

The Apple Tree

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Abstract

Consider an individual whose preferences are changing over time. How do we assess her welfare? We argue that this is an empirically relevant question as preferences are constantly changing over time if the agent has a bias towards the present. We present a simple example where preferences are changing over time (the apple tree) and use it to aid our discussion of welfare. We critically assess the approach that focuses on the decision maker (revealed preference) and the one that ignores the decision maker (long-run self) with reference to some recent literature. Finally, we look at some of the problems that arise when we consider a sovereign individual who aggregates her changing preferences.

1 Introduction

How do you assess the *welfare* of an individual? The answer of course will depend on what is meant by the term welfare. It could for example refer to the level of *happiness*. This would require a well defined concept of happiness and a method for measuring it. Needless to say, this is no simple task. Economists have adopted a concept that is well defined, normatively appealing and that allows us to avoid difficult philosophical questions. We link an individual's welfare to her preferences and consider relative welfare rather than levels of welfare. If x is preferred to y then the individual is better off with x relative to y or her welfare is greater with x than it is with y . Preferences are ascertained through observations of choices. If x is chosen rather than y then x is preferred to y . All that remains to have a well defined concept of welfare is the requirement that preferences and choices are consistent. Difficult social and moral judgements are resolved by the individual and her wellbeing is linked to her resolution of possibly conflicting (inner) objectives. At the heart of the methodology is the sovereignty of the individual in determining her welfare.

We will focus on a conceptual difficulty that arises when preferences are changing over time. We will ignore other methodological questions that arise by assuming that preferences are known. The conceptual problem that arises is simple, we do not have a coherent set of preferences that we can link to welfare. We will begin by arguing that this is an empirically relevant question. In section 2 we argue that temporal preferences are *always* changing over time. A methodology that ignores this is fundamentally flawed. In section 3 we present

a simple example where preferences are changing over time. With the aid of this example we look at whether we can maintain the conceptual link between a sovereign individual and welfare in section 4. Our arguments can be summarised as follows.

The revealed preference approach focuses on the decision maker and assumes that preferences are stationary over time. The shortcomings are obvious when applied to an example where preferences are changing. We also note a methodological problem with this approach, if choices are consistent over time we can neither falsify nor verify the hypothesis that preferences are stationary from observations of choices.

At the other extreme is an approach that ignores the decision maker and focuses instead on the preferences of the long-run self stripped of the short-term impulses that give rise to the intertemporal conflict. We argue that since the short-term self is ever present, we cannot simply ignore these impulses if we are to have a meaningful concept of welfare. We give an example where doing so gives a welfare maximising allocation that is not even Pareto efficient.

Finally, we consider the possibility of using the preferences of a temporally impartial higher self who aggregates the changing preferences. Here we identify another problem, the history dependence of preferences and beliefs. There is no sovereign individual who can compare alternative possible belief systems that arise from different histories as any such judgement requires beliefs. Without an invariable self, we cannot appeal to individual sovereignty to determine what is best for an individual.

2 Temporal Preferences

It is not uncommon for us to consider a past decision with regret. In many cases, the past decision was simply a mistake. If you could do it all again, you would revise your decision. There are also cases where there is a conflict between how you view the past decision now and how you viewed it when it was taken. If you could do it all again, you would not revise your decision. In the first case there is temporal consistency in the view that some other action would have been better for the individual. In this paper, we consider the second case where there is a conflict. A moment's reflection reveals the conflict. Consider the most painful episode, say a head-ache or stomach-ache that you have ever had. You are faced with the following hypothetical choice. You can have this episode now or you can delay it for 20 years, you must choose one or the other. You are to imagine that it is certain that you will still be alive in 20 years and to avoid complications arising from anticipation, if you choose the latter, the fact that you will have this episode in the future will be wiped from your memory. I prefer to delay it for 20 years. Now consider the choice, episode now or 20 years ago. This requires little reflection, I prefer to have had it 20 years ago. No doubt that I will also express this preference in 20 years (over the painful episode then or now). There is a clear inconsistency which arises from the fact that my evaluation of a choice now depends on how I will experience the

consequences of that choice now. Knowledge that I will experience this episode in the future or the memory of having experienced it in the past is not the same as experiencing it now. The same is true of pleasures. This gives rise to a bias towards the present. We will call this present bias. Should the individual resolve the inconsistency that arises from present bias? There is no *internal inconsistency* as these preferences are different at different points in time. The preferences are simply changing and at any point in time the individual chooses the most preferred alternative.

This view is captured by Shackle's term a moment-in-being.¹ At a moment-in-being the individual will have a memory of the past and expectations of the future. Her perceptions and anticipations will be shaped by her history. Any decision that she is faced with will be viewed from this unique and distinct position and her actions will be based on how she will experience the consequences of those actions *now*.²

We have used the term temporal preferences to refer to an individual's ranking of all lifetime alternatives. There seems to be some confusion on what constitutes changing preferences. There may be aspects to an individual's preferences that are not changing over time. For example, in a choice between consuming x today or consuming y today she may always choose the former. Although this gives us useful information about an individual's preferences, it does not tell us that preferences are stationary. It may be insightful to consider the following example which appears in the literature. An individual alternates daily between eating fish and red meat for dinner. The example has been used because there is a simple sense in which her preferences are changing (one day she prefers to eat fish the next red meat) and are stationary (conditional on red meat yesterday she always prefers fish today and vice versa). However, we do not know whether her *preferences* are stationary. It seems more reasonable to imagine that they are not. It may be the case that she prefers a history where she has had fish for the last six days and is having red meat today to one where she has been alternating between fish and red meat. Even if we suppose something like her *satisfactions* today depend only on yesterday's consumption, she may always prefer to be at a point in the cycle where she is having red meat today.

3 The Apple Tree

Isaac lives for three periods and consumes only apples from the apple tree in his garden. An apple tree $A_n \in A$, is characterised by a distribution of apples (a_1, a_2, a_3) where a_i is the number of apples produced in period i and $a_1 + a_2 + a_3 = 100$. Apples cannot be stored from one period to the next and there

¹Shackle [7]

²Parfit [4] (part two) also presents a compelling case in favour of the idea that we take decisions that are in our present best interest (rather than the prevailing assumption that we take decisions that are in our long-term self interest).

are no markets where Isaac can buy or sell apples. Isaac's apple tree is $A_1 = (33\frac{1}{3}, 33\frac{1}{3}, 33\frac{1}{3})$ which by assumption he cannot change. He does nevertheless, have *preferences* over apple trees. In period 2, his most preferred tree is $A_2 = (30.916, 38.168, 30.916)$ and his preferences over apple trees can be represented by the utility function

$$U_2(a_1, a_2, a_3) = 0.9\sqrt{a_1} + \sqrt{a_2} + 0.9\sqrt{a_3}. \quad (1)$$

Given that we know his preferences, can there be any objection to using (1) to represent Isaac's welfare? Well, shouldn't the apple tree $A_3 = (30.916, 30.916, 38.168)$ be welfare equivalent to A_2 ? The normative basis for such a welfare criterion is simply that there should be temporal neutrality in assessing consumption paths. The problem with this is that it ignores Isaac's preferences. Given that Isaac himself would choose A_2 , what position is anyone else in to tell him that he would be just as well off with A_3 . The same criticism will apply to any normative statement concerning Isaac's welfare that ignores his preferences. However, what if Isaac prefers A_3 to A_2 in period 3? This is where we run into a conceptual problem. There is a conflict between the period 2 self and period 3 self which makes it impossible to link welfare directly to preferences. Suppose that Isaac's preferences in periods 1 and 3 can be represented by the utility functions

$$U_1(a_1, a_2, a_3) = \sqrt{a_1} + 0.9\sqrt{a_2} + 0.81\sqrt{a_3}, \quad (2)$$

$$U_3(a_1, a_2, a_3) = 0.81\sqrt{a_1} + 0.9\sqrt{a_2} + \sqrt{a_3}. \quad (3)$$

His most preferred trees are $A_4 = (40.550, 32.845, 26.605)$ in period 1 and $A_5 = (26.605, 32.845, 40.550)$ in period 3. We wish to address the very simple question of how we evaluate Isaac's welfare over the 3 periods. More generally, we want to look at welfare when preferences are changing over time. We will begin by anticipating some objections that the reader may have with this example. Firstly, is there any sense in considering preferences over the past? Time after all is irreversible! This does not prevent us from having preferences over what might have been. One difficulty with this is that we may be mistaken about how we would feel now with a different history given that we have not experienced that history. We suppose that in periods 2 and 3, Isaac is able to imagine perfectly how he feels about a parallel life where he has consumed a different number of apples in the past. Furthermore, his preferences are not history dependent. Secondly, it is apparent from Isaac's utility functions that he discounts the past just as he discounts the future. The rationale for discounting the future is that agents are presently biased. Wouldn't it be ludicrous to imagine that a presently biased agent attaches increasing weight into the past? Temporal consistency requires that an agent does this at the same rate that she discounts the future. In fact, the idea that a presently biased agent may discount the past as well as the future seems so compelling, that it is a surprise that very few authors have looked at its consequences.³ Finally, we have no way of identifying

³Caplin and Leahy ([1]) argue that under such circumstances, a policy maker should use a

preferences that relate to the past. This may be true. However, even if we had complete knowledge of preferences, we would have a conceptual problem with welfare.

4 Welfare

We now turn to the simple question posed in the last section. How do we evaluate Isaac's welfare over the 3 periods.

4.1 The Decision maker

We can focus our attention on the decision maker. This approach recognizes that the knowledge we have of an individual's preferences arises from choice. We can be sure that the decision maker is acting in her self interest so we should use the preferences revealed through her behaviour to evaluate her welfare. If her choices are consistent then it is not possible to falsify the claim that preferences are stationary over time through observations of her choices. However, it is not possible to verify this hypothesis either.

Isaac in the example makes no decision and so we have no choice behaviour to base welfare on. To analyse the decision maker approach, imagine the introduction of a market for apples that effectively allows Isaac to choose any apple tree in a given period subject to the constraint that he cannot change past consumption. We observe the following, in period 1 Isaac chooses to consume 40.550 apples. We also observe from the contracts that he has entered into, that he plans to consume 32.845 apples in period 2 and 26.605 in period 3. We can deduce from this that his most preferred tree is A_4 . In periods 2 and 3 he chooses not to change his previous plans. This behaviour is consistent with the hypothesis that A_4 is always the most preferred tree. If A_4 is the most preferred tree then we can comfortably proclaim that it maximises welfare. However we cannot logically deduce that A_4 maximises welfare as the hypothesis that preferences are stationary has not been verified. In fact, in this case, we know it to be false.

The problem lies with the *as if* approach to utility. This approach has allowed economists to build a coherent theory of consumer choice. The theory relies only on consistency and this makes it remarkably versatile. If it appears that preferences are inconsistent then we can look for consistency in the inconsistency. Once we have consistency, we can hypothesize a utility function that is consistent with choice and model behaviour as if maximising utility. If we want to link welfare to preferences however, we need to know what preferences are. It is not correct to say that the agent acts as if to maximise U_1 and therefore his welfare can be represented by U_1 . Gul and Pesendorfer [2] provide a recent example of this. They extend the standard methodology to resolve *apparent*

discount rate that is less than that of private citizens. Ray and Wang ([6]) consider an agent whose consumption decision at time t depends on a weighted average of his *current utility* and that of a future self who discounts past consumption.

inconsistencies in choice when agents have to deal with *temptations*. The individual acts *as if* she maximises a utility function and the model makes testable predictions about behaviour. However, this consistency is then used to make the link with welfare. Whilst as before the consistency can be used to support the hypothesis that preferences are not changing over time, it does not verify it. Let us take their simple example. Consider an individual who succumbs to the tempting alternative c' if it is included in the menu from which she makes a choice but would rather not have it included in the menu. This gives rise to the following preferences over menus $c \succ (c, c') \sim c'$. If she could commit to a menu prior to making the consumption choice then she would choose the menu c . Suppose that these preferences (over menus) do not change over time and consider this individual over 3 periods. She must make a consumption choice in period 2. If she could commit in period 1 then there is consistency in the view that c' should be removed from the menu. Suppose she cannot commit in period 1. Then she will choose the tempting alternative in period 2. Is this welfare maximising? Well we can hypothesize that the period 1 self and the period 3 self also prefer for the period 2 self to choose c' . This however, cannot be falsified or verified from the decision maker's choice alone. We do not observe the preferences of the other selves in this respect. It may be the case that the period 3 self sympathises sufficiently with the period 2 self's predicament that she does indeed prefer her to choose c' . However, one can imagine that a period 3 self who has to deal with the consequences of the period 2 choice would much rather she chose c in period 2. Given this possible conflict, we need to be sure that it is reasonable to assume that it doesn't arise in proposed applications.⁴

4.2 The Long-Run Self

You are to imagine an individual stripped of her short term impulses that are the root cause of intertemporal conflict. We can use her preferences to evaluate welfare. This approach is at the other extreme, the decision makers are completely ignored!

O'Donoghue and Rabin for example [5] consider a model where an agent discounts utility hyperbolically. With these preferences, the intertemporal conflict is apparent in her inconsistent choices. They assume that the long-run self discounts utility exponentially! It is not clear why the long-run self would have any bias towards the present. One defence is to claim that we can consider a period 0 self who has these preferences. If temporal preferences are changing over time then the normative basis for using the period 0 preferences is dubious.⁵

⁴Gul and Pesendorfer [3] for example apply this method to addiction. They suggest that a price rise of an addictive drug will reduce welfare. Implicit in their argument is the assumption that preferences (over decisions) do not change. This assumption is particularly implausible when we are dealing with a decision that gives immediate pleasure but entails harmful long-term consequences.

⁵We should point out that their analysis does not depend on making this assumption on welfare, rather it depends simply on intertemporal conflict. They argue that given there is intertemporal conflict, we should not rule out paternalistic intervention.

Let us take seriously the idea that at every point in time there is a dual self, one with long term interests that are constant over time and the other with preferences that are biased towards the present. How do we resolve the conflict of the two selves in evaluating the individual's welfare at each point in time? Well is there any reason to question the way the dual self resolves these issues. Given any decision she makes is her resolution of these competing interests it seems reasonable to maintain the link between her preferences and her welfare at that point in time. Using the long-run self however ignores her own resolution of the conflict at each point in time and imposes the long-run self's preferences. To see the perils of this consider Isaac and the Apple Tree with the following preferences

$$\begin{aligned} U_1(a_1, a_2, a_3) &= \sqrt{a_1} + 0.9\sqrt{a_2} + 0.1\sqrt{a_3}, \\ U_2(a_1, a_2, a_3) &= 0.9\sqrt{a_1} + \sqrt{a_2} + 0.9\sqrt{a_3} \\ U_3(a_1, a_2, a_3) &= 0.1\sqrt{a_1} + 0.9\sqrt{a_2} + \sqrt{a_3} \end{aligned}$$

You can imagine that the long-run self's preferences are represented by $\sqrt{a_1} + \sqrt{a_2} + \sqrt{a_3}$ and the short-run self cares only about this period and neighboring periods. The above preferences represent Isaac's resolution to the conflict. Using the long-run self's preferences, the welfare maximising tree is $A_1 = (33\frac{1}{3}, 33\frac{1}{3}, 33\frac{1}{3})$. It doesn't take much reflection to see that this is not even Pareto efficient! If we transfer 1 apple from period 1 and one from period 3 to period 2 then all three decision makers are better off. This example is admittedly contrived. It is designed to illustrate in the most extreme way possible that a welfare criterion that ignores short term impulses is deeply flawed. The same problem arises if we use higher order preferences to evaluate welfare. Suppose that Isaac has the higher order preference in favour of the long-run self. That is, he has the higher order preference to prefer A_1 to any other tree in every period. It remains the case in this example that A_1 is not even Pareto efficient.⁶

More generally, the short-term self is ever present and giving into impulses must be satisfying in some way. Is there any reason for ignoring these satisfactions. Perhaps it is the negative long term consequences. This, however, suggests that we need to consider the aggregate effect of a decision.

4.3 Aggregating Preferences

We can imagine the individual rising above her temporal position and evaluating all life-time plans. If these preferences are stationary then we can return to the methodology that links preferences with welfare. Crucially, there is a sense in which we will have maintained the individual's sovereignty over determining her welfare. We will refer to the temporally impartial individual as the higher self⁷

⁶That is not to say that higher order preferences are not important. They will have a role to play in shaping first-order preferences and any dissonance may have an impact on the *level of welfare*.

⁷The long-run self is also temporally impartial! We have argued that if we are to have a meaningful concept of welfare, we cannot separate welfare from the decision maker who is the subject of experience. The higher self is to be thought of as considering every subject of experience impartially.

and to her beliefs/preferences as higher beliefs/preferences.⁸ Returning to the example in section 3, suppose that higher Isaac has preferences represented by the (utilitarian) utility function

$$U(a_1, a_2, a_3) = 2.71\sqrt{a_1} + 2.8\sqrt{a_2} + 2.71\sqrt{a_3}.$$

We then have an answer to our question, of all the apple trees, the best tree for Isaac is $A_6 = (32.597, 34.801, 32.597)$. There are alas many difficulties with this approach. Leaving aside the practical problem of how we are ever to know what these preferences are, we have only come upon a solution by supposing a solution to exist. Is it reasonable to assume that agents possess higher preferences/beliefs that are not changing over time? Perhaps we can appeal to intransigent beliefs such as a belief in utilitarian welfare. However, utilitarian welfare does not pin down a unique set of preferences. We may further postulate that the preferences represented in section 3 are cardinal and are also in intrapersonally comparable units as viewed by higher Isaac and that none of this changes over time. Aren't we simply moving the difficulty that arises by assuming invariability to another level?

Consider the following example. An individual has to choose between x and y in periods 1 and 2. Her preferences are as follows, in period 1 the most preferred sequence of choices is (x, x) and in period 2 it is (x, x) conditional on the period 1 self choosing x . It would appear that we can make an unambiguous welfare claim that it is optimal for this individual to choose x in period 1 and x in period 2. There is no need to appeal to the higher self who is temporally impartial because temporal preferences are not changing.

Suppose however, that if the individual is *forced*⁹ to choose y in period 1 then her most preferred sequence in period 2 is (y, y) . Which is the welfare maximising sequence? Consider the following story. The choice x is smoking and y is not smoking and parents are in a position to force a choice in period 1. The period 1 self holds the belief that smoking is *good*. The period 2 self whose history is x maintains this belief. If her history is y however, she forms the belief that smoking is *bad*. Suppose further that the belief that smoking is good will give rise to the preference $(x, x) \succ (y, y)$ whereas the belief that it is bad will reverse this preference. There is no reason to presume that preferences and beliefs will be aligned in this way. Let us suppose that they are. In fact, suppose that all higher order preferences are also aligned in this way. That is, the belief that smoking is good gives rise to the second order preference $((x, x) \succ (y, y)) \succ ((y, y) \succ (x, x))$ and so on. If she is forced to choose y in period 1 then there is an intertemporal conflict which is based on a change in beliefs rather than on present bias. The temporally impartial self prefers (y, y) in period 2 and (x, x) in period 1. Also, we no longer have a unique set of period 2 preferences and beliefs. Depending on the history, the period 2 higher

⁸Note the distinction between this and higher order preferences which refers to preferences over preferences.

⁹Should the parent force his/her child to choose y in period 1? The parent's decision will be biased by his/her preferences, beliefs and morality.

self will either believe that (x, x) is welfare maximising or that (y, y) is welfare maximising.¹⁰ An appeal to utilitarian welfare does not help. Let us suppose that there is some objective hedonic calculus that can be used to compare the alternatives and (x, x) gives the greatest hedonic utilitarian welfare. However, we have not supposed that the period 2 higher self with history y prefers (y, y) because it maximises hedonic utilitarian welfare. Her beliefs are such that she prefers (y, y) . As with preferences, we take beliefs as given. Nor does it help to imagine putting the individual in a position where she can compare belief systems. A belief system is part of the entirety of an individual at a given point in time. Any judgement made at any point in time will be made from the unique perspective of the entirety that makes up that individual at that point in time. We appealed to a temporally impartial higher self to deal with the problem of temporal preferences changing. Are we to appeal to a higher, higher self who is independent of changing beliefs? Our systems of thought are shaped by our perceptions, social interactions, past choices and no doubt many other things. Is it possible to rescue any semblance of an individual that is independent of these influences? Are we to suppose that we can consider the preferences of an empty shell?

5 Bibliography

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¹⁰Curiously, if preferences are history dependent, it may be impossible for an individual to ever have her most preferred history. The grass may always be greener on the other side! This however, is not the place to consider this curiosity.