GLS Detrending-Based Unit Root Tests in Nonlinear STAR and SETAR Frameworks*

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Abstract

This paper consider the GLS detrending procedure advanced by Elliott *et al.* (1996) for unit root tests against alternative hypotheses where the time series data under investigation follow either globally stationary SETAR or STAR processes with deterministic components being present. It is found that the proposed testing procedures have considerable power gains against both the standard Dickey-Fuller unit root tests and existing nonlinear unit root tests recently proposed by Kapetanios and Shin (2002) and Kapetanios *et al.* (2003). The empirical application to DM and Yen bilateral real exchange rates against a number of other currencies also confirms that nonlinear unit root tests based on GLS detrending will be more powerful than linear ones. Interestingly, we find that the DM dataset seems to produce more rejections of the null using the GLS detrending-based SETAR tests than using the GLS detrending-based STAR tests, whereas the number of rejections of both tests are similar for the Yen dataset. The different results may arise from the respective liquidity of the DM and Yen Forex markets.

JEL Classification: C12, C22, F31.
Key Words: Unit Root Tests, Nonlinear STAR and SETAR Models, GLS Detrending, Real Exchange Rates.

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1 Introduction

The earlier literature on univariate analysis of nonstationarity against stationarity has focussed on the linear model, implicitly disregarding any possible nonlinearities in the series under investigation. Recently, however, there has been increasing concern that the analysis of a linear model in a single time series may be inappropriate to give satisfactory inferences on important economic hypotheses. For example, the power of Dickey-Fuller (1979) unit root test has been called into question. Theoretical models of nonlinear adjustments have also been proposed in many areas. For instance, in the context of asset markets, the extent of arbitrage trading in response to return differentials is limited by transaction costs, and these costs may lead to a nonlinear relationship between the level of arbitrage activity and the size of the return differentials. Therefore, the speed with which the returns differential reverts towards zero is an increasing function of the size of the returns differential itself. See Sercu *et al.* (1995), Koop *et al.* (1996) and Michael *et al.* (1997). As a response applied economists increasingly turn to nonlinear dynamics to improve estimation and inference. In this regard, Balke and Fomby (1997) have proposed a joint analysis of nonstationarity and nonlinearity in the context of threshold cointegration, where the threshold cointegrating process is defined as globally stationary such that it follows a unit root in the middle regime, but is geometrically ergodic in outer regimes. A growing number of studies have emerged along this line of research, see for example Enders and Granger (1998), Caner and Hansen (2001) and Lo and Zivot (2001).

In particular, Kapetanios and Shin (2002) and Kapetanios *et al.* (2003) analyse the implications of the existence of a particular kind of nonlinear dynamics for unit root tests, and thus provide alternative frameworks for testing the null of a unit root against the alternative under which the time series of interest follow globally stationary processes. More specifically, Kapetanios and Shin (2002) consider self-exciting threshold autoregressive (SETAR) models and Kapetanios *et al.* (2003) examine nonlinear smooth transition autoregressive (STAR) models. Their Monte Carlo experiments clearly show that these types of nonlinear unit root tests are generally more powerful than the standard Dickey-Fuller unit root tests when the data follow either globally stationary SETAR or STAR processes.

However, demeaning or detrending of the data based on OLS estimation causes a significant loss of power of the tests as is the case in Dickey-Fuller unit root tests obtained from linear regression models. Elliott *et al.* (1996) investigate the issue of efficient detrending in the context of linear unit root tests and attempt to derive a point optimal test against specific local alternative hypotheses. It is shown via stochastic simulations that the unit root test obtained after using the GLS detrending procedure is more powerful than the standard Dickey-Fuller test based on OLS detrending. It is therefore important to examine whether GLS detrending procedure is also useful for unit root tests derived in nonlinear models.

In this paper we consider the issue of efficient detrending in the context of unit root tests against particular nonlinear alternatives considered by Kapetanios and Shin (2002) and Kapetanios *et al.* (2003). In general, it is far more complicated to explicitly take into account the nonlinear nature of the alternative hypothesis and derive the appropriate GLS detrending formula. As a result we simply apply the method advanced by Elliott *et al.* (1996) to the nonlinear case and develop the nonlinear unit root test procedure based on the GLS detrended data. We first derive the associated asymptotic distributions analogous to those in Elliott *et al.* (1996), Kapetanios and Shin (2002) and Kapetanios *et al.* (2003). Via Monte Carlo simulations we then provide evidence that the power of the GLS detrended nonlinear unit root tests outperforms that of existing nonlinear tests against either STAR or SETAR alternatives, and that it also dominates the linear Dickey-Fuller unit root tests in many cases where the nonlinear adjustment mechanism would be judged *a priori* to be more important.

We illustrate the usefulness of our proposed tests by examining the stationarity properties of bilateral real exchange rates for Yen and Deutsche Mark against a number of other currencies. The empirical application confirms our idea that nonlinear unit root tests based on GLS detrending will be more powerful. Interestingly, we find that the DM real exchange dataset seems to produce more rejections of the null using the GLS detrending-based SETAR tests (rejecting the null of a unit root 24 times out of 31 cases considered) than using the GLS detrending-based STAR tests (rejecting the null only 13 times), whereas the number of rejections of both tests are similar for the Yen real exchange rate dataset. These different results for SETAR versus STAR-based tests may arise from the respective liquidity of DM and Yen Forex markets. This may imply that a more liquid DM Forex market may make adjustments more sudden as compared to the Yen.

The plan of the paper is as follows: Section 2 describes the underlying models and develops the theoretical results. Section 3 investigates the small sample performance of the suggested tests via Monte Carlo experiments. Section 4 presents the empirical illustration. Section 5 concludes.

2 Theoretical Framework

Elliott, Rothenberg and Stock (1996, hereafter ERS) investigate the issue of efficient detrending in the context of unit root tests in linear models following the previous works by King (1980, 1988) and Dufour and King (1991). Mainly motivated by asymptotic local power considerations, they derive a point optimal unit root test against a sequence of specific local alternative hypotheses. Consider the univariate regression model of the form:

$$x_t = \alpha + \beta t + y_t,\ t = 1, ..., T, \tag{2.1}$$

$$y_t = \rho y_{t-1} + \epsilon_t, \tag{2.2}$$

where ϵ_t is assumed to be an *iid* process with zero mean and finite variance σ^2. Focussing on the sequence of local alternative hypotheses, the autoregressive parameter ρ can be expressed as

$$\rho = 1 - c/T,\ c > 0. \tag{2.3}$$

In particular, ERS consider testing the null hypothesis of a unit root $H_0 : \rho = 1$ against a local alternative of the form,

$$H_{\bar{c}} : \rho = \bar{\rho} = 1 - \bar{c}/T, \tag{2.4}$$

where $\bar{c}$ is a positive constant under which tests are constructed.

The point optimal log-likelihood ratio test can then be obtained by

$$LR = \min_{\boldsymbol{\theta}} \left\{ \ell\left(\bar{\rho}, \boldsymbol{\theta}\right) - \ell\left(1, \boldsymbol{\theta}\right) \right\}, \tag{2.5}$$

where

$$\ell\left(\bar{\rho}, \boldsymbol{\theta}\right) = \left(x_{\bar{\rho}} - \mathbf{z}_{\bar{\rho}}\boldsymbol{\theta}\right)' \Sigma^{-1} \left(x_{\bar{\rho}} - \mathbf{z}_{\bar{\rho}}\boldsymbol{\theta}\right), \tag{2.6}$$

is the log-likelihood function obtained under the local alternative hypothesis, $x_{\bar{\rho}} = (x_1, x_2 - \bar{\rho}x_1, \ldots, x_T - \bar{\rho}y_{T-1})'$, $\mathbf{z}'_{\bar{\rho}} = (\mathbf{z}_1, \mathbf{z}_2 - \bar{\rho}\mathbf{z}_1, \ldots, \mathbf{z}_T - \bar{\rho}\mathbf{z}_{T-1})$, $\mathbf{z}_t = (1, t)'$, $\boldsymbol{\theta} = (\alpha, \beta)'$, $\boldsymbol{\Sigma}$ is the covariance matrix of $(\epsilon_1, \ldots, \epsilon_T)'$, and similarly for $\ell\left(1, \boldsymbol{\theta}\right)$. The term $\left(x_{\bar{\rho}} - \mathbf{z}_{\bar{\rho}}\boldsymbol{\theta}\right)' \Sigma^{-1} \left(x_{\bar{\rho}} - \mathbf{z}_{\bar{\rho}}\boldsymbol{\theta}\right)$ may be viewed as a weighted sum of squared residuals obtained from a constrained GLS regression with the value of $\bar{\rho}$ being imposed. More specifically, ERS suggest a two-step testing procedure. First, one carries out the GLS detrending estimation of (2.1) and get the detrended residuals,

$$\tilde{y}_t = x_t - \tilde{\alpha} - \tilde{\beta}t, \tag{2.7}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are the GLS estimates of α and β obtained from a regression of $x_{\bar{\rho}}$ on $\mathbf{z}_{\bar{\rho}}$. Then, one applies the Dickey-Fuller unit root test to the detrended residuals, $\tilde{y}_t$. Though no rigorous theoretical proof is made on the point optimal property of the test, it is shown mainly via stochastic simulations that the GLS-based unit root test is more powerful than the Dickey-Fuller unit root test based on OLS detrending. It is further shown that the GLS-based unit root test has local power which is practically indistinguishable from that of an optimal test. Based on these simulation findings, ERS suggest selecting the value of $\bar{\rho}$ such that the asymptotic power of the test under the local alternative is equal to 0.5. For example, when the regression (2.1) contains an intercept only, $\bar{c}$ is set to -7, whereas $\bar{c}$ is set to -13.5 when the regression (2.1) contains both intercept and time trend.

Recently, Kapetanios, Shin and Snell (2003, hereafter KSS) and Kapetanios and Shin (2002, hereafter KS) have suggested two alternative testing procedures to distinguish between nonstationary unit root processes, and globally stationary nonlinear processes. KSS consider the case where y_t in (2.1) follows the STAR process under the alternative:

$$\Delta y_t = \gamma y_{t-1} \left\{ 1 - \exp\left(-\theta y_{t-1}^2 \right) \right\} + \epsilon_t, \tag{2.8}$$

where $-2 < \gamma < 0$. They then propose the testing procedure for the null hypothesis $H_0 : \theta = 0$ against the alternative hypothesis $H_1 : \theta > 0$. Notice that under the null y_t follows a linear unit root process, whereas it is a globally ergodic STAR process under the alternative. Since γ is not identified under the null, the direct test of this null is not feasible, see Davies (1987). To overcome this problem they follow Luukkonen *et al.* (1988), and derive a t-statistic for the null $\delta = 0$ against the alternative $\delta < 0$ in the following auxiliary regression:

$$\Delta y_t = \delta y_{t-1}^3 + \epsilon_t. \tag{2.9}$$

KS consider the model where y_t in (2.1) now follows the SETAR process under the alternative:

$$\Delta y_t = \phi_1 y_{t-1} 1_{\{y_{t-1} \leq r_1\}} + \phi_2 y_{t-1} 1_{\{y_{t-1} > r_2\}} + \epsilon_t, \tag{2.10}$$

where $-2 < \phi_i < 0$, $i = 1, 2$, and $1_{\{.\}}$ is an indicator function. Now, the null hypothesis of a unit root is $H_0 : \phi_1 = \phi_2 = 0$ while the alternative hypothesis of globally stationarity becomes $H_1 : \phi_1 < 0$ and/or $\phi_2 < 0$. KS suggest using the Wald statistic for testing the (joint) null hypothesis of $\phi_1 = \phi_2 = 0$, but this test suffers from the Davies (1987) problem since the unknown threshold parameters, r_1 and r_2, are not identified under the null. Therefore, following Andrews and Ploberger (1994) and Hansen (1996), the three most commonly used summary statistics are considered: the supremum, the average and the exponential average of the Wald statistic. Monte Carlo simulation results reported in KSS and KS clearly show that both testing procedures are more powerful than the standard Dickey-Fuller unit root tests that ignore the specific nonlinear nature of the data under the alternative hypothesis of either globally stationary STAR or SETAR processes.

In nonlinear models, the appropriate modelling of intercepts and trends is more complicated. Conventionally, if one aims to derive asymptotically similar tests with respect to intercepts or time trends, OLS demeaning or detrending is routinely applied to the data. The tests derived in both KSS and KS follow this OLS-based demeaning or detrending procedure. However, demeaning or detrending of the data based on the OLS estimation is likely to cause a significant loss of power of the tests like in linear models. As a result it would be equally important to examine whether the GLS detrending procedures that have successfully improved the power of unit root tests in the linear models may also be useful in the nonlinear framework. Generally, it would be far more complicated to explicitly take into account the nonlinear nature of the alternative hypothesis and derive the appropriate GLS detrending procedure. For example, consider the STAR model (2.8), where a local alternative might be constructed by

$$\theta = \frac{c}{T},\ c > 0. \tag{2.11}$$

First, the T^{-1} rate involved in the construction of the local alternative is not necessarily appropriate since θ enters the model exponentially rather than linearly, see also Park and Phillips (2001). Second, it is not clear how to devise a generalised quasi-differencing scheme without prior knowledge as to how to construct demeaned or detrended series comparable to $x_{\bar{\rho}}$ and $\mathbf{z}_{\bar{\rho}}$ for the linear case. Finally and, perhaps, more importantly, any detrending taking account of the alternative nonlinear structure of the model would involve nuisance parameters, such as γ for the STAR alternatives or r_1 and r_2 for the SETAR alternative. As a result, we simply apply ERS' method to the nonlinear case and develop a nonlinear unit root test procedure based on GLS detrending. We note in passing that the linear local-to-null alternative detrending approach may provide a useful approximation to the nonlinear local alternative.

We now give more details of how to construct out proposed tests. Following the literature we consider two cases of deterministic components, $z_t = 1$ and $\mathbf{z}_t = (1, t)'$. Let us denote the GLS-based demeaned or detrended series by $\tilde{y}_t^\mu$ and $\tilde{y}_t^\tau$ estimated for a given value of $\bar{c}$, where the superscripts μ and τ denote demeaning and detrending, respectively. The GLS-based unit root tests against an alternative of a STAR process, denoted t_{STAR}^μ or t_{STAR}^τ, are then obtained as a t-statistic for $\delta = 0$ in the following regression:

$$\Delta\tilde{y}_t^j = \delta\left(\tilde{y}_{t-1}^j\right)^3 + error,\ j = \mu, \tau. \tag{2.12}$$

Similarly, the associated unit root tests against an alternatives of SETAR process are given by the Wald statistics for $\phi_1 = \phi_2 = 0$ in the regression,

$$\Delta\tilde{y}_t^j = \phi_1\tilde{y}_{t-1}^j 1_{\left\{\tilde{y}_{t-1}^j \leq r_1\right\}} + \phi_2\tilde{y}_{t-1}^j 1_{\left\{\tilde{y}_{t-1}^j > r_2\right\}} + error,\ j = \mu, \tau. \tag{2.13}$$

As mentioned earlier, to deal with the Davies problem, we consider the average and the exponential average of the Wald statistics defined respectively by[1]

$$\mathcal{W}_{\text{avg}}^j = \frac{1}{\#\Gamma}\sum_{i=1}^{\#\Gamma}\mathcal{W}_{(r_1,r_2)}^{j(i)},\ \mathcal{W}_{\text{exp}}^j = \frac{1}{\#\Gamma}\sum_{i=1}^{\#\Gamma}\exp\left(\frac{\mathcal{W}_{(r_1,r_2)}^{j(i)}}{2}\right),\ j = \mu, \tau, \tag{2.14}$$

where $\mathcal{W}_{(r_1,r_2)}^{j(i)}$ is the Wald statistic obtained from the i-th point of the nuisance parameter grid, Γ and $\#\Gamma$ is the number of elements of Γ. For more details on the selection of the grid of threshold parameters and its theoretical implications see KS.

We now derive the asymptotic distributions of the GLS-based t_{STAR} and $\mathcal{W}$ tests. First, ERS show that under the local alternative hypothesis (2.4) it follows that

$$T^{-1/2}\tilde{y}_{[rT]}^{\mu} \Rightarrow \sigma W_c\left(a\right),\ a \in \left[0, 1\right], \tag{2.15}$$

$$T^{-1/2}\tilde{y}_{[rT]}^{\tau} \Rightarrow \sigma V_c\left(a, \bar{c}\right),\ a \in \left[0, 1\right], \tag{2.16}$$

where

$$W_c\left(a\right) = \int_0^1 e^{c(a-s)}dW(s), \tag{2.17}$$

$$V_c\left(a, \bar{c}\right) = W_c\left(a\right) - a\left[\lambda W_c\left(1\right) + 3\left(1-\lambda\right)\int_0^1 sW_c\left(s\right)ds\right], \tag{2.18}$$

$\lambda = \left(1-\bar{c}\right)/\left(1-\bar{c}+\bar{c}^2/3\right)$ and $W(a)$ is a standard Brownian motion defined on $a \in [0, 1]$.

Combining these asymptotic results with those in KSS, it follows that the asymptotic null distribution of the $\tilde{t}_{STAR}^{\mu}$ and $\tilde{t}_{STAR}^{\tau}$ test statistics are given by

$$\tilde{t}_{STAR}^{\mu} \Rightarrow \frac{\left\{\frac{1}{4}W\left(1\right)^4 - \frac{3}{2}\int_0^1 W\left(a\right)^2 da\right\}}{\sqrt{\int_0^1 W(a)^6 da}}, \tag{2.19}$$

$$\tilde{t}_{STAR}^{\tau} \Rightarrow \frac{\left\{\frac{1}{4}V_0\left(1, \bar{c}\right)^4 - \frac{3}{2}\int_0^1 V_0\left(a, \bar{c}\right)^2 da\right\}}{\sqrt{\int_0^1 V_0\left(a, \bar{c}\right)^6 da}}, \tag{2.20}$$

where $V_c\left(a, \bar{c}\right)$ is defined in (2.18). Notice that the asymptotic properties of the GLS demeaned series, $\tilde{y}_t^{\mu}$, is the same as that of a random walk with no drift under the null ($c = 0$). Therefore, the asymptotic distribution of the $\tilde{t}_{STAR}^{\mu}$ test obtained using GLS demeaning does not depend on the specific value of $\bar{c}$ and simplifies as shown in (2.19). For the detrended case the asymptotic distribution of the $\tilde{t}_{STAR}^{\tau}$ test depends clearly on the particular value of $\bar{c}$. Following ERS we also select the value of $\bar{c}$ such that the asymptotic power of the test

[1] KS find that the supremum of the Wald tests tend to overreject significantly in small samples. For this reason we will not consider the supremum test.

under the local alternative is equal to 0.5, which gives $\bar{c}$ as -17.5.[2] Hence, the associated 95% critical values for the $\tilde{t}^{\mu}_{STAR}$ and $\tilde{t}^{\tau}_{STAR}t$ are obtained as -2.21 and -2.93, respectively.

Next, combining the asymptotic results in ERS and KS, we have

$$\widetilde{\mathcal{W}}^{\mu}_{\text{avg}} \Rightarrow \widetilde{\mathcal{W}}^{\mu} \equiv \frac{\left\{\int_0^1 1_{\{W(s)\leq 0\}} W(s)dW(s)\right\}^2}{\int_0^1 1_{\{W(s)\leq 0\}} W(s)^2 ds} + \frac{\left\{\int_0^1 1_{\{W(s)>0\}} W(s)dW(s)\right\}^2}{\int_0^1 1_{\{W(s)>0\}} W(s)^2 ds}, \tag{2.21}$$

$$\widetilde{\mathcal{W}}^{\tau}_{\text{avg}} \Rightarrow \widetilde{\mathcal{W}}^{\tau} \equiv \frac{\left\{\int_0^1 1_{\{V_0(s,\bar{c})\leq 0\}} V_0(s,\bar{c})\, dV_0(s,\bar{c})\right\}^2}{\int_0^1 1_{\{V_0(s,\bar{c})\leq 0\}} V_0^2(s,\bar{c})\, ds} + \frac{\left\{\int_0^1 1_{\{V_0(s,\bar{c})>0\}} V_0(s,\bar{c})\, dV_0(s,\bar{c})\right\}^2}{\int_0^1 1_{\{V_0(s,\bar{c})\}} V_0(s,\bar{c})^2\, ds}, \tag{2.22}$$

$$\widetilde{\mathcal{W}}^{\mu}_{\text{exp}} \Rightarrow \exp\left(\widetilde{\mathcal{W}}^{\mu}/2\right),\ \widetilde{\mathcal{W}}^{\tau}_{\text{exp}} \Rightarrow \exp\left(\widetilde{\mathcal{W}}^{\tau}/2\right). \tag{2.23}$$

As described earlier, the asymptotic distributions of the $\widetilde{\mathcal{W}}^{\mu}_{\text{avg}}$ and $\widetilde{\mathcal{W}}^{\mu}_{\text{exp}}$ tests do not depend on the specific value of $\bar{c}$, and their 95% critical values are 7.48 and 42.09, respectively. On the other hand, the 95% asymptotic critical values of the $\widetilde{\mathcal{W}}^{\tau}_{\text{avg}}$ and $\widetilde{\mathcal{W}}^{\tau}_{\text{exp}}$ tests obtained by setting $\bar{c} = -13$ (which is selected to give the asymptotic local power equal to 0.5) are 8.81 and 81.86, respectively.

3 Finite Sample Performance

In the first set of experiments we construct the null model by

$$x_t = \alpha + \beta t + y_t,\ t = 1, ..., T, \tag{3.1}$$

$$y_t = y_{t-1} + \varepsilon_t,\ t = 1, ..., T, \tag{3.2}$$

where ε_t is drawn from the standard normal distribution. Without loss of generality we set the values of α and β to zero since all the tests considered are asymptotically similar.

Secondly, in order to evaluate the power performance against the alternative model where the process follows globally stationary STAR processes, we generate the DGP by

$$\Delta y_t = \gamma y_{t-1}\left[1 - \exp\left(-\theta y_{t-1}^2\right)\right] + \varepsilon_t,\ t = 1, ..., T, \tag{3.3}$$

where $\varepsilon_t \sim N(0,1)$, and we choose a broad range of parameter values for $\gamma = \{-1.5, -1, -0.5, -0.1\}$ and $\theta = \{0.01, 0.05\}$.

The third set of experiments examines the power performance of the tests against the alternative of globally stationary SETAR processes, where the data is generated by

$$y_t = \begin{cases} \phi_1 y_{t-1} + \varepsilon_t & if\ y_{t-1} \leq r_1 \\ y_{t-1} + \varepsilon_t & if\ r_1 < y_{t-1} \leq r_2 \\ \phi_2 y_{t-1} + \varepsilon_t & if\ y_{t-1} > r_2 \end{cases},\ t = 1, ..., T, \tag{3.4}$$

where $\varepsilon_t \sim N(0,1)$. We examine the case of asymmetric adjustments with $\phi_1 = 0.85$ and $\phi_2 = 0.95$, and select five different sets of threshold parameter values, $(r_1, r_2) =$

[2] Local power is evaluated via stochastic simulations which generate the random processes of $T = 1000$ observations to discretely approximate the functionals of Brownian motions with 5000 replications.

$\{(-0.9, 0.9), (-1.65, 1.65), (-2.4, 2.4), (-3.15, 3.15), (-3.9, 3.9)\}$. For each case the grid of either lower or upper threshold parameter comprises of eight equally spaced points between the 10% quantile (lower threshold) or the 90% quantile (upper threshold) of the sample and the mean of the sample.[3]

For each of experiments we have computed the rejection probability of the null hypothesis. The nominal size of each of the tests is set at 0.05, the number of replications at 1000 and the sample size is set to $T = 100, 200$. We consider both versions of the OLS detrending- and the GLS detrending-based nonlinear unit root tests: the former denoted by $\hat{t}^j_{STAR}$, $\widehat{\mathcal{W}}^j_{\text{avg}}$ and $\widehat{\mathcal{W}}^j_{\text{exp}}$, $j = \mu, \tau$, and the latter by $\tilde{t}^j_{STAR}$, $\widetilde{\mathcal{W}}^j_{\text{avg}}$ and $\widetilde{\mathcal{W}}^j_{\text{exp}}$, $j = \mu, \tau$.[4] For comparison purpose we also consider the following tests obtained using the linear models: the standard Dickey-Fuller unit root tests obtained using the OLS detrending denoted as $\hat{t}^j_{DF}$, $j = \mu, \tau$, and those obtained using the GLS detrending, denoted by $\tilde{t}^j_{DF}$, $j = \mu, \tau$.

Simulation results are presented in Tables 1 - 3. As a benchmark, Table 1 gives the empirical size of the tests when the underlying DGP is the random walk process. As shown in ERS, the tests based on GLS demeaning and detrending tend to slightly over-reject especially when $T = 100$. Therefore, in evaluating power performance, we use the size-adjusted empirical critical values for the GLS-based tests that are obtained from the size experiments. For all power experiments, 200 initial observations are discarded to minimise the effect of initial conditions. It is clear from Tables 2 and 3 that the GLS detrending procedure applied to the nonlinear unit root tests against either STAR or SETAR alternatives outperform the existing nonlinear tests. They also dominate linear unit root tests in cases where the nonlinear adjustment mechanism is judged a priori to be more important. For example, our proposed tests are more powerful than linear counterparts when the corridor regime is large when the DGP follow the SETAR processes.

$$Tables\ 1 - 3 \text{ about } here$$

4 Application to Real Exchange Rates

It has been argued that the existence of transaction and transport costs in traded goods and Forex markets may be a reason for the violation of the purchasing power parity (PPP) in the short run and for the persistence of real exchange rates, see Sercu *et al.* (1995). Taking the commodity arbitrage view of PPP (rather than the money homogeneity view) suggests that the extent of goods arbitrage and hence the amount of disequilibrium PPP relationship is inversely related to shipping costs and to a Forex spread. There is no reason to suppose a priori that there should be a linear relationship between these costs and the extent of arbitrage. Quite the opposite may be true in fact. Large transactions are likely to be processed on less favorable terms than for smaller ones because of the existence of adverse selection effects on spreads, see Kyle (1985). A further complication is that spreads

[3] We find that the processes have spent at least 10% of the time in each of the outer regimes even for the largest threshold parameter values considered.

[4] To compute the GLS-demeaned tests $\tilde{t}^\mu_{STAR}$ and $\widetilde{\mathcal{W}}^\mu_{\text{avg}}$ $\left(\widetilde{\mathcal{W}}^\mu_{\text{avg}}\right)$, we find that the relevant values of $\bar{c}$ for the demeaned STAR test and the demeaned SETAR test are -9 and -12, respectively.

themselves narrow following large trades, e.g., such as those that may be initiated by goods arbitrage. Without developing what would be a very complex theoretical model of the relationship between arbitrage, transactions costs and the resulting dynamic adjustment of real exchange rates, we would conjecture that such dynamic adjustments are intrinsically nonlinear.

We now apply our proposed tests to investigate stationary properties of Yen and Deutsche Mark real exchange rates. The bilateral real exchange rate against ith currency at time t, denoted q_{it}, is constructed by

$$q_{it} = s_{it} + p_{ht} - p_{it},\ h = Germany,\ Japan,$$

where s_{it} is the nominal exchange rate defined as the price of foreign currency in terms of home currency, p_{ht} the domestic price level and p_{it} the price level of the foreign country. Thus, a rise in q_{it} implies an appreciation of the home currency against the ith currency. The bilateral nominal exchange rates against currencies other than the US dollar are computed using the US dollar rates. The price levels are consumer price indices for Yen and wholesale price indices for the DM. All the data are (seasonally non-adjusted) quarterly observations over 1960Q1 to 2000Q4, are measured in natural logarithms, and are obtained from the International Monetary Fund's *International Financial Statistics* in CD-ROM. We consider a very large pool of countries in order to make the empirical analysis more comprehensive.

To accommodate possibly serially correlated errors, we follow KSS and KS and use the following augmented models of (2.12) and (2.13):

$$\Delta \tilde{y}_t^{\tau} = \delta \left(\tilde{y}_{t-1}^{\tau}\right)^3 + \sum_{j=1}^{p} \rho_j \Delta \tilde{y}_{t-j}^{\tau} + error, \tag{4.1}$$

$$\Delta \tilde{y}_t^{\tau} = \phi_1 \tilde{y}_{t-1}^{\tau} 1_{\left\{\tilde{y}_{t-1}^{\tau} \le r_1\right\}} + \phi_2 \tilde{y}_{t-1}^{\tau} 1_{\left\{\tilde{y}_{t-1}^{\tau} > r_2\right\}} + \sum_{j=1}^{p} \rho_j \Delta \tilde{y}_{t-j}^{\tau} + error. \tag{4.2}$$

For simplicity we fixed the lag length at 4 for quarterly data. Here we assume the presence of a time trend under the alternative hypothesis. To make the comparison comparable to other existing unit root tests we also consider the DF test based on both OLS and GLS detrending. The size-adjusted empirical critical values obtained from Monte Carlo experiments in the previous section will be used to minimise the impact of over-rejection of the GLS-based tests under the null. In the empirical application we do not consider the average SETAR-based test as the Monte Carlo analysis has shown that it is less powerful that the exponential one.

Test results are presented in Tables 4 and 5. For the Yen real exchange rates the nonlinear unit root tests obtained using the STAR framework seem to reject the null of a unit root more often than any other tests. Looking at the results in Table 4, $\hat{t}_{STAR}$ and $\tilde{t}_{STAR}$ tests reject the null 22 and 26 times respectively out of 37 countries considered, while $\widehat{\mathcal{W}}_{\exp}$ and $\widetilde{\mathcal{W}}_{\exp}$ tests reject the null 15 and 26 times, respectively. On the other hand, $\hat{t}_{DF}$ and $\tilde{t}_{DF}$ tests reject the null only 13 and 15 times, respectively. Turning to the DM real exchange rates and looking at the results in Table 5, the GLS-based $\widetilde{\mathcal{W}}_{\exp}$ test rejects for 24 out of the 31 countries considered, whereas the OLS-based $\widehat{\mathcal{W}}_{\exp}$ test reject the null only 9 times.[5] The

[5]There are fewer countries for the DM application due to data unavailability

number of rejections of $\widetilde{\mathcal{W}}_{\exp}$ is almost double that of any other tests. This clearly provides support for our idea that nonlinear unit root tests based on GLS detrending will be more powerful. On the other hand, there seems less improvement of using GLS detreding in linear Dickey-Fuller tests and the STAR-based tests; $\hat{t}_{STAR}$ and $\tilde{t}_{STAR}$ tests reject the null 14 and 13 times, whereas $\hat{t}_{DF}$ and $\tilde{t}_{DF}$ tests reject the null 10 and 11 times. Overall we find that nonlinear unit root tests are far more powerful than linear ones.

$$Tables\ 4-5\ \text{about}\ here$$

Interestingly, we find that the DM real exchange dataset seems to produce more rejections of the null using the GLS detrending-based SETAR tests (rejecting the null 24 times out of 31 cases) than using the GLS detrending-based STAR tests (rejecting the null only 13 times), whereas the number of rejections of both tests are similar for the Yen real exchange rate dataset. This may be taken to signify the possible presence of particular forms of nonlinearity which are different between two datasets and which can perhaps be picked up more accurately by one or the other classes of tests. In particular sudden step changes in the dynamic evolution of real exchange rates may be better picked up by the SETAR-based tests whereas smoother adjustments may be more amenable to investigation through the STAR-based tests. The different results for STAR versus SETAR-based tests may arise from the respective liquidity of the Yen and DM Forex markets. The more liquid the market (*e.g.*, Germany), the less important are adverse selection effects on the spread. In the limit where liquidity is infinite, the spread will be fixed and will exist only to compensate pure non-information costs. We may tentatively conjecture in the limiting case of a fixed spread that the SETAR model may capture the arbitrary PPP dynamics better than the STAR model. On the other hand, This may partially explain to some degree the different results from the two tests. This may imply that a more liquid DM Forex market may make adjustments more sudden as compared to the Yen, while a reduced liquidity for Yen may make adjustments more gradual relative to the DM. In any case, this difference in performance for the two tests may indicate that the tests may be used complementarily in empirical analysis.

5 Conclusion

It is well-established that inefficient detrending may reduce the power of unit root tests significantly in the linear framework. In this paper we extend the use of efficient GLS detrending mechanism advanced by Elliott *et al.* (1996) to recent works on testing for unit roots against particular forms of nonlinear alternatives, mainly the SETAR alternative proposed by and Kapetanios and Shin (2002) and the STAR alternative considered by Kapetanios *et al.* (2003). We find via Monte Carlo simulations that the GLS detrending applied to the nonlinear unit root tests can may improve the power performance of the existing nonlinear tests. This finding is also borne out in the empirical application where Yen and DM real exchange rates are examined.

Table 1. The size of alternative tests.								
T	$\hat{t}_{DF}$	$\tilde{t}_{DF}$	$\hat{t}_{STAR}$	$\tilde{t}_{STAR}$	$\widehat{\mathcal{W}}_{\text{avg}}$	$\widetilde{\mathcal{W}}_{\text{avg}}$	$\widehat{\mathcal{W}}_{\text{exp}}$	$\widetilde{\mathcal{W}}_{\text{exp}}$
	Demeaned Tests							
100	.046	.095	.066	.101	.048	.079	.040	.107
200	.041	.094	.064	.089	.040	.045	.060	.079
	Detrended Tests							
100	.053	.076	.064	.074	.044	.074	.068	.099
200	.060	.067	.048	.052	.047	.063	.066	.083

Table 2. The power of tests against STAR alternatives									
		Demeaned Tests				Detrended Tests			
T	(θ, γ)	$\hat{t}^{\mu}_{DF}$	$\tilde{t}^{\mu}_{DF}$	$\hat{t}^{\mu}_{STAR}$	$\tilde{t}^{\mu}_{STAR}$	$\hat{t}^{\tau}_{DF}$	$\tilde{t}^{\tau}_{DF}$	$\hat{t}^{\tau}_{STAR}$	$\tilde{t}^{\tau}_{STAR}$
100	$(.01, -1.5)$	.509	.635	.684	.709	.323	.354	.461	.552
	$(.01, -1.0)$	.336	.461	.466	.543	.237	.252	.286	.348
	$(.01, -0.5)$	.190	.265	.238	.303	.144	.140	.149	.205
	$(.01, -0.1)$	.105	.114	.106	.115	.083	.071	.082	.088
	$(.05, -1.5)$	1.0	.980	1.0	.994	.999	.991	.987	.980
	$(.05, -1.0)$	.993	.935	.987	.957	.929	.936	.913	.917
	$(.05, -0.5)$	.729	.775	.751	.757	.432	.525	.530	.614
	$(.05, -0.1)$	.160	.177	.157	.176	.108	.110	.101	.120
200	$(.01, -1.5)$	1.0	.942	.988	.974	.955	.960	.959	.922
	$(.01, -1.0)$	.967	.885	.952	.933	.783	.857	.843	.833
	$(.01, -0.5)$	.618	.724	.720	.749	.349	.505	.461	.525
	$(.01, -0.1)$	.174	.243	.182	.261	.125	.143	.125	.132
	$(.05, -1.5)$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	$(.05, -1.0)$	1.0	.994	1.0	.999	1.0	1.0	1.0	.998
	$(.05, -0.5)$	1.0	.967	.997	.994	.997	.992	.978	.957
	$(.05, -0.1)$	.394	.557	.429	.532	.228	.316	.237	.289

Table 3. The power of tests against SETAR alternatives							
T	(r_1, r_2)	$\hat{t}_{DF}$	$\tilde{t}_{DF}$	$\widehat{\mathcal{W}}_{\text{avg}}$	$\widetilde{\mathcal{W}}_{\text{avg}}$	$\widehat{\mathcal{W}}_{\text{exp}}$	$\widetilde{\mathcal{W}}_{\text{exp}}$
	Demeaned Tests						
100	$(-0.9, 0.9)$	.271	.361	.219	.260	.204	.236
	$(-1.65, 1.65)$	.225	.296	.209	.256	.196	.241
	$(-2.40, 2.40)$	.175	.274	.189	.230	.185	.224
	$(-3.15, 3.15)$	.136	.211	.179	.222	.204	.230
	$(-3.90, 3.90)$	.106	.147	.136	.165	.150	.174
200	$(-0.9, 0.9)$	.663	.676	.604	.703	.586	.670
	$(-1.65, 1.65)$	.619	.639	.564	.682	.569	.650
	$(-2.40, 2.40)$	.546	.586	.529	.635	.548	.624
	$(-3.15, 3.15)$	.433	.512	.512	.634	.552	.622
	$(-3.90, 3.90)$	.279	.386	.355	.512	.419	.524
	Detrended Tests						
100	$(-0.9, 0.9)$	.167	.188	.130	.183	.126	.178
	$(-1.65, 1.65)$	.140	.155	.114	.165	.129	.174
	$(-2.40, 2.40)$	.128	.130	.117	.155	.124	.173
	$(-3.15, 3.15)$	.108	.124	.105	.155	.114	.167
	$(-3.90, 3.90)$	.084	.090	.087	.102	.093	.122
200	$(-0.9, 0.9)$	.430	.466	.353	.453	.371	.430
	$(-1.65, 1.65)$	.384	.402	.327	.436	.352	.415
	$(-2.40, 2.40)$	.308	.357	.304	.407	.320	.395
	$(-3.15, 3.15)$	.294	.316	.292	.396	.332	.409
	$(-3.90, 3.90)$	.193	.193	.195	.257	.246	.278

Table 4. Unit root test results for bilateral real exchange rate (Yen)						
Augmented models with fixed lag of 4						
Country	$\hat{t}^{\tau}_{DF}$	$\tilde{t}^{\tau}_{DF}$	$\hat{t}^{\tau}_{STAR}$	$\tilde{t}^{\tau}_{STAR}$	$\widehat{\mathcal{W}}^{\tau}_{\exp}$	$\widetilde{\mathcal{W}}^{\tau}_{\exp}$
US	−2.77	−2.67	−3.93*	−3.59*	1074.2*	832.2*
$Germany$	−2.96	−2.95*	−3.73*	−3.61*	417.0	1131.7*
$France$	−3.12	−3.12*	−3.95*	−4.30*	6678.1*	3978.9*
$Italy$	−2.85	−2.88	−4.61*	−4.40*	196.6	179.6*
UK	−3.05	−2.94*	−3.99*	−3.74*	342.3	795.2*
$Canada$	−3.75*	−3.44*	−3.85*	−3.46*	96848.0*	279923.2*
$Australia$	−3.59*	−3.61*	−5.12*	−5.13*	12583.7*	12518.4*
$Austria$	−1.55	−1.81	−2.67	−3.58*	443.6	57.0
$Belgium$	−2.80	−2.82	−3.31	−3.18*	202.9	159.9*
$Czech$	−0.59	−1.50	−2.64	−2.75	93.0	894.0*
$Denmark$	−3.40	−3.42*	−4.24*	−3.97*	1295.7*	2149.0*
$Finland$	−3.95*	−3.95*	−3.46*	−3.48*	2550.8*	4426.2*
$Greece$	−2.62	−2.49	−2.78	−2.79	22.7	28.5
$Hungary$	−2.77	−2.48	−2.55	−2.39	25.5	113.3
$Ireland$	−2.53	−2.57	−3.08	−3.16*	195.8	202.7*
$Korea$	−3.88*	−2.68	−3.33	−2.48	98.7	3224.6*
$Mexico$	−1.68	−1.57	−4.01*	−4.11*	15.7	43.7
$Netherlands$	−2.87	−2.62	−2.92	−2.67	24.3	130.4
$NewZealand$	−2.88	−2.63	−3.07	−2.62	24.8	55.9
$Norway$	−3.80*	−3.70*	−4.71*	−5.28*	2171362*	3031270*
$Portugal$	−2.77	−2.80	−3.59*	−3.23*	82.5	59.3
$Spain$	−3.94*	−3.82*	−4.75*	−4.76*	1411.8*	2012.4*
$Sweden$	−4.03*	−3.89*	−5.96*	−5.99*	21500.8*	32971.8*
$Switzerland$	−3.81*	−3.64*	−3.39*	−3.68*	1239.3*	1405.9*
$Turkey$	−3.32	−1.88	−3.96*	−3.16*	12.6	378.6*
$HongKong$	−2.03	−2.11	−4.23*	−3.99*	15.4	21.8
$Singapore$	−3.44*	−3.12*	−3.61*	−3.54*	123.6	651.1*
$Malaysia$	−3.57*	−3.58*	−3.33	−3.39*	947.7*	1352.2*
$Indonesia$	−2.76	−2.47	−4.62*	−3.90*	3794.3*	1062.7*
$Thailand$	−3.28	−3.31*	−3.78*	−3.78*	149.9	156.7*
$Philippines$	−4.22*	−2.29	−5.04*	−3.30*	37.0	42499.3*
$SriLanka$	−1.28	−1.46	−1.76	−1.87	5.9	6.0
$Argentina$	−4.26*	−4.36*	−3.24	−2.87	1104925*	742.1*
$Bolivia$	−5.67*	−1.28	−11.26*	−3.34*	10238196*	26113851*
$Chile$	−2.00	−1.54	−1.99	−1.70	4.2	9.1
$Colombia$	−2.34	−2.34	−2.94	−2.76	26.6	42.1
$Venezuela$	−2.02	−2.33	−0.50	−1.84	715.9*	1656.0*
# rejections	13	15	22	26	15	26

Table 5. Unit root test results for bilateral real exchange rate (DM)						
Augmented models with fixed lag of 4						
Country	$\hat{t}^{\tau}_{DF}$	$\tilde{t}^{\tau}_{DF}$	$\hat{t}^{\tau}_{STAR}$	$\tilde{t}^{\tau}_{STAR}$	$\widehat{\mathcal{W}}^{\tau}_{\exp}$	$\widetilde{\mathcal{W}}^{\tau}_{\exp}$
US	−3.00	−2.98*	−2.42	−2.56	631.4*	702.1*
Japan	−3.25	−2.49	−4.10*	−3.52*	27.8	2943.9*
Italy	−2.24	−2.25	−3.32	−3.26*	22.7	29.3
UK	−2.08	−2.08	−2.27	−2.1	13.2	8.7
Canada	−3.29	−3.09*	−2.56	−2.74	122.7	282.5*
Australia	−3.69*	−3.71*	−3.01	−3.34*	302.2	337.1*
Austria	−3.43*	−3.14*	−3.16	−2.81	218.0	422.5*
Belgium	−2.53	−2.75	−3.53*	−3.84*	155.9	124.4
Denmark	−3.53*	−3.55*	−3.54*	−3.33*	3011.9*	2492.4*
Finland	−3.31	−3.33*	−4.21*	−4.2*	1866.8*	1737.2*
Greece	−2.68	−2.74	−7.68*	−7.62*	171.8	246.4*
Hungary	−1.74	−1.79	−2.28	−2.29	40.1	43.0
Ireland	−2.63	−2.65	−3.54*	−3.56*	180.4	211.1*
Korea	−3.06	−1.86	−4.40*	−1.92	6.8	225.7*
Mexico	−4.75*	−4.71*	−4.09*	−4.09*	315031.4*	275743.3*
Netherlands	−2.57	−2.43	−2.38	−2.04	371.0	230.2*
Norway	−3.55*	−3.65*	−3.09	−2.92	1159.3*	878.8*
Spain	−2.83	−2.66	−3.45*	−2.69	759.0*	1615.2*
Sweden	−3.88*	−3.84*	−4.18*	−4.17*	1360.9*	1587.8*
Switzerland	−3.06	−2.94*	−4.28*	−3.62*	682.2*	1606.0*
Turkey	−2.26	−2.30	−2.02	−2.19	30.64	37.6
Singapore	−2.43	−2.46	−2.10	−2.10	170.1	255.5*
Malaysia	−3.15	−2.35	−3.86*	−3.57*	41.7	748.7*
Indonesia	−2.29	−1.64	−2.40	−1.82	64.0	229.4*
Thailand	−3.42*	−3.23*	−2.44	−2.22	849.0*	1688.4*
Philippines	−3.83*	−1.85	−4.41*	−1.99	59.831	5858.6*
SriLanka	−3.50*	−2.77	−3.97*	−2.85	250.6	4455.7*
Argentina	−5.70*	−1.69	−2.97	−1.55	4.0	10064985*
Chile	−1.99	−1.83	−2.60	−3.54*	97.1	9551.0*
Colombia	−2.27	−2.29	−1.82	−1.85	28.0	27.4
Venezuela	−2.22	−2.28	−2.21	−2.48	28.0	21.1
# rejections	10	11	14	13	9	24

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