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A Model of Participatory Democracy: Understanding the Case of Porto Alegre.*

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Abstract

Participatory Democracy is a process of collective decision making that combines elements from both Direct and Representative Democracy: Citizens have the ultimate power to decide on policy and politicians assume the role of policy implementation. The aim of this paper is to understand how Participatory Democracy operates, and to study its implications over the behavior of citizens and politicians and over the final policy outcomes. To this end, we explore a formal model inspired in the experience of Participatory Budgeting implemented in the Brazilian city of Porto Alegre and that builds on the model of meetings with costly participation by Osborne, Rosenthal, and Turner (2000).

JEL codes: D7, H0, R5.

Keywords: Participatory Democracy, Porto Alegre, assembly, legislator.

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"We are not selling the illusion of the direct democracy in the Greek plaza which, let us bear in mind, was not the democracy of the all but the democracy of the best."

Olivio Dutra, first Workers Party mayor of Porto Alegre.

1 Introduction

Participatory Democracy is a process of collective decision making that combines elements from both Direct and Representative Democracy: Citizens have the ultimate power to decide on policy and politicians assume the role of policy implementation. Since politicians are deprived of the right to alter citizen's proposals, the electorate can easily monitor their performance, and that reduces their discretion over the final outcome. In this system, the extent to which citizens can affect policy and determine their social priorities is directly aligned with the degree to which they choose to involve themselves in the process.[1]

The aim of this paper is to understand how Participatory Democracy operates, and to study its influence over the behavior of citizens and politicians and over the final policy outcomes. To this end, we explore a formal model of Participatory Democracy inspired in the experience of Participatory Budgeting implemented in the Brazilian city of Porto Alegre.

During the last decade, Participatory Budgeting has been implemented in more than one hundred Brazilian cities, including some state capitals such as Porto Alegre (with a population of 1.4 million inhabitants), Belo Horizonte (2 million) and Recife (1.5 million). There are two elements that explain the introduction of this system in Brazil during this period: 1) the signing of the new Brazilian Constitution in 1988 that modified the balance of political power in the country and signalled a political and administrative decentralization, and 2) the increasing number of active community associations that were politically involved[2]. In this context, proposals for popular participation became more attractive challengers to the established clientelism in the municipal institutions.

Participatory Budgeting in all these cities can be described as an annual

[1] Although inspired by earlier figures such Rousseau or John Stuart Mill, the first theoretical formulations of Participatory Democracy were made during the 70s by Pateman (1970) and MacPherson (1977). An excellent discussion of the main features of this model of democracy can be found in Held (1987). Surprisingly enough, it is extremely difficult to find in any of these works a clear definition of Participatory Democracy.

[2] Today about six hundred community associations are established and active in Porto Alegre.

cyclical process that consists of three different stages: a deliberation stage, a negotiation stage, and a monitoring stage. In the deliberation stage citizens may participate in assemblies to decide on the investment priories of their neighborhood and to vote for the representatives who will present and defend the assemblies' decisions in front of the city government. In the negotiation stage, the city government and the representatives from all neighborhoods determine the city investment plan. After the Municipal Budget has been approved the representatives of each neighborhood monitor the execution of the investment plan.

The experience of Participatory Democracy is not limited to Brazilian cities only. A very similar local government system is used in West Bengal and Kerala, India, where development plans and budgets are proposed in meetings called Gram Sabhas (2.5 million people attended these meetings in 1997) and implemented by elected representatives. In Chicago, a participatory system operates at the school level: The Local School Councils, formed by parents, teachers and community members, are in charge of developing annual School Improvement Plans concerning budget, infrastructure and staff issues, and are required by law to select principals and monitor their performance.[3]

1.1 Participatory Budgeting in Porto Alegre

Porto Alegre's system of Participatory Budgeting (*Orçamento Participativo),* referred to as OP, is the best known and most successful experiment of local management based on Participatory Democracy. It was introduced in 1989 when the Workers Party (*Partido dos Trabalhadores,* PT henceforth) won the local elections[4].

OP is a pyramidal system whose main elements are: the regional and thematic assemblies, the Fora of Delegates, and the Council of the OP (COP).

Regional assemblies, called *rodadas,* take place in each of the sixteen regions of the city between April and May. These assemblies are the principal forum for popular participation; they are totally open and any citizen may attend. In these meetings, each region evaluates the executive's performance, defines its priorities and demands, and elects delegates for the Forum of Delegates and councillors for the COP. Prior to the *rodada,* preparatory meetings organized by the community take place.[5]

[3] For a more detailed description of these cases, see Fung and Wright (2001) and the references therein.

[4] The description of Participatory Budgeting in Porto Alegre builds on Santos (1998) and Marquetti (2000) and (2003).

[5] Until 2002 there were two rounds of *rodadas*; the second round was supressed because

Public scrutiny and control of the municipal government is the main issue at the early meetings. The municipality accounts for the implementation of the previous year Investment Plan. In the ensuing meetings, discussions focus on setting a consensual rank of priorities for each region and a list of hierarchical demands inside each priority. Each region selects as priorities five out of the thirteen issue areas available[6]. All decisions are taken by majority rule. The choices of each region are ranked according to three criteria: lack of the service or infrastructure, population, and regional and city ranking of the priority.

Thematic assemblies (*tematicas*) take place after along with the *rodada* and cover six areas (Health and Social Welfare, Transportation and Circulation, City Organization and Urban Development, Culture and Leisure, Education and Economic Development and Taxation). Participation depends on the interest that citizens might have in the area. Decisions are also taken by majority rule.

The Fora of Delegates is formed by about one thousand delegates. They are elected during the popular meetings according to criteria based on the number of participants. Their role is to serve as intermediaries between the COP (see below) and the citizens. They supervise the implementation of the budget and inform the population. Delegates are typically leaders of community organizations, so citizens not integrated in these structures are hardly elected.

Finally, the COP is a body composed by 44 counselors: two counselors for each region assembly (32), two for each thematic assembly (10), one representative of the Residents Association Union of Porto Alegre, and one from the City Hall's Attendants Labor Union. It is constituted in July of each year and its role is to design and submit to the city government a detailed budget proposal based on the priorities decided in the regional assemblies, and to monitor the execution of the approved public works.

The OP is an example of Participatory Democracy because it reconciles two democratic models: Direct democracy embedded in associations and meetings and Representative democracy at the urban level. This duality of power is the key characteristic of participatory institutions. In Porto Alegre, assemblies coexist with two elected bodies who hold the formal municipal power: The Mayoralty or executive body (*Prefeitura)* and the Chamber of Deputies or legislative body (*Câmara de Vereadores).* The COP submits

it was increasingly seen as redundant.

[6] The issue areas are: basic sanitation, land-property and human settlement regulation, transportation and circulation, social assistance, education, health services, street paving (including water and sewage disposal systems), city organization, leisure and sports, parks, culture, and economic development.

the budget proposal to the Chamber of Deputies who has total autonomy to amend or defeat it. Interestingly, the PT has a majority in the Mayoralty but it does not control de Chamber of Deputies. However, since the proposal has been approved by citizens, assemblies, and community organizations, the political cost of turning it down is very high, and the Chamber has never done it.

The relationship between the "formal" elected representatives and popular movements has not been without problems. In fact, the conflict between them has been one of the main political issues in Porto Alegre. On the one hand, the OP has been criticized because the PT community leaders seem to have helped the party to "capture" the process by making regions' political agenda fit into the PT's one. On the other hand, the executive has been accused of abusing of its privileged position when resorting to "technical reasons" in order to challenge the budget proposal; councillors and delegates have reported that they have been denied relevant information by the city technical staff in some occasions. This problem was serious enough to prompt the COP to start training seminars.

Nevertheless, the city has witnessed a remarkable improvement regarding the behavior of the politicians and community leaders who, as in the rest of Brazil, were used to clientelism. Now, the city councilors and potential candidates face a more informed population and more politicized grass-root organizations, so there is little space for the "gift exchange" that characterizes clientelism and corruption. The high degree of accountability of the administration has reduced corruption and rent seeking behavior.[7]

Another remarkable success of the OP has been the massive engagement of citizens in the process. The number of participants in the meetings has continuously increased even though in relative terms no more than the 5% of citizens are directly involved[8] (see Figure 1a). However, one can argue that the quality of representation of the delegates is higher than that of the deputies since the former are closely linked to citizens. Moreover, the increase of popular support for the PT (it won in 1989 with 34% of the votes and in 1996 with 56%) seems to guarantee the legitimacy of the process.

But the most important fact is that participation in the OP is massive for those segments of population typically disengaged from the institu-

[7]It is also worth noticing the massive redistributive effects since the implementation of the OP: In 1989 only 49% of the population was covered by basic sanitation; in the 1996, 85% of the population was covered. During the same period the number of students enrolled in elementary and secondary schools increased by 240%.

[8]The City Hall calculated that if the unofficial preparatory and intermediate meetings were considered, the number of citizens involved in the OP process in 1999 would be 100.000, representing the 8% of the voting age population.

tions of Representative Democracy. This can be seen in Figure 1b showing the income profile of both participants and inhabitants of Porto Alegre. A large majority of participants in the OP structures has a household income below the average. Since a typical middle-class family in Porto Alegre had, in 1996, an income of ten minimum wages, it becomes clear that the less wealthy citizens are overrepresented in the OP. Results regarding the education level of the participants follow the same pattern: 56.5% of the participants have completed less than 8 years of schooling. Finally, middle-class segments participate more in the OP at higher levels, so the composition of the COP is closer to a random sample of the population.

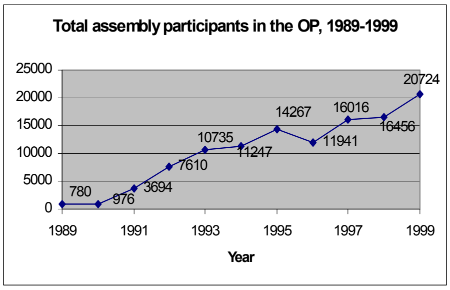

Figure 1a

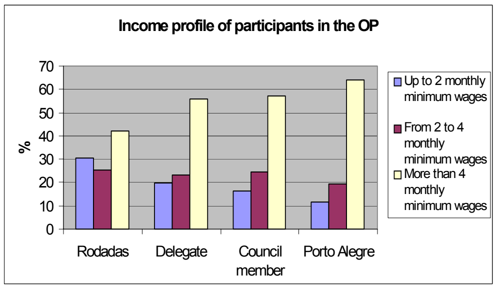

Figure 1b

It can be argued that this pattern of participation in the OP institutions is due to the time-consuming nature of meetings and assemblies: meetings last about two hours, sometimes longer. In fact, housewives, unemployed, and retired people represented in 1998 approximately 40% of participants in the OP institutions. But this is not the unique explanation: the most predominant occupations of participants in the meetings are unskilled service and teachers.

1.2 Overview of the model and results

In this paper, we analyze the potential of Participatory Democracy as a model of collective decision-making. We build a model inspired in the experience of the city of Porto Alegre where the system of direct democracy coexists with a system of parties and local elections: citizens have to make a budget proposal but they also have to elect the city executive and legislative bodies.

Our goal is to explore the effect of combining elements of representative democracy with processes of direct democracy over citizens' participation, in terms of number and characteristics of the participants, and over the final policy outcome of the game played between citizens and representatives.

We construct a formal model that builds on the model of pure direct democracy by Osborne, Rosenthal, and Turner (2000)[9]. In Osborne's model, the members of a society decide independently whether to attend, at a cost, or not to attend a meeting. The policy decision taken in this meeting is a compromise among the attendees' ideal positions. Attendance is based on a cost-benefit calculation where agents compare the cost of participation with the impact that their presence will have on the compromise.

We extend Osborne's model by considering the existence of a representative or *legislator* who is in charge of implementing the policy proposed by the assembly. We assume that the legislator has her own preferences over policies and she also cares about reelection. The tension between the legislator and the citizens introduces the possibility of distortion.

Notice that the roles played here by citizens and representatives differ from the roles they play in a standard model of representative democracy. In our model of Participatory Democracy citizens are the first to move by making a policy decision, and representatives have to react to it, deciding whether to implement it or not. In a standard model of representative democracy the policy decision is made by the elected representatives and the electorate reacts to it, approving or disapproving the policy choice with their vote in future elections[10].

We represent the system of Participatory Democracy as a game in three stages. In the first stage, each citizen decides whether to attend or not to attend a meeting in which a policy proposal will be decided. In the second stage, citizens that attend the meeting come out with a policy representing their interests and their delegates make a proposal to the legislator aiming to induce the legislator to implement the assembly's choice. In the third stage, the legislator decides the policy to be implemented. In order to find the Subgame Perfect Equilibrium strategies of this game, we analyze the optimal choices of the players by backward induction. First, we analyze the optimal reaction of the legislator in terms of policy choices, to a given proposal made by the delegates. Then, we analyze the optimal proposal of the assembly's delegates, for a given distribution of preferences of the attendees, and taking into account the optimal reaction of the legislator. Finally, we analyze the optimal decision of each citizen regarding whether to attend the meeting, given an optimal play of all agents in the continuation of the game.

[9] In the rest of the paper the model by Orborne, Rosenthal, and Turner (2000) is referred to as Osborne's model.

[10] In order to emphasize this difference we can draw an analogy to Stackelberg's model of Oligopoly: in the model of Participatory Democracy citizens play the role of the leader and representatives play the role of the follower, while in a model of Representative Democracy citizens are followers and representatives are leaders.

We assume that all citizens care about the policy implemented, and we also assume that attending the meeting is costly. The legislator cares about the policy implemented and also about holding office in the future. Citizens monitor the decision of the legislator, and punish her if they do not approve of her performance by not coordinating their votes for her in future elections. This assumption captures the high degree of accountability in Participatory Democracy, accountability that stems from the direct involvement of citizens in the process. For instance, in Porto Alegre there are committees formed by elected delegates whose function is to supervise the implementation of the budget. Since they have the right to ask the Mayorality for detailed explanations on each investment work, any deviation that cannot be explained by sound technical or economic criteria may have straightforward electoral consequences.

We find that the set of policies that can be implemented in equilibrium is a subset of the policy space that contains the ideal point of the legislator. That is, the legislator will only implement policies that are close to her ideal point up to a maximal compromise policy, at which point the legislator is indifferent between jeopardizing her reelection by implementing her ideal policy, or guaranteeing her reelection by satisfying the assembly. We show that the more the legislator cares about holding office the larger is the set of policies that can be implemented in equilibrium. The intuition is clear: a legislator that does not care so much about policy is willing to accept proposals further from her ideal point in order to guarantee a sure win in a future election. On the other hand, the softer is the threat of punishment, the smaller the size of the set of implementable policies. That is, a legislator that believes that her chances of being reelected will be very low unless she follows the policy proposed by the assembly, will be willing to implement a larger set of policies.

As in Osborne's model if the cost of attending the meeting is high enough there is a unique equilibrium in which nobody attends the meeting, but otherwise in equilibrium there is always some attendance. In our model, the legislator's ideal policy plays a role similar to the default policy in Osborne's: It is the policy selected if no citizen attends the meeting. There exists however a crucial difference. In our model, the legislator has *decision power* and in equilibrium she will never compromise her policy preferences more than what is needed to ensure her reelection. We find that in equilibrium only citizens that are far enough from the legislator's ideal point do attend the meeting. But they are not necessarily extreme in the usual (spatial) sense: when the legislator has extreme policy preferences, citizens that are moderate relative to the spectrum of tastes in the society may actually have strong incentives to participate. *Moderation becomes thus a relative concept.*

When we assume that the distribution of preferences in the society is symmetric and the legislator's ideal point coincides with the policy most preferred by the society, our model becomes a special case of Osborne's. Otherwise, our model is an extension of their model, showing the importance of two factors: 1) the *alignment* between the policy preferences of the legislator and the policy preferences of the society; and 2) the degree of *extremism* of the legislator.

On the one hand, we show that when the most preferred outcome of the society lies relatively close to the legislator's ideal point, that is, when the society and the legislator's preferences are aligned, in equilibrium the assembly is able to implement its ideal policy. This equilibrium obtains only for a certain subset of distribution of preferences.

On the other hand, we find that for any distribution of preferences if the legislator is extremist relative to the spectrum of preferences of the society there is an equilibrium in which the legislator implements the maximal compromise policy: on her left if her ideal point is to the right of the policy space and on her right if her ideal point is to the left of the policy space. In this equilibrium only one citizen attends the meeting in equilibrium: a leftist one if the legislator is rightist, and a rightist one if the legislator is leftist. This is driven by the fact that the legislator will never compromise her position more than necessary and only one citizen, maybe a moderate, is enough to force her to a maximal compromise.

Therefore, we have that if the legislator is extremist and her preferences are not aligned with the society's this equilibrium is unique. If the legislator is extremist and her preferences are aligned with the society for some distribution of preferences we may have another equilibrium in which the policy outcome coincides with the most preferred choice of the assembly. Finally, if the legislator is relatively moderate and her preferences are not aligned with the society's we find that there is no equilibrium in pure strategies. Therefore, with polarized or extremist societies and moderate legislators, the process of Participatory Democracy may generate unstable outcomes.

The rest of the paper proceeds as follows. The next section describes the formal model. Section 3 presents the sequential derivation of optimal policy choices. Section 4 analyzes the citizens' choice about participation in the assemblies. Finally Section 5 contains some concluding remarks.

2 The model

The policy space is continuous and one dimensional, and represented by the interval $[0,1]$. There is a finite number N of citizens with single-peak preferences over the interval $[0,1]$. The citizens' ideal points are distributed according to a probability distribution $F(\theta_i)$ with support in $[0,1]$. We will assume that there is always at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

At the first stage of the game, citizens have to decide whether to attend a meeting in which a policy will be proposed. Attendance implies that their opinion will be taken into account in the elaboration of policy proposals but it also involves an individual cost $0 < c < \frac{1}{2}$. This cost includes the opportunity cost of the time spent in the assembly, and the cost of discerning their own preferences. The welfare of an individual i with ideal point θ_i depends on the policy implemented and on whether he attends the meeting and it is given by the following expression:

$$V_i(x, a_i) = -\left|x - \theta_i\right| - a_i c,$$

where x is the policy implemented, and a_i represents the decision of citizen i on whether to attend the meeting: if $a_i = 1$, i attends the meeting and pays a cost c, if $a_i = 0$, i does not attend the meeting and pays no cost.

Following the functioning of the OP, where regions rank and select their priorities according to three pre-established criteria, we assume that the citizens that attend the meeting aggregate their preferences according to some previously decided aggregation rule, and the policy selected by the aggregation rule is the policy that the assembly would like to see implemented. Let X denote the list of ideal points of those citizens who attend the meeting and let $\theta^*(X)$ denote the assembly's most preferred policy. The aggregation rule we consider is: the median of the ideal points of the attendees $\theta^*(X) = median(X)$ if the number of attendees is odd, and the mean of the two medians' ideal points $\theta^*(X) = \frac{m_1+m_2}{2}$ if the number of attendees is even.[11]

Given a distribution of ideal points of the citizens, $F(\theta_i)$, let θ^{**} denote the society's most preferred policy defined according to the corresponding aggregation rule. Notice that the policy chosen by the assembly at the meeting does not depend on $F(\theta_i)$, but on the distribution of ideal points of the citizens that decide to attend the meeting, X.

After the assembly, a proposal, considered as the "general will", is transmitted to a *legislator* who is in charge of implementing the final policy. In Porto Alegre, this is done by the Forum of Delegates and the COP. Following

[11] We are of course aware of the fact that with an even number of attendees the choice of this policy cannot be rationalized by a voting process within the assembly. However, this assumption allows us to pin down a unique compromise regardless of the number of participants .

this, we assume the existence of an intermediate body of *delegates* between the citizens and the legislator who elaborates the policy proposal, denoted by x^*. Delegates simply carry the wishes of the assembly and try to induce the legislator to implement a policy as close as possible to θ^*. Hence, the proposal made to the legislator does not need to coincide with the assembly's most preferred policy.

The welfare of the legislator depends on her own policy preferences and on the probability of being reelected, and it is represented by a convex combination as follows:

$$V_L(x^*, x) = (1-\alpha)P(x^*, x) - \alpha\left|x - \theta_L\right|.$$

where $\theta_L \in [0,1]$ represents the ideal point of the legislator, $\alpha \in [0,1]$ is an exogenous parameter that represents the intensity of the policy preferences of the legislator relative to her preferences for holding office. From the point of view of the legislator, $P(x^*, x)$ is interpreted as the probability with which she will be reelected, and it depends on the amount of support that she will be able to obtain from the population, which in turn depends on whether the citizens approve of her performance. Since the legislator does not know either θ^* nor θ^{**}, her performance can only be judged by how close her choice x is from the proposal she received, x^*. We assume thus that the probability of reelection $P(x^*, x)$ is a step function of the distance between the implemented policy x and the mandate of the citizens' assembly x^*:

$$P(x^*, x) = \begin{cases} 1 & \text{if } |x - x^*| \leq B \\ \varepsilon & \text{otherwise} \end{cases}$$

where $B > 0$ is the degree of discretion of the legislator, that may account for financial or technical circumstances unforeseen by the citizens. In Participatory Democracy, legislators still control this knowledge and have privileged access to it.[12] So if the difference between the policy proposed by the delegates and the policy implemented by the legislator is not larger than this degree of discretion B, citizens will approve the legislator's performance and they will likely reelect her in future elections. Otherwise, the reelection of the legislator is compromised and we assume that she will only win future elections with probability ε, with $0 < \varepsilon < 1$. We assume that ε takes small values, reflecting the high degree of accountability of the participatory process.

Note that this last assumption is equivalent to assume an implicit commitment on the part of the majority of citizens to punish legislators who do

[12] In Porto Alegre, there have been substantial efforts to train delegates and councillors aiming to reduce this inevitable degree of discretion.

not take their proposal into account. In Porto Alegre, citizens, councillors and delegates have very often expressed their fears of a return to clientelism and corruption. In fact, they follow very closely the debate on the budget proposal at the Chamber of Deputies: Councillors put pressure on the legislators, meeting with them individually; mobilize communities to attend the debates and organize rallies. Hence, it seems plausible to assume that if the legislator deviates more than B, delegates will inform the population and this deviation will be socially regarded as a reason to vote her out.

Finally, if nobody attends the meeting the legislator can implement her ideal point and she is reelected with probability one.

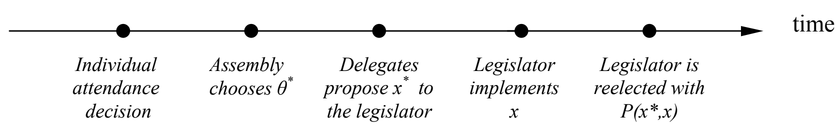

Figure 2: Timing of the game

So far, we have constructed a game in three stages. In the first stage, citizens decide whether to attend or not to attend the meeting. In the second stage citizens that attend the meeting decide a policy and the delegates make a proposal to the legislator. In the third stage the legislator decides a policy to be implemented.

3 Optimal policy choices

In order to find the Subgame Perfect Equilibrium strategies of this game, we analyze the optimal choices of the players by backward induction. First, we analyze the optimal reaction of the legislator in terms of policy choices, to a given proposal made by the delegates. Then, we analyze the optimal proposal of the assembly's delegates, for a given distribution of preferences of the attendees, taking into account the optimal reaction of the legislator. Finally, we analyze the optimal decision of each citizen regarding whether to attend the meeting, given an optimal play of all agents in the continuation of the game.

3.1 The optimal choice of the legislator

In order to choose the policy that will be finally implemented, x, the legislator maximizes her payoff function, given a policy proposed by the delegates, x^*.

$$\begin{aligned} \max_x \; V_L(x^*, x) &= (1-\alpha)P(x^*, x) - \alpha\,|x - \theta_L| \\ s.t. \quad P(x^*, x) &= \begin{cases} 1 & \text{if } |x - x^*| \leq B \\ \varepsilon & \text{otherwise} \end{cases} \end{aligned}$$

Let us define $b = \frac{(1-\alpha)}{\alpha}(1-\varepsilon)$. Notice that $(1-\varepsilon)$ represents the probability with which the legislator is not reelected when the citizens feel deceived, and $\frac{(1-\alpha)}{\alpha}$ represents the value of holding office. Thus, b represents a measure of the cost that the legislator has to pay when she is punished by the electorate. To characterize the best response of the legislator consider four cases:

(i) if $|x^* - \theta_L| \leq B$, then $x = \theta_L$ and $V_L(x^*, \theta_L) = 1 - \alpha$;

(ii) if $|x^* - \theta_L| \geq b + B$, then $x = \theta_L$ and $V_L(x^*, \theta_L) = (1-\alpha)\,\varepsilon$;

(iii) if $B \leq |x^* - \theta_L| \leq b + B$ and $x^* \leq \theta_L$, then $x = x^* + B$ and $V_L(x^*, x^* + B) = 1 - \alpha - \alpha\,(\theta_L - (x^* + B))$;

(iv) if $B \leq |x^* - \theta_L| \leq b + B$ and $x^* \geq \theta_L$, then $x = x^* - B$ and $V_L(x^*, x^* - B) = 1 - \alpha - \alpha\,((x^* - B) - \theta_L)$.

In the first case, the delegates' proposal is very close to the legislator's ideal point, so that the legislator can implement her ideal point without compromising her reelection. In case (ii) the delegates' proposal is very far away from the legislator's ideal point, and the legislator is better off by ignoring the proposal and implement her ideal point sacrificing the probability of winning future elections. In cases (iii) and (iv), the proposal of the delegates is far enough from the legislator's ideal point, so that she cannot implement her ideal point without compromising her approval, but it is close enough for the legislator to prefer to compromise her policy preferences and still guarantee a sure victory in the future.

Thus the optimal policy choice of the legislator will be the legislator's ideal point θ_L, if the proposal of the assembly is not further than a distance B from it, or if the legislator cares mostly about policy (b is sufficiently small). Otherwise the legislator will choose a policy that is located exactly B away from the proposal of the assembly. That is,

$$x\left(x^*\right) = \begin{cases} x^* + B & \text{if } B \leq |x^* - \theta_L| \leq b + B \;\; and \;\; x^* \leq \theta_L \\ x^* - B & \text{if } B \leq |x^* - \theta_L| \leq b + B \;\; and \;\; x^* \geq \theta_L \\ \theta_L & \text{otherwise} \end{cases}$$

3.2 The optimal choice of the delegates

The policy proposal x^* is made by a small group of delegates. Since they are elected by the assembly we assume they are committed to force the legislator to implement θ^* or the closest possible policy[13]. But they are aware of the preferences of the legislator so they choose their policy proposal x^* strategically, knowing that the only policies that can be finally implemented are either the legislator's ideal point or policies that are exactly B away from their proposal. The optimal policy choice of the delegates can be characterized as follows:

(i) if $\theta_L - b \leq \theta^* \leq \theta_L$, then $x^* = \theta^* - B$ and $x = \theta^*$;

(ii) if $\theta_L \leq \theta^* \leq \theta_L + b$, then $x^* = \theta^* + B$ and $x = \theta^*$;

(iii) if $\theta^* \leq \theta_L - b$, then $x^* = \theta_L - b - B$ and $x = \theta_L - b$;

(iii) if $\theta_L + b \leq \theta^*$, and $\theta^* > \theta_L$ then $x^* = \theta_L + b + B$ and $x = \theta_L + b$.

In the first two cases, the assembly's most preferred policy is very close to the legislator's ideal point, and the delegates can induce the legislator to implement the assembly's most preferred policy. In the last two cases, the assembly's most preferred policy is far away from the legislator's ideal point, and the best the delegates can do is to induce a compromise. The best choice in this case is to propose a policy that makes the legislator indifferent between implementing her ideal point and jeopardizing the next election, and implementing the compromise policy that still assures her approval, that is, a policy of maximal compromise. Thus, we can write the delegates' optimal choice function as:

[13]The strength of the bond between the delegates, the councillors and the regions they represent has been another source of unrest in Porto Alegre: CIDADE, a NGO monitoring the OP, has reported problems and discussions about councillors who allegedly took positions without consulting their constituencies or who failed to report back the decisions of the COP. Reelection rules as a way to avoid this have prompted many debates. We refer the interested reader to Santos (2001) pp. 489-90.

$$x^*(\theta^*) = \begin{cases} \theta_L - b - B & \text{if } \theta^* \leq \theta_L - b \\ \theta^* - B & \text{if } \theta_L - b \leq \theta^* \leq \theta_L \\ \theta^* + B & \text{if } \theta_L \leq \theta^* \leq \theta_L + b \\ \theta_L + b + B & \text{if } \theta_L + b \leq \theta^* \end{cases}$$

3.3 The equilibrium policy choice

Combining the optimal choices of delegates and legislator we can characterize the policies that will be implemented in equilibrium as a function of the legislator's ideal point and the most preferred policy of the assembly. The next Proposition characterizes the policies that can be implemented in equilibrium for all values of θ^*.

Proposition 1 *In equilibrium* $x \in [\theta_L - b, \theta_L + b]$.

All proofs can be found in the Appendix.

The introduction of a legislator's choice in a process of pure direct democracy has an immediate consequence: not all policies are implementable in equilibrium. In fact, from the combination of the optimal choice of citizens and legislator we obtain a function that describes the possible policy outcomes:[14]

$$x(\theta^*, \theta_L) = \begin{cases} \theta^* & \text{if } \theta^* \in [\theta_L - b, \theta_L + b] \\ \theta_L - b & \text{if } \theta^* < \theta_L - b \\ \theta_L + b & \text{if } \theta^* > \theta_L + b \end{cases}$$

When the policy most preferred by the assembly is relatively close to the legislator's ideal point, the policy finally implemented coincides exactly with the preferences of the assembly. Otherwise, citizens cannot induce the legislator to implement their most preferred policy. They can at most induce a compromise between the policy preferences of the assembly and the legislator.

Observe that the size of the set of policies that can be implemented in equilibrium is given by $b = \frac{(1-\alpha)}{\alpha}(1-\varepsilon)$. This raises two remarks:

1. The set of implementable policies is decreasing in the value that the legislator attaches to holding office (α). This is an intuitive result, since legislators that do not care so much about policy are willing to accept proposals further from their ideal point in order to stay in office.

[14] The derivation of this function can be found in the proof of Proposition 1.

2. The size of the set of implementable policies is decreasing in the probability with which the legislator's performance is approved independently of the policy implemented (ε). That is, a legislator that believes that her chances of being reelected will be very low, unless she follows the policy proposed by the assembly, will be willing to implement a larger set of policies. Hence, if citizens could commit to a certain degree of punishment, in terms of the probability represented by ε, their optimal choice should be $\varepsilon = 0$, the maximal degree of accountability: never reelect a legislator that deviated too much (more than B) from the policy proposal.

Hence, the more the legislator cares about policy and the smaller the probability with which she is being punished by the citizens, the smaller the chances that the assembly can achieve its most preferred policy. In fact, if the values of α and ε are sufficiently small (so that b is large enough) the set of implementable policies may be the whole policy space.

4 Endogenous participation

Finally, we analyze the first stage of the game: the choice of the citizens regarding their participation to the assembly. In this subgame, an equilibrium is a list of values for $\{a_1, ..., a_N\}$ with $a_i \in \{0, 1\}$ such that for all i, a_i is a best response against $\{a_1, ..., a_{i-1}, a_{i+1}, ..., a_N\}$. We show that when the cost of attendance is large enough, relative to the parameters of the objective function of the legislator, there is a unique equilibrium in which nobody attends the meeting.

Proposition 2 (Non Attendance) *If $c > b$ there is a unique equilibrium in which nobody attends the meeting.*

As in Osborne's model, here citizens perform a cost benefit analysis in order to decide whether to attend the meeting. If the cost of attending is larger than the benefit they will obtain from the impact that their presence at the meeting will have on the final policy, they decide not to attend. We have assumed that if nobody attends the meeting the legislator can implement her ideal point without compromising her future reelection. Thus, in this model the size of b represents the maximal effect that any citizen can have on the policy implemented. Clearly, when the cost of attendance is larger than b nobody has any incentive to attend the meeting.

Next we show that when this is not the case, that is, if the cost of attendance is small enough, in equilibrium there must be some positive attendance.

Proposition 3 *If $c < b$, then nobody attends the meeting is not an equilibrium.*

In fact, the proof of Proposition 3 shows not only that nobody attending is not an equilibrium, but also that if nobody attends the meeting any citizen whose ideal point is more than a distance c away from the legislator's ideal point would be better off attending. The intuition of this result comes again from the cost-benefit analysis that citizens perform: any citizen that attends the meeting can induce the legislator to implement either the citizen's ideal point or a maximal compromise policy (a policy that is a distance b away from the legislator's ideal point). In both cases, if the ideal point of the citizen is more than a distance c away from θ_L, the benefit for the citizen is larger than c, which is the cost of attending the meeting.[15]

Next we characterize some equilibrium strategies when $c < b$, that is, when there are some citizens that attend the meeting in equilibrium.

4.1 Aligned moderate legislators

First we analyze the equilibrium strategies of the game, assuming that the distribution of ideal points of the society, $F(\theta_i)$, is symmetric around the legislator's ideal point. In this case the legislator's policy preferences qualify of moderate and aligned with the society's policy preferences.[16]

Given the symmetry assumed in the structure of the game, we will look for symmetric equilibria. We define a symmetric equilibrium as a list $\{a_1, ..., a_N\}$ that satisfies the equilibrium condition and such that the distribution of the ideal points of the attendees is symmetric around θ^*. If $\theta_L = \frac{1}{2}$, in a symmetric equilibrium the policy desired by the assembly coincides with the policy proposed, with the ideal point of the legislator, and with the implemented policy: $\theta^* = x^* = \theta_L = x = \frac{1}{2}$. If the distribution of ideal points of the society is also symmetric, in addition we have that $x = \theta^{**} = \frac{1}{2}$.

First we show that when the cost of attendance is relatively small, and the distribution of the ideal points of the citizens is symmetric around θ_L we can completely characterize the strategies in a symmetric equilibrium.

[15] When $c = b$ some citizens (or all, depending on the distribution of ideal points of the population) could be indifferent between attending and not attending the meeting when nobody else is attending. Existence of equilibrium in this case would depend on how indifferences are resolved.

[16] Notice that if we assume a distribution of citizens' ideal points that is symmetric around θ_d, we must have that $\theta_d = \frac{1}{2}$ given that we have assumed that there is at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

Proposition 4 (Symmetric Equilibrium) *For any finite number of citizens with distribution of ideal points symmetric around* $\theta_L = \frac{1}{2}$*, if* $c <$*, there is a symmetric equilibrium in which* $X = \{\theta_i : |\theta_L - \theta_i| > c\}$ *and* $x = \frac{1}{2}$.

On the one hand, citizens that are close to the legislator's ideal point ($|\theta_L - \theta_i| < c$) become the median if they were to attend, but the improvement of the policy outcome would be less than c. Hence, they prefer to stay home. On the other hand, citizens that are far away from the legislator's ideal point lose more than c from withdrawing. Thus, in a symmetric equilibrium all citizens whose ideal points are more of a distance c away from the legislator's ideal point will attend the meeting[17]. In this specific configuration, our results become a particular case of the result obtained by Osborne for more general utility functions; both models coincide only when the legislator is moderate and aligned with the society's preferences, since in this case the legislator's ideal point is equivalent to the default policy of Osborne's model.

4.2 Biased legislators, skewed populations

Next we analyze the attendance equilibrium strategies when the ideal point of the legislator takes values other than $\frac{1}{2}$. Recall that we have assumed that there is at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$. Therefore, independently of the shape of the distribution, the fact that the legislator's ideal point is away from $\frac{1}{2}$ introduces some asymmetries. In this case, two factors arise as critical: *how aligned* the legislator is with the society's preferences and *how extremist* (in the spatial sense) she is with respect to $\frac{1}{2}$.

First we analyze the case in which the legislator is relatively extremist with respect to $\frac{1}{2}$, that is, $\left|\theta_L - \frac{1}{2}\right| > b - c$. In this case we find that for any distribution of citizens' ideal points there is an equilibrium in which the legislator implements the maximal compromise policy on her left if her ideal point is to the right of $\frac{1}{2}$, and she implements the maximal compromise policy on her right if her ideal point is to the left of $\frac{1}{2}$. Furthermore, only one citizen attends the meeting in this equilibrium: a leftist one if the legislator is rightist, and a rightist one if the legislator is leftist.

[17] If there was $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$, in a symmetric equilibrium they both would have a weak preference to attend, and a weak preference not to attend, in case both are attending and also in case only one of them attends. Furthermore, if they both would attend, then in equilibrium all $\theta_j \notin \left(\frac{1}{2} - c, \frac{1}{2} + c\right)$ would have a weak preference for attending. If they both would not attend, then all $\theta_j \notin \left(\frac{1}{2} - c, \frac{1}{2} + c\right)$ would have a strong preference for attending. Thus, in this case there is also an equilibrium in which only $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$ attend.

Proposition 5 (Maximal Compromise Equilibrium) *If $c < b$:*

(a) $x = \theta_L + b$ is an equilibrium outcome if and only if $\theta_L < \frac{1}{2} - b + c$. Moreover, only one citizen with ideal policy $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c)\}$ attends.

(b) $x = \theta_L - b$ is an equilibrium outcome if and only if $\theta_L > \frac{1}{2} + b - c$. Moreover, only one citizen with ideal policy $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$ attends.

Observe that this result holds for any distribution of the citizens' ideal points.

When the legislator is extremist, citizens far from the legislator's ideal point have a strong incentive to participate. Moreover, one citizen is enough to induce the maximal compromise. But in this particular case, there is no response from those citizens who are extremist *and* close to the legislator since that maximum compromise is not too far from them and they are already well represented by her.

With extreme legislators we have an equilibrium in which only one citizen attends the meeting. This result can be thought of as an extreme case of Osborne's *Low participation* result. But it also challenges Osborne's result on *Non participation of the moderates*: If the legislator preferences are too extreme, citizens in the center of the political spectrum have strong incentives to participate and moreover they are able to force her to implement the maximal compromise outcome in their favor. Note that this type of equilibria arise even if the legislator's policy preferences are aligned with the view of a majority of the society.

Observe that the previous Proposition also implies that when the legislator's policy preferences are not extreme with respect to the center of the policy space a maximal compromise policy is never implemented in equilibrium.

Corollary 1 (Moderate legislator) *If $\left|\theta_L - \frac{1}{2}\right| \leq b - c$, in equilibrium we must have $x \in (\theta_L - b, \theta_L + b)$* and therefore $x(\theta^*, \theta_L) = \theta^*$.

If the legislator is relatively moderate, citizens located at both extremes have high stakes in attending the meeting, therefore the policy proposed will be relatively centrist. In equilibrium the choice of the legislator must be a policy in interior of the set if implementable policies, and given the optimal play of the delegates and the legislator in the continuation of the game, the latter will implement the assembly's ideal policy.

Proposition 6 *Suppose that $c < b$ and $x \in (\theta_L - b, \theta_L + b)$ is the equilibrium outcome: if θ_i does not attend then θ_i leaves the same number of attendees' ideal points on each side.*

In equilibrium we have two sets of ideologically extreme attendees separated by one set of non-attendees. The most preferred policy of the assembly, according to the median, will always be the average of the two attendees that surround the set of non-attendees. Let $\theta_l = \max\{\theta_i \in X : \theta_i < \theta^*(X)\}$ denote the most moderate leftist attendee, and let $\theta_r = \min\{\theta_i \in X : \theta_i > \theta^*(X)\}$ denote the most moderate rightist attendee. Then, the most preferred policy by the assembly according to the median is given by $\theta^*(X) = \frac{\theta_l + \theta_r}{2}$.

Indeed we can completely characterize the set of citizens that attend the meeting in equilibrium: exactly all those citizens whose ideal points are further than c from the policy outcome.

Proposition 7 (Non-participation of the represented) *If $c < b$ and $x \in (\theta_L - b, \theta_L + b)$ is the equilibrium outcome, then θ_i attends if and only if $|\theta_i - x| > c$.*

Since incentives to attend are given by the impact of attendance decisions on the final outcome, those citizens already represented by the legislator prefer to stay home. Thus, we generalize the result on attendance provided in Osborne's. Note that this Proposition and the two previous ones, are generalizations of Proposition 4 to values of the legislator's ideal point different from $\frac{1}{2}$, and to any distribution of the citizens' policy preferences.

Observe that the results stated in Propositions 6 and 7 hold for any value of the legislator's ideal point. Therefore, the interior equilibrium described here may exist for any location of the legislator's ideal point. These results offer a broader picture of Osborne's *Non participation of the moderates* result. Those citizens who are already represented by the predicted outcome or by the legislator's ideology, have no incentive to participate and will not attend the meeting. And they are not necessarily moderates in the political spectrum. Interestingly enough, this is consistent with the observed lack of participation of unions in the Porto Alegre's OP process. Marquetti (2000) argues that one of the reasons explaining this fact is that unionists feel that they are already represented in the OP, given the leftist leaning of the municipal administration.

Observe that the conditions to be satisfied in an interior equilibrium are rather strong:

(i) There must be an identical number of attendees on both sides of the set of non-attendees, so $\theta^*(X) = \frac{\theta_l + \theta_r}{2}$;

(ii) All attendees must be at more than a distance c of the policy outcome.

These two conditions imply that the existence of equilibrium will depend largely on the shape of the distribution of the ideal points of the population, and they pose a stronger requirement on the set of policies that can be implemented in an interior equilibrium.

Proposition 8 *If* $c < b$ *and* $x \in (\theta_L - b, \theta_L + b)$ *is an equilibrium outcome, then* $x \in (\theta_L - b + c, \theta_L + b - c)$ *.*

Next we show that a necessary condition for existence of an interior equilibrium for any value of the legislator's ideal point is that the policy preferences of the legislator must be aligned with those of the society.

Proposition 9 (Alignment is needed) *If* $c < b$ *and* $\theta^{**} \notin (\theta_L - b, \theta_L + b)$ *then there is no interior equilibrium.*

Therefore, we can conclude that when an interior equilibrium exists, the policy implemented coincides with the most preferred policy of the assembly, and it is very close to the most preferred policy of the society. Nevertheless, the previous results have a rather negative implication about existence of this type of equilibrium:

Corollary 2 (Equilibrium existence) *If* $c < b$ *and* $\theta^{**} \notin (\theta_L - b, \theta_L + b)$ *there is no equilibrium if the legislator is moderate. If the legislator is extremist, maximal compromise is the unique equilibrium.*

Therefore, when the preferences of the legislator are relatively moderate, the alignment between the legislator's and the society's preferences is a necessary condition for the existence of an equilibrium in pure strategies. The following example illustrates non existence of equilibrium in pure strategies when the median of the society is not well aligned with the ideal point of the legislator, and the legislator is moderate.

Example: Non-existence of equilibrium

Consider a society with only three citizens located at 0, $\frac{1}{3}$, and 1. Suppose that the legislator is located at $\frac{3}{5}$ and suppose that $b = \frac{1}{5}$. In this case, the ideal point of the median cannot be implemented in equilibrium. Suppose that the cost of attending is $c < \frac{1}{10}$. We will show that there is no pure strategy equilibrium.

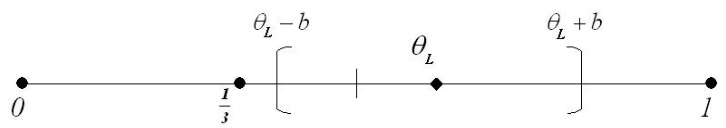

Figure 3: Non-Existence of Equilibrium.

It is clear that all citizens want to attend when nobody is attending, since the attendance cost c is smaller that the impact each one of them has on the final outcome. If all of them were attending, the median of the assembly would be $\frac{1}{3}$, and the final outcome $\theta_L - b = \frac{2}{5}$. Since the legislator will not compromise beyond that, the citizen located at 1 would be better off withdrawing. If the only attendees were the two leftist citizens, any of them would be better off not attending. If the two extremist citizens were the ones attending the meeting, then the citizen at $\frac{1}{3}$ would prefer to attend since his utility would increase by $\frac{1}{2} - (\theta_L - b) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} > c$. Finally, if $\frac{1}{3}$ and 1 were the ones to attend, the citizen at 0 would prefer to attend since his utility would increase by $\frac{2}{3} - (\theta_L - b) = \frac{2}{3} - \frac{2}{5} = \frac{4}{15} > \frac{1}{10} > c$. Hence, there is no equilibrium in pure strategies.

We have shown that in a system of Participatory Democracy, citizens, under certain conditions, may obtain their most preferred policy in equilibrium. But we have also shown that otherwise the policy outcomes may imply instability. Next we characterize general conditions on the distribution of citizens' ideal points that provide the necessary alignment and guarantee the existence of an interior equilibrium.

Let us define the set:

$$S = \{x \in (\theta_L - b, \theta_L + b) : x = \tfrac{\theta_l(x)+\theta_r(x)}{2} \text{ for some } \theta_l(x) \text{ and } \theta_r(x)$$
$$\text{s.t. } \theta_l(x), \theta_r(x) \in (\theta_L - b, \theta_L + b) \text{ and } |\{\theta_i : \theta_i \leq \theta_l(x)\}| = |\{\theta_i : \theta_i \geq \theta_r(x)\}|\}.$$

From previous results we know that an equilibrium outcome must belong to the set of implementable policies, $[\theta_L - b, \theta_L + b]$, and that an interior equilibrium outcome has to be the average of the ideal points of the two attendees delimiting the set of non-attendees. Thus, the set S contains policies that are candidates for equilibrium outcomes. Notice that if $\theta^{**} \notin (\theta_L - b, \theta_L + b)$ then the set S is empty. The next proposition describes a sufficient condition under which the elements of S can be implemented in equilibrium.

Proposition 10 *If $c < b$, then for each $x \in S$ there exist $\underline{c}(x)$ and $\overline{c}(x)$ such that if $c \in (\underline{c}(x), \overline{c}(x))$, x is an equilibrium outcome.*

These conditions ensure that the division of citizens in "extreme" sets of attendees separated by a set of non-attendees is stable. This comes from Proposition 8: the cost should be sufficiently small to offer to relatively extreme citizens enough incentives to attend the meeting but it must be high enough to discourage citizens with ideal points relatively close to the equilibrium policy outcome from attending. Under such conditions, the assembly's most preferred policy can be implemented in equilibrium. Recall that in this case the assembly most preferred policy is very close to the society's most preferred policy.

5 Conclusions

In this paper, we have proposed a model of Participatory Democracy inspired by the system of participatory budgeting implemented in the city of Porto Alegre. This experiment, now extended to many other cities worldwide, shows that a participatory system at the local level is indeed possible and can successfully, but not without problems, govern large communities.

We have analyzed Participatory Democracy by introducing a legislator, with the role of policy implementation, in a formal model of Direct Democracy, and we have shown that this political system is characterized by a relative autonomy between citizens and representatives, since the former will not be able to implement any policy, and the latter have to acquire calculated compromises.

We have found two different kinds of equilibria: a maximal compromise equilibrium and an interior equilibrium. A maximal compromise equilibrium always exists when the preferences of the legislator are extreme with respect to the policy space. In this equilibrium only one citizen, with preferences opposed to those of the legislator's, attends the meeting and the policy outcome is relatively moderate. An interior equilibrium only exists when the preferences of the legislator are aligned with the majoritarian views of the population. This equilibrium may exist with an ideologically extreme or moderate legislator, but only for a certain subset of distributions of citizens' ideal points. In this equilibrium the policy implemented is the most preferred by the assembly and it is close to the policy most preferred by the society. The number of attendees depends on the distribution of the preferences of the society, and the composition of the assembly is characterized by two groups of equal size and opposed preferences.

These two kinds of equilibria represent two different experiences occurred in Porto Alegre.

At the early stages of the implementation of the Participatory Budgeting system, participation was very low and the priorities selected at the rodadas referred to issues of interest of the working class: all the investment budget was devoted to cover basic needs. For this case, the best interpretation of our unidimensional policy space is the population's wealth distribution, as it can be thought as a representation of the citizens' preferences over basic needs. Since elected legislators normally belong to high-income segments, when the policy preferences are given by the wealth distribution, the legislator's preferences are expected to be extreme. Our prediction in this case is given by the maximal compromise equilibrium: very few citizens with preferences opposed to the legislator's will attend the meeting and they will force the legislator to implement a policy close to their most preferred one.

At later stages of the implementation of the Participatory Budgeting system, basic needs had been covered, and the priorities selected at the rodadas shifted to issues that were also attractive to middle class citizens and the composition of the attendance to the rodadas also changed: The percentage of participants in rodadas with up to 4 minimum wages fell from 62% in 1995 to 56% in 1998 (Marquetti, 2000). In our model this change can be thought of as a change of the relevant policy space, and therefore we have to consider a change in the distribution of the citizens' preferences. Thus, it is reasonable to expect that the legislator's policy preferences are now less extreme. Our prediction in this case is given by the interior equilibrium if the policy preferences of the legislator and those of the society are aligned. In this case, participation is expected to be higher and the policy implemented should be close to the most preferred policy by the assembly. Otherwise, if the policy preferences of the legislator and those of the society are not aligned, there is no equilibrium and we should expect instability. The political scenario in Porto Alegre supports the former case as the rising figures of participants in the rodadas suggest. The political affinities between the citizens and the municipal government and a high level of accountability have definitively helped to create a broad enough spectrum of implementable policies.

More ambiguous are the effects derived from the strong identification between the participatory budgeting system and the PT: The executive seems to have sometimes captured the OP as a way to better serve its own interests but at the same time this has definitively linked its electoral success to the well-functioning of the process. In the overall, the OP has been so far successful in avoiding the main risk of Participatory Democracy, namely instability, while keeping its main virtue: The policies implemented have represented the citizens' majoritarian opinions.

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A Appendix

Proof of Proposition 1: Implementable Policies in Equilibrium.

In order to find the policies implemented by the legislator in equilibrium we combine the optimal choice functions of the legislator and the assembly.

$$x(x^*) = \begin{cases} x^* + B & if\ B \leq |x^* - \theta_L| \leq b + B \ \ and \ \ x^* \leq \theta_L \\ x^* - B & if\ \ B \leq |x^* - \theta_L| \leq b + B \ \ and \ \ x^* \geq \theta_L \\ \theta_L & \text{otherwise} \end{cases}$$

$$x^*(\theta^*) = \begin{cases} \theta_L - b - B & if\ \ \theta^* \leq \theta_L - b \\ \theta^* - B & if\ \theta_L - b \leq \theta^* \leq \theta_L \\ \theta^* + B & if\ \theta_L \leq \theta^* \leq \theta_L + b \\ \theta_L + b + B & if\ \theta_L + b \leq \theta^* \end{cases}$$

We have to consider four different cases:

(i) if $\theta^* \leq \theta_L - b$, then the best response of the assembly is to propose $x^* = \theta_L - b - B$. In this case, $\theta_L - x^* = b + B$ and $x^* \leq \theta_L$, thus the legislator implements $x = x^* + B = \theta_L - b$.

(ii) if $\theta_L - b \leq \theta^* \leq \theta_L$, then the best response of the assembly is to propose $x^* = \theta^* - B$. In this case, $B = \theta^* - x^* \leq \theta_L - x^* = \theta_L - \theta^* + B \leq b + B$ and $x^* \leq \theta_L$, thus the legislator implements $x = x^* + B = \theta^*$.

(iii) if $\theta_L \leq \theta^* \leq \theta_L + b$, then the best response of the assembly is to propose $x^* = \theta^* + B$. In this case, $B = x^* - \theta^* \leq x^* - \theta_L = \theta^* + B - \theta_L \leq b + B$ and $x^* \geq \theta_L$, thus the legislator implements $x = x^* - B = \theta^*$.

(iv) if $\theta_L + b \leq \theta^*$, then the best response of the assembly is to propose $x^* = \theta_L + b + B$. In this case, $\theta_L - x^* = b + B$ and $x^* \geq \theta_L$, thus the legislator implements $x = x^* - B = \theta_L + b$.

Therefore, in equilibrium the legislator will implement the most preferred policy of the assembly only when it belongs to the interval $[\theta_L - b, \theta_L + b]$, otherwise the only policies that will be implemented in equilibrium are $\theta_L - b, \theta_L + b$. ■

Proof of Proposition 2: Non Attendance.

First we show that when $c > b$ there is an equilibrium in which nobody attends the meeting.

Suppose that nobody attends the meeting. In this case the legislator will implement her ideal point, thus $x = \theta_L$. Consider a citizen i such that $\theta_i \leq \theta_L - b$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i| = \theta_i - \theta_L$. If he were to attend, he would be the only participant. Since his ideal point is not within the interval $[\theta_L - b, \theta_L + b]$ the best he could obtain is $x = \theta_L - b$, thus his payoff would be $V_i(\theta_L - b, 1) = -|\theta_L - b - \theta_i| - c = \theta_i - \theta_L + b - c$. Since $c > b$ we have that $V_i(\theta_L, 0) > V_i(\theta_L - b, 1)$, so he is better off not attending.

Now consider a citizen i such that $\theta_L - b \leq \theta_i \leq \theta_L$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i|$. If he were to attend, he would be the only participant. Since his ideal point is within the interval $[\theta_L - b, \theta_L + b]$ he would be able to obtain his ideal point θ_i as the policy output, thus his payoff would be: $V_i(\theta_i, 1) = -|\theta_i - \theta_i| - c = -c$. Since $c > b > |\theta_L - \theta_i|$, we have that $V_i(\theta_L, 0) = -|\theta_L - \theta_i| > V_i(\theta_i, 1) = -c$, so he is better off not attending. Using a symmetric argument we can also prove that all citizens with $\theta_i \geq \theta_L$ are better off not attending the meeting when nobody else is attending.

Next we will show that nobody attending the meeting is the unique equilibrium of the game when $c > b$.

Let $x(X) = x(\theta^*(X))$ denote the policy implemented as a function of the set of citizens that attend the meeting, X, given that citizens and legislator are playing their best responses, the last two stages of the game.

Observe that for any set of attendees in equilibrium X, and for any $\theta_i \in X$ we must have that $V_i(x(X), 1) = -|x(X) - \theta_i| - c > V_i(x(X - \{\theta_i\}), 0) = -|x(X - \{\theta_i\}) - \theta_i|$, that is $|x(X) - x(X - \{\theta_i\})| > c$. Since $x \in [\theta_L - b, \theta_L + b]$ we have that:

(i) If $\theta_i \leq \theta_L$, this condition implies that $\theta_L - b \leq x(X) \leq \theta_L + b - c < \theta_L$.

(ii) If $\theta_i \geq \theta_L$, this condition implies that $\theta_L < \theta_L - b + c \leq x(X) \leq \theta_L + b$.

These two conditions cannot hold simultaneously. These implies that in equilibrium we must have that either $X = \{\theta_i : \theta_i \leq \theta_L\}$ or $X = \{\theta_i : \theta_i \geq \theta_L\}$. Suppose that $X = \{\theta_i : \theta_i \leq \theta_L\}$, then since all $\theta_i \geq \theta_L$ do not attend the meeting we must have that $x(X) \leq \theta_L$ and $x(X - \{\theta_i\}) \leq \theta_L$ for all $\theta_i \in X$. But this implies that $|x(X) - x(X - \{\theta_i\})| < b < c$ which contradicts the condition that makes attendance optimal. Similarly we can prove that $X = \{\theta_i : \theta_i \geq \theta_L\}$ leads to a contradiction. Therefore, if $c > b$ there cannot be an equilibrium in which some citizens attend.■

Proof of Proposition 3: Attendance in Equilibrium.

We will show that if nobody attends the meeting any citizen with ideal point θ_i such that $|\theta_i - \theta_L| > c$ would be better off attending the meeting. Since we have assumed that there is at least a citizen with $\theta_i = 0$ and at least a citizen with $\theta_i = 1$, we will always have someone that has a profitable deviation. Suppose that $c < b$ and nobody attends the meeting, that is, $X = \varnothing$. Consider a citizen with θ_i such that $b > |\theta_L - \theta_i| > c$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i|$. If he were to attend, he would be the only participant. Since his ideal point is within the interval $[\theta_L - b, \theta_L + b]$ he would be able to obtain his ideal point θ_i as the policy output, thus his payoff would be $V_i(\theta_i, 1) = -|\theta_i - \theta_i| - c = -c$. Since $c < |\theta_L - \theta_i|$, we have that $V_i(\theta_L, 0) = -|\theta_L - \theta_i| < V_i(\theta_i, 1) = -c$. Therefore, he would be better off attending.

Now consider a citizen with θ_i such that $\theta_i \leq \theta_L - b$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i| = \theta_i - \theta_L$. If he were to attend, he would be the only participant. Since his ideal point is not within the interval $[\theta_L - b, \theta_L + b]$ the best policy he could obtain is $x = \theta_L - b$, thus his payoff would be $V_i(\theta_L - b, 1) = -|\theta_L - b - \theta_i| - c = \theta_i - \theta_L + b - c$. Since $c < b$ we have that $V_i(\theta_L, 0) < V_i(\theta_L - b, 1)$. Therefore, he would be better off attending.

Similarly we can prove that any citizen with $\theta_i \geq \theta_L + b$ would be better off attending the meeting. Therefore, nobody attending the meeting cannot be an equilibrium.■

Proof of Proposition 4: Symmetric Equilibrium.

We are assuming that $F(\theta_i)$ is symmetric around $\theta_L = \frac{1}{2}$ and $c < b$. We will show that there is a symmetric equilibrium in which $X = \{\theta_i : |\theta_i - \theta_L| > c\}$, by proving that all $\theta_i \in X$ are better off attending and all $\theta_i \notin X$ are better off not attending.

Suppose that $X = \{\theta_i : |\theta_i - \theta_L| > c\}$ and consider a citizen $\theta_i \in X$ such that $\theta_i < \frac{1}{2} - c$. Since he attends the meeting his payoff is $V_i(\frac{1}{2}, 1) = -\left|\theta_i - \frac{1}{2}\right| - c = \theta_i - \frac{1}{2} - c$. If he would not attend his utility would be $V_i(x(X - \{\theta_i\}), 0) = -|\theta_i - x(X - \{\theta_i\})| = \theta_i - x(X - \{\theta_i\})$. Thus he is better off attending if and only if $x(X - \{\theta_i\}) > \frac{1}{2} + c$. We will show that this is always the case.

Since $x(X - \{\theta_i\}) = \min\left\{\theta^*(X - \{\theta_i\}), \frac{1}{2} + b\right\}$, and we have that $\theta^*(X - \{\theta_i\}) = median(X - \{\theta_i\}) > \frac{1}{2} + c$ and $b > c$, then we must have that $x(X - \{\theta_i\}) > \frac{1}{2} + c$.

Now consider a citizen $\theta_i \notin X$ such that $\frac{1}{2} - c < \theta_i < \frac{1}{2}$. Since he does not attend his utility is $V_i(\frac{1}{2}, 0) = -\left|\theta_i - \frac{1}{2}\right| = \theta_i - \frac{1}{2}$. If he was to attend he would obtain $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c = -c$ since in this case he would become the median of the assembly, that is, $\theta^*(X \cup \{\theta_i\}) = median(X \cup \{\theta_i\}) = \theta_i$ and since $\theta_i \in [\theta_L - b, \theta_L + b]$ his ideal point would be implemented $x(X \cup \{\theta_i\}) = \theta_i$. Thus, he is better off not attending because $\frac{1}{2} - c < \theta_i$.■

Proof of Proposition 5: Maximal Compromise equilibrium.

We will only prove part (a). The proof of part (b) is almost identical and is left to the reader.

First we show that if $x(X) = \theta_L + b$ is the equilibrium outcome, then X must be a singleton. Suppose that the equilibrium outcome is $x(X) = \theta_L + b$. Then for all $\theta_i \in X$ such that $\theta_i < \theta_L - b$ we have that $x(X - \{\theta_i\}) = x(X)$, therefore we must have that $V_i(x(X - \{\theta_i\}), 0) > V_i(x(X), 1)$. Therefore, they would be better off not attending and in equilibrium we must have that $X \subseteq \{\theta_i : \theta_i \geq \theta_L + b\}$.

Given that, for all $\theta_i \in X$ such that $\theta_i \geq \theta_L + b$ we have that

$$x(X - \{\theta_i\}) = \begin{cases} x(X) & if\ |X| > 1 \\ \theta_L & if\ |X| = 1 \end{cases}$$

This implies that $V_i(x(X - \{\theta_i\}), 0) > V_i(x(X), 1)$ as long as $|X| > 1$. Therefore, they would be better off not attending as long as $|X| > 1$. Therefore, the only possibility for equilibrium is $|X| = 1$.

Next we show that there is an equilibrium with $x(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. We already know that if $x(X) = \theta_L + b$ is the equilibrium outcome, we must have $X = \{\theta_i\}$ with $\theta_i \geq \theta_L + b$. Observe that if a citizen θ_i with $\theta_i \geq \theta_L + b$ is the only citizen that attends the meeting his best proposal is $x^* = \theta_L + b + B$, the policy implemented is $x = \theta_L + b$ and he obtains $V_i(\theta_L + b, 1) = -|\theta_i - \theta_L - b| - c = \theta_L + b - \theta_i - c$. If he was not to attend he would obtain $V_i(\theta_L, 0) = -|\theta_i - \theta_L| = \theta_L - \theta_i$. Thus he is better off attending since $b > c$.

Suppose that $X = \{\theta_i\}$ with $\theta_i \geq \theta_L + b$. Observe that for any $\theta_j \geq \theta_L + b, \theta_j \neq \theta_i$ we have that $x(X \cup \{\theta_j\}) = \theta_L + b$, thus they would gain nothing by attending, and they would have to pay the cost. Therefore, they are all better off not attending.

Now consider a θ_j with $\theta_j < \theta_L + b$. If we show that $\theta_j = 0$ prefers not to attend the meeting, we will have that all $\theta_j < \theta_L + b$ prefer not to attend.

If $\theta_j = 0$, if he does not attend the meeting he obtains $V_0(x(X), 0) = -|\theta_L + b| = -\theta_L - b$; if he were to attend the policy outcome would be $x(X \cup \{0\}) = \max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}$ and thus he would obtain

$$V_0(x(X \cup \{0\}), 1) = -\left|\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}\right| - c = -\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} - c$$

We have that he is better off not attending iff $-\theta_L - b > -\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} - c$ iff $\theta_L + b - c < \max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}$.

Notice that if $\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} = \theta_L - b$ the inequality is never satisfied. Thus it is necessary and sufficient that we have $\frac{\theta_i}{2} > \theta_L + b - c$, that is $\theta_i > 2(\theta_L + b - c)$. And there would be such a citizen if and only if $2(\theta_L + b - c) < 1$ iff $\theta_L < \frac{1}{2} - b + c$.

Thus we have shown that there is an equilibrium with $x(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. And in this equilibrium we have that $X = \{\theta_i\}$ with $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c).\}$.

Similarly we could show that there is an equilibrium with $x(X) = \theta_L - b$ iff $\theta_L > \frac{1}{2} + b - c$. And in this equilibrium we would have that $X = \{\theta_i\}$ with $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$.■

Proof of Proposition 6: Symmetric Attendance.

This result is directly implied by the two following lemmata with their symmetric counterparts.

Lemma 1: *For any distribution of citizens' ideal points, if $c < b$, the equilibrium outcome satisfies $x(X) \in (\theta_L - b, \theta_L + b)$, and there is a $\theta_i < x(X)$ (or a $\theta_i > x(X)$) that is not attending, then he must leave either the same number of attendees at each side or one more attendee to her right (left).*

Proof of Lemma 1:

Suppose that $\theta_i < x(X)$ is not attending, and there are k attendees on her left and $k + l$ attendees on her right with $l > 1$. Let $\theta_1, ..., \theta_l$ denote the first l attendees on the right of θ_i, then $x(X) = median(\theta_1, ..., \theta_l)$.

Suppose that l is odd. Then we have that $x(X) = \theta_{\frac{l+1}{2}}$ and $x(X \cup \theta_i) = x\left(X - \theta_{\frac{l+1}{2}+1}\right) = \max\left\{\frac{\theta_{\frac{l+1}{2}-1} + \theta_{\frac{l+1}{2}}}{2}, \theta_L - b\right\}$. But we need $x(X) - x\left(X - \theta_{\frac{l+1}{2}+1}\right) >$

c because $\theta_{\frac{l+1}{2}+1}$ is attending, and $x(X) - x(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction .

Now suppose that l is even. Then we have that $x(X) = \frac{\theta_{\frac{l}{2}} + \theta_{\frac{l}{2}+1}}{2}$ and $x(X \cup \theta_i) = x\left(X - \theta_{\frac{l}{2}+1}\right) = \max\left\{\theta_{\frac{l}{2}}, \theta_L - b\right\}$. But we need $x(X) - x\left(X - \theta_{\frac{l}{2}+1}\right) > c$ because $\theta_{\frac{l}{2}+1}$ is attending, and $x(X) - x(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction. Thus, we have proved that in equilibrium we must have $l \leq 1$.■

Lemma 2: *For any distribution of citizens' ideal points, if* $c < b$, *the equilibrium outcome satisfies* $x(X) \in (\theta_L - b, \theta_L + b)$, *and there is a* $\theta_i < x(X)$ *that is not attending, then all* θ_j *such that* $\theta_i < \theta_j \leq x(X)$ *are not attending.*

Proof of Lemma 2:

Suppose that $\theta_i < x(X)$ is not attending and there is a θ_j with $\theta_i \leq \theta_j < x(X)$ that is attending. By the previous lemma θ_i must leave either k attendees on both sides or k attendees to his left and $k+1$ attendees to his right.

In the first case, it implies that θ_j must leave k attendees to his left and $k-1$ attendees to his right. Which would imply that $x(X) = median(X) < \theta_j$. A contradiction since $\theta_j < x(X)$.

In the second case, it implies that θ_j must leave k attendees to his left and k attendees to his right. Which would imply that $x(X) = median(X) = \theta_j$. A contradiction since $\theta_j < x(X)$.■

Proof of Proposition 7: Non-participation of the represented.

We will show that if any $\theta_i < x(X) - c$ does not attend the meeting, the equilibrium conditions are not satisfied. A similar reasoning can be used to show the symmetric counterpart for any $\theta_i > x(X) - c$.

Suppose that there is a $\theta_i \notin X$ such that $\theta_L - b < \theta_i < x(X) - c$. His utility is $V_i(x(X), 0) = -|\theta_i - x(X)| = \theta_i - x(X) < -c$. Since he is not attending the meeting, by Proposition 6 there must be half of the attendees' ideal points to his right and half to his left θ_i. If he was to attend he would become the median of the attendees and $x(X \cup \{\theta_i\}) = \theta_i$, thus his utility would be $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c = -c$. Therefore, he would be better off attending, and this is a contradiction.

Now suppose that there is a $\theta_i \notin X$ such that $\theta_i < \theta_L - b < x(X) - c$. His utility is $V_i(x(X), 0) = -|\theta_i - x(X)| = \theta_i - x(X)$ and since he is not attending the meeting, by Proposition 6 he must also have that half of the attendees' ideal points to his right and half to his left. Thus, if he was to attend he would become the median of the attendees and $x(X \cup \{\theta_i\}) = \theta_L - b$. Thus his utility would be $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c =$

$\theta_i - (\theta_L - b) - c > \theta_i - (x(X) - c) - c = \theta_i - x(X)$. Therefore, he would be better off attending, and this is a contradiction.$\blacksquare$

Proof of Proposition 8:

Suppose that $x(X) \in (\theta_L - b, \theta_L + b)$. From Proposition 6 we know that we must have $x(X) = \frac{\theta_l + \theta_r}{2}$ for some θ_l and θ_r that are attending the meeting. Since θ_l and θ_r are attending the meeting, by Proposition 7 we must also have that $\theta_l, \theta_r \notin (x(X) - c, x(X) + c)$.

Since θ_r is attending his utility is $V_r(x(X), 1) = -|\theta_r - x(X)| - c = x(X) - \theta_r - c$. If θ_r was not attending, θ_l would be the median of the attendees, thus $\theta^*(X - \{\theta_r\}) = \theta_l$. Since $\theta_l \leq x(X) - c < \theta_L - b$ we have that the policy that legislator would implement in this case is $x(X - \{\theta_r\}) = \theta_L - b$. Thus his utility in this case would be $V_r(x(X - \{\theta_r\}), 0) = -|\theta_r - \theta_L + b| = \theta_L - b - \theta_r$. Since we assumed that $x(X) - c < \theta_L - b$, we have that θ_r would be better off not attending the meeting, which is a contradiction. Similarly we can prove that in equilibrium we must have $x(X) + c < \theta_L + b$. $\blacksquare$

Proof of Proposition 9: Alignment is needed.

By Proposition 7 we know that in equilibrium all citizens with ideal points to the left of $x(X) - c$ will attend, and by Proposition 8 we know that $\theta_L - b < x(X) - c$. This implies that when $\theta^{**} < \theta_L - b$ we have more than half of the population to the left of $x(X) - c$, and they are all attending the meeting. Combining Propositions 7 and 8 there should be exactly half of the attendees to the left of $x(X) - c$. This is a contradiction. $\blacksquare$

Proof of Proposition 10:

Suppose that $x \in S$, that is, $x = \frac{\theta_r + \theta_l}{2}$ for some θ_l and θ_r such that $|\{\theta_i : \theta_i \leq \theta_l\}| = |\{\theta_i : \theta_i \geq \theta_r\}|$. Observe that if θ_l and θ_r attend the meeting their payoffs are $V_R(x(X), 1) = V_L(x(X), 1) = \frac{\theta_l - \theta_r}{2} - c$. If one of them decides not to attend his payoff is $V_i(x(X - \{\theta_i\}), 0) = \theta_l - \theta_r$ for $i = L, R$. Thus, in order to have both citizens attending in equilibrium we must have $\frac{\theta_l - \theta_r}{2} - c > \theta_l - \theta_r$. Hence we need c to satisfy the following condition $c < \overline{c} = \frac{\theta_r - \theta_l}{2}$. Observe that $\overline{c} < b$. If θ_l and θ_r are attending, then all the other citizens such that either $\theta_i \leq \theta_l$ or $\theta_i \geq \theta_r$ are better off attending.

Next we will show that all citizens such that $\theta_l < \theta_i < \theta_r$, are better off not attending the meeting: Let θ_{l+1} denote the ideal point of the voter next to θ_l on his right and let θ_{r-1} denote the ideal point of the voter next to θ_r on his left. If they are not attending the meeting their payoffs are $V_{L+1}(x(X), 0) = -\left|\theta_{l+1} - \frac{\theta_l + \theta_r}{2}\right|$ and $V_{R-1}(x(X), 0) = -\left|\theta_{r-1} - \frac{\theta_l + \theta_r}{2}\right|$. If one of them decides to attend his payoffs is $V_i(x(X \cup \{\theta_i\}), 1) = -c$ for $i = l+1, r-1$. Thus, in order to have both not attending in equilibrium we must have $\left|\theta_{l+1} - \frac{\theta_l + \theta_r}{2}\right| < c$ and $\left|\theta_{r-1} - \frac{\theta_l + \theta_r}{2}\right| < c$. Hence we need c to satisfy the following condition $c > \underline{c} = \max\left\{\left|\theta_{l+1} - \frac{\theta_l + \theta_r}{2}\right|, \left|\theta_{r-1} - \frac{\theta_l + \theta_r}{2}\right|\right\}$.

Observe that $0 \leq \underline{c} < \overline{c}$. If θ_{l+1} and θ_{r-1} are better off not attending, then so are all the other citizens such that either $\theta_i \in [\theta_{l+1}, \theta_{r-1}]$. Thus, if $c \in (\underline{c}, \overline{c})$ there is an equilibrium with $X = \{\theta_i : \theta_i \leq \theta_l\} \cup \{\theta_i : \theta_i \geq \theta_r\}$ and $x(X) = \frac{\theta_r + \theta_l}{2}$. ■