Growth Prospects

Figure 4 shows the estimates of the recession probability, $\Pr(B_{T+h} \mid \Omega_T)$ over the forecast horizons $h = 1, 2, ..., 24$. For this event, the probability estimates that allow for parameter uncertainty (i.e. $\widetilde{\pi}$) exceed those that do not (i.e. π) at shorter horizons, but the opposite is true at longer horizons. Having said this, however, π and $\widetilde{\pi}$ are very similar in size across the different forecast horizons and suggest a very low probability of a recession: based on the $\widetilde{\pi}$ estimate, for example, the probability of a recession occurring in 2001q2 is estimated to be around zero, rising to 0.09 in 2002q1. However, as shown in Table 4, the probability that UK faces poor growth prospects is much higher, in the region of 0.35 at the end of 2001, falling to 0.3 in 2003q1 according to the $\widetilde{\pi}$ estimates.

Single events are clearly of interest but very often decision makers are concerned with joint events involving, for example, both inflation and output growth outcomes. As examples here, we consider the probability estimates of the two joint events, $A_{T+h} \cap \overline{B}_{T+h}$, and $A_{T+h} \cap \overline{C}_{T+h}$ over the forecast horizons $h = 1, 2, ..., 24$. Probability estimates of these events (based on $\widetilde{\pi}$) are presented in Table 4. Both events are of policy interest as they combine the achievement of the inflation target with alternative growth objectives. For the event $A_{T+h} \cap \overline{B}_{T+h}$, the joint probability forecasts are similar in magnitude to those that for $\Pr(A_{T+h} \mid \Omega_T)$ alone at every time horizon. This is not surprising since the probability of a recession is estimated to be small at most forecast horizons and therefore the probability of avoiding recession is close to one. Nevertheless, the differences might be important since even relatively minor differences in probabilities can have an important impact on decisions if there are large, discontinuous differences in the net benefits of different outcomes. The probability forecasts for $A_{T+h} \cap \overline{C}_{T+h}$ are, of course, considerably less than those for $\Pr(A_{T+h} \mid \Omega_T)$ alone.

Figure 5 plots the values of the joint event probability over the forecast horizon alongside a plot of the product of the single event probabilities; that is $\Pr(A_{T+h} \mid \Omega_T) \times \Pr(\overline{B}_{T+h} \mid \Omega_T)$, $h = 1, 2, ..24$. This comparison provides an indication of the degree of dependence/independence of the two events. As it turns out, there is a gap between these of just under 0.1 at most forecast horizons. But the probabilities are relatively close, indicating little dependence between output growth prospects and inflation outcomes. This result is compatible with the long-term neutrality hypothesis that postulates independence of inflation outcomes from output growth outcomes in the long-run.

Figure 6 also plots the probability estimates of the joint event $A_{T+h} \cap \overline{B}_{T+h}$, but illustrates the effects of taking into account model uncertainty. The Figure shows three values of the probability of the joint event over the forecast horizon, each calculated without taking account of parameter uncertainty. One value is based on the benchmark model, but the other two show the weighted average of the probability estimates obtained from the fourteen alternative models described in the model evaluation exercise of the previous section. The weights in the latter two probability estimates are set equal in one of the estimates and are the in-sample posterior probabilities of the models approximated by the Akiake weights in the other. The plots show that estimated probabilities from the benchmark model are, by and large, quite close to the 'equal weights' estimate, but these are both lower than the AIC-weighted average, by more than 0.1 at some forecast horizons. Again, the extent to which these differences are considered large or important will depend on the nature of the underlying decision problem.

5 Concluding Remarks

One of the many problems economic forecasters and policy makers face is conveying to the public the degree of uncertainty associated with point forecasts. Policy makers recognize that their announcements, in addition to providing information on policy objectives, can themselves initiate responses which effect the macroeconomic outcome. This means that Central Bank Governors are reluctant to discuss either pessimistic possibilities, as this might induce recession, or more optimistic possibilities, since this might induce inflationary pressures. There is therefore an incentive for policy makers to seek ways of making clear statements regarding the range of potential macroeconomic outcomes for a given policy, and the likelihood of the occurrence of these outcomes, in a manner which avoids these difficulties.

In this paper, we have argued for the use of probability forecasts as a method of characterizing the uncertainties that surround forecasts from a macroeconomic model believing this to be superior to the conventional way of trying to deal with this problem through the use of confidence intervals. We argue that the use of probability forecasts has an intuitive appeal, enabling the forecaster (or users of forecasts) to specify the relevant "threshold values" which define the event of interest (e.g. a threshold value corresponding to an inflation target range of 1.5% to 3.5%). This is in contrast to the use of confidence intervals which define threshold values only implicitly, through the specification of the confidence interval widths, and these values may or may not represent thresholds of interest. A further advantage of the use of probability forecasts compared with the use of confidence intervals and over other more popular methods is the flexibility of probability forecasts, as illustrated by the ease with which the probability of joint events can be computed and analyzed. Hence, for example, we can consider the likelihood of achieving a stated inflation target range whilst simultaneously achieving a given level of output growth, with the result being conveyed in a single number. In situations where utility or loss functions are non-quadratic and/or the constraints are non-linear the whole predictive probability distribution function rather than its mean is required for decision making. This paper shows how such predictive distribution functions can be obtained in the case of long-run structural models, and illustrates its feasibility in the case of a small macro-econometric model of the UK.

The empirical exercise of the paper provides a concrete example of the usefulness of event probability forecasting both as a tool for model evaluation and as a means for conveying the uncertainties surrounding the forecasts of specific events of interest. The model used represents a small but comprehensive model of the UK macro-economic which incorporates long-run relationships suggested by economic theory so that it has a transparent and theoretically-coherent foundation. The model evaluation exercise not only demonstrates the statistical adequacy of the forecasts generated by the model but also highlights the considerable improvements in forecasts obtained through the imposition of the theory-based long-run restrictions. The predictive distribution functions relating to single events and the various joint event probabilities presented in the paper illustrate the flexibility of the functions in conveying forecast uncertainties and, from the observed independence of probability forecasts of events involving inflation and growth, in conveying information on the properties of the model. The model averaging approach also provides a coherent procedure to take account of parameter and model uncertainties as well as future uncertainty.

The various probability forecasts presented in the paper are encouraging from the point of view of the government's inflation objectives. Taking account of future as well as parameter and model uncertainties, the probability of inflation falling within the target range is quite high in the short run, accompanied with only a small probability of a recession. Over a longer forecast horizon the probability of inflation falling within the target range starts to decline, primarily due to a predicted rise in the probability of inflation falling below 1.5%, the lower end of the target range. Overall, however, based on information available at the end of 2001q1, the probability that the inflation objective is achieved with moderate output growths in the medium term is estimated to be reasonably high, certainly higher than the probabilities of inflation falling above or below the target range.

A Appendix: Computation of Probability Forecasts by Stochastic Simulation

This Appendix describes the steps involved in calculation of probability forecasts based on a vector error correction model using stochastic simulation techniques. The VAR model underlying the vector error correction model, (26), is given by

$$\mathbf{z}_t = \sum_{i=1}^{p} \Phi_i \mathbf{z}_{t-i} + \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{v}_t,\ t = 1, 2, ..., T, \tag{27}$$

where $\Phi_1 = \mathbf{I}_m - \boldsymbol{\alpha}\boldsymbol{\beta}' + \Gamma_1$, $\Phi_i = \Gamma_i - \Gamma_{i-1}$, $i = 2, ..., p-1$, $\Phi_p = -\Gamma_{p-1}$, $\mathbf{a}_0 = \mathbf{a}_y - \boldsymbol{\alpha}_y \mathbf{b}_1$, $\mathbf{a}_1 = \boldsymbol{\alpha}_y \mathbf{b}_1$ and $\mathbf{v}_t$ is assumed to be a serially uncorrelated *iid* vector of shocks with zero means and a positive definite covariance matrix, Σ. In what follows, we consider the calculation of probability forecasts first for given values of the parameters, and then taking into account parameter uncertainty.

A.1 Forecasts in the absence of parameter uncertainty

Suppose that the ML estimators of Φ_i, $i = 1, \ldots, p$, $\mathbf{a}_0$, $\mathbf{a}_1$ and Σ are given and denoted by $\hat{\Phi}_i$, $i = 1, \ldots, p$, $\hat{\mathbf{a}}_0$, $\hat{\mathbf{a}}_1$ and $\hat{\Sigma}$, respectively. Then the point estimates of the h-step ahead forecasts of $\mathbf{z}_{T+h}$ conditional on Ω_T, denoted by $\hat{\mathbf{z}}_{T+h}$, can be obtained recursively by

$$\hat{\mathbf{z}}_{T+h} = \sum_{i=1}^{p} \hat{\Phi}_i \hat{\mathbf{z}}_{T+h-i} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 (t+h),\ h = 1, 2, \ldots, \tag{28}$$

where the initial values, $\mathbf{z}_T, \mathbf{z}_{T-1}, \ldots, \mathbf{z}_{T-p+1}$, are given. To obtain probability forecasts by stochastic simulation, we simulate the values of $\mathbf{z}_{T+h}$ by

$$\mathbf{z}_{T+h}^{(r)} = \sum_{i=1}^{p} \hat{\Phi}_i \mathbf{z}_{T+h-i}^{(r)} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1(t+h) + \mathbf{v}_{T+h}^{(r)},\ h = 1, 2, \ldots;\ r = 1, 2, ..., R, \tag{29}$$

where superscript '(r)' refers to the r^{th} replication of the simulation algorithm, and $\mathbf{z}_T^{(r)} = \mathbf{z}_T$, $\mathbf{z}_{T-1}^{(r)} = \mathbf{z}_{T-1}, \ldots$, $\mathbf{z}_{T-p+1}^{(r)} = \mathbf{z}_{T-p+1}$ for all r. The $\mathbf{v}_{T+h}^{(r)}$'s can be drawn either by parametric or nonparametric methods as described in A.3 below. The probability that $\boldsymbol{\varphi}_\ell\left(\mathbf{z}_{T+1}^{(r)}, \ldots, \mathbf{z}_{T+h}^{(r)}\right) < \mathbf{a}_\ell$, is computed as

$$\pi_R\left(\mathbf{a}_\ell, h; \boldsymbol{\varphi}_\ell(.), \widehat{\boldsymbol{\theta}}\right) = \frac{1}{R}\sum_{r=1}^{R} I\left(\mathbf{a}_\ell - \boldsymbol{\varphi}_\ell\left(\mathbf{z}_{T+1}^{(r)}, \ldots, \mathbf{z}_{T+h}^{(r)}\right)\right),$$

where $I(A)$ is an indicator function which takes the value of unity if $A > 0$, and zero otherwise. To simplify the notations we denote $\pi_R\left(\mathbf{a}_\ell, h; \boldsymbol{\varphi}_\ell(.), \widehat{\boldsymbol{\theta}}\right)$ by $\pi_R(\mathbf{a}_\ell)$. The predictive probability distribution function is now given by $\pi_R(\mathbf{a}_\ell)$ as the threshold values, $\mathbf{a}_\ell$, are varied over the relevant regions.

A.2 Forecasts in the presence of parameter uncertainty

To allow for parameter uncertainty, we use the boot-strap procedure and first simulate S (*in-sample)* values of $\mathbf{z}_t$, $t = 1, 2, ..., T$, denoted by $\mathbf{z}_t^{(s)}$, $s = 1, ..., S$, where

$$\mathbf{z}_t^{(s)} = \sum_{i=1}^{p} \hat{\Phi}_i \mathbf{z}_{t-i}^{(s)} + \hat{\mathbf{a}}_0 + \hat{\mathbf{a}}_1 t + \mathbf{v}_t^{(s)},\ t = 1, 2, ..., T, \tag{30}$$

realizations are used for the initial values, $\mathbf{z}_{-1}, \ldots, \mathbf{z}_{-p}$, and $\mathbf{v}_t^{(s)}$'s can be drawn either by parametric or nonparametric methods. Having obtained the S set of simulated in-sample values, $\left(\mathbf{z}_1^{(s)}, \mathbf{z}_2^{(s)}, \ldots, \mathbf{z}_T^{(s)}\right)$, the $VAR(p)$ model (27) is estimated S times to obtain the ML estimates, $\hat{\Phi}_i^{(s)}$, $i = 1, 2, \ldots, p$, $\hat{\mathbf{a}}_0^{(s)}$, $\hat{\mathbf{a}}_1^{(s)}$ and $\hat{\Sigma}^{(s)}$, $s = 1, 2, ..., S$.

For each of these boot-strap replications, R replications of the h-step ahead point forecasts are computed as

$$\mathbf{z}_{T+h}^{(r,s)} = \sum_{i=1}^{p} \hat{\Phi}_i^{(s)} \mathbf{z}_{T+h-i}^{(r,s)} + \hat{\mathbf{a}}_0^{(s)} + \hat{\mathbf{a}}_1^{(s)}(t+h) + \mathbf{v}_{T+h}^{(r,s)},\ h = 1, 2, \ldots;\ r = 1, 2, ..., R, \tag{31}$$

and the predictive distribution function is then obtained by

$$\pi_{R,S}(\mathbf{a}_\ell) = \frac{1}{SR}\sum_{r=1}^{R}\sum_{s=1}^{S} I\left(\mathbf{a}_\ell - \boldsymbol{\varphi}_\ell\left(\mathbf{z}_{T+1}^{(r,s)}, \ldots, \mathbf{z}_{T+h}^{(r,s)}\right)\right),$$

A.3 Generating Simulated Errors

There are two basic ways that the in-sample and future errors, $\mathbf{v}_t^{(s)}$ and $\mathbf{v}_{T+h}^{(r,s)}$ respectively, can be simulated so that the contemporaneous correlations that exist across the errors in the different equations of the VAR model are taken into account. The first is a *parametric* method where the errors are drawn from an assumed probability distribution function. Alternatively, one could employ a *non-parametric* procedure. These are slightly more complicated and are based on re-sampling techniques in which the simulated errors are obtained by a random draw from the observed errors (See, for example, Hall (1992)).

A.3.1 Parametric Approach

Under this approach we assume that the errors are drawn from a multivariate distribution with zero means and the covariance matrix, $\hat{\Sigma}$. To obtain the simulated errors for m variables over h periods we first generate mh draws from an assumed *i.i.d.* distribution which we denote by $\boldsymbol{\varepsilon}_{T+i}^{(r,s)}$, $i = 1, 2, ..., h$. These are then used to obtain $\left\{\mathbf{v}_{T+i}^{(r,s)}, i = 1, 2, ...h\right\}$ computed as $\mathbf{v}_{T+h}^{(r,s)} = \hat{\mathbf{P}}^{(s)}\boldsymbol{\varepsilon}_{T+h}^{(r,s)}$ for $r = 1, 2, ..., R$ and $s = 1, 2, ..., S$, where $\hat{\mathbf{P}}^{(s)}$ is the lower triangular Choleski factor of $\hat{\Sigma}^{(s)}$ such that $\hat{\Sigma}^{(s)} = \hat{\mathbf{P}}^{(s)}\hat{\mathbf{P}}^{(s)\prime}$, and $\hat{\Sigma}^{(s)}$ is the estimate of Σ in the s^{th} replication of the boot-strap procedure set out above. In the absence of parameter uncertainty $\mathbf{v}_{T+h}^{(r)} = \hat{\mathbf{P}}\boldsymbol{\varepsilon}_{T+h}^{(r)}$ with $\hat{\mathbf{P}}$ being the lower triangular Choleski factor of $\hat{\Sigma}$. In our applications, for each r and s, we generate $\boldsymbol{\varepsilon}_{T+i}^{(r,s)}$ as $IIN(0, \mathbf{I}_m)$, although other parametric distributions such as the multi-variate Student t can also be used.

A.3.2 Non-Parametric Approaches

The most obvious non-parametric approach to generating the simulated errors, $\mathbf{v}_{T+h}^{(r,s)}$, which we denote 'Method 1', is simply to take h random draws with replacements from the in-sample residual vectors $\left\{\hat{\mathbf{v}}_1^{(s)}, \ldots, \hat{\mathbf{v}}_T^{(s)}\right\}$. The simulated errors thus obtained clearly have the same distribution and covariance structure as that observed in the original sample. However, this procedure is subject to the criticism that it could introduce serial dependence at longer forecast horizons since the pseudo-random draws are made from the same set of relatively small T vector of residuals.

An alternative non-parametric method for generating simulated errors, 'Method 2', makes use of the Choleski decomposition of the estimated covariance employed in the parametric approach. For a given choice of $\hat{\mathbf{P}}^{(s)}$ a set of mT transformed error terms $\left\{\hat{\varepsilon}_1^{(s)}, \ldots, \hat{\varepsilon}_T^{(s)}\right\}$ are computed such that $\hat{\varepsilon}_t^{(s)} = \hat{\mathbf{P}}^{(s)-1}\hat{\mathbf{v}}_t^{(s)}$, $t = 1, 2, ..., T$. The mT individual error terms are uncorrelated with each other, but retain the distributional information (relating to extreme values, and so on) contained in the original observed errors. A set of mh simulated errors are then obtained by drawing with replacement from these transformed residuals, denoted by $\left\{\varepsilon_{T+1}^{(r,s)}, \ldots, \varepsilon_{T+h}^{(r,s)}\right\}$. These are then used to obtain $\left\{\mathbf{v}_{T+1}^{(r,s)}, \ldots, \mathbf{v}_{T+h}^{(r,s)}\right\}$, recalling that $\mathbf{v}_{T+h}^{(r,s)} = \hat{\mathbf{P}}^{(s)}\varepsilon_{T+h}^{(r,s)}$ for $r = 1, 2, ..., R$ and $s = 1, 2, ..., S$. Given that the $\hat{\mathbf{P}}^{(s)}$ matrix is used to generate the simulated errors, it is clear that $\mathbf{v}_{T+h}^{(r,s)}$ again has the same covariance structure as the original estimated errors. And being based on errors drawn at random from the transformed residuals, these simulated errors will also display the same distributional features. Further, given that the re-sampling occurs from the mT transformed error terms, Method 2 also has the advantage over Method 1 that the serial dependence introduced through sampling with replacement is likely to be less problematic.

A.3.3 Choice of Approach

The non-parametric approaches described above have the advantage over the parametric approach that they make no distributional assumptions on the error terms, and are better able to capture the uncertainties arising from (possibly rare) extreme observations. However, they suffer from the fact that they require random sampling *with replacement*. Replacement is essential as otherwise the draws at longer forecast horizons are effectively 'truncated' and unrepresentative. On the other hand, for a given sample size, it is clear that re-sampling from the observed errors with replacement inevitably introduces serial dependence in the simulated forecast errors at longer horizons as the same observed errors are drawn repeatedly. When generating simulated errors over a forecast horizon, therefore, this provides an argument for the use of non-parametric methods over shorter forecast horizons, but suggests that a greater reliance might be placed on the parametric approach for the generation of probability forecasts at longer time horizons.

Table 1
Error Correction Specifications for the Over-identified Model: 1985q1-2001q1

Equation	$\Delta(p_t\text{-}p_t^*)$	Δe_t	Δr_t	Δr_t^*	Δy_t	Δy_t^*	$\Delta(h_t\text{-}y_t)$	$\Delta^2\widetilde{p}_t$
$\hat{\xi}_{1t}$	−.020* (.010)	.136* (.071)	.003 (.004)	.0006 (.001)	.010 (.009)	.002 (.006)	.031* (.017)	−.014* (.008)
$\hat{\xi}_{2t}$	−.775 (.664)	−2.59 (4.63)	−593† (.281)	.117 (.075)	.541 (.592)	.063 (.418)	−1.31 (1.09)	−1.05† (.508)
$\hat{\xi}_{3t}$	.022 (.060)	.073 (.414)	.029 (.025)	−.003 (.007)	−.061 (.050)	.057 (.037)	.271† (.098)	.087* (.045)
$\hat{\xi}_{4t}$	.010* (.006)	.003 (.043)	.004 (.003)	−.001 (.0007)	−.012† (.005)	.0004 (.004)	−.003 (.010)	.005 (.005)
$\hat{\xi}_{5t}$	.131 (.239)	2.04 (1.67)	.007 (.101)	−.014 (.027)	.315 (.203)	.060 (.150)	.257 (.393)	1.26 (.183)
$\Delta(p_{t-1}\text{-}p_{t-1}^*)$	.275 (.176)	−.588 (1.23)	−.030 (.074)	.007 (.020)	.136 (.149)	.031 (.111)	−.066 (.289)	.163 (.134)
Δe_{t-1}	.020 (.022)	.210 (.155)	−.0001 (.009)	.0004 (.003)	.019 (.029)	−.012 (.014)	.059 (.037)	−.025 (.017)
Δr_{t-1}	−.025 (.404)	−3.90 (2.81)	.214 (.171)	.053 (.046)	.190 (.342)	.025 (.254)	−.296 (.665)	.960† (.309)
Δr_{t-1}^*	−.839 (1.23)	5.74 (8.59)	−.120 (.522)	.407† (.139)	.784 (1.05)	−.732 (.775)	−2.42 (2.03)	1.15 (.943)
Δy_{t-1}	−.090 (.177)	−1.47 (1.23)	.009 (.075)	−.017 (.020)	.439† (.150)	.343† (.111)	−.782† (.291)	.252* (.135)
Δy_{t-1}^*	−.052 (.229)	.489 (1.51)	.131 (.097)	.072† (.026)	.351* (.194)	.184 (.053)	.386 (.377)	.147 (.175)
$\Delta(h_{t-1}\text{-}y_{t-1})$	.023 (.086)	−.081 (.588)	−.029 (.036)	−.001 (.010)	−.057 (.073)	−.007 (.053)	−.255* (.141)	−.023 (.066)
$\Delta^2\widetilde{p}_{t-1}$	−.064 (.171)	.860 (1.19)	−.012 (.072)	−.008 (.019)	−.019 (.145)	−.049 (.107)	−.194 (.281)	.017 (.131)
Δp_{t-1}^o	−.005 (.005)	.006 (.036)	−.0001 (.002)	−.0009 (.0006)	.012† (.004)	.005 (.003)	.006 (.009)	.003 (.004)
Δp_{t-1}^o	−.010† (.005)	−.019 (.032)	.002 (.002)	−.0007 (.0005)	−.010† (.004)	−.001 (.003)	−.001 (.007)	.004 (.003)
$\overline{R}^2$	.365	.089	.017	.476	.549	.371	.378	.603
$\hat{\sigma}$	.005	.032	.002	.001	.004	.003	.008	..003
$\chi^2_{SC}[4]$	4.31	3.16	9.40*	1.91	5.74	7.29	7.40	5.89
$\chi^2_{FF}[1]$	3.04	0.76	3.49*	2.26	0.86	2.31	0.02	0.98
$\chi^2_N[2]$	3.53	11.2†	7.13†	0.27	1.91	1.47	33.9†	26.0†
$\chi^2_H[1]$	0.01	0.01	1.08	0.01	0.83	0.84	0.17	.057

Table 2
Evaluation of Probability Forecasts

Models	Allowing for Future Uncertainty			Allowing for Future and Parameter Uncertainties		
	KS_n	$\widehat{P}_n$	PT_n	KS_n	$\widehat{P}_n$	PT_n
Bench (Δp_t^o:eq (25))	0.111	0.679	3.356	0.136	0.617	2.354
Bench (Δp_t^o: Random walk)	0.123	0.642	2.701	0.136	0.605	2.094
Equal Weights	0.062	0.630	2.346	0.111	0.630	2.322
Akaike Weights	0.160	0.642	2.701	0.136	0.630	2.451
Schwarz Weights	0.111	0.605	1.873	0.099	0.617	2.109

Table 3
Single Events Probability Estimates for Inflation

Forecast Horizon	$\Pr(\Delta p < 1.5\%)$		$\Pr(\Delta p < 2.5\%)$		$\Pr(\Delta p < 3.5\%)$		$\Pr(1.5\% < \Delta p < 3.5\%))$	
	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$	π	$\widetilde{\pi}$
2001q2	0.199	0.133	0.975	0.923	1.000	1.000	0.801	0.867
2001q3	0.440	0.277	0.886	0.740	0.995	0.968	0.555	0.691
2001q4	0.543	0.368	0.838	0.689	0.978	0.904	0.435	0.536
2002q1	0.448	0.295	0.688	0.538	0.885	0.766	0.437	0.471
2002q2	0.374	0.248	0.586	0.445	0.776	0.657	0.402	0.409
2002q3	0.404	0.289	0.593	0.486	0.774	0.685	0.370	0.396
2002q4	0.423	0.318	0.607	0.516	0.770	0.706	0.347	0.388
2003q1	0.420	0.325	0.602	0.519	0.748	0.704	0.328	0.379

Table 4
Single and Joint Probability Estimates Involving Output Growth and Inflation

Forecast Horizon	Pr(Recession)	$\Pr(\Delta y<1\%)$	$\Pr(1.5\% < \Delta p < 3.5\%$, No Recession)	$\Pr(1.5\% < \Delta p<3.5\%$, $\Delta y >1\%)$
	$\widetilde{\pi}$	$\widetilde{\pi}$	$\widetilde{\pi}$	$\widetilde{\pi}$
2001q2	0.000	0.045	0.867	0.830
2001q3	0.116	0.330	0.630	0.498
2001q4	0.082	0.349	0.501	0.381
2002q1	0.091	0.377	0.428	0.300
2002q2	0.092	0.313	0.374	0.279
2002q3	0.088	0.313	0.364	0.272
2002q4	0.089	0.304	0.356	0.270
2003q1	0.091	0.294	0.348	0.268

Notes to Table 1: The five error correction terms, estimated over the period 1965q1-2001q1, are given by $\hat{\xi}_{1,t+1}= p_t-p_t^*-e_t-4.8566, \hat{\xi}_{2,t+1} = r_t-r_t^*-0.0057, \hat{\xi}_{3,t+1}= y_t-y_t^*+0.0366, \hat{\xi}_{5,t+1}= r_t-\Delta\tilde{p}_t-0.0037$, and $\hat{\xi}_{4,t+1}= h_t-y_t+\underset{(35.34)}{75.68}r_t+\underset{(0.001)}{0.0068}t + 0.1283$. Standard errors are given in parenthesis. "*" indicates significance at the 10% level, and "†" indicates significance at the 5% level. The diagnostics are chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroscedasticity (H).

Notes to Table 2: The forecast evaluation statistics are based on one-step-ahead forecasts obtained from models estimated recursively, starting with the forecast of events in 1999q1 based on models estimated over 1985q1-1998q4 and ending with forecasts of events in 2001q1. The events of interest are described in Section 3. KS_n is the Kolmogorov-Smirnov statistic. The 5% critical value of KS_n for $n = 81$ is equal to 0.149, $\hat{P}_n$ is the proportion of events correctly forecast to occur, PT_n is the Pesaran and Timmermann (1992) test statistic which has a standard normal distribution.

Notes to Table 3 : The probability estimates for inflation relate to the four quarterly moving average of inflation defined by $400\times(p_{T+h}-p_{T+h-4})$, where p is the natural logarithm of the retail price index. The probability estimates (π and $\widetilde{\pi}$) are computed using the model reported in Table 2, where π only takes account of future uncertainty,and $\widetilde{\pi}$ accounts for both future and parameter uncertainties. The computations are carried out using 2,000 replications. See the Appendix for computational details.

Notes to Table 4: The probability estimates for output growth are computed from the forecasts of per capita output, assuming a population growth of 0.22% per annum. Recession is said to have occured when output growth (measured, quarter on quarter, by $400\times \ln\left(GDP_{T+h}/GDP_{T+h-1}\right)$ becomes negative in two consecutive quarters. Also see the notes to Table 3.

Figure 1: Predictive Distribution Functions for Inflation Using the Benchmark Model and allowing for Parameter Uncertainty

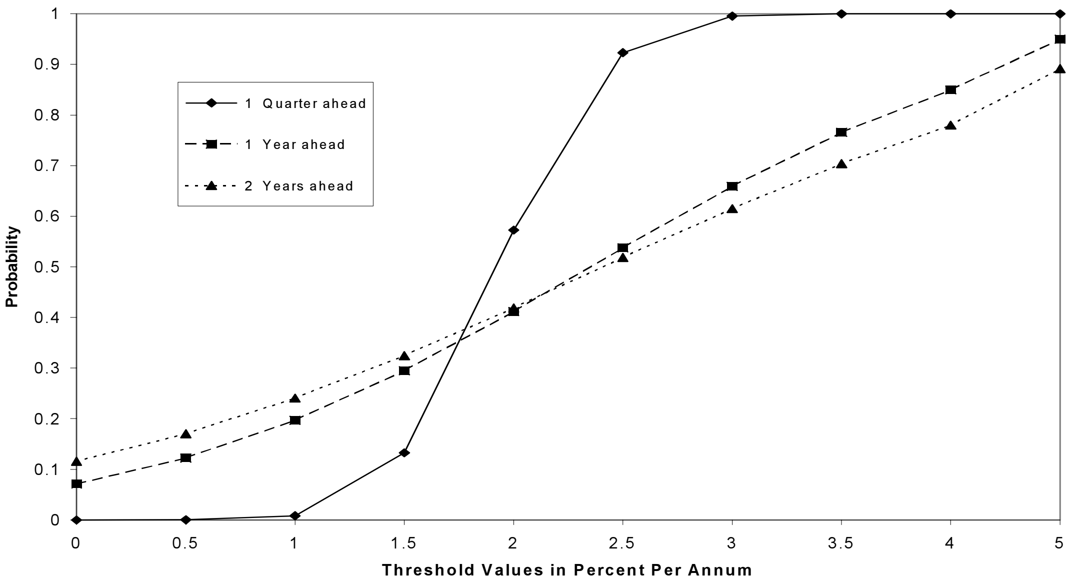


Figure 2: Predictive Distribution Functions for Output Growth Using the Benchmark Model and allowing for Parameter Uncertainty

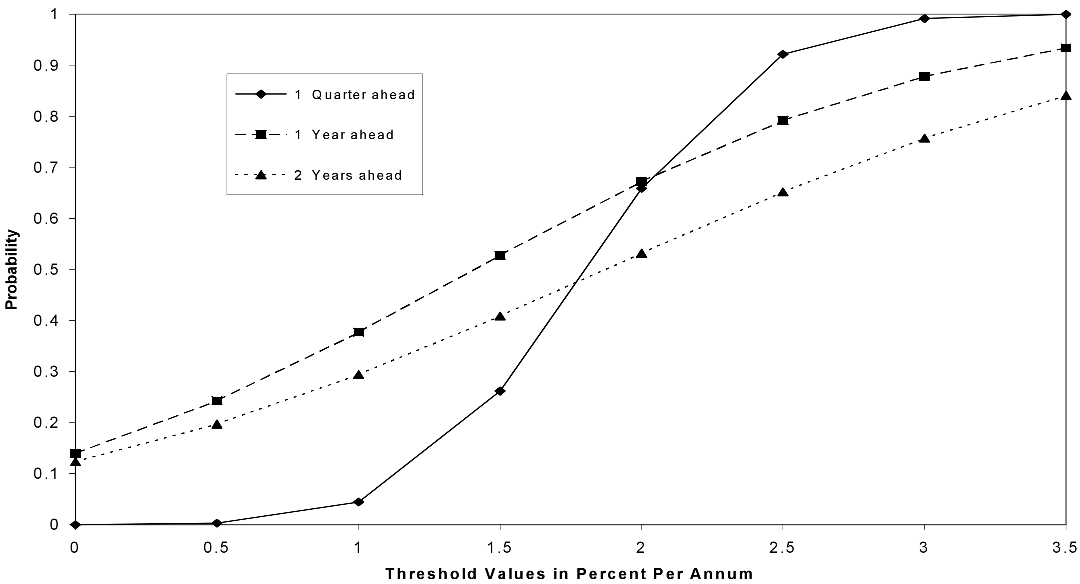


Figure 3: Probability Estimates of Inflation Falling within the Target Range using the Benchmark Model

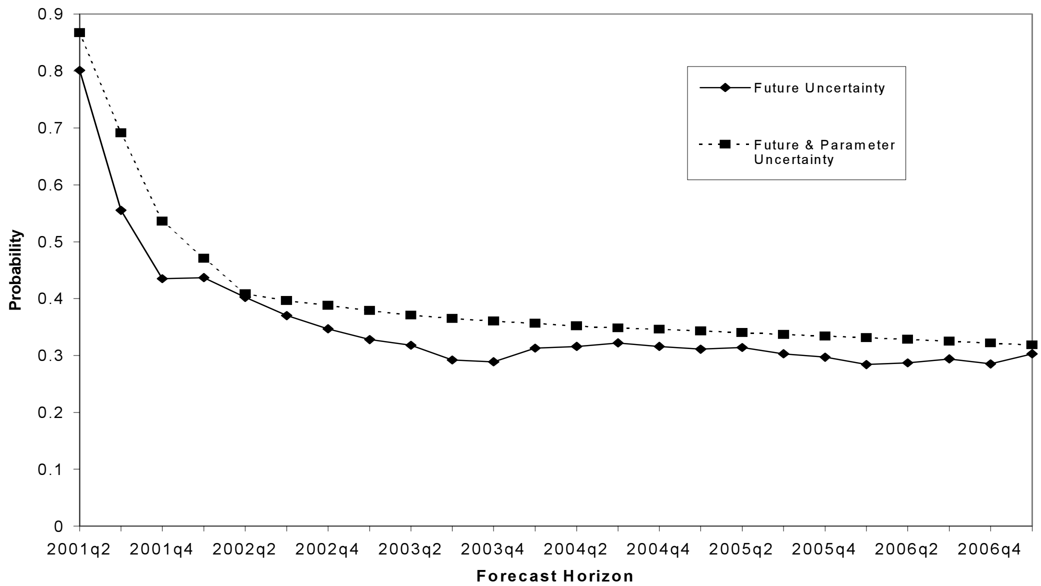


Figure 4: Probability Estimates of a Recession using the Benchmark Model

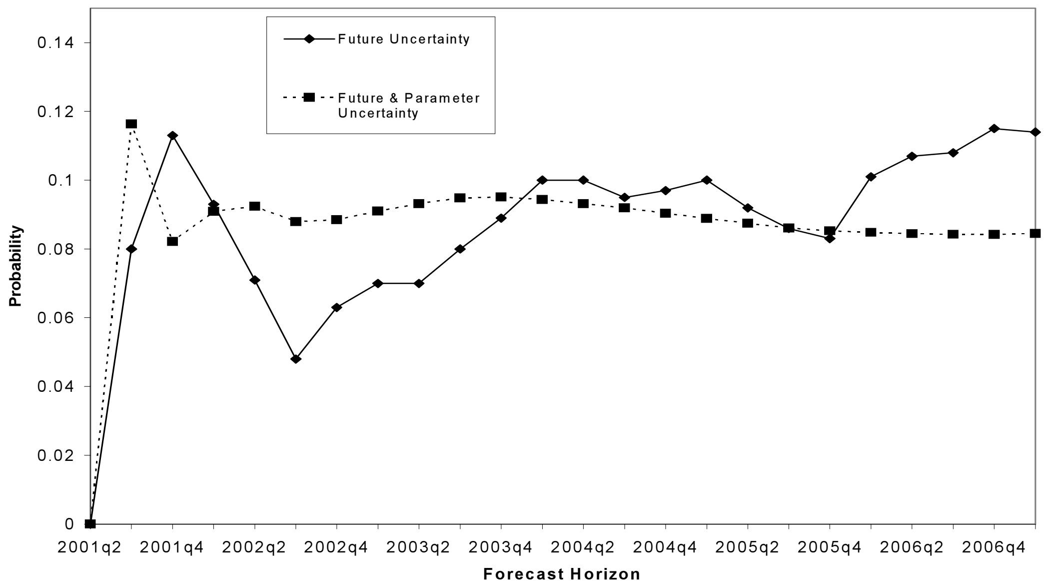
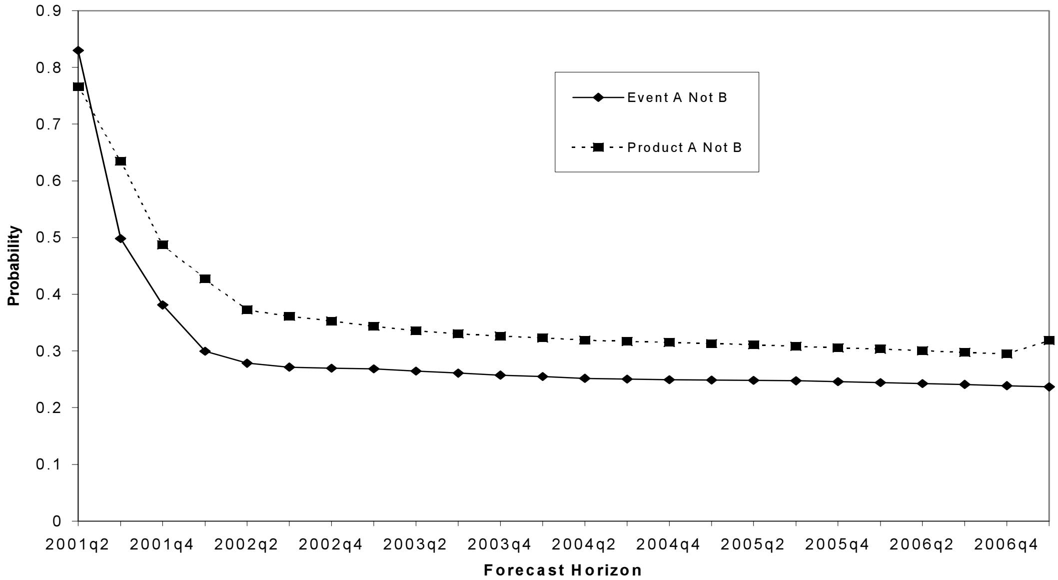
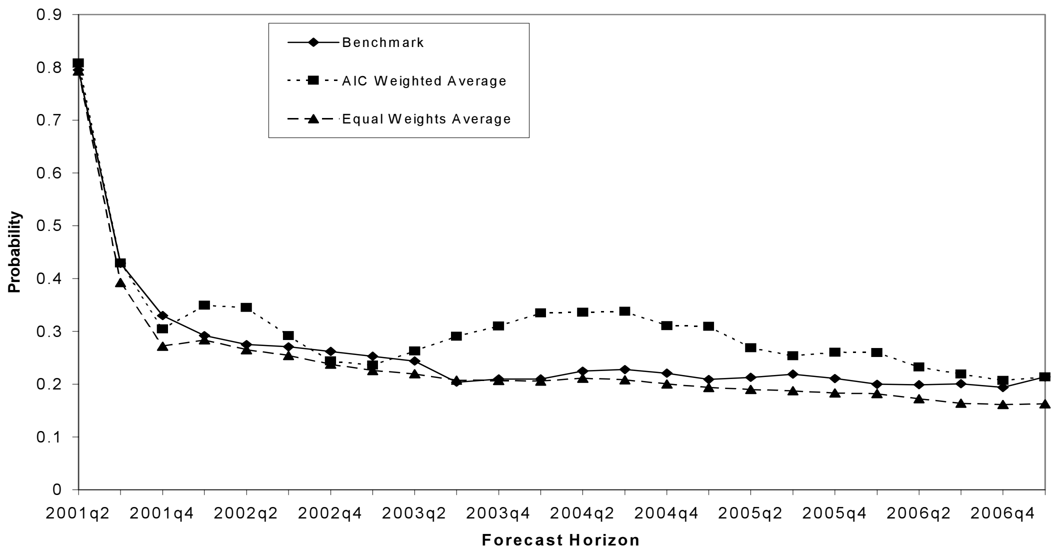


Figure 5: Probability Estimates of Meeting the Inflation Target without a Recession†



† The difference between the product and joint event probabilities measures the degree of independence between events A and $Not\ B$. All probability estimates plotted take into account both future and parameter uncertainty.

Figure 6: Probability Estimates of Meeting the Inflation Target without a Recession computed from Different Models (future uncertainty only)



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