Constrained Communication with Multiple Agents: Anonymity, Equal Treatment, and Public Good Provision

Kohei Kawamura*†

Nuffield College, University of Oxford

This Version: February 2007

Abstract

This paper studies information transmission subject to anonymity requirements and communication in public good provision without transfers. The structure of informative equilibria under anonymity or in public good provision can differ substantially from that of direct one-to-one communication, and in particular we distinguish i) informational distortion caused by the intrinsic divergence of preferences between the decision maker and each agent; and ii) informational distortion caused by the decision maker's weak response to each agent's message due to the equal treatment of all agents that results from anonymity or the nature of public goods. We examine the interaction between these two types of distortion and demonstrate that they may partly offset one another. Information transmission and welfare can be enhanced by introducing the second type of distortion through anonymity when the first type of distortion is severe. In public good provision where the intrinsic preference divergence between the utilitarian decision maker and each agent is absent, as the number of agents becomes larger the quality of communication diminishes and informative equilibria converge to the one that can be played by letting each agent report a binary message (e.g. "yes" or "no") even if their preferences and the decision are continuous.

Keywords: Cheap Talk, Anonymous Communication, Equal Treatment, Public Good Provision

JEL Classifications: D71, D82, D83

1 Introduction

A great deal of information in society is communicated anonymously. In firms or schools, junior members often communicate with senior members (management, teachers) anonymously, through anonymous questionnaires, unions, representatives, or third parties such as external consultants so that the sender of a message may not be known to its receiver. Fraud investigations are often initiated by an anonymous report from a whistleblower in an organization. Are we more likely, or less likely to tell the truth when we are anonymous than otherwise? Why is anonymous communication so widely used?

Let us consider an apparently distinct but related setting. Suppose that a decision maker chooses the quality or quantity of a public good that is consumed by all members of a group with different preferences but no monetary transfers are allowed. Before making her decision, the decision maker may communicate with the members to figure out the optimal provision. For example, a local authority may try to find the optimal public services policy for the community by discussing with the residents, or a teacher may ask his students how fast or how difficult they would like his lectures to be. A regulator may acquire information from firms and their interested parties when choosing a regulation policy. When an altruistic but uninformed decision maker is restricted to impose a uniform decision on all members of a group, are they willing to reveal their private information to the decision maker truthfully? How does the number of agents who consume the public good affect the quality of communication?

This paper addresses these questions by modelling communication as an extension of the standard "cheap talk" model of Crawford and Sobel (1982) to a setting with multiple senders (agents). Each sender receives a private signal and costlessly sends a message to an uninformed receiver (decision maker) who, on the basis of the information received from all agents, makes a decision that affects their utilities. The sender, therefore, tailors the message in order to induce the receiver to take an action closer to that desired by the sender. When the decision concerns provision of a public good but no transfers are available, it must be the same for all senders. When communication is anonymous so that the receiver does not know the sender of each individual message, it is optimal for the receiver to implement the same decision to all (ex ante identical) senders, but otherwise the communication with each sender is the same as in one-to-one communication analyzed by Crawford and Sobel (1982). Since anonymity in effect works as a commitment device for equal treatment of multiple senders, the model can be directly applied to study communication where a decision maker is able to commit to equal treatment. Communication for "uniform allocation" of subsidies or research grants may fall into this category.

Provided that the receiver treats every sender equally, her optimal policy is a function of all messages she has received. Hence, the receiver's response to a particular sender's message will become weaker under anonymity/equal treatment, compared with one-to-one

communication where the receiver can take a tailored action for each individual sender. In other words, each sender has less influence on the decision. Equal treatment of multiple senders certainly changes each sender's communication strategy, because in order to influence the receiver's decision in his favour the sender must also take into account the effects of the other senders' messages on the decision. This gives rise to the type of informational distortion that is qualitatively different from what we call *intrinsic bias*, which represents preference divergence that can be present even in one-to-one communication with a single sender.

In the standard information transmission literature the source of informational distortion in communication is the presence of the sender's bias that reflects the intrinsic divergence of preferences between the sender and the receiver. Typically a positive (negative) intrinsic bias is modelled in such a way that the sender's type is represented in an interval and any type of sender wishes to induce a consistently higher (lower) action than the receiver, if she completely believes the message from the sender. No type fully reveals because doing so always leads to a lower (higher) action than he desires. As a result, in a perfect Bayesian equilibrium the sender's type is at best only partially revealed: the informative equilibria are typically characterized by a partition of the sender's type space into a finite number of intervals, where the types of sender in the same interval induce the same action by the receiver. When the intrinsic bias is too large no information can be transmitted and the receiver's action is based solely on her prior belief.

Weak response to a message due to anonymity/equal treatment leads to certain characteristics in the sender's incentive to reveal information that cannot be found in communication with an intrinsic bias only. First, when the receiver's response to a message is weak, the sender's bias may not be consistently positive or negative. This means that full revelation may lead to higher or lower action by the receiver than the sender's desired action, depending on his type. In order to highlight the effects of weak response, suppose for the moment that there is no intrinsic preference divergence between the receiver and each individual sender, and that the utilitarian decision maker (receiver) simply maximizes the sum of all senders' utilities. An example would be a local authority that seeks to maximize the sum of the residents' utilities. Suppose that the local authority wishes to communicate with the residents regarding the public health service. Since each resident has only a small influence on the final decision, residents who want only a slight increase in the spending (and quality) may not reveal completely truthfully and "overstate" their need by saying they want a huge increase. On the other hand those with private insurance or those who use other public services more often may "understate" their demand by reporting that they want a large cut even if they actually want only a slight reduction. In this example those who want an increase in the spending are positively biased while those who prefer a cut are negatively biased. When many senders are involved in a decision, the receiver must take

into account the senders' incentive to "exaggerate" their types, relative to the average.[1]

Second, when both individual preference divergence (intrinsic bias) and informational distortion caused by weak response are present, they partly offset each other. This suggests that when intrinsic bias is large, introduction of anonymity may improve information transmission and welfare. By introducing anonymity the receiver is unable to take tailored action for each sender and this reduces efficiency if the quality of information the receiver can obtain from each sender is fixed. However when the senders are intrinsically biased weak response as a result of anonymity may encourage them to reveal more information. The intuition behind more revelation is simple. Suppose that the senders have a positive intrinsic bias and the type of a particular sender is low. When anonymous his incentive to "exaggerate" gives rise to a negative bias (since the receiver's response is weak) and this may offset the positive intrinsic bias. Thus low types may reveal more information under anonymity because their interests are more aligned with those of the receiver. In particular we show that when the intrinsic bias is large the benefit of information revelation may exceed the cost of equal treatment.

Finally, in the context of public good provision where there is no intrinsic bias and the receiver is a utilitarian decision maker, any informative equilibria converge to the one that can be played by letting each agent (sender) choose between only two messages as the number of agents who consume the public good becomes larger. Moreover, the informational loss caused by using only two messages (as opposed to using many messages that can be supported in equilibrium) is smaller when there are more agents. This might explain why the "choice between the two" ("yes or no", "agree or disagree", etc.) is a very common way of communicating when many people are involved in a decision, even if neither the agents' preferences nor the decision made after communication is binary (e.g. quality of service, pace of lectures, or tightness of regulations). When an agent can only say whether he agrees or disagrees with a proposal, he cannot express how strongly he agrees or disagrees. A binary message eliminates the possibility of "exaggerating" preferences, which is the chief cause of informational distortion when each sender has weak influence on the decision.

Throughout this paper we are concerned with situations where the receiver *cannot* commit to a complete pre-determined decision rule (mechanism), and she makes her decision after hearing or reading the messages. Although much of the literature on decision making in multi-agent settings assumes that commitment to decision rules is possible and focuses

---

[1] By incentive to "exaggerate", we mean a sender's incentive to misreport in such a way that, if words are taken literally and believed by the receiver, the sender whose type is high (low) "overstates" ("understates") his type by saying his type is even higher (lower). However, in cheap talk games messages used are completely arbitrary and do not have to be taken literally. What matters for the equilibrium outcome and efficiency is the correspondence between each sender's type and the receiver's induced action, so what word (or language) is used to induce a particular action is irrelevant.

on the design of such rules, imperfect commitment or the absence of complete decision rules is prevalent in many situations of interest. Often the receiver of a message is tempted to treat its sender individually even when it discourages truthful communication. For example a manager may be tempted to dismiss a worker who says he is unable to perform an important task, but knowing this the worker may not report his skills truthfully. In public good provision decision makers rarely offer a decision rule that prescribes a decision according to every possible set of received messages. A local authority may ask its residents whether they agree or disagree with a proposal on public services but typically such a proposal is vague. The local authority may decide the details after it has learnt the number of residents who agree with the proposal. Even if a decision rule is offered the decision maker may not necessarily follow it unless it is solicited as a legally binding contract or she has strong reputational concerns for commitment to decision rules. Whereas there are clearly many circumstances where the decision maker is able to commit to decision rules and the design of such rules is of great importance, we focus on communication under limited commitment that seems relevant to a wide variety of decision making environments where multiple agents are involved.

1.1   Relation to the Literature

As anonymity and the concept of equal treatment necessarily entail multiple senders, our model is related to the literature on communication with multiple experts. Krishna and Morgan (2001) study whether senders should be consulted together or sequentially. Battaglini (2002) shows that when there are multiple senders with different biases and the state space is multidimensional, full revelation can be achieved for an arbitrarily large conflict of interest. Battaglini (2004) studies a model where the senders observe imperfect signals. Baliga, Corchon, and Sjöström (1997) investigate a set of social choice rules that can be implemented when agents play a cheap talk game with the decision maker. A common feature of these papers is that the senders observe the same or correlated states of nature while each sender has a different bias, where senders would most naturally be interpreted as experts with different political/business standpoints.

Among models of communication with multiple senders our model is closer to Austen-Smith (1993) and Wolinsky (2002) where senders observe independent signals (types). Austen-Smith (1993) focuses on the comparison between simultaneous and sequential reporting, and Wolinsky (2002) considers information sharing between senders. Thus the questions they address are different from ours and the ways they set up their models are suited particularly to consultation with a relatively small number of experts, while our model can naturally be interpreted in terms of communication with many agents. Also, both Austen-Smith (1993) and Wolinsky (2002) assume that the individual types and messages are binary. Because of the binary structure, the incentive to "exaggerate" types,

which we argue is an important source of informational distortion inherent in many multi-sender settings, cannot be fully incorporated into their models.

It is already known in the literature that the introduction of randomness in messages may facilitate information transmission especially in the presence of large conflict of interest between communicating parties. Myerson (1986) and Forges (1986) have shown this possibility in highly abstract settings, and Krishna and Morgan (2004) and Mitusch and Strausz (2005) have proposed more specific randomization mechanisms that improve information transmission. In this line of research Blume, Board and Kawamura (2007) have shown recently that in the model of Crawford and Sobel (1982) with quadratic utilities and a uniform type distribution, if a certain noise mechanism is introduced into the model it is possible to construct an equilibrium that Pareto dominates any equilibrium without noise as long as the intrinsic bias is present but not too large. Although randomization is typically interpreted as what a "mediator" does, it may not be easily introduced in practice. Hence the present paper introduces anonymity/equal treatment as an alternative device for enhancing communication when there are multiple senders. Moreover while the literature on noisy communication focuses on how communication can be improved in the presence of intrinsic bias, the current paper also offers a detailed analysis of communication where there is no intrinsic bias, which can be applied to study public good provision without transfers.

The literature on public good provision has been concerned with mechanism design problems where agents reveal their preferences (partially or fully) by sending a message on or voting for the provision of a public good (Palfrey and Rosenthal, 1984; Bagnoli and Lipman, 1989; Ledyard, 1995). Typically monetary transfers are allowed and the decision maker is assumed to be a mechanism designer who is able to commit to a mechanism (i.e. a mapping from messages to the decision including transfers). Having received the messages the decision maker implements the provision and compels transfers, according to a pre-specified rule. The main source of moral hazard is the free rider problem, where agents have incentive to "understate" true preferences for the public good, because given the amount of the public good everyone prefers to incur less cost. Without a truthful revelation mechanism the agents are negatively biased in reporting their preferences.

The present paper sheds light on a different set of problems in public good provision. First, we focus on situations where no transfers among members (including the decision maker) are available and therefore each agent's costly contribution is not a concern. As we have suggested earlier, settings with no transfers characterize many important aspects of decision making within organizations including firms and schools as well as certain regulatory and political relationships. In many of these circumstances monetary transfers are often infeasible or considered inappropriate. Second, we assume that the decision maker cannot commit to a mechanism. In other words, the decision maker makes her

decision strategically after hearing or reading the messages, which seems to be relevant to many practical situations, especially where legally binding contracts are unavailable or the decision maker does not have strong reputational concerns. A similar model to ours is studied by Bester and Strausz (2000) but they solely focus on showing that a version of the revelation principle, proposed in Bester and Strausz (2001), does not extend to cheap talk games with multiple senders. Unlike us they do not consider the characteristics of information transmission in equilibrium, but we demonstrate that communication in public good provision can be fruitfully analyzed in our framework.

The structure of informative equilibria we identify is related to that of Melumad and Shibano (1991), Alonso, Dessein and Matouschek (2006) and Gordon (2006) who, like us, study cheap talk models with weak response to a message. However, unlike our model Melumad and Shibano (1991) introduce weak response directly into the receiver's utility function, and Alonso, Dessein and Matouschek (2006) do not consider the interaction between weak response and intrinsic bias. We derive informative equilibria in a simple multi-sender setting that can be applied to a wide variety of circumstances and in particular examine interplay among the quality of information transmission, preference divergence and the number of senders involved. Gordon (2006) provides a general characterization of cheap talk equilibria where the sender's bias is type-dependent and may not be consistently positive or negative. His equilibrium characterization with type-dependent bias can potentially be applied to various types of cheap talk games, including noisy communication and multiple sender/receiver settings, and indeed encompasses communication equilibria in many models that extend Crawford and Sobel (1982) and ones presented in this paper. However, since Gordon (2006) is mainly concerned with equilibrium characterization itself, he either takes the receiver's response function as given, or introduces the source of biases directly into the players' utility functions. In contrast we will illustrate how certain communication environments (such as anonymity or public good provision) lead to weak response by the receiver, and examine how parameters in these environments alter the characteristics of information transmission.

Our model focuses on communication in a one-shot game, but reputational concerns in repeated interactions can give rise to the type of informational distortion different from both intrinsic bias and incentive to "exaggerate" presented in this paper. Ottaviani and Sørensen (2006a,b) have shown that an expert's reputational concerns for his ability create a bias *towards* the uninformed party's prior belief.[2] Interestingly, this type of bias is in stark contrast to incentive to "exaggerate" since the latter can be interpreted as a bias *away from* the prior belief.

---

[2]In the models of Ottaviani and Sørensen (2006a,b) there is no preference divergence as long as the sender of a message is perfectly informed. Other papers on communication with reputational concerns, including Sobel (1986), Morris (2001) and Park (2006), study reputation building for an (unknown) intrinsic bias.

This paper proceeds as follows. Section 2 introduces the model and its applications for constrained communication with multiple senders. In Section 3 we derive informative perfect Bayesian equilibria with quadratic utilities and uniformly distributed sender types, and compare the players' expected utilities in anonymous communication and those in Crawford and Sobel (1982). Section 4 shows that many of our qualitative results hold in a more general setup. Section 5 concludes.

2   Model and Applications

Before we present our multiple sender setting, let us introduce a version of the standard cheap talk model of Crawford and Sobel (1982; henceforth CS). A sender who has private information about his type (or state of nature) $\theta \in [0,1]$ communicates with a receiver. The sender's utility function is $U^S = U(y, \theta, b)$ and the receiver's is $U^R = U(y, \theta, 0)$ where $y \in \mathbb{R}$ denotes the receiver's action. $b \geq 0$ represents the sender's bias or the level of conflict and is common knowledge. $U(\cdot)$ is twice continuously differentiable, with $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$, and $U_1 = 0$ for some y to ensure that the unique maximum with respect to y exists. Let $y^S(\theta)$ and $y^R(\theta)$ be the receiver's actions that maximize U^S and U^R, respectively. The assumptions on the utility functions imply that both $y^S(\theta)$ and $y^R(\theta)$ are strictly increasing θ. Also if $b > 0$ $(b = 0)$ then $y^S(\theta) > y^R(\theta)$ $(y^S(\theta) = y^R(\theta))$. The sender's type θ has a differentiable distribution F with density f on $[0,1]$. Before the receiver takes her action, the sender reports a costless message $m \in M$ where M is a message space that is rich enough to cover all types. Before selecting y the receiver updates her belief on θ according to the message.

CS have shown that, for $b > 0$, the perfect Bayesian equilibria of this game are such that the type space is divided into a finite number of intervals and all types in a particular interval induce the same action. We denote a typical partition of the type space into J intervals by $\{[0, a_1), [a_1, a_2)..., [a_{j-1}, a_j), ..., [a_{J-1}, 1]\}$, where a_j denotes a boundary type within the interval $(0, 1)$. If $a_1 = 0$ the first interval is degenerate and we denote the partition by $\{[0], (0, a_2)..., [a_{j-1}, a_j), ..., [a_{J-1}, 1]\}$. Let $\bar{J}(b)$ be the largest number of intervals that can be supported in equilibrium, which is shown to be a function of b. CS have also shown that there exists an equilibrium with J intervals for any $1 \leq J < \bar{J}(b)$. If $b = 0$ both parties' interests are perfectly aligned and the sender may fully reveal θ.

In CS's "uniform-quadratic" setting, $U^S = -(y - \theta - b)^2, U^R = -(y - \theta)^2$ and θ is uniformly distributed. In this case the first order condition for the receiver's utility maximization gives her best response

$$y(m) = E[\theta \mid m],\tag{1}$$

which is the expected value of the sender's type conditional on the message. Hence the

influence the sender's message has on the receiver's action is captured in (1). In a perfect Bayesian equilibrium the sender's strategy must also be a best response to (1).

2.1 Anonymous Communication

2.1.1 General Formulation

Let us consider an information transmission game between a single receiver and n senders labelled by $\{1, 2, ..., i, ..., n\}$. Each sender has a different type, which is private information to the sender. The receiver can take a different action for each sender, and the receiver's utility function depends on each sender's type and the action taken towards him.

Let sender i's utility function be

$$U^{Si}(y_i, \theta_i, b) = U(y_i, \theta_i, b) \text{ for all } i \tag{2}$$

where $y_i \in \mathbb{R}$ is the receiver's action for sender i, θ_i is the sender's type, and $b \geq 0$ is both symmetric across all senders and common knowledge. The receiver cannot observe $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_i, ..., \theta_n] \in [0,1]^n$. Each sender's type θ_i is independent and identically distributed (i.i.d.) and has a differentiable distribution F with continuous density f. Only sender i observes θ_i, while the receiver and the other senders do not.

The receiver's utility function is given by

$$U^R(\mathbf{y}, \boldsymbol{\theta}) = \sum_{i=0}^{n} U(y_i, \theta_i, 0) \tag{3}$$

where $\mathbf{y} = [y_1, y_2, ...y_i, .., y_n]$ is the vector of actions taken by the receiver. Her utility is the sum of all senders' utilities such that $b = 0$. As we have noted, we interpret y_i as an action taken towards sender i. The utility function implies that if the receiver perfectly knew $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_i, ..., \theta_n]$ she would choose a different action for each sender unless their types coincide.

The intrinsic divergence of preferences between the receiver and each sender is captured by b. Since b is symmetric, ex ante (before the senders observe their types) all senders are identical. Let $m_i \in M$ be the message sender i reports to the receiver, where the message space M has enough elements to cover all sender types, and is shared by all senders. A strategy of a sender is described by $q_i(m_i \mid \theta_i)$, the conditional probability that sender i's message is m_i given that his type is θ_i. Technically speaking, the receiver's action space is multidimensional, while each sender's message and type spaces are unidimensional. Each sender reports his message m_i independently and simultaneously to the receiver, before she selects her action vector $\mathbf{y}$.

Definition 1 Communication between a receiver and n senders is *anonymous* if, having observed all n messages, the rational receiver assigns probability $1/n$ to the event that a message is reported by sender i, for $i = 1, 2, ..., n$.

In anonymous communication, having observed the messages the receiver cannot update her belief on which message corresponds to which sender. Therefore, the probability that a message comes from any sender is $1/n$. There are at least a few ways in which communication with multiple senders is *not* anonymous. First, anonymity is clearly violated when the receiver sees a sender delivering his own message. Second, even if the receiver does not observe the sender of a message, it may include information on both his type and identity. This is the case if the message is multidimensional and includes the "name" of the sender as well as information on his type. Alternatively, even when the message does not contain the "name", the receiver may be able to infer the identity of a sender through his message if the senders adopt different strategies, since a certain message may be used only by a particular sender in equilibrium.

If the identity of the sender of a message is perfectly revealed, the game between that sender and the receiver reduces to the single sender model of CS we have introduced above. Thus unless revelation of identities through messages is ruled out before the game is played, there exist equilibria where anonymity is violated and some (or all) senders' equilibrium strategies replicate those of the single sender model.

On the other hand as long as the receiver does not observe a sender delivering his own message, there also exist equilibria where the identity part of messages is not believed by the receiver or the senders choose not to reveal their identities. That is, in this class of equilibria communication is anonymous in the sense of our definition above, and such equilibria are of particular interest because as we will see later there are situations in which both parties are better off when the identities of the senders are not revealed. Moreover, before the senders learn their types both the receiver and the senders may agree on using some anonymization device that precludes revelation of identities (such as communication through a neutral third party who enforces anonymity). When we refer to *anonymous* communication or communication under *anonymity*, we either focus on the class of equilibria where no senders' identities are revealed, or on the information transmission game where revelation of identities is ruled out by assumption.

Throughout this paper we focus on symmetric sender strategies so that if $\theta_i = \theta_j$ for $i \neq j$, then $q_i(m_i \mid \theta_i) = q_j(m_j \mid \theta_j)$. That is, we assume that any (ex ante identical) senders with the same type report their messages according to the same conditional distribution. As we have suggested already if senders adopt different strategies, the receiver can identify the sender of a message (say sender 1) from the message itself, since the message may not be used in the equilibrium strategies of the other senders. Note that by symmetry we also rule out sender strategies such that the senders reveal their identities directly by telling their names.

Under those assumptions, we establish the effect of anonymity on the receiver's action vector.

Proposition 1 (Equal Treatment) In anonymous communication, the receiver's equilibrium actions are symmetric for all senders, or

$$y_1 = y_2 = ... = y_n.$$

Proof. Consider the receiver's utility maximization problem with respect to y_i. Note that the sender strategy is symmetric $q_1 = q_2 = ... = q_n = q$. Suppose that a message m_j corresponds to sender i. The posterior density function of θ_i is given by

$$p\left(\theta_i \mid m_j\right) = \frac{q\left(m_j \mid \theta_i\right) f(\theta_i)}{\int_0^1 q\left(m_j \mid t\right) f(t)dt}.$$

Since communication is anonymous the message m_j corresponds to any sender with the equal probability $1/n$. Therefore, we have the posterior density function of θ_i given n messages

$$p\left(\theta_i \mid m_1, m_2, ..., m_n\right) = \sum_{j=1}^{n} \frac{1}{n} \frac{q\left(m_j \mid \theta_i\right) f(\theta_i)}{\int_0^1 q\left(m_j \mid t\right) f(t)dt}.$$

Clearly this posterior density function of sender i's type is identical for all senders. Therefore, the maximization problem with respect to y_i is identical for all $y_1, y_2, ..., y_n$. Moreover, since the utility function is strictly concave the solution to the the receiver's maximization problem given $m_1, m_2, ..., m_n$ is unique. Hence, $y_1 = y_2 = ... = y_n$. ∎

Thanks to the equal treatment property we can focus on the relationship between n messages $m_1, m_2, ..., m_n$ and the unidimensional action y to analyze anonymous communication. That is, the receiver's utility maximization problem in anonymous communication is equivalent to the maximization problem where the receiver must treat every sender equally for exogenous reasons.

2.1.2 Uniform-Quadratic Setting

In order to illustrate the effect of anonymity on the receiver's best response and sender strategies clearly, let us consider quadratic utilities and uniform distribution of sender types. In this setting the utility of the receiver is given by $-\sum_{i=1}^{n}(y_i - \theta_i)^2$, while that of sender i is $-(y_i - \theta_i - b)^2$. As above, θ_i is private information to sender i, and independently and uniformly distributed on $[0, 1]$. Before the receiver chooses her action, each sender reports a message m_i on his type, independently, simultaneously, and anonymously.

According to Proposition 1, when the sender strategies are symmetric the receiver's optimal action is the same for every sender. Therefore, without loss of generality we can write the receiver's maximization problem in such a way that she selects a uniform action

11

$y = y_1 = y_2 =, ..., = y_n$ to maximize her expected utility given the received messages:

$$\max_y \ E\left[-\sum_{i=1}^{n}(y - \theta_i)^2 \middle| m_1, m_2, ..., m_n\right] = \sum_{i=1}^{n}\left[-(y - E[\theta_i \mid m_i])^2 - var(\theta_i \mid m_i)\right].$$

In this maximization problem we can ignore the variance term $var(\theta_i \mid m_i)$, which is constant from the receiver's viewpoint. Therefore the first order condition gives the receiver's best response function

$$y(m_1, m_2, ..., m_n) \equiv \frac{1}{n}\sum_{i=1}^{n}E[\theta_i \mid m_i]. \tag{4}$$

From sender i's viewpoint, after sending his message the receiver's action is still a random variable but with quadratic utility functions we can focus on the expected value of the receiver's action to consider the sender's strategy. Since he does not observe the other senders' types or messages, (4) implies that the expected action from the sender's viewpoint conditional on his own message is given by

$$\frac{1}{n}E[\theta_i \mid m_i] + \frac{n-1}{n}E\left[E[\theta_{-i} \mid m_{-i}]\right]$$

where θ_{-i} denotes a sender other than sender i. Using the following fact

$$E\left[E[\theta_{-i} \mid m_{-i}]\right] = E[\theta_{-i}] = \frac{1}{2},$$

and letting $\gamma \equiv 1/n$ to simplify notation, define

$$y_S(m_i) \equiv \gamma E[\theta_i \mid m_i] + (1 - \gamma)\frac{1}{2}. \tag{5}$$

We call $y_S(m_i)$ the *reaction* function, or the receiver's *reaction* to the message from sender i. This is to be distinguished from the receiver's *best response* function $y(m_1, m_2, ..., m_n)$. In the single sender model of CS we have $n = 1 = \gamma$, and indeed both (4) and (5) reduce to (1). Therefore the CS model is a special case of ours, where $n = 1$ and anonymity is irrelevant.

Let us consider the receiver's reaction from a sender's viewpoint. Compared with (1) the sender's message has less influence on the receiver's action in (5) because it is weighted at γ, and the influence becomes weaker as the number of senders becomes larger. Moreover, in expected terms, the reaction is biased towards the unconditional expectation of the senders' types $1/2$.

The change in the reaction function from (1) to (5) may have a great impact on a sender's incentive to reveal information. First, when the receiver's reaction to the message from a sender is given by (5), there may be a "fully revealing type", the type of sender
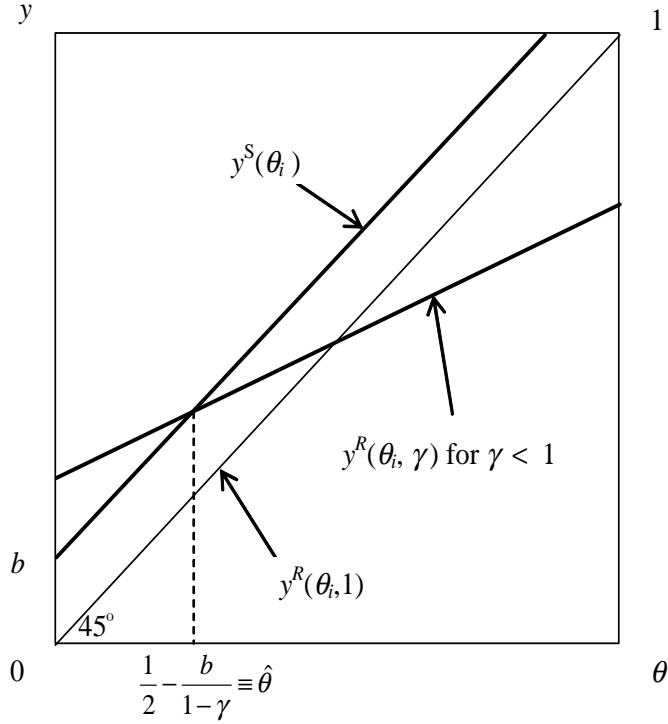
Figure 1: Sender's desired action and receiver's best response

wishes to be fully revealed even for $b > 0$. To find this type, suppose that the sender's type is θ_i. Define

$$
\begin{aligned}
y^S(\theta_i) &\equiv \theta_i + b \\
y^R(\theta_i, \gamma) &\equiv \gamma \theta_i + \frac{1}{2}(1 - \gamma),
\end{aligned}
$$

where $y^S(\theta_i)$ denotes the sender's desired action given his type θ_i, and $y^R(\theta_i, \gamma)$ is the receiver's reaction given that θ_i is perfectly revealed to her. Note that $y_S(m_i)$ denotes the receiver's reaction as a function of the sender's message, while $y^S(\theta_i)$ is the sender's desired action as a function of his type. If the sender's type is such that

$$
\begin{aligned}
y^S(\theta_i) &= y^R(\theta_i, \gamma) \Longleftrightarrow & (6) \\
\theta_i &= \frac{1}{2} - \frac{b}{1 - \gamma} \equiv \hat{\theta}, & (7)
\end{aligned}
$$

the sender may induce his (expected) desired action $\theta_i + b$ by reporting truthfully. Hence, if $\gamma < 1$ ($n \geq 2$) and b is not too large, there may exist $\hat{\theta} \in [0, 1]$ that satisfies (7). The sender's desired action and the receiver's reaction for given θ_i are illustrated in Figure 1, where the horizontal axis represents the sender's type θ_i and the vertical axis represents the receiver's action y. In communication with a single sender ($n = 1$) the receiver's reaction

13

if she knew θ_i is given by $y^R(\theta_i, 1) = \theta_i$, the 45 degree line, which never coincides with the sender's desired action $y^S(\theta_i)$ for $b > 0$. This implies that no type wishes to be fully revealed in one-to-one communication. However, if $n \geq 2$ the receiver's reaction $y^R(\theta_i, \gamma)$ crosses $y^S(\theta_i)$ at $\theta_i = \hat{\theta}$, that is, the sender's desired action and the receiver's reaction coincide at $\hat{\theta}$.

If $b = 0$ and $n = 1$ then $y^R(\theta_i, 1) = y^S(\theta_i) = \theta_i$ so that the sender can induce his desired action simply by revealing truthfully: perfect communication is possible in CS when there is no intrinsic bias. On the other hand if $b = 0$ but $n \geq 2$ then we have $\hat{\theta} = 1/2$. Even if there is no intrinsic bias the sender's desired action and the receiver's response do not coincide except for the "average" type.

Also, when the receiver's reaction is given by (5) the sender's desired action may be higher or lower than the receiver's reaction depending on his type. In communication with a single sender if $b > 0$ then $y^S(\theta_i) > y^R(\theta_i, 1)$ for all θ_i, so that the sender's desired action is consistently higher than the receiver's reaction under perfect revelation. However we have $y^S(\theta_i) < y^R(\theta_i, \gamma)$ ($y^S(\theta_i) > y^R(\theta_i, \gamma)$) if $\theta_i < \hat{\theta}$ ($\theta_i > \hat{\theta}$), in which case the sender's desired action is lower (higher) than the receiver's reaction.

2.1.3 An Alternative Specification of Anonymous Communication

Let us consider a different way of modelling anonymous communication from what we have introduced above. Often a decision is made according to a particular anonymous message from an informed sender rather than anonymous messages from all the senders as presented above. For example an official or investigator may act upon a message from an anonymous whistleblower in a group, rather than anonymous messages from all the members. The above framework can easily be modified to suit circumstances where only a subset of n potential senders report anonymous messages while the others stay "silent".

Suppose that the utility functions and the timing are the same as our original specification, and sender strategies are also symmetric. However, assume that only one of n senders (call him an "informer") observes θ_i from the known distribution on $[0, 1]$ and all the other senders' types, denoted by θ_{-i}, are the same and ex ante known $\theta_{-i} = h \in [0, 1]$. Until the senders learn their types they do not know whether they will be an informer or not, and the types are privately observed.

Also in this specification, unless the receiver observes the informer delivering his message there is an equilibrium where his identity is unknown to the receiver. If so, then equal treatment applies here too. When choosing her action the receiver cannot differentiate the anonymous informer from the others. In the uniform-quadratic setting with $h = 1/2$ ($= E[\theta_i]$), from the informer's viewpoint the receiver's reaction to his anonymous message is given by (5).[3] The receiver's action is taken towards every sender equally but is influenced

---

[3] In the uniform-quadratic setting even if there are l informers such that $l < n$, (5) still holds for each

only by the informer's type. Naturally the messages sent by the senders other than the informer can be interpreted as "silence".[4]

Although this alternative specification may suit certain circumstances better, in order to highlight the common structure with other forms of constrained communication, we will focus on the original specification (where θ_i is i.i.d. for all i) when we consider anonymous communication. However the main theoretical insights in both specifications are almost identical.

2.2 Public Good Provision

The discussion on anonymous communication so far indicates that the essential feature that leads to the reaction function (5) is equal treatment of multiple senders. In certain environments equal treatment is implied in the receiver's decision problem, whether or not the receiver can identify the sender of a message. Consider communication in public good provision with no transfers. Each agent (sender) has a different preference for the decision maker's (receiver's) action y and the utility of an agent is given by $U(y, \theta_i) \equiv U(y, \theta_i, 0)$. Unlike the case of anonymous communication, in this context it would be appropriate to assume $b = 0$, so that there is no intrinsic divergence of preferences between the decision maker and each agent. In other words, the utility functions are such that, if $n = 1$, their interests are completely aligned. The decision maker maximizes the sum of the agents' utilities $\sum_{i=1}^{n} U(y, \theta_i)$, which can be interpreted as a utilitarian social welfare function, but she cannot commit to a mechanism (i.e. a pre-determined mapping from the received messages to y). The agents send messages to the decision maker before she chooses y.

Under these assumptions the decision maker's maximization problem is identical to that of the anonymous communication case. In the uniform-quadratic setting her best response to n messages is represented by (4). Similarly the decision maker's reaction (in expected terms) from an agent's viewpoint is given by (5). Therefore, communication in public good provision with no transfers has a common strategic feature to anonymous communication, and the former can be analyzed as a special case ($b = 0$) of the latter.

2.3 Noisy Communication with a Single Sender

One of the most studied features in the cheap talk literature is the introduction of noise in communication with a single sender. As we suggested earlier a class of equilibria identified under noisy environment in Blume, Board and Kawamura (2007) has a similar structure

---

informer's viewpoint if the types of the informers are independently distributed.

[4]As we noted earlier, in cheap talk games messages are arbitrary. Any anonymous message (including "silence") that induces the receiver's belief that the (anonymous) sender of the message is not an informer, has the same effect on the receiver's reaction.

to communication with multiple senders subject to anonymity/equal treatment presented in this paper. Since we need to specify the sender's equilibrium strategy to point out the formal relationship between our model and noisy communication, we discuss this issue in Appendix II.

The intuition behind the similarity is as follows. Blume, Board and Kawamura (2007) assume that with probability $1 - q$ the receiver observes the message sent by the sender but with probability q the receiver observes a message drawn from a known distribution on the message space. The possibility that the message may not be from the sender himself makes the receiver's response to the observed message weaker than in the situation where the receiver always observes the message from the sender ($q = 0$). This has a similar effect on the sender strategy to that of anonymity/equal treatment.

However, there are significant differences between noisy communication and the present model for communication with multiple senders. Clearly the communication features that lead to the receiver's weak response differ. In the model presented here weak response arises from the presence of multiple senders and an important aim of this paper is to illustrate how the number of senders affects the nature of communication. Moreover, in noisy communication the weight the receiver puts on the received message may change depending on sender strategies. In the uniform-quadratic setting of Blume, Board and Kawamura (2007) the receiver's reaction represented by (5) corresponds to only one of many classes of sender/receiver strategies, and though analytically convenient, this class of strategies is not generally welfare maximizing. On the other hand as we have seen already the receiver's reaction is fully represented by (5) under anonymity/equal treatment and this paper provides a detailed analysis of equilibria that result from (5). Moreover, the case where $b = 0$ is hardly important in the context of noisy communication, because then there is no need to introduce noise as both parties' interests are perfectly aligned. Here we carefully examine this case too, as we have already demonstrated that our multi-sender setting with $b = 0$ can be interpreted as communication in public good provision and therefore be of independent interest from whether and how introduction of randomization or anonymity facilitates information transmission.

3 Equilibrium in the Uniform-Quadratic Case

In this section we study the uniform-quadratic setting introduced earlier, in order to illustrate how anonymity or equal treatment alters the structure of informative equilibria, and then discuss when introduction of anonymity is desired. We come back to the general setting in the following section. By informative equilibrium we mean an equilibrium where with strictly positive probability the receiver's action is different from the action

she chooses based only on her prior belief.[5] In order to simplify notation we drop subscript i when we refer to generic sender i. As in (5) $y_S(m)$ denotes the receiver's reaction to the sender's message (expected action from the sender's viewpoint conditional on his message m). Also, $y^R(\theta, \gamma)$ is the receiver's action from the sender's viewpoint given that the receiver knows θ, and $y^S(\theta)$ is the sender's desired action when his type is θ.

In the following we begin by studying the sender's equilibrium strategy given the receiver's reaction function (5). In the CS model where $n = 1$ it has been shown that, if the receiver's desired action and that of the sender never coincide, any perfect Bayesian equilibrium takes a partitional form, where the type space is divided into a finite number of intervals. In contrast, we will demonstrate that if the fully revealing type $\hat{\theta}$, which is given by (7), exists on $[0, 1]$ there is an equilibrium with an infinite number of intervals, which, however, is not fully revealing.

In order to consider equilibrium sender strategies, let us introduce an alternative representation of the receiver's reaction. Let $\underline{a}$ and $\overline{a}$ be two points in $[0, 1]$ such that $\underline{a} < \overline{a}$. From (5) and the assumption that θ is uniformly distributed

$$E[\theta \mid \theta \in [\underline{a}, \overline{a}]] = \frac{\underline{a} + \overline{a}}{2}.$$

Define

$$\bar{y}_S(\underline{a}, \overline{a}) \equiv \gamma \frac{\underline{a} + \overline{a}}{2} + (1 - \gamma)\frac{1}{2}. \tag{8}$$

$\bar{y}_S(\underline{a}, \overline{a})$ is the expected action from the sender's viewpoint conditional on the receiver's belief that a sender's type is such that $\theta \in [\underline{a}, \overline{a}]$. If $\theta = a$ then we write $\bar{y}_S(a, a)$. While $y_S(m)$ is defined as a function of the sender's message, $\bar{y}_S(\underline{a}, \overline{a})$ is a function of an interval although they both denote the receiver's reaction. Note that the receiver's action is a random variable from the sender's viewpoint. However, the randomness is caused only by messages from the other senders. Hence, the variance of the receiver's action is independent from the sender's strategy (message). The quadratic utility functions imply that, to derive equilibrium sender strategies, we can focus our attention on the expected value of the receiver's action $\bar{y}_S$.

In an equilibrium partition each boundary type $a_j \in (0, 1)$ must satisfy the "arbitrage" condition which says that the sender with $\theta = a_j$ is indifferent between inducing $\bar{y}_S(a_{j-1}, a_j)$ and $\bar{y}_S(a_j, a_{j+1})$. Solving the condition

$$-(\bar{y}_S(a_{j-1}, a_j) - a_j - b)^2 = -(\bar{y}_S(a_j, a_{j+1}) - a_j - b)^2 \tag{9}$$

by using (8) we obtain a second-order difference equation

$$\gamma a_{j+1} - (4 - 2\gamma)a_j + \gamma a_{j-1} = 4b + 2\gamma - 2. \tag{10}$$

---

[5]The uninformative equilibrium refers to the equilibrium where the receiver's action is based only on her prior belief with probability 1.

For a finite number of intervals, substituting $a_0 = 0$ and $a_J = 1$ we can solve the J simultaneous equations with J unknown variables in (10) to obtain the exact equilibrium partition that corresponds to certain γ, b and finite J. From (10) we obtain the following example for $J = 3$.

Example 1 Suppose that $\gamma = 1/2$ and $b = 0$. Then the partitional strategy $\{[0, \frac{3}{7}), [\frac{3}{7}, \frac{4}{7}), [\frac{4}{7}, 1]\}$ supports a perfect Bayesian equilibrium.

Notice that the length of an interval is narrower as it becomes closer to $\frac{1}{2}$. This means that a message from a sender whose type is closer to $\frac{1}{2}$ enables the receiver to infer the sender's type more accurately.

Before discussing applications of the model, let us briefly consider the issue of multiple equilibria. As we have already discussed earlier unless anonymity is imposed to rule out revelation of identities by assumption, there exist equilibria where a single or multiple senders reveal their identities and play the standard information transmission game in the CS model.

Even within the class of equilibria where all senders are anonymous, (10) can generate multiple equilibria. In particular, for any parameter values exists the uninformative equilibrium where all senders messages are ignored and the receiver's action is based only on her prior. However, if we look for the equilibrium where the ex ante (i.e., before the senders learn their types) expected utilities of the receiver and the senders are highest for given n and b within the class, we can focus on the equilibrium that has the largest number of intervals. CS have shown that this holds for the single sender case $n = 1$.

Proposition 2 Under anonymity/equal treatment in the uniform-quadratic setting, for given b and $n \geq 2$, both the receiver and the senders are ex ante better off in an equilibrium with more intervals.

Proof. See Appendix I. ■

In the following we refer to the equilibrium with the largest number of intervals for given parameter values as the "most informative equilibrium".

3.1 Most Informative Equilibrium

Let us consider the partition in the most informative equilibrium. To do so, we first solve (10) with respect to a_j explicitly using $a_0 = 0$, and obtain

$$
\begin{aligned}
a_j &= \hat{\theta} + \frac{\gamma a_1 + 2\hat{\theta}(1 - \gamma - \sqrt{1-\gamma})}{4\sqrt{1-\gamma}} \left(\frac{2 - \gamma + 2\sqrt{1-\gamma}}{\gamma} \right)^j \\
&\quad - \frac{\gamma a_1 + 2\hat{\theta}(1 - \gamma + \sqrt{1-\gamma})}{4\sqrt{1-\gamma}} \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j
\end{aligned}
\tag{11}
$$

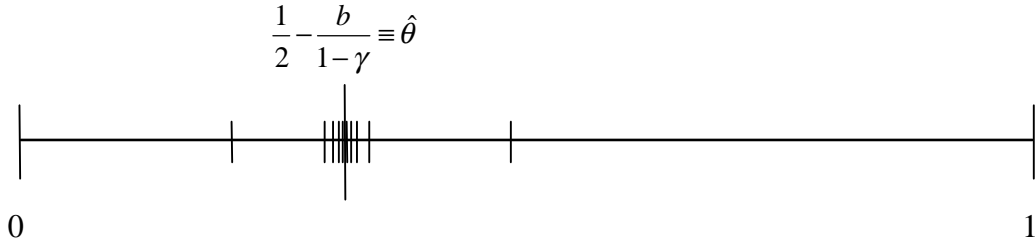$$\frac{1}{2} - \frac{b}{1-\gamma} \equiv \hat{\theta}$$

Figure 2: Most informative equilibrium for $\hat{\theta} \geq 0$

where $\hat{\theta} \equiv 1/2 - \frac{b}{1-\gamma}$. We have

$$\frac{da_j}{da_1} = \frac{\gamma}{4\sqrt{1-\gamma}} \left[\left(\frac{2 - \gamma + 2\sqrt{1-\gamma}}{\gamma} \right)^j - \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j \right] > 0, \qquad (12)$$

which implies that the equilibrium with J intervals is unique, since otherwise the boundary types given by (11) contradict $a_0 = 0$ and $a_J = 0$. By rearranging (11) and letting $J \to \infty$, we obtain

$$a_1 \to 2\hat{\theta} \left(1 - \frac{1 - \sqrt{1-\gamma}}{\gamma} \right) \equiv a_1^*.$$

Substituting a_1^* into a_1 in (11),

$$a_j = \hat{\theta} - \hat{\theta} \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j. \qquad (13)$$

Note that since

$$0 < \frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} < 1 \text{ for } 0 < \gamma < 1,$$

(13) gives a strictly increasing sequence that converges to the fully revealing type $\hat{\theta}$. This converging sequence constitutes a partition in $[0, \hat{\theta})$. Let the sequence of a_j's obtained by (13) be P_0.

It remains to obtain the partition in $(\hat{\theta}, 1]$. Let a_j' be a decreasing sequence such that $a_0' = 1$. Solving (10) with $a_0' = 1$ and $J \to \infty$, we have a strictly decreasing sequence that converges to $\hat{\theta}$

$$a_j' = \hat{\theta} + (1 - \hat{\theta}) \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j. \qquad (14)$$

This sequence constitutes a partition of $(\hat{\theta}, 1]$ with an infinite number of intervals. Let the sequence obtained by (14) be P_1. Define $P \equiv P_0 \cup P_1 \cup \hat{\theta}$. Clearly every boundary type in P satisfies (9), and therefore the partition P supports the most informative equilibrium for $\hat{\theta} \geq 0$ or equivalently $b \leq \frac{1}{2(1-\gamma)}$. Let us summarize the above observations in the following:

Proposition 3 Under anonymity/equal treatment in the uniform-quadratic setting, if the fully revealing type exists on the type space ($\hat{\theta} \geq 0$) then there exists the unique equilibrium with an infinite number of intervals. The boundary types for the intervals are given by (13) and (14) where $\gamma = 1/n$.

From Proposition 2 the equilibrium identified in Proposition 3 Pareto dominates all the other equilibria for $\hat{\theta} \geq 0$. The equilibrium partition is depicted in Figure 2. We can see that in the most informative equilibrium there are an infinite number of intervals in the neighbourhood of the fully revealing type $\hat{\theta}$. The equilibrium partition also takes into account both positive bias for types higher than $\hat{\theta}$ and negative biases for types lower than $\hat{\theta}$. Hence the length of intervals is wider as they are away from $\hat{\theta}$ and is narrower as they are closer to $\hat{\theta}$, which implies that sender types are more accurately inferred when they are closer to the fully revealing type.

Figure 2 also indicates that weak response to a message has just as important implications for the nature of information transmission as intrinsic bias b, which has been the centre of attention in the cheap talk literature. In particular, as long as b is not too large the fully revealing type exists under a wider range of parameters, and this can change the structure of informative equilibria substantially, compared with CS and many models of information transmission where the sole source of informational distortion is intrinsic bias.

Our construction of the equilibrium with an infinite partition has been heuristic, but Gordon (2006) provides a general equilibrium characterization of a large class of cheap talk games that includes the uniform-quadratic setting of our model as a special case. However, due to its generality his paper provides little guidance as to the nature of the equilibrium partition. Our interest here is to study how the nature of the characterized equilibrium changes according to the parameter values. The following proposition follows directly from CS, where there is no fully revealing type and $n = 1$. We can easily extend their result to $n \geq 2$ if there is no fully revealing type on the type space ($\hat{\theta} < 0$).

Proposition 4 (CS) Under anonymity/equal treatment in the uniform-quadratic setting, if $\hat{\theta} < 0$ then the number of intervals in any equilibrium partition is finite. Moreover the largest number of intervals supported in equilibrium is non-increasing in b.

Proof. See Lemma 1 and Lemma 6 in CS. Since $y^S(\theta) \neq y^R(\theta, \gamma)$ for all $\theta \in [0,1]$ and (12) implies that their monotonicity condition (M) is satisfied, we can directly apply the Lemmas for any $n \geq 1$. ∎

If $\hat{\theta} < 0$ any type of sender is only positively biased because $y^S(\theta) > y^R(\theta, \gamma)$ for all $\theta \in [0,1]$. This is the case if b is high, or n is low. In Figure 1 if y^S moves upwards further (for higher b) $\hat{\theta}$ disappears and y^S is consistently higher than y^R. Therefore any informative equilibrium partition must be such that the length of intervals is longer for higher θ, as in the right hand side of the partition shown in Figure 2.

3.2 Public Good Provision and Binary Choice

Before examining the interaction between two types of informational distortion caused by intrinsic bias b and weak response, let us consider the public good provision setting where $b = 0$, so that we can focus on informational distortion caused by weak response. If $b = 0$ we have $\hat{\theta} = 1/2$ and as we can see in Figure 2 for any n the most informative equilibrium features an infinite partition. How do the characteristics of the most informative equilibrium change when the number of agents n increases or decreases? Since

$$\frac{2 - \gamma - 2\sqrt{1 - \gamma}}{\gamma}$$

in (13) and (14) is increasing in γ $(= 1/n)$ for any $0 < \gamma \leq 1$ $(n \geq 2)$, (13) and (14) imply that as n increases, every boundary type except for $a_0 = 0$ and $a_0' = 1$ becomes closer to $\hat{\theta} = 1/2$. Intuitively, as the number of agents becomes larger the intervals in the most informative equilibrium are more concentrated around the fully revealing type $\hat{\theta} = 1/2$ because there are more incentive to "exaggerate" types and messages from sender types away from $1/2$ become less and less precise. In particular, as $n \to \infty$, we have $a_1, a_1' \to 1/2$: as the number of agents goes to infinity the most informative equilibrium converges to the equilibrium with two intervals. Consequently, even if the decision maker and the agents play the equilibrium with an infinite number of intervals, the probability that each agent induces either $\bar{y}(0, a_0)$ or $\bar{y}(a_1', 1)$ may be close to 1. Therefore communication may look as if each agent faces a binary choice of message although other actions are induced with small but positive probabilities.

Note that solving (10) for $a_0 = 0$ and $a_2 = 1$ we obtain the equilibrium with two intervals $\{[0, 1/2), [1/2, 1]\}$ for any n. When an agent can induce one of only two expected reactions $\bar{y}(0, 1/2)$ and $\bar{y}(1/2, 1)$ the informational distortion caused by weak response does not play a role in his incentive to reveal, since a binary choice of message completely invalidates the agents' incentive to "exaggerate" their preferences. This is intuitive because when there are only two alternatives one's choice does not reflect how "strongly" he prefers one to the other.

Proposition 2 implies that the equilibrium with two intervals can never be welfare maximizing when $b = 0$, but Figure 3 suggests that when the number of agents is larger the loss from playing this equilibrium as opposed to the most informative one (with an infinite partition) can be very small. In Figure 3 the number of agents is on the horizontal axis and an agent's expected utility is on the vertical axis. We can see that the difference between the expected utility in the equilibrium with an infinite partition and the expected utility in the equilibrium with only two intervals diminishes as n becomes larger.[6] The

---

[6] We can also do a similar calculation for EU^R and confirm that the difference between EU^R with the infinite partition and EU^R with the binary partition diminishes as n becomes larger.
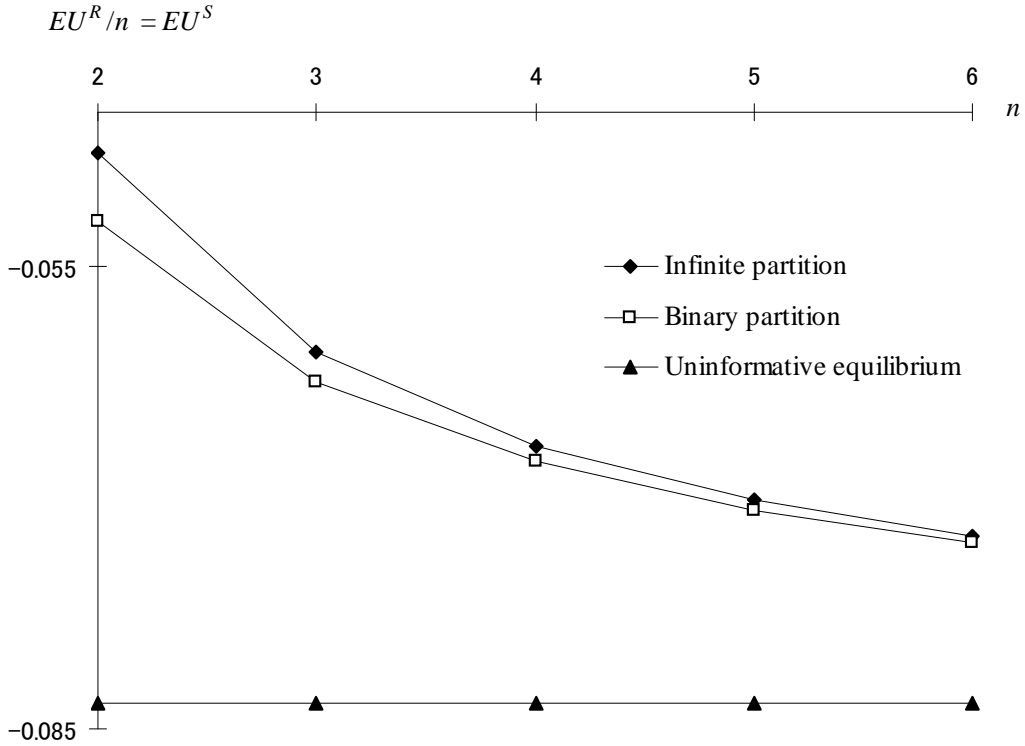
Figure 3: Receiver's expected utility per sender (= sender's expected utility) when $b = 0$.

diminishing difference implies that less is lost by playing the binary choice equilibrium, since messages in the most informative equilibrium become less precise due to the severer incentive to "exaggerate". This might explain why giving each person a "choice between the two" (such as "yes or no" or "agree or disagree") is very common when many people are involved in a decision that affects them, even if neither the preferences nor the decision made after communication is binary.[7] Such an equilibrium has a much simpler structure but approximates the most informative equilibrium when the number of agents is large.

3.3 Large Intrinsic Bias

Let us consider for what value of b an informative equilibrium can be supported. As we have noted, in an informative equilibrium with strictly positive probability the receiver takes a different action from the one based only on her prior belief. Thus the equilibrium with the partition $\{[0], (0, 1]\}$ is not informative because the probability that the type of any senders is $\theta = 0$ (and the receiver's action is chosen accordingly) is 0. The equilibrium

---

[7]The equilibrium with two intervals can be played by letting the senders report one of only two messages, such as "yes" and "no", if we appropriately specify the message space M and off-the-equilibrium beliefs. Of course the same equilibrium can also be played through many more messages if some of them are randomly sent to induce $y_S(0, 1/2)$ and the others induce $y_S(1/2, 1)$.

with the partition $\{[0, a), [a, 1]\}$ such that $a \in (0, 1)$ is informative. Suppose that at least one informative equilibrium exists for $b \in [0, \bar{b})$. From Proposition 4 the largest number of intervals $\bar{J}$ is non-increasing in b. Therefore, in order to find $\bar{b}$ it suffices to consider the condition under which the equilibrium with two non-degenerate intervals can be supported. By solving (10) for $a_0 = 0$, $a_2 = 1$, we obtain $a_1 = \frac{1}{2} - \frac{2b}{2-\gamma}$. In order for this equilibrium to be supported it must be that $a_1 > 0$, which implies

$$b < \bar{b} = \frac{2-\gamma}{4} = \frac{1}{2} - \frac{1}{4n}. \tag{15}$$

In the uniform-quadratic case of CS it has been shown that an informative equilibrium with at least two non-degenerate intervals exists for all $b \in [0, 1/4)$, which can be confirmed here by letting $n = 1$. (15) indicates that by introducing anonymity the possibility of informative communication can be extended beyond $b = 1/4$. From (15) we have that $\bar{b} \to 1/2$ as $n \to \infty$. As we will discuss shortly, this property leads to another important result of this paper. Not only may anonymity change the structure of informative equilibria, but it may also enhance information transmission when b is large.

A wider possibility of information transmission when b is large comes from the fact that anonymity makes the receiver's response to a message weaker. In order to get some intuition, consider Figure 1 again. If $n = 1$, the receiver's reaction for θ, $y^R(\theta, 1)$, is given by the 45 degree line. When the number of senders becomes larger under anonymity, $y^R(\theta, \gamma)$ rotates around $\theta = 1/2$ clockwise and its slope becomes less steep. The change in the receiver's response makes the vertical distance between the sender's desired action $y^S(\theta)$ and the receiver's reaction $y^R(\theta, \gamma)$ smaller (and disappear at $\hat{\theta}$) especially for lower types. The narrowing distance means that both parties' interests are less incongruent and this may encourage information revelation. Thus, for a large value of b, weak response to the message due to anonymity/equal treatment works as if it mitigates intrinsic bias.

3.4 When Should Communication be Anonymous?

In many situations where people communicate anonymously, they could have revealed identities if they wished to. Therefore it is natural to ask when they opt for anonymous communication. Here we show that both parties prefer to communicate anonymously (or commit to equal treatment) when intrinsic bias between the receiver and each sender is large but not too severe.

Let us focus on the most informative equilibrium for given b, and consider when anonymous communication is more desired than direct one-to-one communication. Note that without anonymity or equal treatment the information transmission between the receiver and sender i in our model is equivalent to the single sender game in CS, which is represented by the case where $n = 1$ in our model. When $n \geq 2$ and communication is anonymous Proposition 3 and (15) imply that the most informative equilibrium involves an infinite
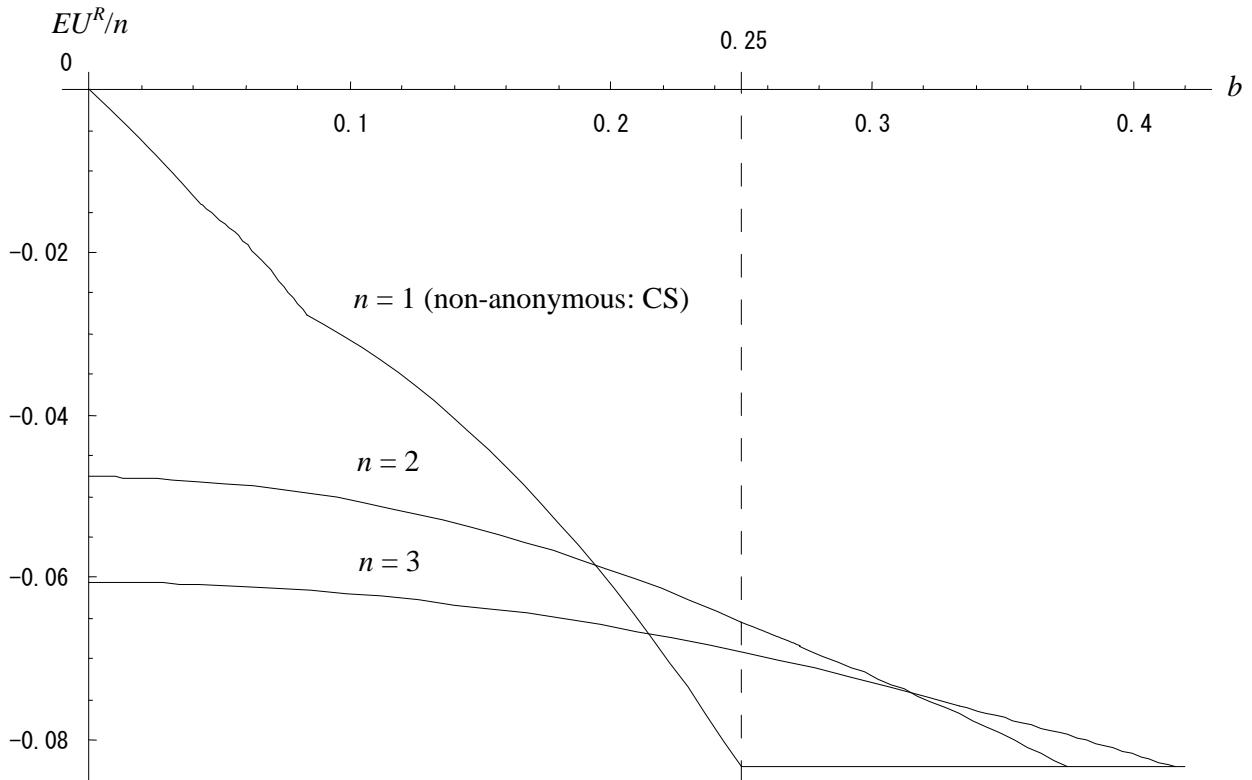
Figure 4: Receiver's normalized expected utility in the most informative equilibrium

number of intervals for $b \leq \frac{1}{4}$ and communication is informative up to $b < \frac{2-\gamma}{4} = \frac{1}{2} - \frac{1}{4n}$. Hence, if $b \in [\frac{1}{4}, \frac{1}{2} - \frac{1}{4n})$, information transmission can occur even if messages are completely uninformative without anonymity. Also, we can see that the more senders there are, the larger the intrinsic bias can be for information transmission (and welfare improvement) to occur in equilibrium.

The comparison among the most informative equilibria is shown in Figure 4. The horizontal axis represents the bias b and the vertical axis represents the per sender expected utility of the receiver (normalized by dividing by n). Let us look at the curves for the CS model ($n = 1$) and anonymous communication for $n = 2$ where the receiver's utility function is given by $U^R = -(y_1 - \theta_1)^2 - (y_2 - \theta_2)^2$. Note that the curve for CS becomes flat for $b \geq 1/4 = 0.25$, indicating that communication is completely uninformative. Both curves are identical for $b \geq 3/8 = 0.385$, where communication is uninformative also in anonymous communication. Thus when b is too large anonymity is irrelevant to information transmission and decision making. We can calculate that for $n = 2$ the receiver prefers anonymous communication if $b \gtrsim 0.194$. A similar graph can be drawn for the senders, and they also prefer anonymous communication when $b \gtrsim 0.194$. When the bias is large but not too severe both parties can benefit from anonymity. The other curve plots the normalized expected utility for $n = 3$. We can see that compared with the case where

24

$n = 2$ anonymity becomes less advantageous for b not very high but more advantageous for b very high.

These welfare characteristics of anonymity are driven by the trade-off between loss of flexibility in the receiver's action and information revelation. Imposing equal treatment itself is costly because it prevents the receiver from choosing her action optimally for each sender. This cost increases with the number of senders because the receiver's action is less likely to suit each sender's type. Indeed we can see in Figure 4 that the normalized expected utility under anonymity becomes lower as n becomes larger when b is low. However, when b is very high, weak response as a result of equal treatment induces more information revelation than in one-to-one communication and the benefit of enhanced communication may outweigh the cost of inflexibility. This is pronounced when the number of senders is large, as we have seen in (15) and also by comparing the curves for $n = 2$ and $n = 3$.

4 General Setting

In the previous section we have focused on quadratic utility functions with uniformly distributed sender types, in order to illustrate the characteristics of information transmission in anonymous communication or public good provision. In this section we show that some of the important qualitative results extend to the general case.

Under anonymity/equal treatment, from a sender's viewpoint his message induces a *distribution* of the receiver's action rather than a certain action. This makes the characterization of informative equilibria difficult except for the uniform-quadratic case, for which we can (as in the previous section) concentrate on the expected value of the receiver's action to study equilibrium strategies. On the other hand in the general setting we need to take into account the entire distribution of the receiver's action because the expected utility is now affected by higher moments. Because of this, preference over distributions of the action, which is key in equilibrium characterization, cannot be determined in as simply as above.[8]

Fortunately, however, as we will show below the equilibrium with two intervals can be characterized by the "arbitrage" condition just as in the uniform-quadratic setting. We focus on this equilibrium, and show that when the intrinsic bias is large anonymity/equal treatment may improve information transmission also in the general setting. Using this the result we also observe that in public good provision ($b = 0$), regardless of the number of senders, there exists an informative equilibrium with two non-degenerate intervals.

---

[8]The argument using the first order stochastic dominance is invalid here because the utility function is non-monotonic in the receiver's action y. The equilibrium characterization provided by Gordon (2006) is not applicable either because he assumes that the receiver's reaction to a message is deterministic. The receiver's action is random also in Blume, Board and Kawamura (2007) but in their model the distribution can have a much simpler structure for which the arbitrage condition is still valid.

4.1 Equilibrium with Two Intervals

Let us review our general setting. Sender i's utility is $U^{Si} = U(y_i, \theta_i, b)$ and that of the receiver is $U^R = \sum_{i=1}^{n} U(y_i, \theta_i, 0)$, where $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$, and $U_1 = 0$ for some $y_i \in \mathbb{R}$. In this section when we study communication under anonymity we simply assume equal treatment $y = y_1 = y_2 =, ..., = y_n$, since Proposition 1 says that in anonymous communication the receiver's best response can be represented by a scalar $y \in \mathbb{R}$.[9] Each sender's type θ_i has a differentiable i.i.d. distribution F with continuous density f on $[0, 1]$. The distribution does not have to be uniform. However, we assume that the utility functions and the type distribution are such that, if a sender and the receiver communicate directly as in CS, any equilibrium partition satisfies their monotonicity condition (M).

The receiver's best action given the messages she has received is given by

$$
\bar{y}\left(a_{1j}, a_{1j+1}; a_{2j}, a_{2j+1}; ...; a_{ij}, a_{ij+1}; ...; a_{nj}, a_{nj+1}\right)
$$
$$
= \arg\max_{y} \; \sum_{i=1}^{n} \frac{\int_{a_{ij}}^{a_{ij+1}} U(y, \theta_i, 0) f(\theta_i) d\theta_i}{\int_{a_{ij}}^{a_{ij+1}} f(\theta_i) d\theta_i} \tag{16}
$$

where $\bar{y}\left(a_{1j}, a_{1j+1}; a_{2j}, a_{2j+1}; ...; a_{ij}, a_{ij+1}; ...; a_{nj}, a_{nj+1}\right)$ is the receiver's best response given the posterior belief that sender i's type $\theta_i \in [a_{ij}, a_{ij+1})$. Let $G_i(y \mid \theta_i \in [\underline{a}, \overline{a}))$ be the distribution function of the receiver's action from sender i's viewpoint, conditional on the receiver's belief that the sender's type (whose identity is may or may not be known) is in $[\underline{a}, \overline{a})$.

Now let us characterize the informative equilibrium with two intervals. Let a be the boundary type for a particular (symmetric) sender strategy $\{[0, a), [a, 1]\}$. If $a = 0$ the lower interval is degenerate so that we denote this partition specifically by $\{[0], (0, 1]\}$. The receiver's best response (16) implies that given the partitional sender strategy $\{[0, a), [a, 1]\}$ the receiver's action conditional on all messages is a function of the number of senders whose types are in the upper (or lower) interval. Let $\tilde{y}(k \mid a)$ denote the receiver's best response given that k senders' types are in the upper interval $[a, 1]$. The probabilities that $\theta_i \in [0, a)$ and that $\theta_i \in [a, 1]$ are given by $F(a)$ and $1 - F(a)$, respectively. Hence the distribution $G_i(y \mid \theta_i \in [0, a))$ has the probability mass function

$$
g_0(y) =
$$
$$
\begin{cases}
\dfrac{(n-1)!}{(n-1-k)!k!}(F(a))^{n-1-k}(1-F(a))^k & \text{for } y = \tilde{y}(k \mid a), \; k = 0, 1, ..., n-1 \\
0 & \text{for } y \neq \tilde{y}(k \mid a)
\end{cases}
$$

---

[9] As a result, as long as we assume equal treatment whether or not the receiver knows the sender of a message is irrelevant in deriving an equilibrium, even when we are primarily concerned with anonymous communication.

where $g_0(\tilde{y}(k \mid a))$ is the probability that $y = \tilde{y}(k \mid a)$ from sender i's viewpoint, conditional on the receiver's belief $\theta_i \in [0, a)$. Note that $n - 1 \geq k \geq 0$ because the receiver's action is based on her belief that at least one sender's type is in the lower interval $[0, a)$. Similarly the probability mass function g_1 for $G_i(y \mid \theta_i \in [a, 1])$ is given by

$$g_1(y) =$$
$$\begin{cases} \dfrac{(n-1)!}{(n-1-k)!k!}(F(a))^{n-1-k}(1-F(a))^k & \text{for } y = \tilde{y}(k+1 \mid a),\ k = 0, 1, ..., n-1 \\ 0 & \text{for } y \neq \tilde{y}(k+1 \mid a). \end{cases}$$

Since the receiver's utility function is supermodular in y and θ_i, the best response given by (16) implies that, for a given a, $\tilde{y}(k \mid a)$ is strictly increasing in k. The more senders are in the upper interval $(a, 1]$, the higher the receiver's action is. Define

$$V(0, a, \theta_i, b) \equiv \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!k!}(F(a))^{n-1-k}(1-F(a))^k \times U(\tilde{y}(k \mid a), \theta_i, b) \qquad (17)$$

and

$$V(a, 1, \theta_i, b) \equiv \sum_{k=0}^{n-1} \frac{(n-1)!}{(n-1-k)!k!}(F(a))^{n-1-k}(1-F(a))^k \times U(\tilde{y}(k+1 \mid a), \theta_i, b). \qquad (18)$$

$V(0, a, \theta_i, b)$ is the expected utility of a sender whose type is θ_i when his message induces $G_i(y \mid \theta_i \in [0, a))$. Likewise $V(a, 1, \theta_i, b)$ is the same sender's expected utility when his message induces $G_i(y \mid \theta_i \in [a, 1])$. The "arbitrage" condition requires

$$V(0, a, a, b) = V(a, 1, a, b),$$

which says that the sender with the boundary type $\theta_i = a$ is indifferent between inducing the two distributions $G_i(y \mid \theta_i \in [0, a))$ and $G_i(y \mid \theta_i \in [a, 1])$.

Proposition 5 Suppose that $a^* \in [0, 1]$ satisfies $V(0, a^*, a^*, b) = V(a^*, 1, a^*, b)$. Then the partition $\{[0, a^*), [a^*, 1]\}$ supports a perfect Bayesian equilibrium.

Proof. See Appendix I. ∎

Proposition 5 establishes that the informative equilibrium with two intervals in the general setting is characterized by the "arbitrage" condition, as in the uniform-quadratic setting we have studied earlier. In what follows we prove the existence of an equilibrium with two non-degenerate intervals by showing that there exists $a^* \in (0, 1)$ such that $V(0, a^*, a^*, b) = V(a^*, 1, a^*, b)$.

4.2 Anonymous Communication/Public Good Provision

Let b' be the level of intrinsic bias such that at least one informative equilibrium exists for all $b \in [0, b')$ in communication with a single sender. A version of our Proposition 4 for the general setting with $n = 1$ guarantees that b' is well-defined.[10] For the uniform-quadratic setting we have seen that $b' = 1/4$. The following proposition says that anonymity extends the possibility of information transmission beyond b'.

Proposition 6 Suppose that $n \geq 2$. Under anonymity/equal treatment, there exists an informative equilibrium with two non-degenerate intervals for $b \in [0, b' + \epsilon(n))$ such that $\epsilon(n) > 0$. Moreover, $\epsilon(n)$ is increasing in n. At least the receiver strictly prefers this equilibrium to the uninformative one.

Proof. See Appendix I. ∎

This Proposition has important implications for both anonymous communication and communication in public good provision. First, anonymity has been shown to enhance information transmission when the intrinsic bias is large in this general setting as well as the uniform-quadratic case. In particular when $b \in [b', b' + \epsilon(n))$ the informative equilibrium does not exist in the CS model of communication with a single sender but by introducing anonymity in this multi-sender setting we can construct an informative equilibrium. The fact that ϵ is increasing in n implies that anonymity enhances communication especially when both the intrinsic bias and the number of senders are large. In the uniform-quadratic setting of the previous section, from (15) we have $\epsilon = \bar{b} - 1/4 = \frac{n-1}{4n}$. As the number of anonymous senders increases the receiver's response to a message becomes weaker. This encourages information revelation for senders with very low types. Despite their positive intrinsic bias ($b > 0$) those senders may want to lower the receiver's action by partially revealing their types, because the receiver's action may well be higher than their desired actions due to the presence of other senders whose types are also likely to be much higher than theirs.

Second, Proposition 6 says that in public good provision ($b = 0$) there exists an informative equilibrium with two intervals regardless of the number of senders n. In other words, however serious the incentive to "exaggerate" caused by the presence of other senders is, this does not completely eliminate the possibility of information transmission. That is, a binary message is coarse but "robust" to this type of informational distortion. The intuition is the same as in the uniform-quadratic setting: when one is given the choice of saying only "yes" or "no" to a proposal he cannot express how strongly he feels for or against it. Thus the incentive to "exaggerate" is irrelevant to the choice of message.

---

[10]See Lemma 6 in CS.

5 Conclusion

Are we more likely or less likely to tell the truth when we are anonymous than otherwise? How is the provision of a public good determined when the decision maker cannot commit to a mechanism and no monetary transfers are available? This paper has studied constrained communication with multiple senders. We have offered a first attempt to analyze anonymity in cheap talk communication, and shown that under anonymity the receiver puts less weight on an individual message in choosing her action, compared with situations where the receiver and a sender communicate directly. Weak response to an individual message gives rise to incentive to "exaggerate", which differs qualitatively from the intrinsic preference divergence between the receiver and each sender.

In the uniform-quadratic setting we have derived informative equilibria and also demonstrated that anonymity/equal treatment may change the structure of informative equilibria significantly. In particular, there exists no fully revealing equilibrium even in the absence of intrinsic bias, but there may exist a type of sender whose desired action coincides with that of the receiver (in expected terms) even in the presence of such bias. The most informative equilibrium may have an infinite number of intervals, although it is not fully revealing. The two types of informational distortion, one caused by the intrinsic bias and the other caused by weak response to a message, may partly offset each other when the intrinsic bias is large. We have argued that communicating parties may choose to ignore the senders' identities or they may agree on introducing some anonymization device (such as a third party who imposes anonymity) in order to enhance communication and welfare. This holds much more generally than in the simple uniform-quadratic setting.

Some of the insights we have obtained for anonymous communication can be directly applied to study communication in public good provision. As the number of agents becomes larger the decision maker's response to each individual message becomes weaker. This gives the agents incentive to "exaggerate" their preferences as in anonymous communication. When the decision maker is a utilitarian welfare maximizer the quality of communication becomes inevitably lower as there are more agents who consume the public good. We have also seen that as the number of agents becomes large the most informative equilibrium converges to the equilibrium with two intervals, which can be played by each agent choosing between only two messages. This might explain why the "choice between the two" is very widely observed when many people are involved in a decision, even when the decision or preference is not binary. This paper contributes to the literature on public good provision by offering an analysis of communication where the decision maker cannot commit to a mechanism and no transfers are available, which seems relevant to a lot of practical stations, including communication in political or regulatory relationships as well as decision making within organizations.

6 Appendix I: Proofs

6.1 Preliminaries to the Proof of Proposition 2

Before we prove the Proposition, we provide some useful lemmas and outline how we construct the main proof. Let us call a sequence $(a_0, a_1, ..., a_J)$ that satisfies the arbitrage condition (9) a "*solution*" to (9). The monotonicity condition (M) in CS requires that, for given γ and b, if we have two solutions a^+ and a^{++} with $a_0^+ = a_0^{++}$ and $a_1^+ > a_1^{++}$, then $a_j^+ > a_j^{++}$ for all $j = 2, 3, ...$ In other words, (M) says that starting from a_0, all solutions to (9) must move up or down together. As we have seen in (12), our uniform-quadratic setting with $1 \geq \gamma > 0$ satisfies (M).

In order to show that the players' expected utility is higher in an equilibrium with more intervals, CS deform the partition with J intervals to that with $J + 1$ intervals, by continuously increasing the player's expected utility throughout the deformation. We follow this method, but we need to proceed by two step deformation, rather than one, because the deformation takes place towards the opposite directions for the right-hand and left-hand sides of $\hat{\theta}$ on $(0, 1]$. Intuitively as the number of interval increases, the each boundary type on the left hand side of $\hat{\theta}$ move to the left (except for $a_0 = 0$) while each boundary type of the right hand side of $\hat{\theta}$ move to the right (except for $a_J = 1$). We need to perform a different comparative statics for each case.

Let $a(J)$ be the equilibrium partition of size J. We show that $a(J)$ can be deformed to $a(J + 1)$ by two steps, continuously increasing the players' expected utility in each step. Here we consider the case where $\hat{\theta} \in (0, 1]$. We omit the case where $\hat{\theta} \notin (0, 1]$ because the Proposition for this case can be proven similarly, by using the first deformation only.

Let the sub-partition of $a(J)$ equal or below $\hat{\theta}$ be $\underline{a}(J) \equiv (a_0(J), a_2(J), ..., a_K(J))$ where $a_0(J) = 0$. Also, suppose that $a_K(J)$ is closer to $\hat{\theta}$ than $a_{K+1}(J)$ is, in other words, $\hat{\theta} - a_K(J) < a_{K+1}(J) - \hat{\theta}$. In the following we proceed in two steps:

1. We fix $a_K(J)$ and make the sub-partition $(a_K(J), a_{K+1}(J), ..., a_J(J))$ deform continuously to $(a_K(J), a_{K+1}(J+1), a_{K+2}(J+1), ..., a_{J+1}(J+1))$, increasing the expected utility.

2. We make the sub-partition $(a_0(J), a_1(J)..., a_K(J))$ deform continuously to $(a_0(J+1), a_2(J+1), ..., a_K(J+1))$, increasing the expected utility.

- If $\hat{\theta} - a_K(J) \geq a_{K+1}(J) - \hat{\theta}$ then the first step deforms $(a_0(J), a_1(J), ..., a_K(J), a_{K+1}(J))$ to $(a_0(J+1), a_1(J+1), ..., a_{K+1}(J+1), a_{K+1}(J))$ while fixing $a_{K+1}(J)$, and the second step deforms $(a_{K+1}(J), a_{K+2}(J), ..., a_J(J))$ to $(a_{K+2}(J+1), a_{K+3}(J+1), ..., a_{J+1}(J+1))$. Except for this the same method and result as the case where $\hat{\theta} - a_K(J) < a_{K+1}(J) - \hat{\theta}$ apply.

Lemma 1 If $a(J)$ and $a(J+1)$ are two equilibrium partitions for the same values of b and γ, then $a_{j-1}(J) < a_j(J+1) < a_j(J)$.

Proof. See Lemma 3 (p.1446) in CS. The proof follows directly from (M). ∎

The first step of deformation is carried out as follows. Let $(a_K^x, a_{K+1}^x, ..., a_j^x, ..., a_{J+1}^x)$ be the sub-partition that satisfies (9) for all $j = K+1, K+2, ..., J$ with $a_K^x = a_K(J)$, $a_J^x = x$ and $a_{J+1}^x = 1$. If $x = a_{J-1}(J)$ then $a_{K+1}^x = a_K^x = a_K(J)$. If $x = a_J(J+1)$ then we have $(a_K(J), a_{K+1}(J+1), ..., a_J(J+1))$, where (9) is satisfied for all $j = K+2, K+3, ..., J$. We are going show that, if $x \in [a_{J-1}(J), a_J(J+1)]$, which is again a non-degenerate interval by Lemma 1, then the sender's expected utility is strictly increasing in x.

In the second step, let $(a_0^z, a_1^z, ..., a_j^z, ..., a_K^z)$ be the sub-partition that satisfies (9) for $j = 1, 2, ..., K-1$, with $a_0^z = 0$ and $a_K^z = z$. If $z = a_K(J)$ then $a_j^z = a_j(J)$ for all $j = 0, 1, ..., K$. If $z = a_K(J+1)$ then $a_j^z = a_j(J+1)$ for all $j = 0, 1, ..., K$.. We will show that when $z \in [a_K(J+1), a_K(J)]$, which is again a non-degenerate interval by Lemma 1, the sender's expected utility is strictly decreasing in z.

Lemma 2 Suppose that $(a_0, a_1, ..., a_j, ..., a_J)$ is a solution to (9). Then for all $j = 1, 2, ..., J-1$ if $a_j > (<)\hat\theta$ then $a_j - a_{j-1} < a_{j+1} - a_j$ $(a_j - a_{j-1} > a_{j+1} - a_j)$. If $a_j = \hat\theta$ then $a_j - a_{j-1} = a_{j+1} - a_j$.

Proof. The sequences that satisfy (9) are described by (10). Rearranging (10) we have

$$(a_{j+1} - a_j) - (a_j - a_{j-1}) = \frac{4a_j + 4b + 2\gamma - 2}{\gamma} - 4a_j. \tag{19}$$

The left hand side $(a_{j+1} - a_j) - (a_j - a_{j-1}) = 0$ if

$$
\begin{aligned}
\frac{4a_j + 4b + 2\gamma - 2}{\gamma} - 4a_j &= 0 \Rightarrow \\
4a_j(1-\gamma) &= -4b - 2\gamma + 2 \Rightarrow \\
a_j &= \frac{1}{2} - \frac{b}{1-\gamma} \equiv \hat\theta.
\end{aligned}
$$

Since the right hand side of (19) is increasing in a_j, if $a_j > \hat\theta$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) > 0$, and if $a_j < \hat\theta$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) < 0$. ∎

The above lemma says that an interval $[a_{j+1}, a_j)$ is longer (shorter) than the previous interval $[a_{j-1}, a_j)$ when $a_j > (<)\hat\theta$. The intuition is captured in Figure 2. The following Lemma is similar but cannot be implied by Lemma 2. Since by definition a_K^x and a_{K+1}^z are fixed throughout the respective deformation, (9) is not satisfied at $a_j = a_{K+1}^x$ or $a_j = a_K^z$.

Lemma 3 $a_{K+1}^x - a_K^x < a_{K+2}^x - a_{K+1}^x$ and $a_K^z - a_{K-1}^z > a_{K+1}^z - a_K^z$.

Proof. From Lemma 2 we have $a_{K+1}^x - \tilde{a}_K < a_{K+2}^x - a_{K+1}^x$ where $\tilde{a}_K$ is defined such that $\{a_{j-1} = \tilde{a}_K, a_j = a_{K+1}^x, a_{j+1} = a_{K+2}^x\}$ satisfies (10). Since $a_K(J+1) < \tilde{a}_K < a_K(J) = a_K^x$ from Lemma 1, we have $a_{K+1}^x - a_K^x < a_{K+2}^x - a_{K+1}^x$. This proves the first part of the Lemma.

Similarly we have $a_K^z - a_{K-1}^z \geq \check{a}_{K+1} - a_K^z$ where $\check{a}_{K+1}$ is defined such that $\{a_{j-1} = a_{K-1}^z, a_j = a_K^z, a_{j+1} = \check{a}_{K+1}\}$ satisfies (10). Lemma 1 implies $a_{K+1}^z = a_{K+1}(J+1) < \check{a}_{K+1} < a_{K+1}(J)$. Hence we have $a_K^z - a_{K-1}^z > a_{K+1}^z - a_K^z$. $\blacksquare$

6.2 Proof of Proposition 2

- Sender

The receiver's action from a sender's viewpoint is a random variable, and since the utility functions are quadratic, we can separate the expected value terms and the variance terms. Let $y_i(m_i)$ be the receiver's (random) action from the sender's viewpoint. The sender's utility in this separated form conditional of his report is given by

$$
\begin{aligned}
&E\left[-(y_i(m_i) - (\theta_i + b))^2 \mid m_i\right] \\
&= -var(y_i(m_i)) - (Ey_i(m_i))^2 + 2(\theta_i + b)Ey_i(m_i) - (\theta_i + b)^2 \\
&= -var(y_i) - (Ey_i(m_i) - (\theta_i + b))^2,
\end{aligned} \tag{20}
$$

where from (5)
$$
Ey_i(m_i) \equiv y_S(m_i) = \frac{1}{n}E\left[\theta_i \mid m_i\right] + \frac{n-1}{n} \times \frac{1}{2}.
$$
The variance term is independent of the sender's message since the randomness is caused by the other senders' messages unobservable to the sender. Let sender i's expected type given his message be $\hat{a}_i(a_j, a_{j+1})$. If a message is sent from $\theta_i \in [a_j, a_{j+1})$, then

$$
\hat{a}_i = \frac{a_j + a_{j+1}}{2}.
$$

From (4) the receiver's action is the mean of all posterior expected types. Hence, from sender i's viewpoint

$$
var(y_i) = var\left(\frac{1}{n}\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right)\right) = \frac{1}{n^2}var\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right) = \frac{n-1}{n^2}var(\hat{a}_i),
$$

where $var(\hat{a}_i)$ is the variance of the expected type of a sender given his equilibrium strategy. The last equality follows from independent type distributions and symmetric sender strategies. In what follows we drop the subscript i.

The expected utility for the first part of deformation is given by

$$EU^S \equiv -\sum_{j=1}^{K} \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}+a_j}{2n} + \frac{n-1}{2n} - b - \theta \right)^2 d\theta$$

$$-\sum_{j=K+1}^{J+1} \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right)^2 d\theta$$

$$-\frac{n-1}{n^2} \left[\sum_{j=1}^{K} (a_j - a_{j-1}) \left(\frac{a_{j-1}+a_j}{2} \right)^2 \right.$$

$$\left. + \sum_{j=K+1}^{J+1} (a_j^x - a_{j-1}^x) \left(\frac{a_{j-1}^x+a_j^x}{2} \right)^2 - \frac{1}{4} \right].$$

It follows that

$$\frac{dEU^S}{dx} \equiv \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \left\{ -\left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x+a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 \right.$$

$$-\frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(\frac{a_j^x+a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right]$$

$$\left. -\left[\frac{n-1}{2n^2}(a_{j+1}^x)^2 - (a_{j-1}^x)^2 + \frac{(a_{j-1}^x+a_j^x)^2 - (a_j^x+a_{j+1}^x)^2}{2} \right] \right\}.$$

For the first line we have[11]

$$-\left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x+a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2$$

$$= \frac{n-1}{2n^2}(a_{j+1}^x - a_{j-1}^x)(1 - 2a_j^x) - \frac{b(a_{j+1}^x - a_{j-1}^x)}{n} + \frac{(a_{j+1}^x - a_{j-1}^x)(a_{j-1}^x - 2a_j^x + a_{j+1}^x)}{4n^2}.$$

Also for the second line,

$$-\frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_j^x+a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right]$$

$$= \frac{n-1}{2n^2} \left[(a_{j+1}^x)^2 - (a_{j-1}^x)^2 - (a_{j+1}^x - a_{j-1}^x) \right] + \frac{b(a_{j+1}^x - a_{j-1}^x)}{n}.$$

---

[11]For $j = K+2, K+3, ..., J-1$ we can use the fact that a_j^x satisfies (9) or

$$-\left(\frac{a_{j-1}^x+a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x+a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 = 0$$

to simplify the calculation, alhtough later exposition will become more complex because this does not apply to $j = K$.

Hence, all terms in the curly brackets can be written

$$-\left(\frac{a^x_{j-1}+a^x_j}{2n}+\frac{n-1}{2n}-b-a^x_j\right)^2+\left(\frac{a^x_j+a^x_{j+1}}{2n}+\frac{n-1}{2n}-b-a^x_j\right)^2$$

$$-\frac{1}{n}\left[\int_{a^x_{j-1}}^{a^x_j}\left(\frac{a^x_{j-1}+a^x_j}{2n}+\frac{n-1}{2n}-b-\theta\right)d\theta+\int_{a^x_j}^{a^x_{j+1}}\left(\frac{a^x_j+a^x_{j+1}}{2n}+\frac{n-1}{2n}-b-\theta\right)d\theta\right]$$

$$-\frac{n-1}{2n^2}\left[(a^x_{j+1})^2-(a^x_{j-1})^2+\frac{(a^x_{j-1}+a^x_j)^2-(a^x_j+a^x_{j+1})^2}{2}\right]$$

$$=\frac{a^x_{j+1}-a^x_{j-1}}{2n}\left[\frac{a^x_{j-1}-2a^x_j+a^x_{j+1}}{2}\right]>0.$$

The inequality follows because from Lemmas 2 and 3, we have $a_j-a_{j-1}<a_{j+1}-a_j\Rightarrow$ $a^x_{j-1}-2a^x_j+a^x_{j+1}>0$ for all $j=K+1,K+2,...,J$. We have $\frac{da^x_j}{dx}>0$ by (M). It follows that

$$\frac{dEU^S}{dx}\equiv\sum_{j=K+1}^{J+1}\frac{da^x_j}{dx}\left\{\frac{a^x_{j+1}-a^x_{j-1}}{2n}\left[\frac{a^x_{j-1}-2a^x_j+a^x_{j+1}}{2}\right]\right\}>0.$$

We have the second part of deformation as follows:

$$\frac{dEU^S}{dz}\equiv\sum_{j=1}^{K}\frac{da^z_j}{dz}\left\{-\left(\frac{a^z_{j-1}+a^z_j}{2n}+\frac{n-1}{2n}-b-a^z_j\right)^2+\left(\frac{a^z_j+a^z_{j+1}}{2n}+\frac{n-1}{2n}-b-a^z_j\right)^2\right.$$

$$-\frac{1}{n}\left[\int_{a^z_{j-1}}^{a^z_j}\left(\frac{a^z_{j-1}+a^z_j}{2n}+\frac{n-1}{2n}-b-\theta\right)d\theta+\int_{a^z_j}^{a^z_{j+1}}\left(\frac{a^z_j+a^z_{j+1}}{2n}+\frac{n-1}{2n}-b-\theta\right)d\theta\right]$$

$$\left.-\frac{n-1}{2n^2}\left[(a^z_{j+1})^2-(a^z_{j-1})^2-\frac{(a^z_{j-1}+a^z_j)^2-(a^z_j+a^z_{j+1})^2}{2}\right]\right\}$$

$$=\sum_{j=1}^{K}\frac{da^z_j}{dz}\left\{\frac{a^z_{j+1}-a^z_{j-1}}{2n}\left[\frac{a^z_{j-1}-2a^z_j+a^z_{j+1}}{2}\right]\right\}<0.$$

The inequality follows because $\frac{da^z_j}{dz}>0$ by (M), and from $a_0,a_1,...,a_K\leq\hat{\theta}$ and Lemmas 2 and 3 we have $a_j-a_{j-1}>a_{j+1}-a_j\Rightarrow a^z_{j-1}-2a^z_j+a^z_{j+1}<0$ for all $j=1,2,...,K$.

Since we have completed the deformation from $a(J)$ to $a(J+1)$ by two steps while increasing the expected utility, we conclude that the sender's expected utility is higher in an equilibrium with more intervals.

- Receiver

Since the receiver's utility is the sum of the senders' utilities without bias ($b=0$), we can apply the above result for a sender's expected utility directly to show that the receiver's expected utility is higher with an equilibrium with more intervals. Q.E.D.

6.3 Proof of Proposition 5

Recall that a^* satisfies $V(0, a^*, a^*, b) - V(a^*, 1, a^*, b) = 0$. Define

$$A(\theta_i) \equiv V(0, a^*, \theta_i, b) - V(a^*, 1, \theta_i, b).$$

$A(\theta_i)$ is the difference between the expected utilities of the sender with θ_i when his message induces $G_i(y \mid \theta_i \in [0, a^*))$ and $G_i(y \mid \theta_i \in [a^*, 1])$, respectively. Note that $\tilde{y}(k+1 \mid a^*) > \tilde{y}(k \mid a^*)$ for any k. Since $U_{12} > 0$, from (17) and (18)

$$\frac{\partial}{\partial \theta_i} V(0, a^*, \theta_i, b) < \frac{\partial}{\partial \theta_i} V(a^*, 1, \theta_i, b).$$

Hence in terms of $A(\theta_i)$ we obtain

$$\frac{dA(\theta_i)}{d\theta_i} < 0. \tag{21}$$

By definition $A(a^*) = 0$. Thus (21) implies that the sender with $\theta_i < a^*$ strictly prefers to induce $G_i(y \mid \theta_i \in [0, a^*))$, while the sender with $\theta_i > a^*$ strictly prefers to induce $G_i(y \mid \theta_i \in [a^*, 1])$. The receiver's best response is implied in these distributions. Therefore the partition $\{[0, a^*), [a^*, 1]\}$ supports a perfect Bayesian equilibrium. Q.E.D.

6.4 Proof of Proposition 6

Let us first define b' formally. Let $\bar{y}(\underline{a}, \bar{a})$ be the receiver's best response when her belief is such that $\theta \in [\underline{a}, \bar{a}]$. CS have shown that an informative equilibrium cannot exist for $b \geq b'$ in communication with a single sender ($n = 1$ in our model), where the critical value b' is given by the following condition

$$U(\bar{y}(0, 1), 0, b') = U(\bar{y}(0, 0), 0, b'). \tag{22}$$

This condition says that the sender with the lowest type $\theta = 0$ is indifferent between being fully revealed and being completely disguised. From the assumptions on the utility functions it is easy to see that $\bar{y}(0, 1) > \bar{y}(0, 0)$ and thus $U(\bar{y}(0, 1), 0, b) > U(\bar{y}(0, 0), 0, b)$ for all $b > b'$.

For notational convenience let us define

$$B(a, b) \equiv V(0, a, a, b) - V(a, 1, a, b).$$

$B(a, b)$ is the difference between the expected utilities of the sender on the cut-off point a ($\theta_i = a$) when his message induces $G_i(y \mid \theta_i \in [0, a))$ and $G_i(y \mid \theta_i \in [a, 1])$, respectively. The "arbitrage" condition implies $B(a, b) = 0$.

The rest of the proof proceeds as follows. We first show that $B(0, b) > 0$ and then $B(1, b) < 0$, for all $b \in [0, b']$. Since $B(a, b)$ is shown to be continuous on $[0, 1]$, by

the intermediate value theorem there exists $a^* \in (0,1)$ such that $B(a^*, b) = 0$. Also by continuity an informative equilibrium exists for all $b \in [0, b' + \epsilon)$ at least with some $\epsilon > 0$. That ϵ is increasing in n is proven similarly.

Suppose that $a = 0$ and $\theta_i = a = 0$. Recall that $\tilde{y}(k \mid a = 0)$ is the receiver's best response when she believes that k senders' types are in the upper interval $(0, 1]$. Since the senders' types are drawn from a continuous density function, almost surely all the other senders types are in $(0, 1]$. That is, if he induces $G_i(y \mid \theta_i \in [0, 1))$ the receiver's action is, almost surely, $\tilde{y}(n - 1 \mid a = 0)$, and if he induces $G_i(y \mid \theta_i = 0)$ the receiver's action is, almost surely, $\tilde{y}(n \mid a = 0)$. Hence

$$V(0, a, \theta_i, b) = V(0, 0, 0, b) = U(\tilde{y}(n - 1 \mid a = 0), 0, b)$$

and

$$V(a, 1, \theta_i, b) = V(0, 1, 0, b) = U(\tilde{y}(n \mid a = 0), 0, b).$$

From (16) we have $\tilde{y}(n \mid a = 0) = \bar{y}(0, 1)$, which says that the action in the uninformative equilibrium is the same for both CS and anonymous communication. (16) also implies $\tilde{y}(0 \mid a = 0) = \bar{y}(0, 0)$: the action when $\theta_i = 0$ for all $i = 1, 2, ..., n$ and the action in CS where $\theta = 0$ are the same too. Since $\tilde{y}(k \mid a)$ is increasing in k,

$$\bar{y}(0, 0) = \tilde{y}(0 \mid a = 0) < \tilde{y}(n - 1 \mid a = 0) < \tilde{y}(n \mid a = 0) = \bar{y}(0, 1). \tag{23}$$

Suppose that $b = b'$. By definition the sender is indifferent between $\bar{y}(0, 0) = \tilde{y}(0 \mid a = 0)$ and $\bar{y}(0, 1) = \tilde{y}(n \mid a = 0)$. By strict concavity of the utility function, the sender is strictly better off with $\tilde{y}(n - 1 \mid a = 0)$ than $\tilde{y}(n \mid a = 0)$. Hence

$$V(0, 0, 0, b') = U(\tilde{y}(n - 1 \mid a = 0), 0, b') > U(\tilde{y}(n \mid a = 0), 0, b') = V(0, 1, 0, b').$$

The assumption $U_{13} > 0$ implies $U(\tilde{y}(n - 1 \mid a = 0), 0, b) > U(\tilde{y}(n \mid a = 0), 0, b)$ for $b \in [0, b']$. Therefore, $B(0, b) = V(0, 0, 0, b) - V(0, 1, 0, b) > 0$ for all $b \in [0, b']$.

Suppose on the contrary that $\theta_i = a = 1$. The receiver's action is, almost surely, $\tilde{y}(0 \mid a = 1)$ if he induces $G_i(y \mid \theta_i \in [0, 1))$ and $\tilde{y}(1 \mid a = 1)$ if he induces $G_i(y \mid \theta_i = 1)$. Suppose that $b = 0$. The sender strictly prefers $\tilde{y}(1 \mid a = 1)$ to $\tilde{y}(0 \mid a = 1)$ since from (16) his desired action is $\tilde{y}(n \mid a = 1)$ and this implies that the sender's utility is strictly increasing for all $y < \tilde{y}(n \mid a = 1)$. Equivalently, we have

$$V(0, 1, 1, 0) = U(\tilde{y}(0 \mid a = 1), 1, 0) < U(\tilde{y}(1 \mid a = 1), 1, 0) = V(1, 1, 1, 0).$$

This and $U_{13} > 0$ imply $U(\tilde{y}(0 \mid a = 1), 1, b) < U(\tilde{y}(1 \mid a = 1), 1, b)$, and therefore $B(1, b) = V(0, 1, 1, b) - V(1, 1, 1, b) < 0$ for all $b \geq 0$.

Since U and f are assumed to be differentiable (16) implies that $\tilde{y}(a, k)$ and consequently $B(a, b)$ are continuous on $a \in [0, 1]$. Therefore, by the intermediate value theorem there

exists $a^* \in (0, 1)$ such that $B(a^*, b) = 0$. From Proposition 5 $\{[0, a^*), [a^*, 1]\}$ supports a perfect Bayesian equilibrium. Moreover, by continuity there exists $\epsilon' > 0$ and $B(0, b' + \epsilon') = 0$. Therefore we have established that an informative equilibrium with two non-degenerate intervals exists for all $b \in [0, b' + \epsilon')$.

Suppose $n' > n$ and define $b'' \equiv b' + \epsilon'$. As above we have $B(0, b'') = 0$ when there are n senders, which implies a sender with $\theta_i = 0$ is indifferent between $\tilde{y}(n - 1 \mid a = 0)$ and $\tilde{y}(n \mid a = 0)$. However, when there are n' senders, inducing $G_i(y \mid \theta_i = 0)$ leads to $\tilde{y}(n' - 1 \mid a = 0)$ and we have $\tilde{y}(n' - 1 \mid a = 0) > \tilde{y}(n - 1 \mid a = 0)$. On the other hand inducing $G_i(y \mid \theta_i \in [0, 1))$ leads to $\tilde{y}(n' \mid a = 0) = \tilde{y}(n \mid a = 0)$. Hence we have

$$\tilde{y}(n - 1 \mid a = 0) < \tilde{y}(n' - 1 \mid a = 0) < \tilde{y}(n' \mid a = 0) = \tilde{y}(n \mid a = 0) = \bar{y}(0, 1).$$

By strict concavity the sender strictly prefers $\tilde{y}(n' - 1 \mid a = 0)$ to $\tilde{y}(n' \mid a = 0)$, so that he induces $G_i(y \mid \theta_i = 0)$ and we have $B(0, b'') > 0$. It is easy to observe formally that when there are n' senders we have $B(0, b'') > 0$ and $B(1, b'') < 0$ using the same method as above, and the rest of the argument follows immediately. Hence when there are n' senders there exists an informative equilibrium for $b \in [0, b' + \epsilon'')$, and $\epsilon'' > \epsilon'$.

The receiver strictly prefers the equilibrium with two intervals to the uninformative equilibrium, since the receiver's expected utility maximization (16) guarantees that conditional on any combination of messages in the informative equilibrium her expected utility is higher. Q.E.D.

7   Appendix II: Formal Relation to Noisy Communication

As we have discussed briefly in the main text the receiver's response to a message may become weaker when some noise is introduced in communication too. In particular, a class of equilibria identified by Blume, Board and Kawamura (2007) in their uniform-quadratic setting has a similar structure to the one we have seen for anonymous communication and public good provision, although the similarity applies only to the class described below.

Consider communication between a single sender and receiver, whose utility functions are $-(y - \theta - b)^2$ and $-(y - \theta)^2$ respectively. As above θ is uniformly distributed on $[0, 1]$. Blume, Board and Kawamura (2007) introduce noise in communication as follows: the sender observes the value of θ, and then sends a message $m \in [0, 1]$; with probability $1 - q$ the receiver observes the message m sent by the sender; otherwise with probability q the receiver observes a message m_0 that is uniformly distributed on $[0, 1]$. Finally, the receiver chooses action $y \in \mathbb{R}$.

Suppose that the sender's strategy is such that if $\theta \in [a_{j-1}, a_j)$ he randomizes his message uniformly over $[a_{j-1}, a_j)$. Let m_R be the message the receiver observes and $m_R \in$

$\{m_0, m\}$. Note that with this sender strategy, upon receiving a message the receiver cannot update her belief on whether the received message is noise ($m_R = m_0$) or comes from the sender himself ($m_R = m$), since a message is drawn from the same uniform distribution in both cases. From the receiver's viewpoint the received message is informative about the sender's type only with probability $1 - q$. Therefore the receiver's best response is given by

$$y_R(m_R) = (1 - q)E\left[\theta \mid m_R = m\right] + q\frac{1}{2},$$

where $E\left[\theta \mid m_R = m\right]$ is the conditional expectation of θ given that the received message is from the sender himself. From the sender's viewpoint we have

$$y_S(m) = (1 - q)E\left[\theta \mid m\right] + q\frac{1}{2}. \tag{24}$$

Clearly the receiver's reaction from the sender's viewpoint has a common feature to that of anonymous communication or equal treatment we have seen in (5). This implies that, using (10), the structure of equilibria with this type of sender strategy in noisy communication can be studied just in the same way as we did for anonymity/equal treatment in the main text. In particular, the noise can be introduced to enhance information transmission when the intrinsic bias b is large.

However, in Blume, Board and Kawamura (2007) the simple form of the receiver's best response (10) applies only to the class of equilibria described above. There are many other types of equilibria in their model and they obtain their strongest welfare result with a (less straightforward) sender strategy where the receiver can update her belief on whether $m_R = m_0$ or $m_R = m$.

References

[1] Alonso, Ricardo, Dessein, Wouter, and Matouschek, Niko (2006) "When Does Coordination Require Centralization?" working paper. Northwestern University.

[2] Austen-Smith, David (1993) "Interested Experts and Policy Advice: Multiple Referrals under Open Rule." *Games and Economic Behavior* 5: 3-43.

[3] Bagnoli, Mark, and Lipman, Barton L. (1989) "Provision of Public Goods: Fully Implementing the Core through Private Contributions." *Review of Economic Studies* 56: 583-601.

[4] Baliga, Sandeep, Corchon, Luis C., and Sjöström, Thomas (1997) "The Theory of Implementation When the Planner Is a Player." *Journal of Economic Theory* 77: 15-33.

[5] Battaglini, Marco (2002) "Multiple Referrals and Multidimensional Cheap Talk." *Econometrica* 70: 1379-1401.

[6] ——— (2004) "Policy Advice with Imperfectly Informed Experts. *Advances in Theoretical Economics* 4: No.1 Article 1.

[7] Bester, Helmut, and Strausz, Roland (2000) "Imperfect commitment and the revelation principle: the multi-agent case." *Economics Letters* 69: 165-171.

[8] ———, ——— (2001). "Imperfect commitment and the revelation principle: the single agent case." *Econometrica* 69: 1077-1098.

[9] Blume, Andreas, Board, Oliver, and Kawamura, Kohei (2007) "Noisy Talk." in preparation.

[10] Crawford, Vincent P. and Sobel, Joel (1982) "Strategic Information Transmission." *Econometrica* 50: 1431-1451.

[11] Forges, Francoise (1986) "An Approach to Communication Equilibria." *Econometrica* 54: 1375-1385.

[12] Gordon, Sidartha (2006) "Informative Cheap Talk Equilibria as Fixed Points." working paper. Université de Montréal

[13] Krishna, Vijay and Morgan, John (2001) "A Model of Expertise." *Quarterly Journal of Economics* 116: 747-775.

[14] ———, ——— (2004) "The Art of Conversation: Eliciting Information from Experts through Multi-stage Communication." *Journal of Economic Theory* 117: 147-179.

[15] Ledyard, John O. (1995) "Public Goods: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, J. Kagel and A. Roth (eds), Princeton.

[16] Melumad, Nahum D. and Shibano, Toshiyuki (1991) "Communication in Settings with No Transfers." *Rand Journal of Economics* 22: 173-198.

[17] Mitusch, Kay and Strausz, Roland (2005) "Mediation in Situations of Conflict and Limited Commitment." *Journal of Law, Economics, and Organization* 21: 467-500.

[18] Morris, Stephen (2001) "Political Correctness." *Journal of Political Economy* 109: 231-265.

[19] Myerson, Roger B. (1986) "Multistage Games with Communication." *Econometrica* 54: 323-358.

[20] Ottaviani, Marco, and Sørensen, Peter Norman (2006a) "Reputational Cheap Talk." *RAND Journal of Economics* 37: 155-175.

[21] ———, ——— (2006b) "The Strategy of Professional Forecasting." *Journal of Financial Economics* 81: 441-466.

[22] Palfrey, Thomas R. and Rosenthal, Howard (1984) "Participation and the Provision of Discrete Public Goods: A Strategic Analysis." *Journal of Public Economics* 24: 7-53.

[23] Park, In-Uck (2005) "Cheap Talk Referrals of Differentiated Experts in Repeated Relationships." *RAND Journal of Economics* 36: 391-411.

[24] Sobel, Joel (1986) "A Theory of Credibility." *Review of Economic Studies* 52: 557-573.

[25] Wolinsky, Asher (2002) "Eliciting Information from Multiple Experts." *Games and Economic Behavior* 41: 141-160.