

# A Long Run Structural Macroeconometric Model of the UK\*

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## Abstract

A new modelling strategy is introduced that provides a practical approach to incorporating long-run structural relationships, suggested by economic theory, in an otherwise unrestricted VAR model. The strategy is applied to construct a small quarterly macroeconometric model of the UK, estimated over 1965q1-1999q4 in nine variables: domestic and foreign outputs, prices and interest rates, oil prices, the nominal effective exchange rate, and real money balances. The aim is to develop a model with a transparent and theoretically coherent foundation. Tests of restrictions on the long-run relations of the model are presented. The dynamic properties of the model are discussed using impulse responses for the effects of an oil price shock and a monetary policy shock on the long-run relations and the endogenous variables. A decision-based approach is used to identify the monetary policy shock as the movements in interest rates beyond those explained by the implementation of an optimal interest rate rule and by oil price, exchange rate and foreign interest rate innovations.

Keywords: Long-Run Structural VAR, A Core UK Model, Macroeconomic Modelling, Monetary Policy Shock, Oil Price Shock.

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# 1 Introduction

Over the past two decades, there has been a growing interest in developing macroeconomic models with transparent theoretical foundations and flexible dynamics that fit the historical time series data reasonably well. The modelling framework described in the present paper, along with the work of King *et al.* (1991), Gali (1992), Mellander *et al.* (1992) and Crowder *et al.* (1999), represent the first steps towards this aim. However, our work is distinguished from these earlier contributions in three respects. *First*, we develop a long-run framework suitable for modelling a small open macroeconomy like the UK. The models of King *et al.*, Gali and Crowder *et al.* are closed economy models suitable for modelling an economy such as the US. The four-variable model of Mellander *et al.* is made open only through a terms of trade variable added to the three-variable consumption-investment-income model analysed by King *et al.* and does not have a rich enough structure for the analysis of many open economy macroeconomic problems of interest. *Second*, we describe a new strategy which provides a practical approach to incorporating the long-run structural relationships suggested by economic theory in an otherwise unrestricted vector autoregressive (VAR) model. And *third*, we employ new econometric techniques in the construction of the model and in the testing of the long-run properties predicted by the theory. The description of the modelling work not only provides one of the first examples of the use of these techniques in an applied context, but also includes a discussion of some bootstrap experiments designed to investigate the small-sample properties of the tests employed. Hence, the paper reinforces the arguments of the papers cited above by emphasising the (statistical and economic) importance of the long-run in macroeconometric modelling. But it also provides a practical demonstration of how this can be done through the development a quarterly “core” macroeconometric model for the UK economy estimated over the period 1965q1-1999q4.

The model contains transparent and theoretically coherent long-run properties of the type exhibited by Real Business Cycle models. The long-run relations are derived rigorously from production, arbitrage, solvency and portfolio balance conditions, and these are then embedded in an otherwise unrestricted VAR model.<sup>1</sup> The model comprises six domestic variables whose developments are widely regarded as essential to a basic understanding of the behaviour of the UK macroeconomy; namely, aggregate output, the ratio of domestic to foreign price levels, inflation, the nominal interest rate, the exchange rate and real money balances. The model also contains foreign output, foreign interest rates and oil prices.

The empirical analysis of the core model provides insights into the functioning of the UK macroeconomy from two perspectives. *First*, the econometric methodology that has been developed provides the means for testing formally the validity of (over identifying) restrictions implied by specific long-run structural relations without imposing “incredible” restrictions on the *short-run* coefficients.<sup>2</sup> The ability to test rigorously the validity of long-run restrictions implied by economic theory within the context of a small and transparent, but reasonably

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<sup>1</sup>The role of the long-run in macroeconometric modelling is discussed in Garratt *et al.* (2000) where the structural cointegrating VAR approach to modelling followed in this paper is compared to other approaches found in the literature.

<sup>2</sup>For a discussion of the econometric methodology see Pesaran and Shin (2001) and Pesaran, Shin and Smith (2000) and the references cited therein.

comprehensive, model of the UK macroeconomy is an important step towards an evaluation of the long-run underpinnings of alternative macro theories. *Second*, our approach allows an investigation of the short-run dynamic responses of the model to shocks, while ensuring that the effects of such shocks on the long-run relations eventually vanish. This property holds irrespective of whether the shocks under consideration are structural or reduced form, and provides an important insight into the dynamics of cointegrating models where shocks have permanent effects on the levels of the individual variables in the model.

In considering the dynamic responses of the macroeconomy captured by our model, we shall discuss how our modelling approach relates to the recent developments in macroeconomics literature, as set out in the papers by Bernanke (1986), Christiano and Eichenbaum (1992), Cochrane (1998), Crowder *et al.* (1999) and Wickens and Motto (2001), for example. In these, the effects of particular types of shocks, typically those associated with monetary policy changes, are considered. Such an analysis requires the use of *a priori* identifying restrictions on the short-run dynamics as well as on the long-run relationships of the model. Different approaches to the identification of the short-run dynamics in macroeconomic models have been attempted, but so far no consensus has emerged.<sup>3</sup> In this paper, we focus on the identification of the dynamic responses of shocks to oil prices and domestic interest rates (viewed as the non-systematic component of the monetary policy). Bernanke *et al.* (1997) discuss the importance of these shocks for a better understanding of the role of monetary policy in postwar US business cycles, and the role of the shocks is equally interesting in the context of the UK economy. Within our modelling framework, the analysis of oil price shocks does not pose particular identification problems since we treat oil prices as the only “long-run forcing variable” of our model.<sup>4</sup> For identification of the effects of monetary shocks, however, suitable *a priori* restrictions are required. We adopt a minimalist strategy, in the sense that we consider restrictions needed to identify the effects of monetary shocks only. No attempts are made in this paper to identify the effects of other shocks such as supply and demand shocks. For identification of the monetary shocks, we adopt a decision-based approach. In this, we explicitly derive an interest rate policy rule by assuming that monetary authorities minimize a quadratic loss function in inflation and output growth subject to a structural model that links the policy instrument (the interest rate) to the target variables conditional on the publicly available information at the start of each period. Ruling out time inconsistency in the conduct of the monetary policy, and assuming that the authorities have no information advantage over the economic agents, we are then able to identify the impulse response function of the monetary shocks consistent with our core model of the UK economy.

It is worth noting that our long-run modelling strategy is not affected by the particular scheme that we will be using for the identification of monetary shocks. For long-run modelling, other types of identification problems need to be addressed; namely, the identification of the cointegrating vectors and the different types of disturbances that are involved. These

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<sup>3</sup>For a recent statement of the problem see, for example, Levchenkova *et al.* (1998) and Pagan (1999).

<sup>4</sup>The concept of long run forcing is discussed in Granger and Lin (1995) and Pesaran, Shin and Smith (2000) and is weaker than the concept of Granger non-causality, in the sense that it allows for lagged changes in the endogenous variables to influence oil prices, although it still rules out shocks to the endogenous variables to have any long-run impacts on oil prices.

include: “long-run structural disturbances”, which measure the extent to which the economy deviates from its long-run equilibrium; “long-run reduced form disturbances”, which are derived as a function of the long-run structural disturbances and which are measured in terms of observable variables; and “structural innovations”, measuring the unexpected movements of particular variables. By highlighting the distinctions between these types of disturbance, the modelling procedure makes explicit some of the difficulties which will arise in formulating the economic theory of the short-run and the problems involved in interpreting the effects of shocks in general, and in the analysis of impulse responses in particular.

The plan of the paper is as follows: Section 2 describes a long-run theoretical framework for macroeconomic modelling of a small open economy such as the UK, and derives testable restrictions on the long-run relations. Section 3 outlines how the long-run relations are embodied in a Vector Error Correction model and discusses the relationship between the “*long run structural*” approach to macroeconometric modelling and the “*structural*” approach which primarily focuses on identification schemes relating to the contemporaneous relationships between variables. Section 4 discusses the identification of the short-run dynamics and set out the decision-based approach to the identification of the effects of the monetary policy shock. Section 5 describes the empirical analysis underlying the construction of the core model and discusses the results obtained from testing its long-run properties. Section 6 reports the estimates of impulse response functions for the oil price and monetary policy shocks and comments on the dynamic properties of the estimated model. Section 7 provides some concluding remarks.

## 2 A Framework for Long-Run Macromodelling

In this section, we outline the theoretical basis of our approach to macroeconomic modelling of a small open economy such as the UK. It begins with a rigorous derivation of the long-run steady-state relationships expected to prevail between the main variables in the core model. The analysis emphasises arbitrage conditions and stock-flow equilibria and, as such, corresponds to many of the long-run properties of the RBC and large macroeconometric models.

There are two main theoretical approaches to the derivation of long-run, steady state relations of a core macroeconomic model. One possibility is to start with the inter-temporal optimization problems faced by “representative” households and firms and solve for the long-run relations using the Euler first-order conditions and the stock-flow constraints. Given the typically non-quadratic form of the utility and production functions and the linear forms of the constraints, the resultant relations of the model economy are generally highly non-linear and are usually approximated by log-linear relations (the real business cycle literature follows this methodology). The long-run relations are then obtained by ignoring expectational errors and assuming that the model economy is stationary and ergodic in certain variables, such as growth rates, capital per effective worker and asset-income ratios. An alternative approach, and the one which is followed here, is to work directly with the arbitrage conditions which provide inter-temporal links between prices and asset returns in the economy as a whole. The arbitrage conditions, however, must be appropriately modified to allow for the risks associated with market uncertainties.

The above two approaches are closely related and yield similar results as far as the long-run relations are concerned. The main difference between them lies in their treatment of short-run dynamics. The inter-temporal optimization approach purports to be based on a complete short-run dynamic theory: the intrinsic dynamics of the model, generated by the model's intertemporal elements, are usually complemented by extrinsic dynamics through the specification of dynamic stochastic processes for taste and technology variables. The strength of the inter-temporal optimization approach lies in the explicit identification of macroeconomic disturbances as innovations (shocks) to processes generating tastes and technology. However, this is achieved at the expense of often strong assumptions concerning the form of the underlying utility and production functions. Further, despite the efforts to capture the short-run dynamics explicitly, the models do abstract from the dynamic effects of many types of adjustment costs, of learning, and of aggregation across agents with heterogeneous information. The evidence presented by Christiano and Eichenbaum (1992) and Kim and Pagan (1995), for example, suggests that many of the parameter restrictions underlying the current vintage of RBC models are not supported by the data. In contrast, the approach advocated in this paper, by focussing on long-run theory restrictions and leaving the short-run dynamics largely unrestricted (in the context of a VAR model), provides a much more flexible modelling strategy.

We begin our derivation of long-run relations with a specification of the sectoral disaggregation that we will be using, and the associated accounting identities.

## 2.1 Stock-Flow Relations and Accounting Identities

To introduce the notations and to define the concepts, we use the following stock identities:

$$\tilde{D}_t = \tilde{H}_t + \tilde{B}_t, \quad (2.1)$$

$$\tilde{F}_t = E_t \tilde{B}_t^* - (\tilde{B}_t - \tilde{B}_t^d), \quad (2.2)$$

$$\tilde{L}_t = \tilde{H}_t + \tilde{B}_t^d + E_t \tilde{B}_t^*, \quad (2.3)$$

where  $\tilde{D}_t$  is net government debt,  $\tilde{H}_t$  is the stock of high-powered money,  $\tilde{B}_t$  is the stock of domestic bonds issued by the government,  $\tilde{F}_t$  is the net foreign asset position of the economy,  $\tilde{B}_t^*$  is the stock of foreign assets held by domestic residents,  $\tilde{B}_t^d$  is the stock of domestic assets held by domestic residents, and  $\tilde{L}_t (= \tilde{D}_t + \tilde{F}_t)$  is the stock of financial assets held by the private sector.<sup>5</sup> All the stocks are measured at the beginning of period  $t$ . Nominal magnitudes are denoted with a ' $\sim$ ', and are expressed in Pounds Sterling, except  $\tilde{B}_t^*$  which is expressed in foreign currency.  $E_t$  is the effective exchange rate, defined as the domestic price of a unit of foreign currency at the beginning of period  $t$ , so that an increase in the exchange rate represents a depreciation of the home country currency. It is assumed that the government holds no foreign assets of its own.

We also have the output-expenditure flow identity:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + (\tilde{X}_t - \tilde{M}_t), \quad (2.4)$$

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<sup>5</sup>It is assumed that foreign asset holdings of domestic residents and domestic holdings of foreign residents are composed of government bonds only.

where  $\tilde{Y}_t$  is gross domestic product,  $\tilde{C}_t$  consumption expenditures,  $\tilde{I}_t$  investment expenditures,  $\tilde{G}_t$  government expenditures,  $\tilde{X}_t$  is expenditures on exports and  $\tilde{M}_t$  expenditures on imports, all are in current market prices and expressed in Pounds Sterling. The private sector disposable income is defined by

$$\tilde{Y}_t^d = \tilde{Y}_t - \tilde{T}_t + R_t \tilde{B}_t^d + E_t R_t^* \tilde{B}_t^*, \quad (2.5)$$

where  $\tilde{T}_t$  represents taxes net of transfers to the private sector,  $R_t$  is the nominal interest rate on domestic assets held from the beginning to the end of period  $t$ , and  $R_t^*$  is the nominal interest rate paid on foreign assets during period  $t$ .

The core model economy's stock-flow relationships are:

$$\Delta \tilde{D}_{t+1} = \tilde{G}_t + R_t \tilde{B}_t - \tilde{T}_t \quad (2.6)$$

$$\Delta \tilde{L}_{t+1} = \tilde{Y}_t^d - \tilde{C}_t - \tilde{I}_t + (E_{t+1}^e - E_t) \tilde{B}_t^* \quad (2.7)$$

$$\Delta \tilde{F}_{t+1} = \tilde{X}_t - \tilde{M}_t + \widetilde{NFA}_t + (E_{t+1}^e - E_t) \tilde{B}_t^* \quad (2.8)$$

where  $\widetilde{NFA}_t = E_t R_t^* \tilde{B}_t^* - R_t(\tilde{B}_t - \tilde{B}_t^d)$  is net factor income from abroad, and  $E_{t+1}^e$  stands for exchange rate expectations formed on the basis of publicly available information at time  $t$ . Hence, the term  $(E_{t+1}^e - E_t) \tilde{B}_t^*$  is the (expected) revaluation of foreign assets held by domestic residents accruing through exchange rate appreciation in period  $t$ .<sup>6</sup> Note that, since  $\tilde{L}_t = \tilde{D}_t + \tilde{F}_t$ , any two of (2.6)-(2.8) implies the third.

## 2.2 Production Technology and Output Determination

We assume that, in the long-run, aggregate output is determined according to the following constant returns to scale production function in labour (denoted by  $N_t$ ) and capital stock (denoted by  $K_t$ ):

$$\frac{\tilde{Y}_t}{P_t} = F(K_t, A_t N_t) = A_t N_t F\left(\frac{K_t}{A_t N_t}, 1\right), \quad (2.9)$$

where  $P_t$ , is a general price index,  $\tilde{Y}_t/P_t$  is real aggregate output, and  $A_t$  stands for an index of labour-augmenting technological progress, assumed to be composed of a deterministic component,  $a_0 + g t$ , and a stochastic mean-zero component,  $\eta_{at}$  :

$$\ln(A_t) = a_0 + g t + \eta_{at}. \quad (2.10)$$

The process generating  $\eta_{at}$  is likely to be quite complex and there is little direct evidence on its evolution. But a few studies that have used patent data or R&D expenditures to directly

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<sup>6</sup>In most formulations of stock-flow relationships the asset revaluation term is either ignored or is approximated by an *ex post* counterpart such as  $(E_t - E_{t-1}) \tilde{B}_t^*$ . But for consistency with the arbitrage (equilibrium) conditions to be set out in Section 2.3 below, we prefer to work with the *ex ante* asset revaluation term.

analyse the behaviour of  $\eta_{at}$  over the course of the business cycle generally find highly persistent effects of technological disturbances on output. (Discussed in Fabiani (1996) and the references cited therein). The indirect evidence on  $\eta_{at}$ , obtained from empirical analysis of aggregate output, also corroborates this finding and generally speaking does not reject the hypothesis that  $\eta_{at}$  contains a unit root. (See, for example, Nelson and Plosser (1982) and, for the UK, Mills (1991)).

We further assume that the fraction of the population which is employed at time  $t$ ,  $\lambda_t = N_t/POP_t$ , is a stationary process such that

$$N_t = \lambda POP_t \exp(\eta_{nt}), \quad (2.11)$$

where  $POP_t$  is population at the end of period  $t$  and  $\eta_{nt}$  represents a stationary, mean-zero process capturing the cyclical fluctuations of the unemployment rate around its steady state value,  $1 - \lambda$ .<sup>7</sup> Under the above assumptions and using the relations (2.9), (2.10) and (2.11) it now readily follows that

$$y_t = a_0 + \ln(\lambda) + g t + \ln(f(\kappa_t)) + \eta_{at} + \eta_{nt}, \quad (2.12)$$

where  $y_t = \ln\left(\frac{\tilde{Y}_t/P_t}{POP_t}\right)$  is the logarithm of real per capita output,  $\kappa_t = K_t/A_t N_t$  is the capital stock per effective labour unit, and  $f(\kappa_t) = F(K_t/A_t N_t, 1)$  is a well behaved function in the sense that it satisfies the Inada conditions. (See, for example, Barro and Sala-i-Martin (1995, p.16)).

Assuming the aggregate saving rate is monotonic in  $\kappa_t$  then, under certain other mild regularity conditions, Binder and Pesaran (1999) show that, irrespective of whether the process generating  $u_{at}$  is stationary or contains a unit root,  $\kappa_t$  converges to a globally attracting, invariant steady state probability distribution; namely  $\kappa_t \rightarrow \kappa_\infty$ , where  $\kappa_\infty$  is a time-invariant random variable with a non-degenerate probability distribution function. Hence, in the long-run the evolution of per capita output will be largely determined by technological process, with  $E(\Delta \ln(y_t)) = g$ . Also whether  $y_t$  contains a unit root crucially depends on whether there is a unit root in the process generating technological progress.

Given the small and open nature of the UK economy, it is reasonable to assume that, in the long-run,  $A_t$  is determined by the level of technological progress in the rest of the world; namely

$$A_t = \gamma A_t^* \exp(\eta_{at}), \quad (2.13)$$

where  $A_t^*$  represents the level of foreign technological progress,  $\gamma$  captures productivity differentials based on fixed, initial technological endowments, and  $\eta_{at}$  represents stationary, mean zero disturbances capturing the effects of information lags or (transitory) legal impediments to technology flows across different countries, for example. Assuming that per capita output in the rest of the world is also determined according to a neoclassical growth model, and using a similar line of reasoning as above, we have

$$y_t - y_t^* = \ln(\gamma) + \ln(\lambda/\lambda^*) + \ln\{f(\kappa_t)/f^*(\kappa_t^*)\} + \eta_{at} + (\eta_{nt} - \eta_{nt}^*), \quad (2.14)$$

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<sup>7</sup>Notice that the assumption that the unemployment rate,  $1 - (L_t/POP_t)$ , is stationary in effect rules out long-run hysteresis effects in the unemployment process. To accommodate such effects in a satisfactory manner requires consideration of non-linear dynamic models which is beyond the scope of the present paper.

where foreign variables are shown with a “star”. Similarly to  $\kappa_t$  the foreign capital stock per effective labour unit,  $\kappa_t^*$ , also tends to a time invariant probability distribution function, and hence under the assumption that  $A_t^*$  (or  $A_t$ ) contain a unit root,  $(y_t, y_t^*)$  will be cointegrated with a cointegrating vector equal to  $(1, -1)$ .

The above stochastic formulation of the neoclassical growth model also has important implications for the determination of the real rate of return, which we denote by  $\rho_t$ . Profit maximisation on the part of firms ensures that, in the steady-state,  $\rho_t$  will be equal to the marginal product of capital, so that

$$\rho_t = f'(\kappa_t), \quad (2.15)$$

where  $f'(\kappa_t)$  is the derivative of  $f(\kappa_t)$  with respect to  $\kappa_t$ . Since  $\kappa_t \rightarrow \kappa_\infty$ , it therefore follows that  $\rho_t \rightarrow f'(\kappa_\infty)$ ; thus establishing that the steady state distribution of the real rate of return will also be ergodic and stationary. This result allows us to write

$$1 + \rho_{t+1} = (1 + \rho) \exp(\eta_{\rho,t+1}), \quad (2.16)$$

where  $\eta_{\rho,t+1}$  is a stationary process normalized so that  $E(\exp(\eta_{\rho,t+1}) | I_t) = 1$ , and where  $I_t$  is the publicly available information set at time  $t$ . This normalization ensures that  $\rho$  is in fact the mean of the steady state distribution of real returns,  $\rho_t$ , given by  $E[f'(\kappa_\infty)]$ .

## 2.3 Arbitrage conditions

The first set of arbitrage conditions to be considered here are included in many macroeconomic models in one form or another. They are the (relative) Purchasing Power Parity (*PPP*), the Fisher Inflation Parity (*FIP*), and the Uncovered Interest Parity (*UIP*) relationships. We consider each of these in turn.

Purchasing Power Parity is based on the presence of goods market arbitrage, and captures the idea that the price of a common basket of goods will be equal in different countries when measured in a common currency. Information disparities, transportation costs or the effects of tariff and non-tariff barriers are likely to create considerable deviations from (absolute) *PPP* in the short-run and, with the likely exception of information disparities, these might persist indefinitely. However, if the size of these influences has a constant mean over time, then the common currency price of the basket of goods in the different countries will rise one-for-one over the longer term, and this is captured by the (weaker) concept of ‘relative *PPP*’. The primary explanation of long-run deviations from relative *PPP* is the ‘Harrod-Balassa-Samuelson (H-B-S) effect’ in which the price of a basket of traded and non-traded goods rises more rapidly in countries with relatively rapid productivity growth in the traded goods sector.<sup>8</sup> Following these arguments, we express relative *PPP* as

$$P_{t+1} = E_{t+1} P_{t+1}^* \exp(\eta_{ppp,t+1}), \quad (2.17)$$

where  $P_t^*$  is the foreign price index and the term in brackets captures the deviations from *PPP*. Here,  $\eta_{ppp,t+1}$  is assumed to follow a stationary (or possibly trend-stationary) process

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<sup>8</sup>See Obstfeld and Rogoff (1996, Ch.4) and Rogoff (1996) for further discussion of this effect and alternative modifications to *PPP*.



capturing short-run variations in transport costs, information disparities, and the effects of tariff and non-tariff barriers. The errors  $\eta_{ppp,t+1}$  could be conditionally heteroscedastic, although this is unlikely to be very important in quarterly macro-models. The effects of differential productivity growth rates in the traded and non-traded goods sectors at home and abroad, accommodating the H-B-S effect, can be captured by assuming that  $\eta_{ppp,t+1}$  contains a trend.

The *FIP* relationship captures the equilibrium outcome of the arbitrage process between holding bonds and investing in physical assets. Denoting the expected real rate of return on physical assets over the period  $t$  to  $t + 1$  by  $\rho_{t+1}^e$ , and denoting inflation expectations over the same period by  $(P_{t+1}^e - P_t)/P_t$ , we have

$$\begin{aligned} (1 + R_t) &= (1 + \rho_{t+1}^e) \left( 1 + \frac{P_{t+1}^e - P_t}{P_t} \right) \exp(\eta_{fip,t+1}) \\ &= (1 + \rho_{t+1}^e) \left( \frac{P_{t+1}^e}{P_t} \right) \left( 1 + \frac{\Delta P_{t+1}}{P_t} \right) \exp(\eta_{fip,t+1}), \end{aligned} \quad (2.18)$$

where  $\eta_{fip,t+1}$  is the risk premium, capturing the effects of money and goods market uncertainties on risk-averse agents. We assume that  $\eta_{fip,t+1}$  follows a stationary process with a finite mean and variance. Also recall that in the context of the neoclassical growth model the real rate of interest (which we take to be the same as the real rate of return on capital) follows a stationary process; see (2.15) and (2.16).

The third arbitrage condition is based on the *UIP* relationship, which captures the equilibrium outcome of the arbitrage process between holding domestic and foreign bonds. In this, any differential in interest rates across countries must be offset by expected exchange rate changes to eliminate the scope for arbitrage. The presence of transactions costs, risk premia and speculative effects provide for the possibility of short-run deviations from *UIP*, and we therefore define the *Interest Rate Parity* relationship (*IRP*) as follows:

$$\begin{aligned} (1 + R_t) &= (1 + R_t^*) \left( 1 + \frac{E_{t+1}^e - E_t}{E_t} \right) \exp(\eta_{uip,t+1}) \\ &= (1 + R_t^*) \left( \frac{E_{t+1}^e}{E_t} \right) \left( 1 + \frac{\Delta E_{t+1}}{E_t} \right) \exp(\eta_{uip,t+1}), \end{aligned} \quad (2.19)$$

where  $\eta_{uip,t+1}$  is the risk premium associated with the effects of bonds and foreign exchange uncertainties on risk-averse agents. As before, we shall assume that  $\eta_{uip,t+1}$  is stationary and ergodic.<sup>9</sup>

In the absence of direct observations on inflation, exchange rate and real rates of return expectations and for the purpose of long-run modelling, we write

$$P_{t+1}^e = P_{t+1} \exp(\eta_{p,t+1}^e), \quad E_{t+1}^e = E_{t+1} \exp(\eta_{e,t+1}^e), \quad \text{and} \quad (1 + \rho_{t+1}^e) = (1 + \rho_{t+1}) \exp(\eta_{\rho,t+1}^e) \quad (2.20)$$

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<sup>9</sup>As noted above, the relationships in (2.18) and (2.19) can also be derived from Euler equations obtained from consumer and producer optimisation in an intertemporal model of an economy with well behaved preferences and technologies.

and assume that the expectations errors  $\eta_{i,t+1}^e$ ,  $i = p, e, \rho$ , follow stationary processes. The assumption that the expectation errors are stationary seems quite plausible and is consistent with a wide variety of hypotheses concerning the expectations formation process.<sup>10</sup>

Using the relations in (2.20) and (2.16), the *FIP* and the *IRP* relations in terms of the observables can be written as:

$$\ln(1 + R_t) = \ln(1 + \rho) + \ln\left(1 + \frac{\Delta P_{t+1}}{P_t}\right) + \eta_{fip,t+1} + \eta_{\rho,t+1} + \eta_p^e + \eta_{\rho,t+1}^e, \quad (2.21)$$

and

$$\ln(1 + R_t) = \ln(1 + R_t^*) + \eta_{\Delta e,t+1} + \eta_{uip,t+1} + \eta_e^e, \quad (2.22)$$

where  $\eta_{\Delta e,t+1} = \Delta \ln(E_{t+1})$ . The log-linear version of the *PPP* relationship in (2.17) is also given by

$$\ln(P_{t+1}) = \ln(P_{t+1}^*) + \ln(E_{t+1}) + \eta_{ppp,t+1}. \quad (2.23)$$

## 2.4 Long-run solvency requirements

In addition to the arbitrage conditions, the economy is also subject to the long-run solvency constraint obtainable from the stock-flow relationships given by (2.6)-(2.8). In order to ensure the long-run solvency of the private sector asset/liability position, we assume

$$\tilde{L}_{t+1}/\tilde{Y}_t = \mu \exp(\eta_{ly,t+1}), \quad (2.24)$$

where  $\eta_{ly,t+1}$  is a stationary process, so that the ratio of total financial assets to the nominal income level is stationary and ergodic. Expression (2.24) captures the idea that domestic residents are neither willing nor able to accumulate claims on, or liabilities to, the government and the rest of the world which are out of line with its current and expected future income. This condition, in conjunction with assumptions on the determinants of the equilibrium portfolio balance of the private sector assets, provides an additional long-run relation which we shall include in our core model of the macroeconomy.

In modelling the equilibrium portfolio balance of private sector assets, we follow Branson's (1977) Portfolio Balance Approach. From (2.3), we note that the stock of financial assets held by the private sector consists of the stock of high-powered money plus the stock of domestic and foreign bonds held by domestic residents. Given this adding-up constraint, we specify two independent equilibrium relationships relating to asset demand; namely, those relating to the demand for high-powered money and for foreign assets. These relationships are characterised in our model by the following:

$$\frac{\tilde{H}_{t+1}}{\tilde{L}_t} = F_h \left( \frac{Y_t}{P_t}, \rho_{b,t+1}^e, \rho_{b,t+1}^{*e}, \frac{\Delta P_{t+1}^e}{P_t}, t \right) \exp(\eta_{h,t+1}), \quad F_{h1} \geq 0, F_{h2} \leq 0, F_{h3} \leq 0, F_{h4} \leq 0, \quad (2.25)$$

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<sup>10</sup>This assumption is consistent with the Rational Expectations Hypothesis (REH), for example. However, it is much less restrictive than the REH, and can accommodate the possibility of systematic expectational errors in the short-run, possibly due to incomplete learning.

and

$$\frac{\tilde{F}_{t+1}}{\tilde{L}_t} = F_f \left( \frac{Y_t}{P_t}, \rho_{b,t+1}^e, \rho_{b,t+1}^{*e}, \frac{\Delta P_{t+1}^e}{P_t}, t \right) \exp(\eta_{f,t+1}), \quad F_{f1} \leq 0, F_{f2} \leq 0, F_{f3} \geq 0, F_{f4} \geq 0, \quad (2.26)$$

where

$$Y_t = \tilde{Y}_t / POP_{t-1}, \quad \rho_{b,t+1}^e = \frac{(1 + R_t)}{\left(1 + \frac{P_{t+1}^e - P_t}{P_t}\right)} - 1, \quad \text{and} \quad \rho_{b,t+1}^{*e} = \frac{(1 + R_t^*) \left(1 + \frac{E_{t+1}^e - E_t}{E_t}\right)}{\left(1 + \frac{P_{t+1}^e - P_t}{P_t}\right)} - 1, \quad (2.27)$$

where the last two are respectively the expected real rates of return on domestic and foreign bonds (both measured in domestic currency),  $\eta_{h,t}$  is a stationary process which captures the effects of various factors that contribute to the short-run deviations of the ratio of money balances to total financial assets from its long-run determinants, and where  $\eta_{f,t}$  is the corresponding stationary process capturing the effects of short-run deviations of the ratio of foreign assets to total financial assets from its long-run position. The determinants of the ratio of money to total financial assets in (2.25) include the real output level, to capture the influence of the transactions demand for money, and the expected real rates of return on the three alternative forms of holding financial assets; namely domestic bonds, foreign bonds and high-powered money. We have also specified a deterministic trend in  $F_h(\cdot)$  to allow for the possible effect of the changing nature of financial intermediation, and the increasing use of credit cards in settlement of transactions on the convenience value of money. One would expect a downward trend in  $H/L$ , reflecting a trend reduction in the proportion of financial assets held in the form of non-interest bearing high-powered money over time. The determinants of the ratio of foreign assets to total financial assets in (2.26) are the same, with the decision to hold assets in the form of bonds mirroring that relating to holding assets in the form of money.

In view of the *IRP* relationship of (2.19), it is clear that, in the steady state, domestic and foreign bonds become perfect substitutes, and their expected rates of return are equal. Similarly, given the *FIP* relationship of (2.18) the real rates of return on (both) domestic and foreign bonds are equal to the (stationary) real rate of return on physical assets in the steady state. Hence, the asset demand relationships of (2.25) and (2.26) can be written equally as:

$$\frac{\tilde{H}_{t+1}}{\tilde{L}_t} = F_{hl} \left( \frac{Y_t}{P_t}, R_t, t \right) \exp(\eta_{hl,t+1}), \quad F_{hl1} \geq 0, F_{hl2} \leq 0, \quad (2.28)$$

and

$$\frac{\tilde{F}_{t+1}}{\tilde{L}_t} = F_{fl} \left( \frac{Y_t}{P_t}, R_t, t \right) \exp(\eta_{fl,t+1}), \quad F_{fl1} \leq 0, F_{fl2} \geq 0, \quad (2.29)$$

where the effects of the short-run deviations from *IRP* and *FIP* are now subsumed into the more general stationary processes  $\eta_{hl,t+1}$  and  $\eta_{fl,t+1}$  and where the effects of the expected real rate of return on non-interest bearing money holdings (i.e. minus the expected inflation

rate) are captured by the domestic nominal interest rate (again making use of (2.18)). Note that this final effect implies that different rates of inflation, and hence different levels of nominal interest rates, could change the equilibrium portfolio composition, depending on the responsiveness of the asset demands to the relative returns on the three assets, so that changes in nominal rates of interest can potentially have lasting real effects.<sup>11</sup>

The solvency condition in (2.24) combined with equation (2.28) now yields

$$\frac{\tilde{H}_{t+1}}{\tilde{Y}_t} = \frac{H_{t+1}}{Y_t} = \mu F_h(Y_t, R_t, t) \exp(\eta_{ly,t+1} + \eta_{hl,t+1}), \quad (2.30)$$

where  $H_t = \tilde{H}_t/POP_{t-1}$ . A similar relationship, based on (2.24) and (2.29), also exists, although foreign asset levels are less frequently the focus of attention in macroeconomic models.<sup>12</sup> Equation (2.30) therefore provides the final long-run relationship to be considered in our core model of the UK macroeconomy.

### 3 Econometric Formulation of the Core Model

For empirical purposes we employ a log-linear approximation of the five long-run equilibrium relationships set out in the previous section in (2.23), (2.22), (2.14), (2.30) and (2.21). These constitute the long-run relationships of the core model and take the following form:

$$p_t - p_t^* - e_t = b_{10} + b_{11}t + \xi_{1,t+1}, \quad (3.1)$$

$$r_t - r_t^* = b_{20} + \xi_{2,t+1}, \quad (3.2)$$

$$y_t - y_t^* = b_{30} + \xi_{3,t+1}, \quad (3.3)$$

$$h_t - y_t = b_{40} + b_{41}t + \beta_{42}r_t + \beta_{43}y_t + \xi_{4,t+1}, \quad (3.4)$$

$$r_t - \Delta p_t = b_{50} + \xi_{5,t+1}, \quad (3.5)$$

where  $p_t = \ln(P_t)$ ,  $p_t^* = \ln(P_t^*)$ ,  $e_t = \ln(E_t)$ ,  $y_t = \ln(Y_t/P_t)$ ,  $y_t^* = \ln(Y_t^*/P_t^*)$ ,  $r_t = \ln(1 + R_t)$ ,  $r_t^* = \ln(1 + R_t^*)$ ,  $h_t - y_t = \ln(H_{t+1}/P_t) - \ln(Y_t/P_t) = \ln(H_{t+1}/Y_t)$  and  $b_{50} = \ln(1 + \rho)$ .<sup>13</sup> We have allowed for intercept and trend terms (when appropriate) in order to ensure that (long-run) reduced form disturbances,  $\xi_{i,t+1}$ ,  $i = 1, 2, \dots, 5$ , have zero means. These disturbances

<sup>11</sup>The possibility of the “super-non-neutrality” of monetary policy arising through this route is discussed in Buiters (1980), for example.

<sup>12</sup>The stock-flow relationship of (2.8) can be used in conjunction with (2.2) to motivate a relationship between net foreign assets, net exports and domestic and foreign interest rates. Assuming that net exports depends on domestic and foreign output and the terms of trade, substitution of these relationships into (2.29) provides the justification for a further possible long run relationship between  $Y_t$ ,  $Y_t^*$ ,  $R_t$ ,  $R_t^*$ , and  $E_t P_t^*/P_t$ .

<sup>13</sup>For expositional simplicity, we have chosen to denote  $\ln(H_{t+1}/P_t)$  by  $h_t$ , rather than  $h_{t+1}$ . Recall that  $H_{t+1}$  relates to the stock of high powered money at the beginning of period  $t + 1$ .

are related to the (long-run) structural disturbances, the  $\eta_i$ 's, in the following manner:<sup>14</sup>

$$\begin{aligned}
\xi_{1,t+1} &= \eta_{ppp,t} - b_{10} - b_{11}t, \\
\xi_{2,t+1} &= \eta_{uip,t+1} + \eta_{e,t+1}^e + \eta_{\Delta e,t+1} - b_{20}, \\
\xi_{3,t+1} &= \eta_{at} + (\eta_{nt} - \eta_{nt}^*) + (\eta_{kt} - \eta_{kt}^*), \\
\xi_{4,t+1} &= \eta_{ly,t} + \eta_{hl,t}, \\
\xi_{5,t+1} &= \eta_{fip,t+1} + \eta_{\rho,t+1} + \eta_{p,t+1}^e + \eta_{\rho,t+1}^e.
\end{aligned} \tag{3.6}$$

The above relationships between the long-run structural disturbances,  $\eta_i$ 's, and the long-run reduced form disturbances,  $\xi_i$ 's, clearly show the difficulties involved in identifying the effects of changes in particular structural disturbances on the dynamic behaviour of the macroeconomy. For example,  $\xi_{5,t+1}$  is composed of the four structural disturbances,  $\eta_{fip,t+1}, \eta_{\rho,t+1}, \eta_{p,t+1}^e, \eta_{\rho,t+1}$ , representing the different factors that could be responsible for disequilibria between inflation and interest rates. In general, without further *a priori* restrictions, the effect of structural disturbances,  $\eta_i$ 's, cannot be identified: firstly, there are many more long-run structural disturbances than there are long-run reduced form disturbances; and, secondly, there is no reason to believe that the  $\eta_i$ 's are not themselves contemporaneously correlated. Empirical analysis at best enables us to identify the effect of changes in the long-run reduced form disturbances on the evolution of the macroeconomy towards its long-run equilibrium, although, as we discuss below, even identification of the effects of specific changes in these long-run reduced form disturbances will typically require further identifying restrictions based on an explicit model of short-run decision-making.

The five long-run relations of the core model, (3.1) - (3.5), can be written more compactly as

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_0 - \mathbf{b}_1(t-1), \tag{3.7}$$

where

$$\mathbf{z}_t = (p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'. \tag{3.8}$$

$$\mathbf{b}_0 = (b_{01}, b_{02}, b_{03}, b_{04}, b_{05})', \quad \mathbf{b}_1 = (b_{11}, 0, 0, b_{14}, 0),$$

$$\boldsymbol{\xi}_t = (\xi_{1t}, \xi_{2t}, \xi_{3t}, \xi_{4t}, \xi_{5t})',$$

and

$$\boldsymbol{\beta}' = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\beta_{42} & 0 & -\beta_{43} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.9}$$

In modelling the short-run dynamics, we follow Sims (1980) and others and assume that departures from the long-run relations,  $\boldsymbol{\xi}_t$ , can be approximated by a linear function of

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<sup>14</sup>In the case of  $\xi_{2,t+1}$ , we have taken account of the effect of exchange rate depreciation on the interest rate differential since, as we shall see below, the hypothesis that  $\eta_{\Delta e,t+1}$  is stationary cannot be rejected.

a *finite* number of past changes in  $\mathbf{z}_{t-1}$ . For estimation purposes we also partition  $\mathbf{z}_t = (p_t^o, \mathbf{y}_t)'$  where  $\mathbf{y}_t = (e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$ . Here,  $p_t^o$  (the logarithm of oil prices) is considered to be a ‘long-run forcing’ variable for the determination of  $\mathbf{y}_t$ , in the sense that changes in  $p_t^o$  have a direct influence on  $\mathbf{y}_t$ , but changes in  $p_t^o$  are not affected by the presence of  $\boldsymbol{\xi}_t$ , which measure the extent of disequilibria in the UK economy. We choose to treat foreign output as endogenous following the arguments advanced in the previous section in which both domestic and foreign output are driven by a common stochastic trend capturing the rate of technological progress. Given the simultaneity of the determination of  $y_t$  and  $y_t^*$ , and the possibility of feedbacks, it is appropriate to model the two output variables symmetrically. Similar arguments also hold for domestic and foreign interest rates. The endogenous treatment of foreign output and interest rates involves loss of efficiency in estimation if they were in fact long-run forcing or strictly exogenous, but we consider this to be less serious than following the alternative route of treating these variables as exogenous if this turned out to be false.

The treatment of oil prices as ‘long-run forcing’ represents a generalisation of the approach to modelling oil price effects in some previous applications of cointegrating VAR analyses (e.g. Johansen and Juselius (1992), or Pesaran and Shin (1996)), where the oil price change is treated as a strictly exogenous  $I(0)$  variable. The approach taken in the previous literature excludes the possibility that there exist cointegrating relationships which involve the oil price level, while the approach taken here allows the validity of the hypothesised restriction to be tested, and for the restriction to be imposed if it is not rejected.

Under the assumption that oil prices are long-run forcing for  $\mathbf{y}_t$  the cointegrating properties of the model can be investigated without having to specify the oil price equation.<sup>15</sup> However, specification of a oil price equation is required for the analysis of the short-run dynamics. For this purpose we shall adopt the following general specification for the evolution of oil prices:

$$\Delta p_t^o = \delta_o + \sum_{i=1}^{s-1} \delta_{oi} \Delta \mathbf{z}_{t-i} + u_{ot}, \quad (3.10)$$

where  $u_{ot}$  represents a serially uncorrelated oil price shock with a zero mean and a constant variance. The above specification ensures oil prices are long-run forcing for  $\mathbf{y}_t$  since it allows lagged changes in the endogenous and exogenous variables of the model to influence current oil prices but rules out the possibility that error correction terms,  $\boldsymbol{\xi}_t$  have any effects on oil price changes. These assumptions are weaker than the requirement of ‘Granger non-causality’ often invoked in the literature.

Assuming that the variables in  $\mathbf{z}_t$  are difference-stationary (as discussed in Section 5 below), our modelling strategy is now to embody  $\boldsymbol{\xi}_t$  in an otherwise unrestricted  $VAR(s-1)$  in  $\mathbf{z}_t$ . Under the assumption that oil prices are long-run forcing, it is efficient (for estimation purposes) to base our analysis on the following *conditional* error correction model

$$\Delta \mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y \boldsymbol{\xi}_t + \sum_{i=1}^{s-1} \Gamma_{yi} \Delta \mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo} \Delta p_t^o + \mathbf{u}_{yt}, \quad (3.11)$$

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<sup>15</sup>See, for example, Pesaran, Shin and Smith (2000).

where  $\mathbf{a}_y$  is an  $8 \times 1$  vector of fixed intercepts,  $\boldsymbol{\alpha}_y$  is a  $8 \times 5$  matrix of error-correction coefficients (also known as the loading coefficient matrix),  $\{\Gamma_{yi}, i = 1, 2, \dots, s-1\}$  are  $8 \times 9$  matrices of short-run coefficients,  $\boldsymbol{\psi}_{yo}$  is an  $8 \times 1$  vector representing the impact effects of changes in oil prices on  $\Delta \mathbf{y}_t$ , and  $\mathbf{u}_{yt}$  is an  $8 \times 1$  vector of disturbances assumed to be  $IID(0, \Sigma_y)$ , with  $\Sigma_y$  being a positive definite matrix, and by construction uncorrelated with  $u_{ot}$ . Using equation (3.7), we now have

$$\Delta \mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y \mathbf{b}_0 - \boldsymbol{\alpha}_y \left[ \boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \Gamma_{yi} \Delta \mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo} \Delta p_t^o + \mathbf{u}_{yt}, \quad (3.12)$$

where  $\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)$  is  $5 \times 1$  vector of error correction terms. The above specification embodies the economic theory's long-run predictions by construction, in contrast to the more usual approach where the starting point is an unrestricted VAR model, with some vague priors about the nature of the long-run relations.

Estimation of the parameters of the core model, (3.12), can be carried out using the long-run structural modelling approach described in Pesaran and Shin (2001) and Pesaran, Shin and Smith (2000). It is based on a modified and generalised version of Johansen's (1991,1995) maximum likelihood approach to the problem of estimation and hypothesis testing in the context of vector autoregressive error correction models. With this approach, having selected the order of the underlying VAR model (using model selection criteria such as the Akaike Information Criterion (*AIC*) or the Schwarz Bayesian Criterion (*SBC*)), we test for the number of cointegrating relations among the 9 variables in  $\mathbf{z}_t$ . When performing this task, and in all the subsequent empirical analysis, we work in the context of a VAR model with unrestricted intercepts and restricted trend coefficients.<sup>16</sup> In terms of (3.12), we allow the intercepts  $\mathbf{c}$  to be freely estimated but restrict the trend coefficients so that  $\boldsymbol{\alpha}_y \mathbf{b}_1 = \Pi_y \boldsymbol{\gamma}$ , where  $\Pi_y = \boldsymbol{\alpha}_y \boldsymbol{\beta}'$  and  $\boldsymbol{\gamma}$  is an  $9 \times 1$  vector of unknown coefficients. These restrictions ensure that the solution of the model in levels of  $\mathbf{z}_t$  will not contain quadratic trends.<sup>17</sup> We then compute Maximum Likelihood (*ML*) estimates of the model's parameters subject to exact and over-identifying restrictions on the long-run coefficients. Assuming that there is empirical support for the existence of five long-run relationships, as suggested by theory, exact identification in our model requires five restrictions on each of the five cointegrating vectors (each row of  $\boldsymbol{\beta}$ ), or a total of twenty-five restrictions on  $\boldsymbol{\beta}$ . These represent only a subset of the restrictions suggested by economic theory as characterized in (3.9), however. Estimation of the model subject to all the (exact- and over-identifying) restrictions given in (3.9) enables a test of the validity of the over-identifying restrictions, and hence the long-run implications of the economic theory, to be carried out.

<sup>16</sup>This is referred to as Case IV in Pesaran, Shin and Smith (2000).

<sup>17</sup>By an analogous argument, it should be noted that the analysis of a model in which oil price inflation is taken as strictly exogenous has the undesirable property that the effects of oil prices on the model differs according to the number of cointegrating relations assumed to exist among the variables. The treatment of oil prices in this paper, which allows oil prices to enter into the cointegrating vectors, eliminates this possibility. This provides a further important argument in favour of the approach adopted in this paper as compared to the treatment of oil prices in some of the earlier papers discussed above.

## 4 Identification of Short-Run Dynamics

The modelling strategy described above has the two-fold advantage that it accommodates the relationships that hold between variables in the long-run as suggested by an explicit macroeconomic theory and that the estimated model reflects parsimoniously the complex dynamic relationships that exist between variables at shorter horizons. For the analysis of the short-run dynamic responses of the system to economic shocks of interest further restrictions on the contemporaneous relationships amongst the variables are required. As a first step towards deriving such a “structural” form, we first combine the oil price equation (3.10) and the conditional model (3.12) to obtain the reduced form specification

$$\Delta \mathbf{z}_t = \begin{pmatrix} \Delta p_t^o \\ \Delta \mathbf{y}_t \end{pmatrix} = \mathbf{a} - \boldsymbol{\alpha} \left[ \boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{v}_t, \quad (4.1)$$

where

$$\begin{aligned} \mathbf{a} &= \begin{pmatrix} \delta_o \\ \psi_{yo} \delta_o + a_y - \alpha_y b_0 \end{pmatrix}, \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \boldsymbol{\alpha}_y \end{pmatrix}, \Gamma_i = \begin{pmatrix} \delta_{oi} \\ \boldsymbol{\psi}_{yo} \boldsymbol{\delta}_{oi} + \Gamma_{yi} \end{pmatrix}, \\ \mathbf{v}_t &= \begin{pmatrix} 1 & \mathbf{0} \\ \boldsymbol{\psi}_{yo} & \mathbf{I}_8 \end{pmatrix} \begin{pmatrix} u_{ot} \\ \mathbf{u}_{yt} \end{pmatrix} = \begin{pmatrix} u_{ot} \\ \boldsymbol{\psi}_{yo} u_{ot} + \mathbf{u}_{yt} \end{pmatrix}. \end{aligned}$$

Under standard assumptions, all the reduced form coefficients can be consistently estimated from the ML estimates of the parameters of the conditional model (3.12), and the OLS regression of the oil price equation (3.10). In particular, the vector of the reduced form errors  $\mathbf{v}_t$ , and its covariance matrix,  $\Sigma$ , can be estimated consistently from the reduced form parameters.

The “structural” Vector Error Correction Model (VECM) associated with the *long run structural* macroeconometric model defined by (3.10) and (3.12) is given by:

$$\mathbf{A} \Delta \mathbf{z}_t = \tilde{\mathbf{a}} - \tilde{\boldsymbol{\alpha}} \left[ \boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \tilde{\Gamma}_i \Delta \mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_t, \quad (4.2)$$

where  $\mathbf{A}$  represents the  $9 \times 9$  matrix of contemporaneous structural coefficients,  $\tilde{\mathbf{a}} = \mathbf{A}\mathbf{a}$ ,  $\tilde{\boldsymbol{\alpha}} = \mathbf{A}\boldsymbol{\alpha}$ ,  $\tilde{\Gamma}_i = \mathbf{A}\Gamma_i$ , and  $\boldsymbol{\varepsilon}_t = \mathbf{A}\mathbf{v}_t$  are the associated structural shocks which are serially uncorrelated and have zero means and the positive definite variance covariance matrix,  $\Omega = \mathbf{A}\Sigma\mathbf{A}'$ .

For the identification of the structural coefficients, a large variety of *a priori* restrictions have been entertained in the literature, most of which are in the form of restrictions on the elements of  $\mathbf{A}$  (or its inverse  $\mathbf{A}^{-1}$ ) and/or  $\Omega$ . The so-called structural VAR approach considers restrictions on the (structural) long-run multiplier matrix,  $\tilde{\Pi} = \tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}'$ . The remaining structural coefficients, defined by  $\tilde{\mathbf{a}}$ ,  $\tilde{\boldsymbol{\alpha}}\mathbf{b}_1$ , and  $\tilde{\Gamma}_i$ ,  $i = 1, 2, \dots, s-1$ , are generally left unrestricted. For exact identification of all the structural coefficients in our model,  $9^2 = 81$  restrictions are required. The familiar recursive structure pioneered by Sims (1980) achieves this by requiring  $\mathbf{A}$  to be a triangular matrix and  $\Omega$  to be a diagonal matrix. This imposes a recursive, Wold-causal ordering on the contemporaneous relationships amongst the endogenous variables of the model. Non-recursive restrictions on  $\mathbf{A}$  have also been considered in



the literature, often by imposing restrictions directly on  $\tilde{\Pi}$ , and/or on the coefficients of the exogenous or forcing variables,  $\psi_{y_0}$ , when such variables are included in the analysis. See, for example, Blanchard and Watson (1986), Blanchard and Quah (1989), Bernanke (1986), Sims (1986), Christiano and Eichenbaum (1992), Cochrane (1998), Crowder *et al.* (1999) and Wickens and Motto (2001). A useful review, in the context of the liquidity premium puzzle, is provided in Pagan and Robertson (1998).<sup>18</sup>

It is worth noting that, in order to impose binding restrictions on  $\tilde{\Pi}$ , it is important that the loading coefficients,  $\tilde{\alpha} = \mathbf{A}\alpha$ , are restricted directly. For example, since  $\mathbf{A}$  is non-singular, cointegrating rank restrictions on  $\tilde{\Pi}$  do not restrict  $\tilde{\alpha}$ . Similarly, the imposition of exact identifying restrictions on  $\beta$  does not help in the identification of  $\mathbf{A}$  if  $\tilde{\alpha}$  is left unrestricted. As pointed out in Pesaran and Shin (2001), in the absence of restrictions on  $\tilde{\alpha}$ , the choice of an identification scheme for the long-run parameters in  $\beta$  has no bearing on the identification of  $\mathbf{A}$  (and *vice versa*). However, if restrictions were to be placed on  $\tilde{\alpha}$  then, as a consequence, there will be inter-relationships between the identification of the cointegrating vectors and the (structural) impulse responses.

In the case of the core model presented in this paper, we have not been able to motivate strong arguments in favour of the imposition of identifying restrictions on the loading coefficients and/or on the coefficients of the oil price changes. Therefore, without *a priori* restrictions on  $\mathbf{A}$  and/or  $\Omega$  it would not be possible to give economic meanings to the estimates of the loading coefficients,  $\tilde{\alpha}$ , or to identify economically meaningful impulse response functions to shocks.<sup>19</sup> The restrictions on  $\mathbf{A}$  that are necessary for identification of these structural effects requires a tight description of the decision-rules followed by the public and private economic agents, incorporating information on agents' use of information and the exact timing of the information flows. To address the identification problem relevant to all the structural shocks will be beyond the scope of the present paper and requires a detailed analysis of the consumption and investment decision problems of private agents as well as the reaction functions of the monetary authorities. Here we focus on the identification of the effects of oil and monetary policy shocks, taking as given the decision rules of the private agents as embodied in the structural relations bearing on output and inflation in the structural VECM, (4.2).

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<sup>18</sup>In the absence of a fully-specified economic model of the short-run, impulse response analysis is best conducted using the Generalised Impulse Response function, developed in Koop *et al.* (1996) and Pesaran and Shin (1998), which allows for the interdependence of shocks but does not place a structural interpretation on the effect of shocks.

<sup>19</sup>The structural shocks,  $\varepsilon_{it}$ 's, are quite distinct from the long run structural disturbances,  $\eta_{it}$ 's, of (3.6). While both have a clear economic interpretation, the former measure the (typically white noise) deviations of choice variables from the value suggested by the corresponding decision rule. The latter measure the deviations from long-run relationships in which the equilibrating pressures are identified by economic theory. The mechanics of the equilibrating processes are not necessarily described by economic theory (involving unspecified adjustment costs, rigidities, coordination issues and so on), but theory explains why the long run structural disturbances are stationary.

## 4.1 Identification of Monetary Policy Shocks

For identification of the monetary policy shocks, we need to formally articulate the decision problem of the monetary authorities. We assume that, at the start of each period, the monetary authorities try to influence the market interest rate,  $r_t$ , by setting the base rate,  $r_t^b$  that they have under their direct control. The difference between the market rate and the base rate, the term premium, is determined by unanticipated factors such as oil price shocks, unexpected changes in foreign interest rates and exchange rates. These four variables, namely  $p_t^o$ ,  $e_t$ ,  $r_t^*$ , and  $r_t - r_t^b$  are likely to be contemporaneously determined in the market place on a daily (even intra-daily) basis. The remaining variables,  $y_t$ ,  $y_t^*$ ,  $\Delta p_t$ ,  $p_t - p_t^*$ , and  $h_t - p_t$ , are much less frequently observed, often with relatively long delays, and their contemporaneous values can be reasonably excluded from the determination of the term premium although, as we shall see below, lagged values of these variables can still affect the term premium.<sup>20</sup> Accordingly, we shall assume that the term premium equation has the following form

$$\begin{aligned} r_t - r_t^b &= \rho_{t-1} + a_{rr^*} [r_t^* - E(r_t^* | \mathfrak{J}_{t-1})] \\ &\quad + a_{re} [e_t - E(e_t | \mathfrak{J}_{t-1})] + a_{ro} [p_t^o - E(p_t^o | \mathfrak{J}_{t-1})] + \varepsilon_{rt}, \end{aligned} \quad (4.3)$$

where  $\mathfrak{J}_{t-1}$  is the information set of the monetary authorities at the end of  $t - 1$ ,  $\rho_{t-1}$  is the predictable component of the term premium, which could be a general function of one or more elements in the information set  $\mathfrak{J}_{t-1}$ ,  $r_t^b$  is the systematic component of monetary policy, and  $\varepsilon_{rt}$  the monetary policy shock.<sup>21</sup> In addition to  $\varepsilon_{rt}$ , the unexpected component of the term premium is assumed to vary linearly with the unanticipated changes in oil prices, the exchange rate, and the foreign interest rate. We shall assume that the monetary policy shocks,  $\varepsilon_{rt}$ , satisfy the following standard orthogonality condition

$$E(\varepsilon_{rt} | \mathfrak{J}_{t-1}) = 0,$$

and the associated time-varying expected term premium is given by

$$E(r_t - r_t^b | \mathfrak{J}_{t-1}) = \rho_{t-1}.$$

The term premium equation (4.3) can be written equivalently as

$$\begin{aligned} \Delta r_t &= r_t^b - r_{t-1} + \rho_{t-1} + a_{rr^*} [\Delta r_t^* - E(\Delta r_t^* | \mathfrak{J}_{t-1})] \\ &\quad + a_{re} [\Delta e_t - E(\Delta e_t | \mathfrak{J}_{t-1})] + a_{ro} [\Delta p_t^o - E(\Delta p_t^o | \mathfrak{J}_{t-1})] + \varepsilon_{rt}. \end{aligned} \quad (4.4)$$

Under expectations formation mechanisms consistent with the reduced form VECM (4.1), the expectational variables  $E(\Delta r_t^* | \mathfrak{J}_{t-1})$ ,  $E(\Delta e_t | \mathfrak{J}_{t-1})$ , and  $E(\Delta p_t^o | \mathfrak{J}_{t-1})$  can be replaced by

<sup>20</sup>The issue of timing is clearly important here. Rotemberg and Woodford (1999) make the point that even data which is reported concurrently needs to be processed before its message about the economy becomes clear, especially considering the revisions which occur in output data and prices. Equally, there may be difficulties in responding to contemporaneous variables even if they are observable immediately given that the political process of responding to them takes time.

<sup>21</sup>In the past, monetary shocks have been measured by innovations to monetary aggregates, but several researchers, including Sims (1992) and Bernanke *et al.* (1997), have argued that innovations to short-term interest rates are preferable in this respect.

the error correction terms  $\beta' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)$  and the lagged changes  $\Delta z_{t-i}$ ,  $i = 1, 2, \dots, s-1$ . This would yield

$$\begin{aligned} & \Delta r_t - a_{rr^*} \Delta r_t^* - a_{re} \Delta e_t - a_{ro} \Delta p_t^o \\ &= r_t^b - r_{t-1} + \rho_{t-1} + \phi_r^* \left[ \beta' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \phi_{zi}^* \Delta z_{t-i} + \varepsilon_{rt}. \end{aligned} \quad (4.5)$$

where the parameters  $\phi_r^*$  and  $\phi_{zi}^*$  are functions of the  $a_{rr^*}$ ,  $a_{re}$ ,  $a_{ro}$  and the coefficients in the rows of (4.1) associated with  $\Delta r_t^*$ ,  $\Delta e_t$  and  $\Delta p_t^o$ .

For the derivation of  $r_t^b$ , we follow the literature on inflation targeting and assume that it is derived as the solution to the following optimization problem<sup>22</sup>

$$\min_{r_t^b} \{E [C(\mathbf{w}_t, r_t) \mid \mathfrak{I}_{t-1}]\}, \quad (4.6)$$

where  $C(\mathbf{w}_t, r_t)$  is the loss function of the monetary authorities, assumed to be quadratic so that

$$C(\mathbf{w}_t, r_t) = \frac{1}{2}(\mathbf{w}_t - \mathbf{w}_t^\dagger)' \mathbf{Q}(\mathbf{w}_t - \mathbf{w}_t^\dagger) + \frac{1}{2}\theta(r_t - r_{t-1})^2, \quad (4.7)$$

where  $\mathbf{w}_t = (y_t, \Delta p_t)'$  and  $\mathbf{w}_t^\dagger = (y_t^\dagger, \pi_t^\dagger)'$  are the target variables and their desired values, respectively. Since the target variables are both assumed to be  $I(1)$  in our model, the desired target values,  $\mathbf{w}_t^\dagger$  also need to be  $I(1)$  for the optimization problem to be controllable; otherwise the solution to the control problem will not be consistent with the assumed structural model. A simple yet realistic specification for  $\mathbf{w}_t^\dagger$  is given by

$$\mathbf{w}_t^\dagger = \begin{pmatrix} y_{t-1} + g_y^\dagger \\ \Delta p_{t-1} - g_\pi^\dagger \end{pmatrix} = \mathbf{w}_{t-1} + \Delta \mathbf{w}^\dagger, \quad (4.8)$$

where  $\Delta \mathbf{w}^\dagger = (g_y^\dagger, -g_\pi^\dagger)'$ ,  $g_y^\dagger > 0$  is the fixed target level for output growth, and  $g_\pi^\dagger > 0$  is the desired amount of reduction in the rate of inflation. Using (4.8), the monetary authorities' objective function can now be written in terms of stationary variables. We have

$$C(\mathbf{w}_t, r_t) = \frac{1}{2}(\Delta \mathbf{w}_t - \Delta \mathbf{w}^\dagger)' \mathbf{Q}(\Delta \mathbf{w}_t - \Delta \mathbf{w}^\dagger) + \frac{1}{2}\theta(\Delta r_t)^2. \quad (4.9)$$

The  $2 \times 2$  matrix  $\mathbf{Q}$  characterizes the authorities' short-term trade off between output growth and a reduction in the rate of inflation. The final term in (4.9) is intended to capture the institutional and political costs of changes to the interest rate.

The solution to the above optimization problem requires the specification of a model linking the target variables,  $\mathbf{w}_t$ , to the policy instrument,  $r_t^b$ . Within our framework, such a model can be derived as a sub-system of the general long-run structural model specified

<sup>22</sup>For recent accounts, see Blinder (1998), Bernanke *et al.* (1999) and Svensson (1999), for example.

in (4.2) with (4.5) as its structural interest rate equation. Subject to this sub-model, and assuming that the first order condition for the minimization of (4.9) is given by<sup>23</sup>

$$E \left[ \left( \frac{\partial \Delta \mathbf{w}_t}{\partial r_t^b} \right)' \mathbf{Q} (\Delta \mathbf{w}_t - \Delta \mathbf{w}^\dagger) + \theta \left( \frac{\partial r_t}{\partial r_t^b} \right) \Delta r_t \mid \mathcal{I}_{t-1} \right] = 0, \quad (4.10)$$

the outcome is a feedback rule (or reaction function) of the following form:

$$r_t^b = r_{t-1} - \rho_{t-1} + \phi^\diamond + \phi_r^\diamond \left[ \beta' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \phi_{z_i}^{\diamond'} \Delta \mathbf{z}_{t-i}, \quad (4.11)$$

where  $\phi^\diamond$ ,  $\phi_r^\diamond$  and  $\phi_{z_i}^\diamond$  are functions of the preference parameters of the monetary authorities,  $g_y^\dagger$ ,  $g_\pi^\dagger$ ,  $\mathbf{Q}$  and  $\theta$ , and the parameters of the econometric model.<sup>24</sup>

Using (4.11) in (4.5) now yields the structural equation for the interest rate

$$\begin{aligned} & \Delta r_t - a_{rr^*} \Delta r_t^* - a_{re} \Delta e_t - a_{ro} \Delta p_t^o \\ &= \phi^\diamond + (\phi_r^\diamond + \phi_r^*) \left[ \beta' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} (\phi_{z_i}^\diamond + \phi_{z_i}^*)' \Delta \mathbf{z}_{t-i} + \varepsilon_{rt}, \end{aligned} \quad (4.12)$$

where we identify and define  $\varepsilon_{rt}$  as the monetary policy shock. Note that this structural equation is consistent with the long-run properties of the general structural model specified in (4.2).<sup>25</sup> In particular, although changes in the preference parameters of the monetary authorities affect the magnitude and the speed with which interest rates respond to economic disequilibria, such changes have no effect on the long-run coefficients,  $\beta$ , that are determined by general arbitrage conditions. It is also easily shown that, while changes in the trade-off parameter matrix,  $\mathbf{Q}$ , affect all the short-run coefficients of the interest rate equation, changes to the desired target values,  $g_y^\dagger$  and  $g_\pi^\dagger$ , affect only the intercept term,  $a_b$ .<sup>26</sup>

## 4.2 Impulse Response Functions

The structural interest rate equation (4.12) can now be used, in conjunction with certain other *a priori* restrictions, to derive the impulse response functions of the monetary policy

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<sup>23</sup>The problem of the credibility of the monetary policy, discussed in the literature by Barro and Gordon (1983), Rogoff (1985) and, more recently, by Svensson (1997), for example, is resolved in our application by the common knowledge assumption and the information symmetry. It is also worth noting that the extension of the decision problem to an inter-temporal setting will complicate the analysis but does not materially alter our main conclusions.

<sup>24</sup>Details of the exact dependence of the parameters of the feedback rule on the preference parameters and the parameters of the econometric can be obtained from the authors on request.

<sup>25</sup>Note that the interest rate decision would not have been consistent with the assumed underlying structural model if a fixed inflation rate target were specified; i.e. if it was required that  $\pi_t^\dagger = g_\pi^\dagger$  as opposed to our specification where  $\pi_t^\dagger = \Delta p_{t-1} - g_\pi^\dagger$ .

<sup>26</sup>These properties could form the basis of an empirical test of recent developments in the conduct of monetary policy in the UK. But such an analysis is beyond the scope of the present work.

shocks,  $\varepsilon_{rt}$ . To this end we decompose  $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t})'$ , where  $\mathbf{z}_{1t} = (p_t^o, e_t, r_t^*, r_t)'$  and  $\mathbf{z}_{2t} = (\Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$ ,<sup>27</sup> and partition the structural model (4.2) accordingly:

$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{z}_{1t} \\ \Delta \mathbf{z}_{2t} \end{pmatrix} = \boldsymbol{\mu}_{t-1} + \begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{pmatrix},$$

where

$$\boldsymbol{\mu}_{t-1} = \tilde{\mathbf{a}} - \tilde{\boldsymbol{\alpha}} \left[ \boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1) \right] + \sum_{i=1}^{s-1} \tilde{\Gamma}_i \Delta \mathbf{z}_{t-i},$$

and

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{pmatrix} \sim iid \left[ \mathbf{0}, \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \right].$$

Our primary concern is with identification of the impulse responses associated with the shocks to the individual elements of  $\boldsymbol{\varepsilon}_{1t}$ . For this purpose, we adopt the following sets of restrictions:

$$\mathbf{A}_{12} = \mathbf{0}, \tag{4.13}$$

$$\mathbf{A}_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -a_{eo} & 1 & 0 & 0 \\ -a_{r^*o} & -a_{r^*e} & 1 & 0 \\ -a_{ro} & -a_{re} & -a_{rr^*} & 1 \end{pmatrix}, \tag{4.14}$$

and

$$\Omega_{11} = \begin{pmatrix} \omega_{oo} & 0 & 0 & 0 \\ 0 & \omega_{ee} & 0 & 0 \\ 0 & 0 & \omega_{r^*r^*} & 0 \\ 0 & 0 & 0 & \omega_{rr} \end{pmatrix}. \tag{4.15}$$

The first set of restrictions, (4.13), are justified on the grounds that the variables in  $\mathbf{z}_{2t}$  are much less frequently observed than those in  $\mathbf{z}_{1t}$ , and hence are unlikely to contemporaneously affect them. The lower triangular form of  $\mathbf{A}_{11}$  is motivated by the our theoretical derivation of the structural interest rate equation in the previous sub-section, plus the assumption that the UK exchange rate has a contemporaneous impact on foreign interest rates and not *vice versa*.<sup>28</sup> The final set of restrictions, (4.15), identify the structural shocks associated with  $\mathbf{z}_{1t}$  by assuming that these shocks are orthogonal to each other. For the sub-system containing  $\mathbf{z}_{1t}$ , the assumptions (4.14) and (4.15) are the familiar type of exact identifying restrictions employed in the literature, and together impose  $4^2$  restrictions needed for the exact identification of the impulse responses of the shocks to  $\boldsymbol{\varepsilon}_{1t}$ . However, as demonstrated below, the impulse responses associated with  $\boldsymbol{\varepsilon}_{1t}$  are invariant to the identification of the

<sup>27</sup>As shown below, the impulse response functions of the monetary policy shocks are invariant to the ordering of the variables in  $\mathbf{z}_{2t}$ .

<sup>28</sup>Recall that we are also assuming that oil prices can contemporaneously affect the macroeconomic variables, but is not itself contemporaneously affected by them.

rest of the system and, in particular, does not require  $\Omega_{12} = 0$ , or  $\mathbf{A}$  to be a lower-triangular matrix. It is also possible to show that in our set up the impulse responses of the monetary policy shocks are invariant to a re-ordering of the variables  $p_t^o, e_t$ , and  $r_t^*$  in  $\mathbf{z}_{1t}$ .<sup>29</sup> Hence, once the position of the monetary policy variable in  $\mathbf{z}_t$  is fixed (in our application after  $p_t^o, e_t$ , and  $r_t^*$ ), the impulse response functions of the monetary policy shocks will be invariant to the re-ordering of the variables before and after  $r_t$  in  $\mathbf{z}_t$ .

To derive the impulse responses, partition the reduced form errors defined by (4.1),  $\mathbf{v}_t = (\mathbf{v}'_{1t}, \mathbf{v}'_{2t})'$  conformably with  $\mathbf{z}_t$ , and note that

$$\Omega_{11} = Cov(\boldsymbol{\varepsilon}_{1t}), \Sigma_{11} = Cov(\mathbf{v}_{1t}), \mathbf{v}_{1t} = \mathbf{A}_{11}^{-1} \boldsymbol{\varepsilon}_{1t}.$$

Then, under (4.13),  $\Sigma_{11} = \mathbf{A}_{11}^{-1} \Omega_{11} \mathbf{A}_{11}'^{-1}$  and the 10 unknown coefficients in  $\mathbf{A}_{11}$  and  $\Omega_{11}$  can be obtained uniquely from the 10 distinct elements of  $\Sigma_{11}$ . A consistent estimate of  $\Sigma_{11}$  can be computed from the reduced form residuals,  $\hat{\mathbf{v}}_{1t}$ , namely  $\hat{\Sigma}_{11} = T^{-1} \sum_{t=1}^T \hat{\mathbf{v}}_{1t} \hat{\mathbf{v}}'_{1t}$ . Under the identification scheme in (4.14)-(4.15), the impulse response functions of the effects of a unit shock to the structural errors on  $\mathbf{z}_t$  can now be obtained following the approach set out in Koop *et al.* (1996), and discussed further in Pesaran and Shin (1998).<sup>30</sup> Let  $\mathbf{g}(n, \mathbf{z} : \varepsilon_i)$ ,  $i = o, e, r^*, r$ , be the generalized impulse responses of  $\mathbf{z}_{t+n}$  to a unit change in  $\varepsilon_{it}$ , measured by one standard deviation, namely  $\sqrt{\omega_{ii}}$ . Then, at horizon  $n$  we have

$$\mathbf{g}(n, \mathbf{z} : \varepsilon_i) = E(\mathbf{z}_{t+n} | \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathcal{J}_{t-1}) - E(\mathbf{z}_{t+n} | \mathcal{J}_{t-1}), \quad i = o, e, r^*, r.$$

Since all the shocks are assumed to be serially uncorrelated with zero means, (4.1) provides the following recursive relations in  $\mathbf{g}(n, \mathbf{z} : \varepsilon_i)$ :

$$\Delta \mathbf{g}(n, \mathbf{z} : \varepsilon_i) = \Pi \mathbf{g}(n-1, \mathbf{z} : \varepsilon_i) + \sum_{i=1}^{s-1} \Gamma_i \Delta \mathbf{g}(n-i, \mathbf{z} : \varepsilon_i), \quad \text{for } n = 1, 2, \dots, \quad (4.16)$$

with the initialization  $\mathbf{g}(n, \mathbf{z} : \varepsilon_i) = 0$  for  $n < 0$ , where  $\Delta \mathbf{g}(n, \mathbf{z} : \varepsilon_i) = \mathbf{g}(n, \mathbf{z} : \varepsilon_i) - \mathbf{g}(n-1, \mathbf{z} : \varepsilon_i)$ . In the case of  $n = 0$  (i.e. the impact effects), we have

$$\mathbf{g}(0, \mathbf{z} : \varepsilon_i) = E(\Delta \mathbf{z}_t | \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathcal{J}_{t-1}) - E(\Delta \mathbf{z}_t | \mathcal{J}_{t-1}). \quad (4.17)$$

Under (4.2) and conditional on  $\mathcal{J}_{t-1}$ ,  $(\varepsilon_{it}, \Delta \mathbf{z}'_t)'$  is distributed with mean

$$\begin{pmatrix} 0 \\ \mathbf{A}^{-1} \boldsymbol{\mu}_{t-1} \end{pmatrix},$$

and the covariance matrix

$$\begin{pmatrix} \omega_{ii} & E(\varepsilon_{it} \mathbf{v}'_t) \\ E(\varepsilon_{it} \mathbf{v}_t) & E(\mathbf{v}_t \mathbf{v}'_t) \end{pmatrix}.$$

<sup>29</sup>A proof is available from the authors on request.

<sup>30</sup>For more details and the application of the approach to structural simultaneous equation models see Pesaran and Smith (1998).

In the case where conditional on  $\mathfrak{J}_{t-1}$ ,  $\Delta \mathbf{z}_t$  is normally distributed, using familiar results on conditional expectations of multivariate normal densities, we have<sup>31</sup>

$$E(\Delta \mathbf{z}_t \mid \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathfrak{J}_{t-1}) = \mathbf{A}^{-1} \boldsymbol{\mu}_{t-1} + \frac{E(\varepsilon_{it} \mathbf{v}_t)}{\omega_{ii}} \sqrt{\omega_{ii}}.$$

But under (4.13),

$$\mathbf{v}_t = \begin{pmatrix} \mathbf{A}_{11}^{-1} \boldsymbol{\varepsilon}_{1t} \\ \mathbf{v}_{2t} \end{pmatrix},$$

and hence, using (4.17), we have

$$\mathbf{g}(0, \mathbf{z} : \varepsilon_i) = \frac{E(\varepsilon_{it} \mathbf{v}_t)}{\sqrt{\omega_{ii}}} = \frac{1}{\sqrt{\omega_{ii}}} \begin{bmatrix} \mathbf{A}_{11}^{-1} E(\varepsilon_{it} \boldsymbol{\varepsilon}_{1t}) \\ E(\varepsilon_{it} \mathbf{v}_{2t}) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11}^{-1} \Omega_{11}^{\frac{1}{2}} \boldsymbol{\tau}_i \\ E\left(\frac{\varepsilon_{it} \mathbf{v}_{2t}}{\sqrt{\omega_{ii}}}\right) \end{bmatrix}, \quad (4.18)$$

where  $\boldsymbol{\tau}_i$  is a  $4 \times 1$  selection vector for  $i = o, e, r^*, r$ . For the oil price shock  $\boldsymbol{\tau}_o = (1, 0, 0, 0)'$ , and for the monetary policy shock the selection vector is defined by  $\boldsymbol{\tau}_r = (0, 0, 0, 1)'$ . Under the identification restrictions (4.14) and (4.15), a consistent estimate of  $\mathbf{A}_{11}^{-1} \Omega_{11}^{\frac{1}{2}}$  can be obtained uniquely by the lower triangular Cholesky factor of  $\hat{\Sigma}_{11}$ . To consistently estimate  $E\left(\frac{\varepsilon_{it} \mathbf{v}_{2t}}{\sqrt{\omega_{ii}}}\right)$ , we note that, under the same restrictions,  $\omega_{ii}$ ,  $i = o, e, r^*, r$ , and the unknown elements of  $\mathbf{A}_{11}$  can also be consistently estimated using  $\hat{\Sigma}_{11}$ . It, therefore, remains to obtain a consistent estimate of  $E(\varepsilon_{it} \mathbf{v}_{2t})$ . Recall that  $\boldsymbol{\varepsilon}_{1t} = \mathbf{A}_{11} \mathbf{v}_{1t}$ . Hence  $E(\varepsilon_{it} \mathbf{v}_{2t})$  can be consistently estimated by the  $i$ -th row of

$$T^{-1} \sum_{t=1}^T \hat{\mathbf{A}}_{11} \hat{\mathbf{v}}_{1t} \hat{\mathbf{v}}'_{2t},$$

where  $\hat{\mathbf{A}}_{11}$  is a consistent estimate of  $\mathbf{A}_{11}$ . It is clear that the impulse response functions of shocks to the structural errors,  $\varepsilon_{it}$ ,  $i = o, e, r^*$ , and  $r$ , are invariant to the way the structural coefficients associated with the second block,  $\mathbf{z}_{2t}$ , in (4.2) are identified.

## 5 Estimation and Testing of the Model

In order to embody the long-run relations (3.1) to (3.5) within a suitable macroeconomic model, it is important that the variables used in the empirical analysis can reasonably be argued to be  $I(1)$  variables. The variables under consideration are  $y_t$ ,  $y_t^*$ ,  $r_t$ ,  $r_t^*$ ,  $e_t$ ,  $h_t - y_t$ ,  $p_t$ ,  $\tilde{p}_t$ ,  $p_t^*$  and  $p_t^o$ . Note that in the empirical analysis, to ensure a more satisfactory match between theoretical and empirical concepts, producer price indices are used to construct deviations between the domestic and foreign price levels in the PPP relationship, and the retail price index is used to measure domestic inflation in the FIP relationship. Hence we define  $\tilde{p}_t = \ln(\tilde{P}_t)$ , where  $\tilde{P}_t$  is the consumer price index. A detailed description of these variables is given in Table 1. The data is quarterly and seasonally-adjusted and covers the

<sup>31</sup>This result provides an optimal linear approximation when the errors are not normally distributed.

period 1963q1-1999q4, although to ensure that all regressions are comparable (irrespective of the order chosen for the underlying dynamic model), all estimations are carried out over the period 1965q1-1999q4 (140 observations). The Augmented Dickey-Fuller (ADF) test statistics, computed over the period 1965q1-1999q4, for the levels and first differences of the core variables are reported in Table 2. The results suggest that it is reasonable to treat  $y_t$ ,  $y_t^*$ ,  $e_t$ ,  $r_t$ ,  $r_t^*$ ,  $h_t - y_t$  and  $p_t^o$  as  $I(1)$  variables. For these variables the unit root hypotheses is rejected when applied to their first differences, but provide no evidence with which to reject the unit root hypothesis when the tests are applied to their levels. We obtained similar results using the Phillips and Perron (1988) tests.<sup>32</sup> There is, however, some ambiguity regarding the order of integration of the price variables. Application of the *ADF* tests to  $\Delta p_t$ ,  $\Delta \tilde{p}_t$  and  $\Delta p_t^*$  yields mixed results: the hypothesis that there is a unit root in the domestic and foreign inflation rates is rejected for low orders of augmentation (namely, for  $s = 0$  and 1), but not for higher orders. The application of the Phillips and Perron test rejects the unit root hypothesis when applied to  $\Delta p_t^*$ , but not when applied to  $\Delta p_t$  and  $\Delta \tilde{p}_t$ . Overall the available data is not informative as to whether domestic and foreign prices are  $I(1)$  or  $I(2)$ .

These results raise interesting issues concerning the use of economic theory and statistical evidence in macroeconometric modelling. Starting from the long-run theory set out in Section 2, the validity of the Fisher equation requires that inflation and interest rates have the same order of integration. The theoretical literature generally assumes that these series are  $I(0)$ , but as we have seen above the empirical evidence is mixed with the interest rate behaving as an  $I(1)$  variable and the inflation rate being a border line case. There is, therefore, a trade-off between the demands of theory and econometrics. Our approach to this dilemma is a pragmatic one, aiming to adequately capture the statistical properties of the data in a modelling framework which, at the same time, is coherent with our underlying analytic account of how the economy operates. For these reasons, in our work, we treat  $r_t$ ,  $r_t^*$ ,  $\Delta p_t$ ,  $\Delta \tilde{p}_t$  and  $\Delta p_t^*$  to be  $I(1)$  variables. This allows the empirical model to adequately represent the statistical features of the series over the sample period within our model and provides the scope for accommodating in the model the long run relationships described in the previous section of the paper.

Of course, domestic and foreign prices appear in their *level* in the PPP relationship of (3.1) and this raises the potential difficulty of mixing  $I(1)$  and  $I(2)$  variables. Haldrup's (1998) review of the econometric analysis of  $I(2)$  variables warns of the dangers of the inappropriate application of econometric methods designed for use with  $I(1)$  variables and suggests that it is often useful to transform time series *a priori* to obtain variables that are unambiguously  $I(1)$  rather than dealing with mixtures of  $I(1)$  and  $I(2)$  variables directly. In the case of the core variables under consideration, this is achieved by working with the relative price variables  $p_t - p_t^*$  rather than the two separate price levels  $p_t$  and  $p_t^*$ . As shown in Table 2, the relative price term is unambiguously  $I(1)$  according to the ADF statistics and it is therefore appropriate to apply the econometric methods described in Pesaran, Shin and Smith (2000)

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<sup>32</sup>The results for the Phillips and Perron unit root tests are available from the authors on request.



to a VAR model in the nine variables in  $\mathbf{z}_t = (p_t^o, e_t, r_t^*, r_t, \Delta\tilde{p}_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$ .<sup>33 34</sup>

## 5.1 Testing and Estimation of the Long Run Relations

The first stage of our modelling sequence is to select the order of the underlying VAR in these variables. Here we find that a VAR of order two appears to be appropriate when using the *AIC* as the model selection criterion, but that the *SBC* favours a VAR of order one. We proceed with the cointegration analysis using a VAR(2), on the grounds that the consequences of over-estimation of the order of the VAR is much less serious than under-estimating it (see Kilian (1997)).

Using a VAR(2) model with unrestricted intercepts and restricted trend coefficients, and treating the oil price variable,  $p_t^o$ , as weakly exogenous for the long-run parameters (or ‘long-run forcing’), we computed Johansen’s ‘trace’ and ‘maximal eigenvalue’ statistics.<sup>35</sup> These statistics, together with their associated 90% and 95% critical values, are reported in Table 3. The maximal eigenvalue statistic indicates the presence of just two cointegrating relationships at the 95% significance level, which does not support our *a priori* expectations of five cointegrating vectors. However, as shown by Cheung and Lai (1993), the maximum eigenvalue test is generally less robust to the presence of skewness and excess kurtosis in the errors than the trace test. Given that we have evidence of non-normality in the residuals of the VAR model used to compute the test statistics, we therefore believe it is more appropriate to base our cointegration tests on the trace statistics. As it happens the trace statistics reject the null hypotheses that  $r = 0, 1, 2, 3$  and 4 at the 5 per cent level of significance but cannot reject the null of hypothesis that  $r = 5$ . This is in line with our *a priori* expectations based on the long-run theory of Section 2. Hence we proceed under the assumption that there are five cointegrating vectors.

With five cointegrating relations we require five restrictions on each relationship to exactly identify them. In view of the underlying long-run theory as encapsulated in the relations (3.6) we impose the following 25 exact-identifying restrictions on the cointegrating matrix:

$$\beta' = \begin{pmatrix} \beta_{11} & \beta_{12} & 0 & 0 & \beta_{15} & 0 & 1 & \beta_{18} & 0 \\ \beta_{21} & 0 & \beta_{23} & 1 & \beta_{25} & 0 & 0 & 0 & \beta_{29} \\ \beta_{31} & 0 & 0 & 0 & 0 & 1 & \beta_{37} & \beta_{38} & \beta_{39} \\ \beta_{41} & 0 & 0 & \beta_{44} & \beta_{45} & \beta_{46} & 0 & 1 & 0 \\ \beta_{51} & 0 & 0 & \beta_{54} & -1 & 0 & 0 & \beta_{58} & \beta_{59} \end{pmatrix}, \quad (5.1)$$

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<sup>33</sup>The decision to include domestic prices in the model in two forms,  $(p_t - p_t^*)$  and  $\Delta\tilde{p}_t$  does not create difficulties of inconsistency either algebraically or economically (and would not do so even if we used  $\Delta p_t$  in place of  $\Delta\tilde{p}_t$  in the model). The associated structural model of the form in (4.2) contains nine equations in eight endogenous variables. One of the nine equations corresponds to the determination of domestic prices and one corresponds to the determination of foreign prices.

<sup>34</sup>There is considerable evidence, both on the basis of our own analysis and elsewhere, that the various alternative measures of inflation that are available are pairwise cointegrated with a cointegrating vector of  $(1, -1)$  and a zero constant. The use of two measures of prices,  $p_t$  and  $\tilde{p}_t$ , in the analysis has no impact on the long run properties of the model, therefore, but is likely to capture the short run dynamics more accurately.

<sup>35</sup>An account of the algorithms used for the computation of cointegration test statistics in the presence of  $I(1)$  exogenous variables can be found, for example, in Pesaran, Shin and Smith (2000).

that corresponds to  $\mathbf{z}_t = (p_t^o, e_t, r_t^*, r_t, \Delta\tilde{p}_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$ . The first vector (the first row of  $\beta'$ ) relates to the *PPP* relationship defined by (3.1) and is normalised on  $p_t - p_t^*$ ; the second relates to the *IRP* relationship defined by (3.2) and is normalised on  $r_t$ ; the third relates to the “output gap” relationship defined by (3.3) and is normalised on  $y_t$ ,<sup>36</sup> the fourth is the money market equilibrium condition defined by (3.4) and is normalised on  $h_t - y_t$ ; and the fifth is the real interest rate relationship defined by (3.5), normalised on  $\Delta\tilde{p}_t$ .

Having exactly identified the long-run relations, we then tested the over-identifying restrictions predicted by the long-run theory outlined in Section 2. There are twenty unrestricted parameters in (5.1), and two in (3.9), yielding a total of eighteen over-identifying restrictions. In addition, working with a cointegrating VAR with restricted trend coefficients, there are potentially five further parameters on the trend terms in the five cointegrating relationships. The economic theory of Section 2 provided no justification for a time trend in the *IRP*, *FIP* or output gap relationships, and the imposition of zeros on the trend coefficients in these relationships provides a further three over-identifying restrictions. The absence of a trend in the *PPP* relationship is also consistent with the theory of Section 2, as is the restriction that  $\beta_{46} = 0$  (so that equation (3.4) is effectively a relationship explaining the velocity of circulation of money). These final two restrictions, together with those which are intrinsic to the theory of Section 2, mean that there are just two parameters to be freely estimated in the cointegrating relationships and provide a total of twenty-three over-identifying restrictions on which the core model is based and with which the validity of the economic theory can be tested. The estimated core model is described in detail in Section 5.2 and the long run relationships are given in (5.2)-(5.6).

The log-likelihood ratio (*LR*) statistic for jointly testing the twenty-three over-identifying restrictions takes the value 71.49. In view of the relatively large dimension of the underlying VAR model, the number of restrictions considered and the available sample size, we proceed to test the significance of this statistic using critical values which are computed by means of bootstrap techniques.<sup>37</sup> In our application the bootstrap exercise is based on 3000 replications of the *LR* statistic testing the twenty-three restrictions. For each replication, an artificial data set is generated (of the same length as the original data set) on the assumption that the estimated version of the core model is the true data-generating process, using the observed initial values of each variable, the estimated model, and a set of random innovations.<sup>38</sup> The test of the over-identifying restrictions is carried out on each of the

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<sup>36</sup>Our use of the term ‘output gap relationship’ to describe (3.3) should not be confused with the more usual use of the term which relates more specifically to the difference between a country’s actual and potential output levels (although clearly the two uses of the term are related).

<sup>37</sup>The extent of the small sample bias of chi-squared tests in (small) cointegrating VAR models is illustrated, for example, by Gredenhoff and Jacobson (1998) who suggest the use of bootstrap methods for this reason. It has also been shown by Gonzalo (1994), Haug (1996) and Abadir *et al.* (1999), for example, that asymptotic critical values may not be valid for VAR models with a relatively large number of variables, unless samples are sufficiently large.

<sup>38</sup>These innovations can be obtained as draws from a multivariate normal distribution chosen to match the observed correlation of the estimated reduced form errors (termed a ‘parametric bootstrap’) or by resampling with replacement from the estimated residuals (a ‘non-parametric bootstrap’). In light of the evidence of non-normality of residuals, in this exercise we apply the non-parametric bootstrap. The cointegrating matrix subject to the over-identifying restrictions is estimated for each data set using the Simulated Annealing routine by Goffe *et al.* (1994).

replicated data sets and the empirical distribution of the test statistic is derived across all replications. This shows that the relevant critical values for the joint tests of the twenty-three over-identifying restrictions are 67.51 at the 10 per cent significance level and 73.19 at the 5 per cent level.<sup>39</sup> The *LR* statistic of 71.49 therefore is not sufficiently large that we reject the over-identifying restrictions implied by the long-run theory of Section 2.

## 5.2 The Vector Error Correction Model

The estimates of the long-run relations and the reduced form error correction specification are provided in Table 4. The long-run relations, which incorporate all the restrictions suggested by the theory in Section 2, are summarised below:

$$(p_t - p_t^*) - e_t = 4.588 + \widehat{\xi}_{1,t+1} \quad (5.2)$$

$$r_t - r_t^* = 0.0058 + \widehat{\xi}_{2,t+1} \quad (5.3)$$

$$y_t - y_t^* = -0.0377 + \widehat{\xi}_{3,t+1} \quad (5.4)$$

$$h_t - y_t = 0.5295 - \frac{56.0975}{(22.2844)} r_t - \frac{0.0073}{(0.0012)} t + \widehat{\xi}_{4,t+1} \quad (5.5)$$

$$r_t - \Delta \widetilde{p}_t = 0.0036 + \widehat{\xi}_{5,t+1} \quad (5.6)$$

The bracketed figures are asymptotic standard errors. The first equation, (5.2), describes the *PPP* relationship and the failure to reject this in the context of our core model provides an interesting empirical finding. Of course, there has been considerable interest in the literature examining the co-movements of exchange rates and relative prices, and the empirical evidence on *PPP* appears to be sensitive to the data set used and the way in which the analysis is conducted. For example, the evidence of a unit root in the real exchange rate found by Darby (1983) and Huizinga (1988) contradicts *PPP* as a long-run relationship, while Grilli and Kaminsky (1991) and Lothian and Taylor (1996) have obtained evidence in favour of rejecting the unit root hypothesis in real exchange rates using longer annual series. In work investigating *PPP* using cointegration analysis, the results seem to be sensitive to whether the model is a trivariate one (including  $e_t$ ,  $p_t$  and  $p_t^*$  in the VAR as separate variables) or a bivariate one (including  $e_t$  and  $(p_t - p_t^*)$  as two separate variables) since the null of no cointegration is rejected more frequently in trivariate than in bivariate analyses.<sup>40</sup> The finding here that *PPP* can be readily incorporated into the model is a useful contribution to this literature, indicating that the empirical evidence to support the relationship is stronger in a more complete model of the macroeconomy incorporating feedbacks and interactions omitted from more partial analyses.

The second cointegrating relation, defined by (5.3), is the *IRP* condition. This includes an intercept, which can be interpreted as the deterministic component of the risk premia

<sup>39</sup>Simulation here is used to find the probability of rejection for one point in  $H_0$ . The classical significance level is the *maximum* of the rejection probabilities over  $H_0$ . By using a single point, the observed critical values potentially understate the true rejection level therefore. In other words, our observed failure to reject at the 5% level might provide more compelling evidence to support the validity of the restrictions than it appears to provide.

<sup>40</sup>See Taylor (1988) and Mark (1990) for illustrations of further work in this area, and Froot and Rogoff (1995) and MacDonald (1995) for a review of the literature.

associated with bonds and foreign exchange uncertainties. Its value is estimated at 0.0058, implying a risk premium of approximately 2.3 per cent per annum. The empirical support we find for the *IRP* condition, namely that  $r_t - r_t^* \sim I(0)$  is in accordance with the results obtained in the literature, and is compatible with *UIP*, defined by (2.22). However, under the *UIP* hypothesis it is also required that a regression of  $r_t - r_t^*$  on  $\Delta \ln(E_{t+1})$  has a unit coefficient, but this is not supported by the data.

The third long-run relationship, given by (5.4), is the Output Gap (*OG*) relationship with per capita domestic and foreign output (measured by the total OECD output) levels moving in tandem in the long-run. It is noteworthy that the co-trending hypothesis cannot be rejected; *i.e.* the coefficient of the deterministic trend in the output gap equation is zero. This suggests that average long-run growth rate for the UK is the same as that in the rest of the OECD. This finding seems, in the first instance, to contradict some of the results obtained in the literature on the cointegrating properties of real output across countries. Campbell and Mankiw (1989), Cogley (1990) and Bernard and Durlauf (1995), for example, consider cointegration among international output series and find little evidence that outputs of different *pairs* of countries are cointegrated. However, our empirical analysis, being based on a single foreign output index, does not necessarily contradict this literature, which focuses on pairwise cointegration of output levels. The hypothesis advanced here, that  $y_t$  and  $y_t^*$  are cointegrated, is much less restrictive than the hypothesis considered in the literature that *all* pairs of output variables in the OECD are cointegrated.<sup>41</sup>

For the money market equilibrium (*MME*) condition, given by (5.5), we could not reject the hypothesis that the elasticity of real money balances with respect to real output is equal to unity, and therefore (5.5) in fact represents a M0 velocity equation. The *MME* condition, however, contains a deterministic downward trend, representing the steady decline in the money-income ratio in the UK over most of the period 1965-1999, arising primarily from the technological innovations in financial intermediation. There is also strong statistical evidence of a negative interest rate effect on real money balances. This long-run specification is comparable with the recent research on the determinants of the UK narrow money velocity reported in, for example, Breedon and Fisher (1996).

Finally, the fifth equation, (5.6), defines the *FIP* relationship, where the estimated constant implies an annual real rate of return of approximately 1.67 per cent. While the presence of this relationship might appear relatively uncontentious, there is empirical work in which the relationship appears not to hold; see, for example Macdonald and Murphy (1989) and Mishkin (1992). In La Cour and Macdonald (2000), evidence of a cointegrating relationship between interest rates and inflation was obtained in an analysis of financial data series from the Euro-area and US. However, the *FIP* relationship itself, with coefficients of  $(1, -1)$  on the interest rate and inflation, was observed in the two zones only when the financial variables were incorporated into a larger macro-system. Our results support the *FIP* relationship and again highlights the important role played by the *FIP* relationship in a model of the macroeconomy which can incorporate interactions between variables omitted from more partial analyses.

The short-run dynamics of the model are characterized by the error correction specifications given in Table 4. The estimates of the error correction coefficients (also known as

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<sup>41</sup>See Lee (1998) for further discussion of cross-country interdependence in growth dynamics.

the loading coefficients) show that the long-run relations make an important contribution in most equations and that the error correction terms provide for a complex and statistically significant set of interactions and feedbacks across commodity, money and foreign exchange markets. The results in Table 4 also show that the core model fits the data well and has satisfactory diagnostic statistics. In order to evaluate the in-sample fit of the individual equations in the model, we estimated ‘benchmark’ univariate  $ARIMA(s, q)$ ,  $s, q = 0, 1, \dots, 4$  models for each of the eight endogenous variables. The orders  $(s, q)$  were selected using the  $AIC$ .<sup>42</sup> The  $\bar{R}^2$  of the benchmark models are also given in Table 4 and show that the core model equations outperform the benchmark models in all cases and by considerable margins in the case of output and inflation equations.<sup>43</sup> Similarly, the diagnostic statistics of the equations in Table 4 are generally satisfactory as far as the tests of the residual serial correlation, functional form and heteroskedasticity are concerned. The assumption of normally distributed errors is rejected in all the error correction equations which is understandable if we consider the three major hikes in oil prices experienced during the estimation period. Generally speaking, therefore, the diagnostic statistics are satisfactory, and the equations of Table 4 appear to capture well the time series properties of the main macroeconomic aggregates in the UK over the period since the mid-1960s.

## 6 Impulse Response Analysis

Using the identification scheme discussed in Section 4, here we report the estimates of impulse response functions of oil price and monetary policy shocks on the endogenous variables of the model.<sup>44</sup> We carry out this analysis for a unit (one standard error) increase in oil prices and a contractionary monetary policy shock. The macro-economic analyses of the effects of these shocks have been of special interest and help provide further insights into the short-run dynamic properties of our model. The theoretical basis of our approach to the identification of the monetary policy shock and the algorithms necessary for the computation of the associated impulse responses are set out in Section 4.

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<sup>42</sup>These univariate time series models provide theory-free benchmarks for evaluation of the in-sample fit of the error correction equations in the core model. Of course, such an evaluation does not take into account the value of the structural interpretation and understanding provided through the use of a model based on explicit economic theory.

<sup>43</sup>Following a comment by a referee we also compared the in-sample fit of our model measured by  $\bar{R}^2$  using as benchmarks univariate  $ARIMA(s, q)$ ,  $s, q = 0, 1, \dots, 4$  specifications with  $s$  and  $q$  selected by the Schwarz Bayesian Criterion (SBC) and obtained similar results. Many other in-sample comparisons are also possible, but they should be carried out with care. For example, comparing the SBC of the individual equations of our model with the SBC of  $ARIMA$  benchmarks selected by the SBC can be misleading, since in the case of the benchmark models the SBC is then used both for model selection and model evaluation, while the same is not true of the error correction equations whose specifications are left unrestricted. A more satisfactory approach would be to evaluate the models by their out-of-sample forecast performance. Such an exercise for our model specification is carried out in Garrett *et al.* (2001a).

<sup>44</sup>Another important use of the model developed in this paper could be for short- to medium-term forecasting. Garratt *et al.* (2001a) provide such an exercise and show that the recursive forecasts generated from the model over the period 1999q1-2000q1 out-perform the forecasts based on random walk models in predicting turning points. The analysis also shows that the cointegrating VAR model that imposes over-identifying restrictions on the long-run relations perform significantly better than its exactly identified counter part.

We shall also consider the time profile of the effects of shocks on the long-run relationships. Recall that despite the integrated properties of the underlying variables, the effects of shocks on the long-run relations can only be temporary and should eventually disappear. But it is interesting to see how long such effects are likely to last. These types of impulse response functions are referred to as “persistence profiles” and, as shown in Pesaran and Shin (1996), they shed light on the equilibrating mechanisms embedded within the model.

To compute the impulse response functions, we need an estimate of the oil price equation specified by (3.10). We decided to exclude domestic variables from (3.10) since we would not expect a small open economy such as the UK to have any significant influence on oil prices. The resultant oil price equation, estimated over the period 1965q1-1999q4, is given by:

$$\begin{aligned}\Delta p_t^o &= \begin{matrix} -.0039 \\ (.0352) \end{matrix} + \begin{matrix} .04787 \\ (.1070) \end{matrix} \Delta p_{t-1}^o + \begin{matrix} 2.7731 \\ (2.6818) \end{matrix} \Delta y_{t-1}^* + \begin{matrix} 0.4199 \\ (1.8572) \end{matrix} \Delta p_{t-1}^* \\ &+ \begin{matrix} 2.4855 \\ (11.635) \end{matrix} \Delta r_{t-1}^* + \hat{\varepsilon}_{ot}, \quad (6.1) \\ \sqrt{\widehat{\omega}_{oo}} &= .1661, \chi_{SC}^2[4] = 1.86, \chi_N^2[2] = 6558.9.\end{aligned}$$

None of the coefficients are statistically significant at the conventional levels, although there is some evidence of a positive effect from past changes in foreign output. The hypothesis that the residuals are serially uncorrelated cannot be rejected either. But, not surprisingly, there is a clear evidence of non-normal errors, primarily reflecting the three major oil price shocks experienced during the period under consideration. These results are in line with the widely held view that oil prices follow a geometric random walk, possibly with a drift. Therefore, we base our computations of impulse responses on the following simple model:

$$\begin{aligned}\Delta p_t^o &= \begin{matrix} .0173 \\ (.0139) \end{matrix} + \hat{\varepsilon}_{ot}, \quad (6.2) \\ \sqrt{\widehat{\omega}_{oo}} &= .16485, \chi_{SC}^2[4] = 2.19, \chi_N^2[2] = 6399\end{aligned}$$

## 6.1 Effects of an Oil Price Shock

Over the past three decades, oil price changes have had a significant impact on the conduct of monetary policy in the UK and elsewhere. Unexpected increases in oil prices have often been associated with rising prices, falling output and a tightening of monetary policy which has in turn contributed to further output falls. It is important that special care is taken to separate the output and inflation effects of an oil price shock from those of a monetary shock as they are likely to be positively correlated. In our framework, this is achieved by treating oil prices as long-run forcing, and by explicitly modelling the contemporaneous dependence of monetary policy shocks on the oil price shocks, as well as on shocks to exchange rates and foreign interest rates.<sup>45</sup>

Figure 1 provides the persistence profiles of the effects of a one standard error increase in oil prices on the five long run relationships. Figure 2 gives the impulse responses of

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<sup>45</sup>For an alternative identification scheme, applied to the US economy, see Bernanke, *et al.* (1997).

the oil price shock on the levels of all the eight endogenous variables in the model. Both figures also provide bootstrapped 95% confidence error bands.<sup>46</sup> All the persistence profiles converge towards zero, thus confirming the cointegrating properties of the long-run relations. In addition, the persistence profiles provide useful information on the speed with which the different relations in the model, once shocked, will return to their long-run equilibria. The results are generally in line with those found in the literature, with PPP and output gap relations showing much slower rates of adjustments to shocks. The effect of the oil price shock on the output gap takes some ten years to complete. This is rather slow, but is comparable to those implied by Barro and Sala-i-Martin's (1995) analyses of international output series.<sup>47</sup> Similarly, deviations from *PPP* are relatively long lived, but the speed of convergence towards equilibrium in this relationship is again consistent with existing results which put the half life of deviations from *PPP* at about four years for the major industrialised countries.<sup>48</sup> Convergence to the *FIP*, *IRP* and *MME* relationships is more rapid than that to *PPP* and *OG*, reflecting the standard view that arbitrage in asset markets functions much faster than in the goods markets in restoring equilibria.

Turning to the impulse response functions in Figure 2, the oil price shock has a permanent effect on the level of the individual series, reflecting their unit root properties. Its effect on output has the expected negative sign, reducing domestic output by approximately 0.24% below its base after 2.5 years. Foreign output also declines to the same long run value but at a much slower speed. On impact the oil price shock raises the domestic rate of inflation by 0.20%, and by 0.82% after 1 quarter before gradually falling back close to zero after approximately 3 years. Despite the higher domestic prices the oil price shock generates a small appreciation of the nominal exchange rate as can be seen from Figure 2g. This initial movement is then followed by further appreciations, although the process starts to reverse after approximately one year. In the long run, the nominal exchange rate fully adjusts to the change in relative prices with *PPP* restored but, as noted above, the speed of adjustment is relatively slow.

The oil price shock is accompanied by increases in both domestic and foreign interest rates, suggesting a possible tightening of the monetary policy in response to the rise in oil prices. Domestic interest rates increase by some 9 basis points on impact, rising to 16 basis points after approximately 3 quarters, and then falling to a long run values of 8 basis points above its pre-shock level. The oil price shock affects real money balances both directly and

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<sup>46</sup>The 95% confidence intervals are obtained with the same bootstrap procedure used to calculate the small sample critical values for the test of the over-identifying restrictions. Kilian (1998, 1999) has suggested the use of bias corrections in the context of the analysis of impulse responses. We do not pursue this route here, in drawing our inferences, for two reasons. First, even in constructing error bands for impulse responses, no evaluation (theoretical or by simulation) has been made to compare the relative performance of alternative methods for large VAR models with cointegration. Second, as noted by Kilian, there is little gain in using the bias corrected bootstrap in cointegrating VAR models with drifts. The intervals are presented for completion and are less reliable as indicators of the precision of the estimates at long horizons (i.e. beyond the first one or two years).

<sup>47</sup>However, Barro and Sala-i-Martin assume that output series are trend stationarity and study convergence to a common trend growth rate. The present study assumes the output series are difference stationary and tests for cointegration between UK and OECD output series. For further discussion, see Lee *et al.* (1997, 1998).

<sup>48</sup>See, for example, Johansen and Juselius (1992), Pesaran and Shin (1996), or Rogoff (1996).

indirectly through its impact on interest rates. The overall outcome is to reduce real money balances by around 1% in the long-run. This is indicative of the presence of a strong liquidity effect in our model, which we shall discuss below in more detail. The oil price shock also causes the real rate of interest to fall, initially by 0.1% and then by 0.7%, before gradually returning to its equilibrium value of zero.

## 6.2 Effects of a Monetary Policy Shock

Before discussing the impulse response results for the monetary policy shock, it is important to recall that under our identification scheme set out in Section 4 the monetary policy shock refers to the non-systematic (or unanticipated) component of the policy. More specifically, it is defined by  $\varepsilon_{rt}$ , the shock in the structural equation for the market interest rate (given by (4.12)), and allows oil prices, exchange rates and foreign interest rates to have contemporaneous effects on  $r_t$ . For the rationale behind the identification scheme and further details see Section 4.

Figure 3 presents the persistence profiles of the effects of one standard error unexpected increase in the interest rate (i.e. a rise of 91 basis points) on the five long-run relations of the model. As with the oil price shock, the effects of the monetary policy shock on these relations disappear eventually, but the speed with which this occurs varies considerably across the different arbitrage conditions. The interest parity condition is the quickest to adjust followed by the Fisher inflation parity, the monetary equilibrium condition, purchasing power parity and the output gap. In our model the long-run equilibrium condition for interest rate parity rules out the phenomena observed in Eichenbaum and Evans (1995), where a contractionary monetary policy shock could result in a permanent shift in the interest rate differential.

On impact the effect of the monetary policy shock is most pronounced on the money market equilibrium condition, resulting in a 12.7% unexpected excess supply of money. With foreign interest rates unchanged on impact (by construction), the shock raises the domestic interest rate above the foreign interest rate by 91 basis points, but it also raises the real interest by 59 basis points while leaving the real exchange rate unchanged. The output gap is initially left intact, reflecting a lagged response of real output to interest rate changes. However, the contractionary impact of the shock on domestic output (relative to the foreign output) begins to be seen after the second quarter, with domestic output falling below foreign output by 0.29% after two years.

The impulse response functions for the effects of the monetary shock on the various endogenous variables in the model are given in Figure 4. Most of these exhibit familiar patterns. After the initial impact, the domestic interest rate declines at a steady rate settling down after approximately four years at an equilibrium value of 5 basis points above the base line value. In tandem with the fall in the interest rate, the excess supply of money declines to approximately 8.6% after one year, then to 5.0% after two years, reaching its equilibrium after approximately five years. These results clearly show the presence of a sizeable “liquidity effect” in our model following the unexpected tightening of the monetary policy.<sup>49</sup>

The monetary policy shock has little immediate effects on the real side of the economy. The contractionary effects of the policy begin to be felt on output and real money balances

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<sup>49</sup>See, for example, the analysis of liquidity effect in Pagan and Robertson (1998).



after one quarter. The impulse responses of domestic and foreign output are given in Figures 4d and 4e, each showing a relatively smooth decline to around 0.46% and 0.17% respectively below base after two and half years. The speed of adjustments of the two series differ, however, as was seen clearly from the persistence profile of the output gap presented in Figure 3c. Figure 4f provides evidence of the well known “price puzzle”, as inflation increases in immediate response to the contractionary monetary shock, falling back to close to zero after three years. Note however, that with the exception of the first few quarters the inflation responses are insignificantly different from zero so that, insofar as the puzzle is apparent, the underlying long-run relations ensure that the anomaly are observed in the short run only.

The impact effect of the monetary policy shock on the nominal exchange rate is zero, by construction. But as can be seen from Figure 4g, after one quarter the effect of the monetary shock is to bring about an appreciation of the exchange rate with the rate of appreciation reaching the maximum value of 0.66% after seven quarters, before depreciating back to its long-run value. Hence, the UIP is only briefly violated during which time a positive interest rate differential coincides with an appreciating exchange rate. There is little evidence of the “exchange rate puzzle” observed by Eichenbaum and Evans (1995).

There are, of course, a variety of other impulse response analyses that can be conducted using our model. Recent work by Bernanke *et al.* (1997) and Cochrane (1998), for example, suggest counter-factual exercises aimed at distinguishing the effects of systematic changes to monetary policy rules from those that influence the economy’s intrinsic propagation mechanisms. As a second example, one might consider impulse response functions associated with a once-and-for-all shift in the intercept of the interest rate equation. This would help, for example, identify the time profile of the effects of shifts in the target variables for output growth or inflation reduction, i.e.  $\Delta \mathbf{w}^\dagger$  in (4.8). The model presented in this paper provides a potentially fruitful framework with which to investigate these and other counter-factual policy exercises.<sup>50</sup>

## 7 Concluding Remarks

This paper provides an account of how macro-modelling can be undertaken following the long-run structural VAR modelling approach. It outlines a theoretical framework for the long-run analysis of an open macroeconomy; introduces a new modelling strategy providing a practical approach to incorporating theory-based long-run relationships in an otherwise unrestricted VAR; and presents the estimates and the tests we have carried out to construct a “core” macroeconometric model of the UK. The description of our approach to modelling starts with an explicit statement of a set of long-run relationships between the variables of interest, as derived from macroeconomic theory, taking account of a number of key long-term arbitrage and solvency conditions. These long-run relationships are embedded within an otherwise unrestricted VAR model in nine “core” variables, augmented appropriately

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<sup>50</sup>It is worth noting that the impulse responses presented in this section are invariant to the re-ordering of the variables  $p_t^o, r_t^*, e_t$  in  $\mathbf{z}_{1t} = (p_t^o, r_t^*, e_t, r_t)$ . In particular, switching the orders of  $r_t^*$  and  $e_t$  so that  $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t})'$  with  $\mathbf{z}_{1t} = (p_t^o, r_t^*, e_t, r_t)$  and  $\mathbf{z}_{2t} = (\Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)$  will yield identical results to those depicted in Figure 4. Of course, they are not invariant to a re-ordering of  $r_t$  in  $\mathbf{z}_t$ .

by intercepts and linear trends. The VAR model is estimated over the period 1965q1-1999q4, subject to the theory restrictions on the long-run coefficients using recently developed econometric techniques. Such a long-run structural model has the advantage shared by all VAR models that it is able to capture complicated dynamic relationships in the data, but it also incorporates theory-consistent long-run properties in a transparent manner. Hence, the approach is capable of providing a structural understanding of the macroeconomy through the estimation of a small and transparent, but reasonably comprehensive, model of the UK macroeconomy.

An important feature of our modelling approach is that it provides the means for testing formally the validity of restrictions suggested by economic theory in the context of a complete macroeconomic model. The economic theory elaborated in the paper, and the statistical considerations for the empirical application, suggest that there are five long-run cointegrating relationships among the nine core variables of the macro-model, and the statistical tests provided little evidence with which to reject this view. Under the assumption that there are indeed five long-run relationships, we obtained a model in which the freely-estimated parameters take sensible signs and are of plausible orders of magnitude, including the intercept estimated for the Interest Rate Parity relationships and the parameters of the real money balance relationship. Further, taking into account their small sample properties, the likelihood ratio tests did not reject the over-identifying restrictions suggested by economic theory, so that we conclude that the estimated model is both theory and data consistent, at least as far as our data sample is concerned.

The estimated model is then used for the analysis of the effects of an exogenous oil price shock and an endogenous monetary policy shock identified as the movements in interest rates not explained by the systematic movements suggested by a monetary policy rule, derived as a solution to an optimization problem, and abstracting from movements associated with unexpected changes in oil prices, exchange rates and foreign interest rates. The paper reports familiar impulse response functions for the effects of shocks on the levels of the model's eight endogenous variables as well as the persistence profiles that identify the effects of shocks on the model's five long-run relations. Standard results are obtained for the oil price shock. In particular, it is shown that an unexpected rise in oil prices increases domestic and foreign interest rates, reduces real money balances, has a moderate contractionary impact on real output, increases the inflation rate, reduces the real interest rate, and results in an appreciation of the nominal and real exchange rates. The persistence profiles of the shocks to the long-run relations also show reasonably rapid adjustments in the case of the uncovered interest parity, and the money market equilibrium conditions (around two and half years) but a rather slow rate of adjustment in the case of the output gap and the purchasing power parity conditions (as much as 7-10 years). The results for the monetary policy shock are qualitatively similar in the case of output, real money balances and inflation, although these effects are now stronger. In both sets of responses, the model illustrates a number of the dynamic features identified elsewhere in the literature, including a strong liquidity effect and the inflation puzzle. But the cointegrating properties of our model ensure that the effects of the oil or monetary shocks on the long-run relations will not be permanent. This means that the inflation puzzle is a short run phenomena only and, for example, that the possibility that a contractionary monetary policy shock can permanently shift the interest rate differential,

a phenomenon reported in Eichenbaum and Evans (1995), is ruled out entirely.

**Table 1: List of Variables and their Descriptions in the Core Model**

$y_t$  : natural logarithm of the UK real per capita GDP at market prices (1995 = 100).

$p_t$  : natural logarithm of the UK Producer Price Index (1995 = 100).

$\tilde{p}_t$  : natural logarithm of the UK Retail Price Index, All Items (1995 = 100).

$r_t$  : is computed as  $r_t = 0.25 \ln(1 + R_t/100)$ , where  $R_t$  is the 90 day Treasury Bill average discount rate per annum.

$h_t$  : natural logarithm of UK real per capita M0 money stock (1995 = 100).

$e_t$  : natural logarithm of the nominal Sterling effective exchange rate (1995 = 100).

$y_t^*$  : natural logarithm of the foreign (OECD) real per capita GDP at market prices (1995 = 100).

$p_t^*$  : natural logarithm of the foreign (OECD Producer Price Index) (1995 = 100).

$r_t^*$  : is computed as  $r_t^* = 0.25 \ln(1 + R_t^*/100)$ , where  $R_t^*$  is the weighted average of 90 day interest rates per annum in the United States, Germany, Japan and France.

$p_t^o$  : natural logarithm of oil prices, measured as the Average Price of Crude Oil.

$t$  : time trend, taking the values 1, 2, 3, ..., in 1965Q1, 1965Q2, 1965Q3, ..., respectively.

Notes: For the data sources and a detailed description of the construction of foreign prices and interest rates see the Data Appendix in Garratt *et al.* (2000c).

Table 2: Augmented Dickey-Fuller Unit Root Test Applied to Variables  
in the Core Model; 1965q1- 1999q4

(i) For the First Differences					
Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)
$\Delta y_t$	-11.94	-8.06	-5.40 <sup>a</sup>	-5.18	-4.81
$\Delta y_t^*$	-7.43	-5.28 <sup>a</sup>	-4.53	-4.22	-4.11
$\Delta r_t$	-20.36	-14.35	-11.65	-10.08	-8.99 <sup>a</sup>
$\Delta r_t^*$	-7.06 <sup>a</sup>	-6.19	-4.89	-4.85	-4.54
$\Delta e_t$	-9.81 <sup>a</sup>	-7.89	-6.45	-5.52	-5.39
$\Delta(h_t - y_t)$	-12.22 <sup>a</sup>	-8.62	-6.33	-5.40	-3.89
$\Delta p_t$	-3.50 <sup>a</sup>	-3.19	-2.67	-2.44	-2.43
$\Delta \tilde{p}_t$	-4.41	-3.05 <sup>a</sup>	-2.97	-2.42	-2.23
$\Delta p_t^*$	-5.06	-3.47	-2.73 <sup>a</sup>	-2.75	-2.90
$\Delta p_t^o$	-11.05 <sup>a</sup>	-8.71	-6.41	-5.68	-5.71
$\Delta^2 p_t$	-13.32	-10.74 <sup>a</sup>	-8.80	-7.15	-6.95
$\Delta^2 \tilde{p}_t$	-17.37	-10.22	-9.48	-8.06	-7.43
$\Delta^2 p_t^*$	-17.82	-12.75 <sup>a</sup>	-8.63	-6.65	-6.40
$\Delta(p_t - p_t^*)$	-6.69	-4.91	-3.72 <sup>a</sup>	-3.60	-3.32

(ii) For the Levels					
Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)
$y_t$	-2.32	-2.33	-2.46	-3.14 <sup>a</sup>	-3.06
$y_t^*$	-3.37	-3.18	-3.22 <sup>a</sup>	-3.24	-3.22
$r_t$	-2.23	-2.57 <sup>a</sup>	-2.65	-2.32	-2.46
$r_t^*$	-1.24	-2.47 <sup>a</sup>	-2.46	-2.86	-2.71
$e_t$	-1.03	-1.45 <sup>a</sup>	-1.32	-1.33	-1.37
$h_t - y_t$	1.41	1.82 <sup>a</sup>	2.00	1.83	1.86
$p_t$	2.21	-0.39 <sup>a</sup>	-0.48	-0.76	-0.90
$\tilde{p}_t$	2.12	-0.03	-0.61 <sup>a</sup>	-0.57	-0.88
$p_t^*$	1.82	-0.07	-0.73	-1.20 <sup>a</sup>	-1.13
$p_t^o$	-1.43 <sup>a</sup>	-1.53	-1.38	-1.49	-1.44
$p_t - p_t^*$	0.47	-0.40	-0.66	-1.01 <sup>a</sup>	-0.96

Notes: When applied to the first differences, augmented Dickey-Fuller (1979, ADF) test statistics are computed using ADF regressions with an intercept and  $s$  lagged first-differences of dependent variable, while when applied to the levels, ADF statistics are computed using ADF regressions with an intercept, a linear time trend and  $s$  lagged first-differences of dependent variable, with the exception of the following variables:  $\Delta p_t$ ,  $\Delta p_t^*$ ,  $\Delta \tilde{p}_t$ ,  $\Delta p_t^o$ ,  $r_t$  and  $r_t^*$  where only an intercept was included in the underlying ADF regressions. The relevant lower 5 per cent critical values for the ADF tests are -2.88 for the former and -3.45 for the latter. The symbol “a” denotes the order of augmentation in the Dickey-Fuller regressions chosen using the Akaike Information Criterion, with a maximum lag order of four.

**Table 3: Cointegration Rank Statistics for the Core Model**

$$(p_t - p_t^*, e_t, r_t, r_t^*, y_t, y_t^*, h_t - y_t, \Delta \tilde{p}_t, p_t^o)$$

**(a) Trace Statistic**

$H_0$	$H_1$	Test Statistic	95% Critical Values	90% Critical Values
$r = 0$	$r = 1$	324.75	199.12	192.80
$r \leq 1$	$r = 2$	221.16	163.01	157.02
$r \leq 2$	$r = 3$	161.88	128.79	123.33
$r \leq 3$	$r = 4$	116.14	97.83	93.13
$r \leq 4$	$r = 5$	78.94	72.10	68.04
$r \leq 5$	$r = 6$	48.71	49.36	46.00
$r \leq 6$	$r = 7$	22.46	30.77	27.96
$r \leq 7$	$r = 8$	6.70	15.44	13.31

**(b) Maximum Eigenvalue Statistic**

$H_0$	$H_1$	Test Statistic	95% Critical Values	90% Critical Values
$r = 0$	$r = 1$	103.59	58.08	55.25
$r \leq 1$	$r = 2$	59.27	52.62	49.70
$r \leq 2$	$r = 3$	45.75	46.97	44.01
$r \leq 3$	$r = 4$	37.20	40.89	37.92
$r \leq 4$	$r = 5$	30.23	34.70	32.12
$r \leq 5$	$r = 6$	26.25	28.72	26.10
$r \leq 6$	$r = 7$	15.76	22.16	19.79
$r \leq 7$	$r = 8$	6.70	15.44	13.31

Notes: The underlying VAR model is of order 2 and contains unrestricted intercepts and restricted trend coefficients, with  $p_t^o$  treated as exogenous  $I(1)$  variable. The statistics refer to Johansen's log-likelihood-based trace and maximal eigenvalue statistics and are computed using 140 observations for the period 1965q1-1999q4. The asymptotic critical values are taken from Pesaran, Shin and Smith (2000).

Table 4: Reduced Form Error Correction Specification for the Core Model

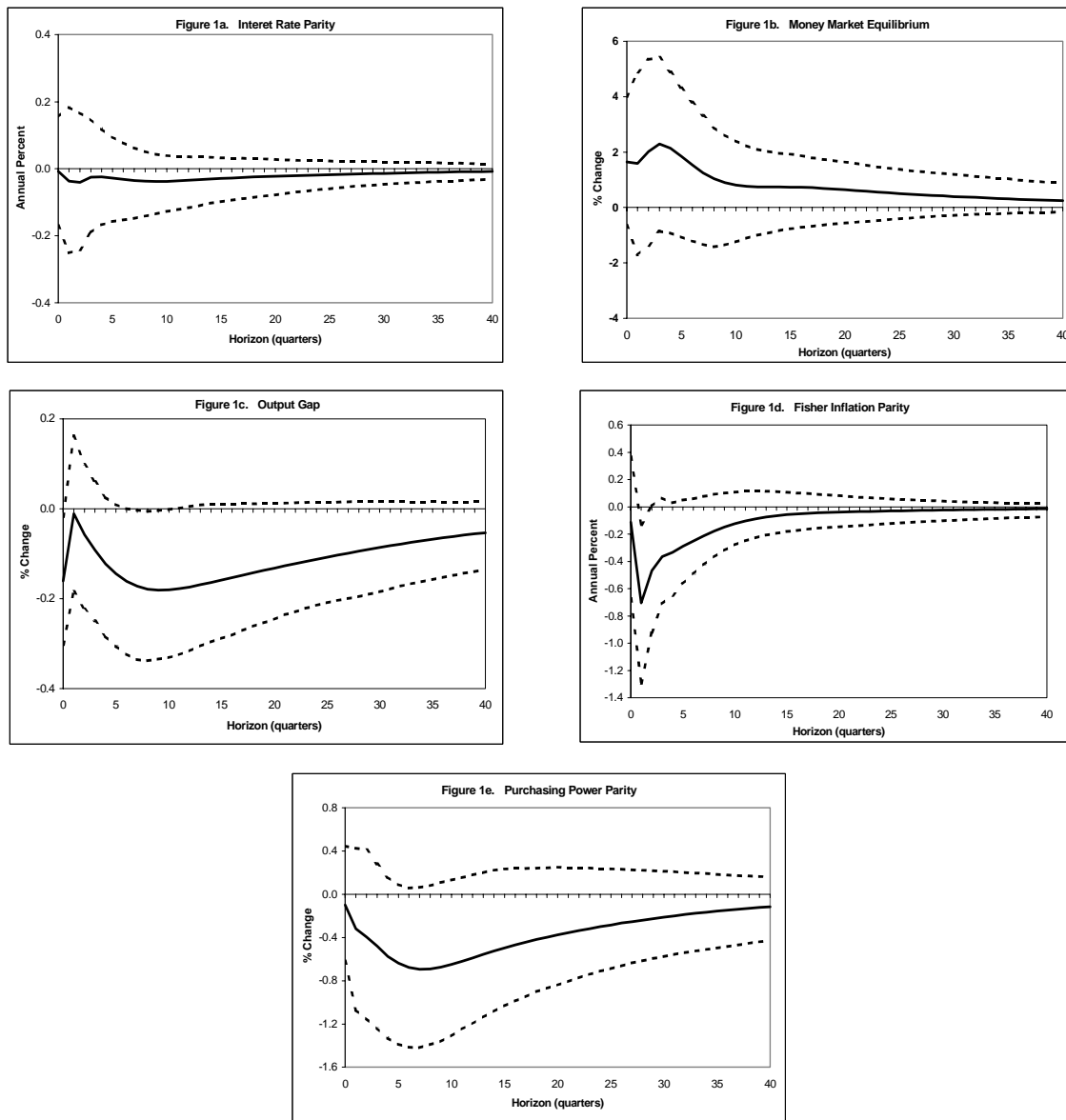
Equation	$\Delta(p_t-p_t^*)$	$\Delta e_t$	$\Delta r_t$	$\Delta r_t^*$	$\Delta y_t$	$\Delta y_t^*$	$\Delta(h_t-y_t)$	$\Delta(\Delta\tilde{p}_t)$
$\hat{\xi}_{1,t}$	-.015 <sup>†</sup> (.007)	.060 <sup>†</sup> (.029)	.002 (.002)	.002 (.001)	.017 <sup>†</sup> (.008)	.021 <sup>†</sup> (.004)	-.024* (.013)	-.005 (.004)
$\hat{\xi}_{2,t}$	-.840 <sup>†</sup> (.301)	1.42 (1.28)	.049 (.107)	.130* (.043)	1.34 <sup>†</sup> (.353)	.891 <sup>†</sup> (.181)	-.721 (.576)	-.811 <sup>†</sup> (.297)
$\hat{\xi}_{3,t}$	.062 <sup>†</sup> (.029)	-.210* (.121)	-.013 (.010)	-.006 (.004)	-.165 <sup>†</sup> (.034)	-.021 (.017)	.106* (.055)	.034 (.028)
$\hat{\xi}_{4,t}$	.018 <sup>†</sup> (.005)	-.029 (.020)	-.003* (.002)	-.001* (.001)	-.027 <sup>†</sup> (.005)	-.016 <sup>†</sup> (.003)	-.003 (.009)	.009* (.005)
$\hat{\xi}_{5,t}$	-.149* (.083)	-.244 (.353)	-.054* (.028)	-.024 <sup>†</sup> (.012)	-.099 (.098)	-.119 <sup>†</sup> (.050)	.408 <sup>†</sup> (.159)	.451 <sup>†</sup> (.082)
$\Delta(p_{t-1} - p_{t-1}^*)$	.459 <sup>†</sup> (.095)	.150 (.404)	-.039 (.032)	-.028 <sup>†</sup> (.014)	-.136 (.111)	-.013 (.057)	.046 (.182)	.436 <sup>†</sup> (.094)
$\Delta e_{t-1}$	.051 <sup>†</sup> (.022)	.216 <sup>†</sup> (.092)	-.005 (.007)	-.001 (.003)	.021 (.025)	.013 (.013)	.007 (.042)	-.022 (.021)
$\Delta r_{t-1}$	.416 <sup>†</sup> (.294)	-1.31 (1.25)	.125 (.098)	-.067 (.042)	.467 (.345)	.204 (.177)	-.677 (.562)	.974 <sup>†</sup> (.290)
$\Delta r_{t-1}^*$	-0.810 (.617)	2.75 (2.62)	-0.606 <sup>†</sup> (.205)	.430 <sup>†</sup> (.088)	.306 (.723)	.573 (.371)	-.267 (1.18)	.166 (.606)
$\Delta y_{t-1}$	.083 (.089)	.072 (.381)	.017 (.030)	.015 (.013)	-.044 (.105)	.031 (.053)	-.168 (.172)	.356 <sup>†</sup> (.089)
$\Delta y_{t-1}^*$	.010 (.161)	-.630 (.683)	-.050 (.054)	.040* (.023)	-.073 (.188)	.069 (.097)	.602* (.307)	-.010 (.158)
$\Delta(h_{t-1} - y_{t-1})$	.116 (.054)	.331 (.228)	.026 (.018)	.006 (.008)	.069 (.063)	-.014 (.032)	-.253 <sup>†</sup> (.103)	.140 <sup>†</sup> (.053)
$\Delta(\Delta\tilde{p}_{t-1})$	-.151 <sup>†</sup> (.073)	.321 (.302)	.016 (.024)	.010 (.011)	.125 (.086)	-.082* (.044)	.012 (.140)	-.244 <sup>†</sup> (.072)
$\Delta p_t^o$	-.018 <sup>†</sup> (.004)	-.024 (.018)	.001 (.001)	.001 <sup>†</sup> (.0005)	-.010 <sup>†</sup> (.005)	.0001 (.002)	.024 <sup>†</sup> (.008)	.003 (.004)
$\Delta p_{t-1}^o$	.010 <sup>†</sup> (.005)	-.013 (.019)	-.002 (.002)	-.0001 (.0001)	.006 (.005)	.002 (.003)	-.011 (.009)	.016 <sup>†</sup> (.004)
$\overline{R}^2$	.484	.070	.115	.345	.260	.367	.257	.445
Benchmark $\overline{R}^2$	.316	.026	.007	.213	.022	.196	.00	.191
$\hat{\sigma}$	.007	.032	.002	.001	.009	.004	.014	.007
$\chi_{SC}^2[4]$	2.79	0.96	2.43	17.13 <sup>†</sup>	6.71	.79	8.37 <sup>†</sup>	5.63
$\chi_{FF}^2[1]$	8.57 <sup>†</sup>	0.13	4.34 <sup>†</sup>	6.70 <sup>†</sup>	0.04	5.28 <sup>†</sup>	.033	0.01
$\chi_N^2[2]$	12.53 <sup>†</sup>	13.98 <sup>†</sup>	17.15 <sup>†</sup>	19.9 <sup>†</sup>	112.4 <sup>†</sup>	10.84	31.45 <sup>†</sup>	118.9 <sup>†</sup>
$\chi_H^2[1]$	6.13 <sup>†</sup>	1.97	4.53 <sup>†</sup>	5.2 <sup>†</sup>	0.88	0.93	0.19	4.55 <sup>†</sup>

Notes: The five error correction terms are given by

$$\begin{aligned} \hat{\xi}_{1,t+1} &= p_t - p_t^* - e_t - 4.588, \\ \hat{\xi}_{2,t+1} &= r_t - r_t^* - 0.0058, \quad \hat{\xi}_{3,t+1} = y_t - y_t^* + 0.0377, \\ \hat{\xi}_{4,t+1} &= h_t - y_t + \frac{56.0975}{(22.2844)} r_t + \frac{0.0073}{(0.0012)} t + 0.05379, \\ \hat{\xi}_{5,t+1} &= r_t - \Delta\tilde{p}_t - 0.0036. \end{aligned}$$

Standard errors are given in parenthesis. “\*” indicates significance at the 10% level, and “†” indicates significance at the 5% level. The diagnostics are chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroscedasticity (H). The benchmark  $\overline{R}^2$  statistics are computed based on univariate ARMA(s,q), s,q=0,1,...,4 specifications with the s and q orders selected by AIC.

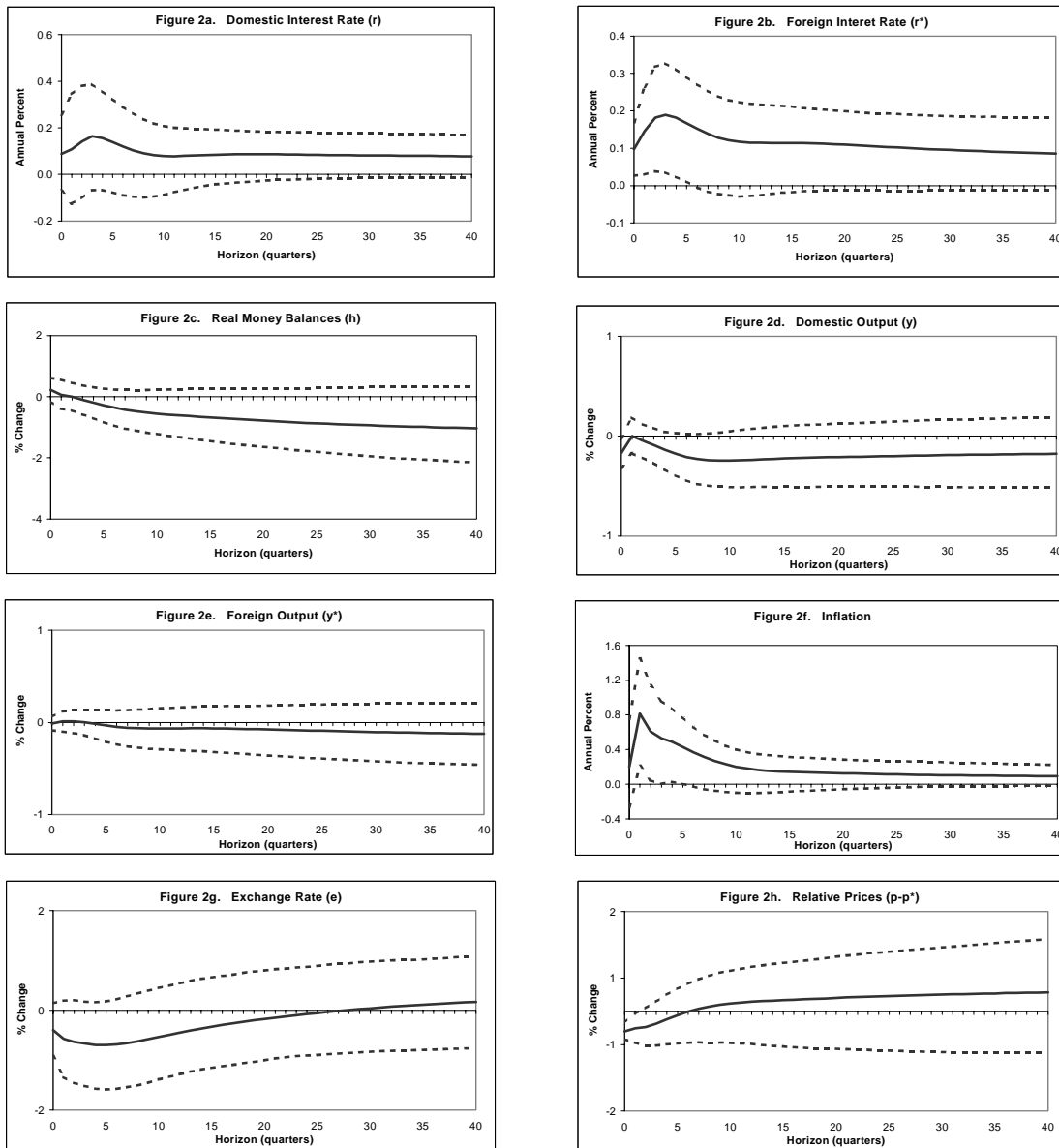
**Figure 1:** Persistence Profiles of the Long-Run Relations to a Unit Oil Price Shock



Notes: The graphs define the Long-Run relationships in the following way, Interest Rate Parity:  $r_t - r_t^*$ , Money Market Equilibrium Condition:  $h_t - y_t + 56.1r_t$ , the Output Gap:  $y_t - y_t^*$ , Fisher Inflation Parity:  $r_t - \Delta p_t$  and the PPP (real exchange rate):  $e_t + p_t^* - p_t$ . The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the standard error bands.

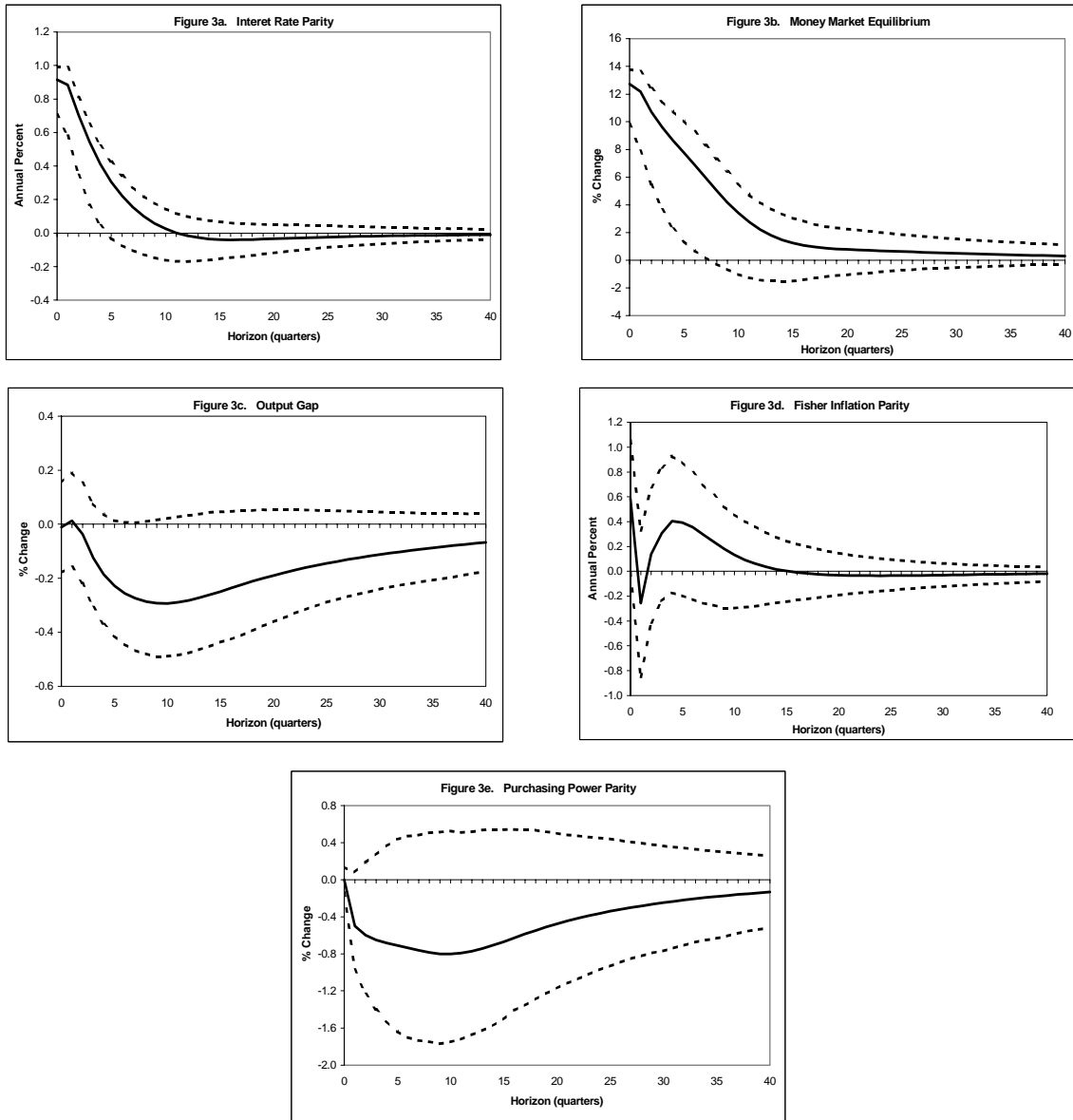


**Figure 2:** Impulse Responses to a Unit Oil Price Shock



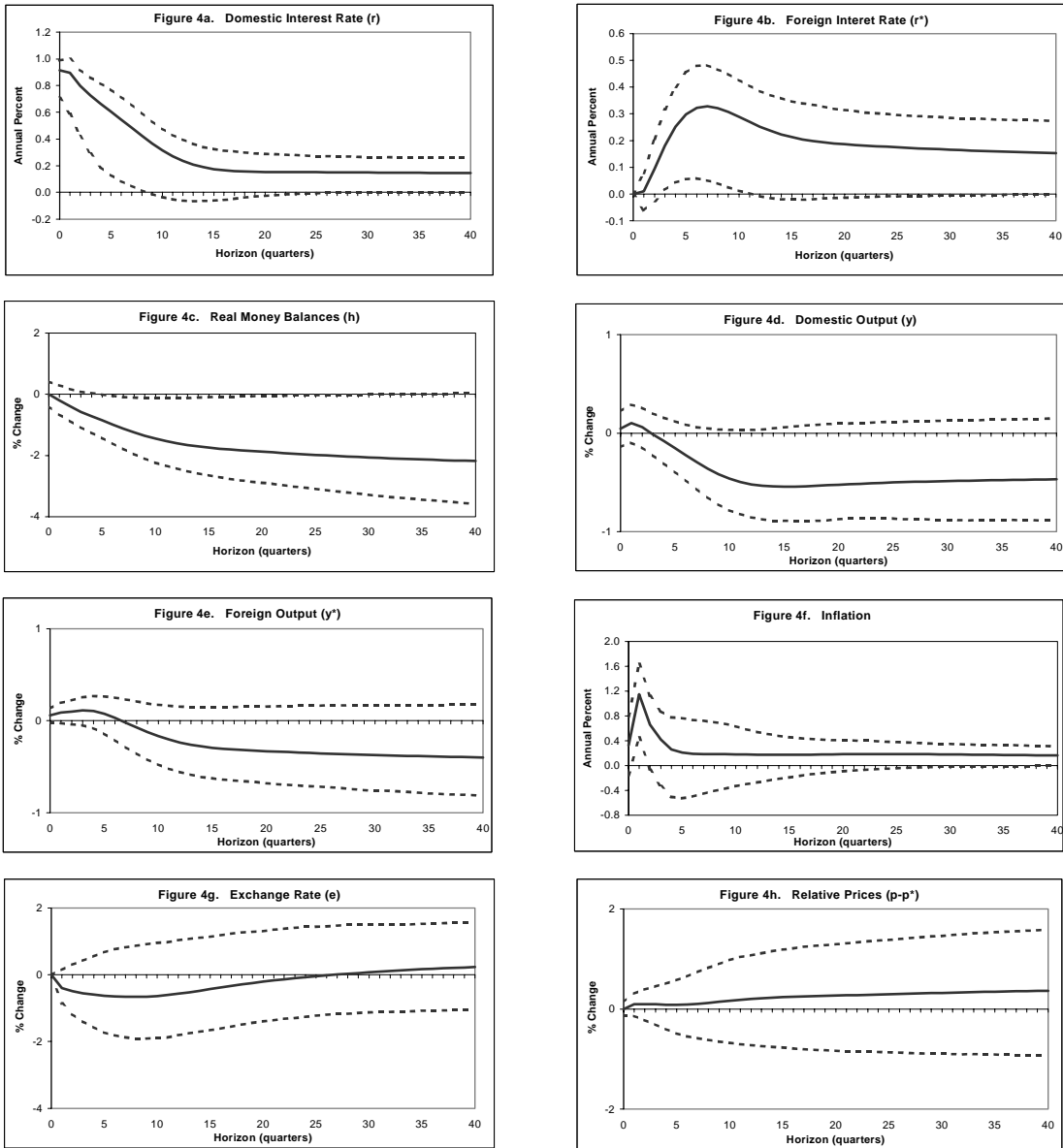
Notes: The solid lines plot the mean values of the empirical distributions of the impulse responses generated from the bootstrap procedure used to calculate the standard error bands.

**Figure 3:** Persistence Profiles of the Long Run Relationships to a Unit Monetary Policy Shock



See the notes to Figure 1.

**Figure 4:** Impulse Responses of a Unit Monetary Policy Shock



See the notes to Figure 2.

## References

- [1] Abadir, K.M., K. Hadri, and E. Tzavalis (1999), "The Influence of VAR Dimensions on Estimator Biases," *Econometrica*, 67, 163-181.
- [2] Barro, R. and D. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91, 589-610.
- [3] Barro, R.J. and X. Sala-i-Martin (1995), *Economic Growth*, Advanced Series in Economics. McGraw-Hill, New York, London and Montreal.
- [4] Bernanke, B.S. (1986), "Alternative Exploration of the Money-Income Correlation," *Carnegie Rochester Conference Series in Public Policy*, 25, 49-100.
- [5] Bernanke, B.S., M. Gertler, and M.W. Watson (1997), "Systematic Monetary Policy and the Effects of Oil Price Shocks," *Brookings Papers on Economic Activity*, 1, 91-142.
- [6] Bernanke, B.S, T. Laubach, F.S. Mishkin and A.S. Posen (1999), *Inflation Targeting: Lessons from the International Experience*, Princeton University Press, Princeton.
- [7] Bernard, A.B. and S.N. Durlauf (1995), "Convergence in International Output," *Journal of Applied Econometrics*, 10, 97-108.
- [8] Binder M. and M.H. Pesaran (1999), "Stochastic Growth Models and Their Econometric Implications," *Journal of Economic Growth*, 4, 139-183.
- [9] Blanchard, O.J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, 79, 655-673.
- [10] Blanchard, O.J. and M. Watson (1986), "Are Business Cycles Alike?" in *The American Business Cycle: Continuity and Change*, ed. by R.J. Gordon. University of Chicago Press, Chicago.
- [11] Blinder A.S. (1998), *Central Banking in Theory and Practice*, MIT Press, Cambridge.
- [12] Branson, W.H. (1977), "Asset Markets and Relative Prices in Exchange Rate Determination," *Sozialwissenschaftliche Annalen*, 1, 69-89.
- [13] Breedon, F.J. and P.G. Fisher (1996), "M0: Causes and Consequences," *The Manchester School*, 64, 371-387.
- [14] Buiter, W. (1980), "The Macroeconomics of Dr. Pangloss: A Critical Survey of the New Classical Macroeconomics," *Economic Journal*, 90, 34-50.
- [15] Campbell, J.Y. and N.G. Mankiw (1989), "International Evidence of the Persistence of Economic Fluctuations," *Journal of Monetary Economics*, 23, 319-333.
- [16] Cheung, Y.W. and K.S. Lai (1993), "Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration," *Oxford Bulletin of Economics and Statistics*, 55, 313-328.
- [17] Christiano, L. and M. Eichenbaum (1992), "Current Real Business Cycle Theories and Aggregate Labour Market Fluctuations," *American Economic Review*, 82, 432-450.
- [18] Cochrane, J.H. (1998), "What do VARs Mean ? Measuring the Output Effects of Monetary Policy," *Journal of Monetary Economics*, 41, 277-300.
- [19] Cogley, J. (1990), "International Evidence on the Size of the Random Walk in Output," *Journal of Political Economy*, 98, 501-18.
- [20] Crowder, W.J., D.L. Hoffman and R.H. Rasche (1999), "Identification, Long-Run Relations, and Fundamental Innovations in a Simple Cointegrated System," *Review of Economics and Statistics*, 81, 109-121.

- [21] Darby, M.R. (1983), "Movements in Purchasing Power Parity: The Short and Long Runs," in *The International Transmission of Inflation*, eds. by M.R. Darby and J.R. Lothian. University of Chicago Press (for National Bureau of Economic Research), Chicago.
- [22] Dickey, D.A. and W.A. Fuller (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," *Journal of the American Statistical Association*, 74, 427-431.
- [23] Eichenbaum, M and C.L. Evans (1995), "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *Quarterly Journal of Economics*, 110, 975-1010.
- [24] Fabiani, S. (1996), *Technological Change and Output Fluctuations: An Empirical Analysis for the G7 Countries*, unpublished Ph.d dissertation, University of Cambridge.
- [25] Froot, K.A. and K. Rogoff (1995), "Perspectives on PPP and Long-Run Real Exchange Rates," *Handbook of International Economics*, Volume 3, eds. by G.M. Grossman and K. Rogoff. North-Holland, Amsterdam.
- [26] Gali, J. (1992), "How well does the IS/LM Model fir Postwar US Data?", *Quarterly Journal of Economics*, 107, 709-738.
- [27] Garratt, A., K. Lee, M.H. Pesaran and Y. Shin (2000), "The Structural Cointegrating VAR Approach to Macroeconometric Modelling," Chapter 5 in *Econometric Modelling; Techniques and Applications*, eds. by S. Holly and M. Weale. Cambridge University Press, Cambridge.
- [28] Garratt, A., K. Lee, M.H. Pesaran and Y. Shin (2001a), "Forecast Uncertainties in Macroeconometric Modelling: An Application to the UK Economy," University of Cambridge *DAE Working Paper* No. 0004, available on <http://www.econ.cam.ac.uk/faculty/pesaran>.
- [29] Garratt, A., K. Lee, M.H. Pesaran, and Y. Shin (2001b), *A Structural Cointegrating Macroeconomic Model of the UK*, Book under preparation.
- [30] Goffe, W.L., G.D. Ferrier and J. Rodgers (1994), "Global Optimisation of Statistical Functions with Simulated Annealing," *Journal of Econometrics*, 60, 65-99.
- [31] Gonzalo, J. (1994), "Five Alternative Methods of Estimating Long-Run Equilibrium Relationships," *Journal of Econometrics*, 60, 203-233.
- [32] Granger, C.J.W. and J.L. Lin (1995), "Causality in the Long-Run," *Econometric Theory*, 11 530-536.
- [33] Gredenhoff, M. and T. Jacobson (1998), "Bootstrap Testing and Approximate Finite Sample Distributions for Tests of Linear Restrictions on Cointegrating Vectors," unpublished Paper, Department of Economic Statistics, Stockholm School of Economics.
- [34] Grilli V. and G. Kaminsky (1991), "Nominal Exchange Rate Regimes and the Real Exchange Rate: Evidence from the United States and Great Britain, 1885-1986," *Journal of Monetary Economics*, 27, 191-212.
- [35] Haldrup, N. (1998), "A Review of the Econometric Analysis of I(2) Variables," *Journal of Economic Surveys*, 12, 595-650.
- [36] Haug, A.A. (1996), "Tests for Cointegration: a Monte Carlo Comparison," *Journal of Econometrics*, 71, 89-115.
- [37] Huizinga J. (1988), "An Empirical Investigation of the Long-Run Behavior of Real Exchange Rates," *Carnegie-Rochester Conference Series Public Policy*, 27, 149-214.
- [38] Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models," *Econometrica*, 59, 1551-1580.
- [39] Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press, Oxford.
- [40] Johansen, S. and K. Juselius (1992), "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and UIP for UK," *Journal of Econometrics*, 53, 211-244.

- [41] Kilian, L. (1997), "Impulse Response Analysis in Vector Autoregressions with Unknown Lag Order," unpublished manuscript, University of Michigan.
- [42] Kilian, L. (1998), "Small-Sample Confidence Intervals For Impulse Response Functions," *Review of Economics and Statistics*, 80, 186-201.
- [43] Kilian, L. (1999), "Finite-Sample Properties of Percentile and Percentile-t Bootstrap Confidence Intervals for Impulse Responses," *Review of Economics and Statistics*, 81, 652-660.
- [44] Kim, K. and A.R. Pagan (1995), "The Econometric Analysis of Calibrated Macroeconomic Models," Chapter 7 in *Handbook of Applied Econometrics: Macroeconomics*, eds. by M.H. Pesaran and M. Wickens. Basil Blackwell, Oxford.
- [45] King R.G., C.I. Plosser, J.H. Stock, and M.W. Watson (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 81, 819-840.
- [46] Koop, G., M.H. Pesaran, and S.M. Potter (1996), "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74, 119-47.
- [47] La Cour, L. and R. Macdonald (2000), "Modeling the ECU against the Dollar: A Structural Monetary Interpretation," *Journal of Business and Economic Statistics*, 18, 436-450.
- [48] Lee, K.C. (1998), "Cross-Country Interdependence in Growth Dynamics: A Model of Output Growth in the G7 Economies, 1960-1994," *Weltwirtschaftliches Archiv*, 134, 3 367-403.
- [49] Lee, K.C., M.H. Pesaran, and R.P. Smith (1997), "Growth and Convergence in a Multi-Country Empirical Stochastic Solow Model," *Journal of Applied Econometrics*, 12, 357-392.
- [50] Lee, K.C., M.H. Pesaran and R.P. Smith (1998), "Growth Empirics: A Panel Approach - A Comment," *Quarterly Journal of Economics*, 113, 319-323.
- [51] Levchenkova S., A.R. Pagan, and J.C. Robertson (1998), "Shocking Stories," *Journal of Economic Surveys*, 12, 507-532.
- [52] Lothian J.R. and M.P. Talyor (1996), "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Last Two Centuries," *Journal of Political Economy*, 104, 488-509.
- [53] MacDonald, R. (1995), "Long-Run Exchange Rate Modeling: A Survey of the Recent Evidence," *International Monetary Fund Working Paper*, No. WP/95/14.
- [54] MacDonald, R. and P.D. Murphy (1989), "Testing for the Long Run Relationship Between Nominal Interest Rates and Inflation using Cointegration Techniques," *Applied Economics*, 21, 439-447.
- [55] Mark, N.C. (1990), "Real and Nominal Exchange Rates in the Long Run: An Empirical Investigation," *Journal of International Economics*, 28, 115-136.
- [56] Mellander, E., A. Vredin, and A. Warne (1992), "Stochastic Trends and Economic Fluctuations in a Small Open Economy," *Journal of Applied Econometrics*, 7, 369-394.
- [57] Mills, T. (1991), "Are Fluctuations in UK Output Transitory or Permanent?" *Manchester School*, 59, 1-11.
- [58] Mishkin, F.S. (1992), "Is the Fisher Effect For Real?", *Journal of Monetary Economics*, 30, 195-215.
- [59] Nelson, C.R. and C.I. Plosser (1982), "Trends and Random Walks in Macro-Economic Time Series," *Journal of Monetary Economics*, 10, 139-162.
- [60] Obstfeld, M. and K. Rogoff (1996), *Foundations of International Macroeconomics*. Cambridge: Massachusetts Institute of Technology Press.
- [61] Pagan, A.R. (1999), "The Getting of Macroeconomic Wisdom," unpublished manuscript, Australian National University.

- [62] Pagan, A.R. and J.C. Robertson (1998), "Structural Models of the Liquidity Effect," *Review of Economics and Statistics*, 80, 202-217.
- [63] Pesaran, M.H. and Y. Shin (1996), "Cointegration and Speed of Convergence to Equilibrium," *Journal of Econometrics*, 71, 117-43.
- [64] Pesaran, M.H. and Y. Shin (1998), "Generalized Impulse Response Analysis in Linear Multivariate Models," *Economics Letters*, 58, 17-29.
- [65] Pesaran, M.H. and Y. Shin (2001), "Long-Run Structural Modelling," forthcoming in *Econometric Reviews*, available on <http://www.econ.cam.ac.uk/faculty/pesaran/> (Under Revision).
- [66] Pesaran, M.H. and R.P. Smith (1998), "Structural Analysis of Cointegrating VARs," *Journal of Economic Surveys*, 12, 471-506.
- [67] Pesaran, M.H., Y. Shin, and R.J. Smith (2000), "Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables", *Journal of Econometrics*, 97, 293-343.
- [68] Phillips. P.C.B. and P. Perron (1988), "Testing For Unit Roots in Time Series Regressions," *Biometrika*, 75, 335-46.
- [69] Rogoff, K. (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target," *Quarterly Journal of Economics*, 100, 1169-1190.
- [70] Rogoff, K. (1996), "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 34, 647-668.
- [71] Rotemberg, J.J. and M. Woodford (1999), "Interest Rate Rules in an Estimated Sticky Price Model," in *Monetary Policy Rules*, ed. by J. Taylor. National Bureau of Economic Research, University of Chicago Press, Chicago.
- [72] Sims, C. (1980), "Macroeconomics and Reality," *Econometrica*, 48, 1-48.
- [73] Sims, C. (1986), "Are Forecasting Models Usable for Policy Analysis?," *Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 2-16.
- [74] Sims, C. (1992), "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy," *European Economic Review*, 36, 975-1000.
- [75] Svensson, L.E.O. (1997), "Optimal Inflation Targets, Conservative Central Banks, and Linear Inflation Contracts," *American Economic Review*, 87, 98-114.
- [76] Svensson, L.E.O. (1999), "Inflation Targeting as a Monetary Policy Rule," *Journal of Monetary Economics*, 43, 607-654.
- [77] Taylor, M.P. (1988), "An Empirical Examination of Long Run Purchasing Power Parity Using Cointegration Techniques," *Applied Economics*, 20, 1369-1381.
- [78] Wickens, M.R. and R. Motto (2001), "Estimating Shocks and Impulse Response Functions," *Journal of Applied Econometrics*, 16, 371-387.