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Chaos in credit–constrained emerging economies with Leontief technology

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Abstract

This work provides a framework to analyze the role of financial development as a source of endogenous instability in emerging economies subject to moral hazard problems. We study a piecewise linear dynamic model describing a small open economy with a tradable good produced by internationally mobile capital and a country specific production factor, using Leontief technology. We demonstrate that emerging markets could be endogenously unstable when large capital in–flows increase risk and exacerbate asymmetric information problems, according to empirical evidence. Using bifurcation and stability analysis we describe the properties of the system attractors, we assess the plausibility for complex dynamics and we find out that border collision bifurcations can emerge.

Key words: Endogenous instability, emerging economies, moral hazard, complex dynamics, border collision bifurcations, piecewise linear map.

1 Introduction

The facts leading to the financial crisis in the emerging markets of South–East Asia in summer 1997, have shown how a crisis can emerge after a boom in the fundamentals, therefore they open new theoretical approachs to financial crises and a need for new explanations. In the case of emerging markets we are witnessing to a new phenomenon because, differently from past crises (like Mexico 1994 or European Monetary System 1992), such crisis was characterized by:

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Preprint submitted to Elsevier Science

- large capital inflows with borrowing excess [1] in a financial liberalization context,
- fast economic growth driven by fundamentals with poverty reduction (Asian miracle),
- increase in the financial risk assumed without prudential regulation and a financial supervision system. [2]

According to such considerations we think that a model that can explain such *modern* financial crisis must prove that an inversion in the real aggregates with a fall in investment and save, is not only possible but can also appear in an unpredictable and sudden way if creditors face moral hazard problems [3] when the economy goes through financial development. This kind of explanation needs to account for the process leading to crisis so it must be dynamic, comparing states at different times. Furthermore, the unpredictability and the instability of the asymptotic dynamics are explained in terms of chaotic properties of the system attractors.

In this work we present a framework that provides an explanation to these peculiar events according to the balance–sheet view to crises [4], underlying the real aspects of the economic environment and proving that such economies are endogenously unstable. The macroeconomic model here studied is a dynamic, open economy with a tradeable good produced by internationally mobile capital and a country–specific production factor, using the Leontief production function. The two key ingredients of the model are the following.

(1) Firms face credit constraint, in the sense that there is a maximum proportion, μ, of their current wealth level, W_t, the entrepreneurs can borrow from banks, due to moral hazard considerations, as proved in [7]. [5]
(2) The level of financial development reached by the economy appears as an explicit parameter because a high μ represents a well developed financial sector, while a low μ represents an undeveloped one.

Using the discrete dynamic system theory we prove that economies are endogenously unstable when going through a phase of financial development.

[1] The fact that macroeconomic factors, especially a boom in lending, played a key role in the vulnerability of emerging markets to financial crises, has been discussed in [24]. Furthermore in [11], it is stated that in Thailand the boom in lending caused problems in its financial sector.
[2] In Hong Kong and Singapore the development has been accompanied by strong risk supervision and control, so the financial crisis was prevented.
[3] The relevance of asymmetric information problems in such crises has been largely recognized in [14].
[4] Contributions to this line of research are in [1–3,9,10,18].
[5] The fact that the level cash flow of the firm plays an important role in the investment, is widely recognized in [8] and in [16].

In similar works, authors considered that financial constraints on firms due to asymmetric information considerations can play a role in the propagation of the business cycle. For instance in [6] and [19], the authors studied a closed economy and they showed that credit constraints can lead to oscillations.

Differently, considering open economies, in [2] the authors studied a credit constrained model where firms have debt both in domestic and foreign currency, and they prove that the economy can easily suffer a financial crisis. A revised version of this monetary model is offered in a later work of the same authors, [4], where they proved that the existence of nominal price rigidities can lead to multiple equilibria.

While the models in [2,4] focused on the monetary sector, we study a real model of the kind considered in [1,5].

In [5] the authors developed a simple macroeconomic model where the combination between moral hazard problems in capital markets and unequal access to investment opportunities across individuals generate endogenous and permanent fluctuations in aggregate GDP, investment and interest rates. In their work the endogenous cycles are the product of two separate forces: high investment begets high profits and high investment but, at the same time, high investment pushes up interest rates and reduces future profits and investment. We will consider a similar mechanism in which high investment pushes up the price of the constant country–specific factor and reduces future profits and investment. The basic mechanism we describe is a combination of two opposite forces deriving from an increase in the investment level. Firstly, a greater investment leads to grater output and profits. Higher profits improve creditworthiness and fuel borrowing thus leading to grater investment. Simultaneously, this boom increases the demand for country–specific factor and rises its relative price. This rise in input prices leads to lower profits and reduce creditworthiness, borrowing and investment with a subsequent fall in aggregate output. So we will be able to conclude that financial development may destabilize economies that start from an intermediate level of financial development according to the experience documented in a number of countries. [6]

Finally, in [1] the authors studied the model that inspired our work. Basically they developed a dynamic, open, real economy but, without using the discrete dynamic system theory, they are only able to conclude that:

- financial underdeveloped or very developed economies are stable because the fixed point is stable when μ is *high* or *small*;

[6] For example in the years leading up to the crisis of the early 1980's in Southern Cone countries there is evidence that profits in the tradable sector sharply deteriorated due to a rise in domestic input prices. See [13,15,22].

- at an intermediate level of financial development, economies may be unstable because having fixed an intermediate value of μ, the attractor is a two stable cycle;
- more complex dynamics, perhaps chaotic, are likely to emerge;
- shocks have large and persistent effects on the final dynamics.

Studying a discrete dynamic system, we prove Propositions that support or refute their conclusions in the following senses:

- there exist values of μ, we say μ^m and μ^M, depending on the other parameters values, such that economies with very developed, $\mu \geq \mu^M$, or very undeveloped, $\mu < \mu^m$, financial markets have a unique globally structurally stable, fixed point;
- emerging markets, that are economies with $\mu \in [\mu^m, \mu^M)$, could be stable because it does exist a $\mu^\star \in [\mu^m, \mu^M)$ such that the unique fixed point is still globally structurally stable $\forall \mu \in [\mu^m, \mu^\star)$. We also prove that at $\mu = \mu^\star$ the system has an a–typical bifurcation (label border collision [7]) that opens a two-piece chaotic region;
- when entering in the a–periodic region for μ a *slightly bigger* then $\mu^\star$, the chaotic properties of the attractor make the evolution of the system sensitively depending on the initial condition, the dynamics is unpredictable and structurally unstable, so perturbations on the parameters (exogenous shocks) produce large and persistent effects;
- in the chaotic region we observe also periodic windows so that the dynamics are predictable even though the period of the periodic orbit could be so high as to make impossible the distinction between such a cycle and a proper a–periodic orbits;
- when such periodic attractors are hyperbolic, the system is structurally stable so an exogenous shock does not affect the final asymptotic dynamics.

The properties we demonstrate allow us to argue that when going through a phase of financial development, the dynamics shown by the system could drastically change and pass from a s table fixed point to chaotic, aperiodic, unpredictable behaviour.

The endogenous explanation we pursue in this work is consistent with the experience of several emerging markets where the liberalization process has taken place (like South–East Asia) where, as a result of a rapid financial liberalization process, capital in–flowed in large quantities allowing rapid growth in lending and a boom in investment. When large capital inflows is associated with growing imbalances, the crisis came and most of these forces got reversed: capital flowed out, currency collapsed, real estate prices dropped,

[7] About such kinds of non–canonical route to chaos, see [21].

lending stopped and investment collapsed.[8] It is however important to emphasize that the aim of this paper is not to explain exactly what happened in some specific country but rather to propose and study a unified, dynamic, macroeconomic model that awards a central role to financial constraints and financial development.

The paper is organized as follows. In Section 2 we present the model. In Section 3 we study the qualitative dynamics of the model: we prove the global stability of economies with low or high financial development and we assess the plausibility for instability of economies at an intermediate level of financial development. In Section 4 we present numerical simulations that enforce the results we proved in Section 3. We give our conclusions in Section 5.

2 The model

We consider a small open economy with a single tradeable good produced by internationally mobile capital K and a country–specific production factor Z whose price p is expressed in terms of produced goods (so it can be seen as the real exchange rate). We assume that the supply of Z is constant. Let $\alpha \in (0,1)$ be the consumption rate so $(1-\alpha)$ is the constant fraction the consumers save of their own wealth. In such an economy there are two categories of individuals: lenders who lend their wealth at an international equilibrium interest rate $r > 1$ but they cannot invest directly in the production, and borrowers that are the entrepreneurs investing in the production. The total output, in time t, produced in the economy is given by using the following Leontief technology

$$y_t = \min\left\{\frac{K_t}{a}, Z\right\} \tag{1}$$

where $\frac{1}{a} > r$ is the capital productivity.[9]

Asymmetric information considerations generate moral hazard so, according to the results reached in [7], entrepreneurs can borrow at most a proportional amount $\mu \geq 0$ of their own wealth at time t, W_t, that is μW_t. Let L be the amount that entrepreneurs can borrow then, in time t, $L_t \leq \mu W_t$. The parameter μ represents the level of financial development reached by the economy. As a limit case, when $\mu = 0$, entrepreneurs can invest only their own wealth, while the bigger μ is, as the more possibility they have to borrow from the

[8] See [23] for a description of the link between capital–flow reversal and currency crises.

[9] This hypothesis is necessary because otherwise the entrepreneurs have no incentive in investing in the production.

capital market, and so the financial system is well–developed.[10] Since the maximum amount entrepreneurs can borrow from capital markets is fixed, the total investment I_t in each time is upper–bounded by

$$I_t = (1 + \mu)W_t. \tag{2}$$

At each period entrepreneurs maximize their profits and this program determines their optimal demand z_t of the input Z. Given the production function (1) the profit maximization implies that the entrepreneurs' optimal demand of the country–specific factor is $z_t = \frac{K_t}{a}$, where

$$K_t = I_t - p_t z_t \tag{3}$$

that is the difference between the total amount invested and the cost of the production factor demanded. In an equilibrium situation it must be $(I_t - p_t z_t) = a z_t$ so we reach the following equation:

$$I_t - p_t z_t = a z_t. \tag{4}$$

Since Z is constant, each of the following cases can be verified:

$Z > \frac{K_t}{a}$. There is an excess in the supply of Z, so its price (in terms of produced goods) is equal to zero. In this case, production is given by substituting equation (3) in equation (1), placing $p_t = 0$, and considering that the credit constraint (2) holds with equality. So we have that:

$$y_t = \frac{1}{a}(1 + \mu)W_t. \tag{5}$$

$Z \leq \frac{K_t}{a}$. There is an excess in the demand of Z (its price is positive) and production in bounded by $y_t = Z$. In this case we can derive the equilibrium price p_t of the production factor Z considering that both relations (4) and the credit constraint (2) hold and placing $z_t = Z$. Finally we have

$$p_t = \frac{(1 + \mu)W_t - aZ}{Z}. \tag{6}$$

Relation (6) states a positive relationship between p_t and W_t. It depends on the existence of the credit constraint (2) in the sense that grater wealth implies greater investment via credit constraint and so greater demand of Z and, consequently, its price increases.

[10] In [17] it is considered the direct relationship between capital market rules and the level of the financial development due to moral hazard considerations.

Now we can derive the dynamic model describing the economy. Considering that the price of the country specific factor p_t, the investment, I_t, and the production, y_t, are all expressed in terms of entrepreneurs wealth W_t, the dynamic system is the net wealth produced by entrepreneurs and saved by consumers, available for the next period, is given by:

$$W_{t+1} = (1-\alpha)(e + y_t - r\mu W_t) \tag{7}$$

where $e \geq 0$ is an exogenous income. This relation can be understood considering that at time t entrepreneurs borrow, invest, produce and pay their debt $r\mu W_t$ while consumers save. Now we have to take consider the role played by the credit constraint. To do this we need to study three different cases.

(1) If the financial system is well developed, entrepreneurs invest in the production only up to the point in which the productive investment return is equal to the capital market return so:

$$(y_t - r\mu W_t) = rW_t. \tag{8}$$

In this case there is no pure profit, and substituting (8) in the dynamic equation (7) we obtain the following increasing function of the entrepreneurs' wealth

$$W_{t+1} = (1-\alpha)(e + rW_t) \tag{9}$$

that holds for well–developed economies.

(2) If the financial system is underdeveloped the investment, which is constrained, does not absorb the total supply of the country specific factor Z so its relative price is zero. Greater current wealth implies greater investment, and therefore greater production and, because of $p = 0$, grater profits and future wealth. In this case the dynamic system is given by substituting equation (5) in the wealth dynamic equation (7) so we obtain the following increasing function

$$W_{t+1} = (1-\alpha)\left[e + \left(\frac{1+\mu}{a} - r\mu\right) W_t\right] \tag{10}$$

that holds for less–developed economies.

(3) Finally, if the financial system is at an intermediate level of development, the investment absorbs the supply of the country–specific factor Z and, according to the Leontief production function, the production $y_t = Z$. Substituting such equation in the dynamic relation (7), we have the following decreasing function

$$W_{t+1} = (1-\alpha)\left[e + Z - r\mu W_t\right] \tag{11}$$

that holds for intermediate financial developed economies.

Equations (10), (11) and (9) describe the dynamic system for low, intermediate, and high level of financial development of the economy respectively and, similarly, low, intermediate, and high level of entrepreneurs' wealth.

From the previous considerations we derive the map $W_{t+1} = f(W_t)$ that is given by (10) for $W_t \in [0, W^M)$, by (11) for $W_t \in [W^M, W^m)$ and by (9) for $W_t \in [W^m, +\infty)$ where the turning points W^M and W^m are given by:

$$W^M = \frac{Za}{1+\mu} \tag{12}$$

and

$$W^m = \frac{Z}{r(1+\mu)}. \tag{13}$$

The dynamic model we want to study is given by the following continuous first–order piecewise linear map, whose iterates describe the dynamics of entrepreneur wealth we investigate.

$$f(W_t) = \begin{cases} f_1(W_t) = (1-\alpha)\left[e + \left(\frac{1+\mu}{a} - r\mu\right) W_t\right], & 0 \le W_t < W^M; \\ f_2(W_t) = (1-\alpha)\left[e + Z - r\mu W_t\right], & W^M \le W_t < W^m; \\ f_3(W_t) = (1-\alpha)(e + rW_t), & W^m \le W_t, \end{cases} \tag{14}$$

where $\alpha \in (0,1)$, $\mu \ge 0$, $\frac{1}{a} > r > 1$, $e \ge 0$ and $Z > 0$ are the economically plausible definition domains of the parameters.

3 Qualitative dynamics

In this Section we study the qualitative dynamics of the continuous bimodal piecewise linear map given by (14) when varying its parameters. In particular we consider the case of $e > 0$ [11] and $(1-\alpha)r < 1$. [12] It must be remembered that f is increasing on $D_1 = [0, W^M)$ and on $D_3 = [W^m, +\infty)$ while it is decreasing on $D_2 = [W^M, W^m)$. [13] Furthermore its constant slopes in each of

[11] The study of the special case $e = 0$ needs a partially different approach because the system would also have a fixed point at the origin even if it is always unstable.
[12] The hypothesis $(1-\alpha)r < 1$ is economically plausible considering that $\alpha << 1$ while $r = 1 + \epsilon$, ϵ is sufficiently low.
[13] Note that if $\mu = 0$, f_2 is constant on D_2.

such pieces, are respectively given by:

$$f_1'(W_t) = (1-\alpha)\left(\frac{1+\mu}{a} - r\mu\right), \tag{15}$$

$$f_3'(W_t) = (1-\alpha)r \tag{16}$$

and

$$f_2'(W_t) = -(1-\alpha)r\mu. \tag{17}$$

The following Proposition gives sufficient conditions on the parameter μ with respect to the other parameters of the model such that the fixed point lies on each of the three linear pieces of the map (14).

Proposition 1 *Let f given by (14) and let $e > 0$ and $(1-\alpha)r < 1$. Then:*

(a) $\forall \mu \in [0, \mu^m)$, f has a unique positive fixed point $W_1^\star \in D_1$;
(b) $\forall \mu \in [\mu^m, \mu^M)$, f has a unique positive fixed point $W_2^\star \in D_2$;
(c) $\forall \mu \in [\mu^M, +\infty)$, f has a unique positive fixed point $W_3^\star \in D_3$;

where $\mu^m = \frac{Za-(1-\alpha)(Z+e)}{(1-\alpha)(e+Z+Zar)}$, $\mu^M = \frac{Z(1-(1-\alpha)r)}{(1-\alpha)er} - 1$ and $\mu^M > \mu^m > 0$.

PROOF. Let $g(W_t) = f(W_t) - W_t$.

To prove part (a) we consider that $g(0) = (1-\alpha)e > 0$ while $g(W^M) = \frac{\mu(1-\alpha)(e+Z-rZ)+[(1-\alpha)(e+Z)-Za]}{1+\mu}$ where $g(W^M) < 0, \forall \mu < \mu^m$. So f has at least one fixed point in D_1. The uniqueness in D_1 follows because f is linear in D_1 with slope other than one because we are assuming $(1-\alpha)r < 1$. In this case the fixed point is given by

$$W_1^\star = \frac{(1-\alpha)e}{1-(1-\alpha)\left[((1+\mu)/\alpha) - r\mu\right]}. \tag{18}$$

Furthermore it is the unique fixed point of f in $\Re_+$ because f is continuous, f_2 is a decreasing function and f_3 is an increasing function with slope lesser than one for the hypothesis $(1-\alpha)r < 1$.

To prove part (b) we first consider that if $\mu = \mu^m$ then $g(W^M) = 0$ so the existence of the fixed point in D_2 is proven. Otherwise, if $\mu \in (\mu^m, \mu^M)$ then the same arguments we use to prove part (a) show that $g(W^M) > 0$ while $g(W^m) = \frac{\mu[re(1-\alpha)]+[r(1-\alpha)(e+Z)-Z]}{1+\mu}$ where $g(W^m) < 0, \forall \mu \in (\mu^m, \mu^M)$. So f

has at least one fixed point in D_2. The uniqueness in D_2 follows because f is decreasing in D_2. In this case the fixed point is given by

$$W_2^\star = \frac{(1-\alpha)(e+Z)}{1+(1-\alpha)r\mu}. \tag{19}$$

Furthermore it is the unique fixed point of f in $\Re_+$ because f is continuous, f_1 is an increasing function with positive intercept for the hypothesis $e > 0$ and f_3 is an increasing function with slope lesser than one for the hypothesis $(1-\alpha)r < 1$.

To prove part (c) we first consider that if $\mu = \mu^M$ then $g(W^m) = 0$ so the existence of the fixed point in D_3 is proven. Otherwise, if $\mu \in (\mu^M, +\infty)$ then the same arguments we use to prove part (b) show that $g(W^m) > 0$ while it does exist a value of W_t, for instance $\bar{W} = \frac{(1-\alpha)e}{1-(1-\alpha)r}$, where $\bar{W} > W^m$ because $\mu > \mu^M$, such that $g(\bar{W}) < 0$. So f has at least one fixed point in D_3. The uniqueness in D_3 follows because f is linear in D_3 with slope lesser than one for the hypothesis $(1-\alpha)r < 1$. In such a case the fixed point is given by

$$W_3^\star = \frac{(1-\alpha)e}{1-(1-\alpha)r}. \tag{20}$$

Furthermore it is the unique fixed point of f in in $\Re_+$ because f is continuous and piecewise linear and f_1 is an increasing function with positive intercept for the hypothesis $e > 0$. $\square$

The following Proposition states the global stability of economies at high or low financial development levels.

Proposition 2 *Let f given by (14) and let $e > 0$ and $(1-\alpha)r < 1$. Then*

(a) $\forall \mu \in [0, \mu^m)$, $W_1^\star$ is a globally stable fixed point.
(b) $\forall \mu \in [\mu^M, +\infty)$, $W_3^\star$ is a globally stable fixed point.

PROOF. To prove statement (a) we first consider that $\forall \mu \in [0, \mu^m)$, the map f has a unique fixed point $W_1^\star$ such that $W_1^\star \in D_1$, as proved in proposition 1 part (a). Furthermore f is a linear increasing function in D_1 and $f(0) > 0$ (because of $e > 0$)while $f(W^M) < W^M$, so its multiplier, given by (15), $f_1' \in (0,1), \forall W_t \in D_1$. Then the fixed point $W_1^\star \in D_1$ is asintotically stable in D_1. Considering that this case set D_1 is a positively invariant set, then we conclude that $W_1^\star \in D_1$ is globally stable.

To prove statement (b) we first consider that $\forall \mu \in [\mu^M, +\infty)$, the map f has a unique fixed point $W_3^\star \in D_3$ as proved in Proposition 1 part (c). Now we have to consider two different cases. First, if $\mu \in (\mu^M, +\infty)$ then $W_3^\star \in D_3'$ where $D_3' = D_3 \setminus \{W^m\}$. Furthermore its multiplier, given by (16), $f_3' \in (0,1)$ for the hypothesis $(1-\alpha)r < 1$, so the fixed point $W_3^\star$ is asintotically stable in the set D_3'. The global stability depends on the positive invariance of the set D_3'. Secondly, if $\mu = \mu^M$ then $W_3^\star = W^m$ that is a no differentiable point so we cannot calculate its eigenvalue. However W^m is asintotically stable from the right in the sense that $\forall W_0 \in D_3'$ the sequence of the iteratives converges to W^m. Furthermore, as previously said, D_3' is positively invariant. So we conclude that $W_3^\star \in D_3$ is globally stable $\forall \mu \in [\mu^M, +\infty)$. $\square$

As we proved, the dynamics exhibited by economies at high or low levels of financial development are tame: the generic orbit converging to the unique positive fixed point is definitively monotone. Furthermore the economy is structurally stable, because of the hyperbolicity of the fixed point, so its behaviour is predictable.

Now we have to consider the case of $\mu \in [\mu^m, \mu^M)$ that is the case of economies at an intermediate level of financial development. First we consider that in such a case the generic orbit that eventually converges to the fixed point (or to another attractor invariant set) is not monotone. In fact the only fixed point $W_2^\star$ belongs to the decreasing piece of f, that is the set D_2, so every point at the right of $W_2^\star$ is mapped to its left and vice–versa. So, even though the fixed point is stable, the dynamics of the trajectory is definitively oscillating.

The following Proposition proves the stability of economies at an intermediate level of financial development when μ is small enough.

Proposition 3 *Let f given by (14) and let $e > 0$ and $(1-\alpha)r < 1$. Then the fixed point $W_2^\star$ is globally stable $\forall \mu \in [\mu^m, \mu^\star)$ where $\mu^\star = \frac{1}{(1-\alpha)r} \in [\mu^m, \mu^M)$.*

PROOF. As we proved in Proposition 1 Part (b), if $\mu \in [\mu^m, \mu^M)$ then the unique fixed point $W_2^\star$ belongs to D_2 so its multiplier is given by (17) that belongs to $(-1, 0) \Leftrightarrow \mu < \mu^\star$. The fixed point is also globally stable because the set D_2 is positively invariant. $\square$

Now we have to study the case of $\mu \in [\mu^\star, \mu^M)$. A previous consideration is that if $\mu = \mu^\star$ then the unique fixed point $W_2^\star$ is not hyperbolic: its multiplier, given by (17), is in fact $f_2'(W_2^\star) \mid_{\mu=\mu^\star} = -1$. So, when $\mu = \mu^\star$ the map exhibits a bifurcation: its fixed point becomes unstable and we have to identify the new attractor that is eventually born.

Before studying this case we consider that the map f is piecewise linear so it is only piecewise smooth and it can exhibit non–canonical bifurcation phenomena. While in the well–known period–doubling route to chaos when a fixed point becomes unstable we observe a period–two stable orbit, in such a case this does not happen even if we find out a cycle-2 owned by the map as proved in the following Proposition.

Proposition 4 *Let f given by (14) and let $e > 0$ and $(1-\alpha)r < 1$. Let also $\mu = \mu^\star$. Then there exists a period–2 orbit, say O_2, that involves the maximum point, given by (12), that is $O_2 = \{W^M, f(W^M)\}$.*

PROOF. The proof is straightforward as it is only based on the computation that $f^2(W^M) \mid_{\mu=\mu^\star} = W^M \mid_{\mu=\mu^\star}$. $\square$

Once known that f has a cycle–2 involving the maximum point for the bifurcation value of $\mu = \mu^\star$, we are interested in knowing if such invariant set is stable. However, since the map is not differentiable in W^M, we cannot compute its multiplier so we cannot study its stability in the typical way.

The discontinuity in the first derivative of the map implies it can jump without crossing the bifurcation value -1, so an attractor could die without a double–period one being born. Furthermore, as proved in [21], border collision bifurcations are possible so that the map could pass from a stable fixed point to a variety of attractors like a period-m attractor ($m \geq 2$), or a $2m$–piece chaotic attractor, or a m–piece chaotic attractor or finally a *one*–piece chaotic attractor.

In order to study the stability of the cycle–2, O_2, we found out for $\mu = \mu^\star$, we have to consider that the point W^M that belongs to O_2 has no derivative (it has two one–sided derivatives). However the following Proposition proves that the bifurcation parameter value $\mu^\star$ opens a chaotic region.

Proposition 5 *Let f given by (14) and let $e > 0$ and $(1-\alpha)r < 1$. Let also $\mu = \mu^\star$. Then W^M is a Misiurewicz point.* [14]

PROOF. $\forall W_t \in [W^M, f(W^M)]$ we have that $f^2(W_t) = W_t$, as it has been proved by verifying that in such set $f^2(W_t) = f_2 \circ f_2 \mid_{\mu=\mu^\star} = W_t$. So each point in the set $I_2 = [W^M, f(W^M)]$ is a fixed point for the second iterate, f^2. Then each point in I_2, except $W_2^\star$, is involved by a cycle–2 and each of such periodic orbits must be unstable, so also O_2 is a period–two orbit unstable. Because O_2

[14] A pre–periodic point is usually called a Misiurewicz point.

involves the maximum W^M, as we proved in Proposition 4, then the critical point is attracted by an unstable orbit so it is a Misiurewicz point. $\square$

Since the topological entropy at the Misiurewicz point is greater than 1, this signales that we have entered into a (a–periodic) chaotic region.[15] In particular, after the bifurcation occurred at $\mu^\star$, the map is 2-piece chaotic.

Here we cannot prove other results with respect to all the parameters of the system however, since other qualitative dynamics that could eventually emerge strictly depend on the fixed values of the parameters, in the following Section 4 we use the numerical analysis to support the results we derived in this section and we present numerical simulation fixing all the parameters but μ at economically plausible values. In such a way we pursue numerical results about the dynamics exhibited by the model. The quantitative analysis allow us to show the dynamic evolution of the system and to conclude about its properties.

4 Quantitative dynamics

In this Section we provide some numerical simulations by fixing the values of all the parameters of the model but μ. In such a way we are able to prove quantitatively the qualitative results reached in the Section 3 and also to pursue other results that cannot be proved rigorously. Let $\alpha = 0.8$, $r = 1.02$, $a = 0.5$ so $\frac{1}{a} = 2$, $e = 10$ and $Z = 100$[16]. [17]

In Figure 1 we present the scheme of f for different values of μ. As we proved in Proposition 1, the fixed point can belong to each of the pieces of f depending on μ, where $\mu^m \simeq 2.37$ while $\mu^M \simeq 39.02$. [18]

As we proved in Proposition 2 the economy is globally stable for $0 \leq \mu < \mu^m$ and $\mu \geq \mu^M$ and the generic orbit definitively converges monotonically to $W_1^\star$ or $W_3^\star$, as determined in (18) and (20), that are increasing points of f. In fact the Koenigs Lemerary staircase diagram in Figures 2 and 3 shows the converging trajectory for an arbitrary initial condition.

[15] At the pre–periodic point we have no attracting cycles since they cannot capture the critical point, which is pre–periodic.
[16] As it can be proved the chosen value of Z only affects the quantitative dynamics, that is the width of the invariant interval where the dynamics are exhibited, but not the qualitative dynamics, that is the bifurcation sequence occurs at the same parameter values of μ.
[17] We choose these parameter values according to what considered in [1].
[18] We are approximating with an error lesser than 10^{-2}.

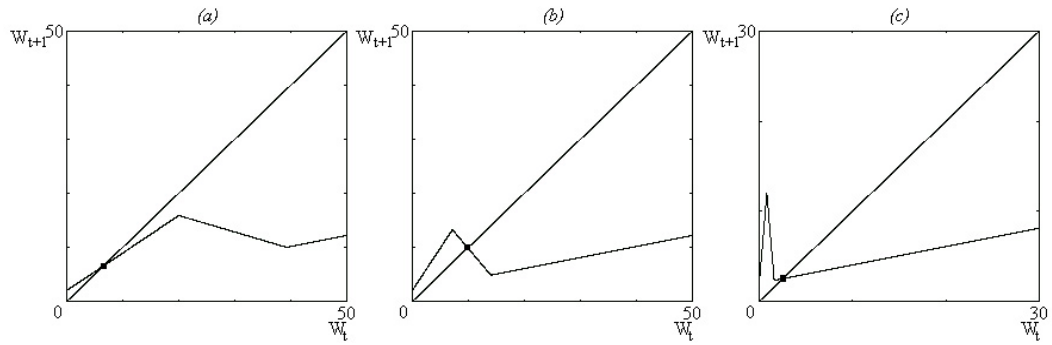

Fig. 1. Scheme of f as defined in (14) in the three cases of localization of the fixed point: in $(a), \mu = 1.5$, in $(b), \mu = 6$ and in $(c), \mu = 60$.

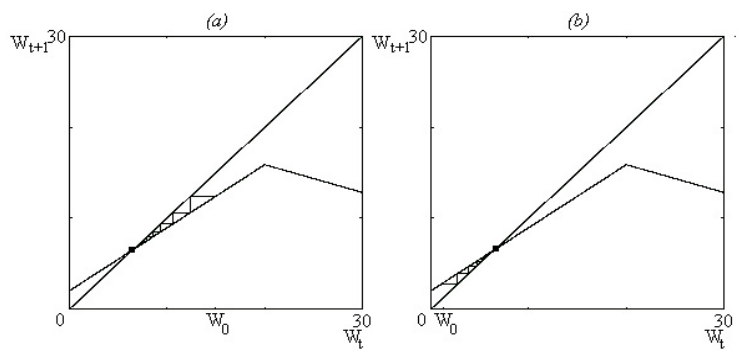

Fig. 2. Koenigs Lemerary staircase diagram for two different initial conditions: in $(a), W_0 = 15 \in (W_1^\star, W^M)$ while in $(b), W_0 = 1 < W_1^\star$. In both cases $\mu = 1.5$.

As we proved in Proposition 1 the fixed point $W_2^\star$ belongs to the decreasing piece when $\mu^m \leq \mu < \mu^M$ that is the case of economies going through a phase of financial development. Furthermore, because of the bimodality of f, there is a stretching and folding action that could generate complex dynamics like cycles of every period and chaos. However, as we stated in Proposition 3, if $\mu < \mu^\star$, where $\mu^\star \simeq 4.9$, the fixed point is still globally stable even if the generic orbit is asymptotically oscillating.

In case $\mu = \mu^\star$ the map exhibits a bifurcation and it gives rise to an infinite number of repelling period–2 cycles. In fact, Propositions 4 and 5 show such evidence. Note that for such value of μ the fixed point is not hyperbolic while each point $W_t \in [W^M, f(W^M)]$ is fixed for the second iterate of f, as it is clear when looking at Figure 4, (b). So all the period–2 orbits are unstable. Furthermore the set $[W^M, f(W^M)]$ is positively invariant so every initial condition will converge to one of such repelling periodic orbit.

Numerical computations also show that all these cycles–2 are of the kind

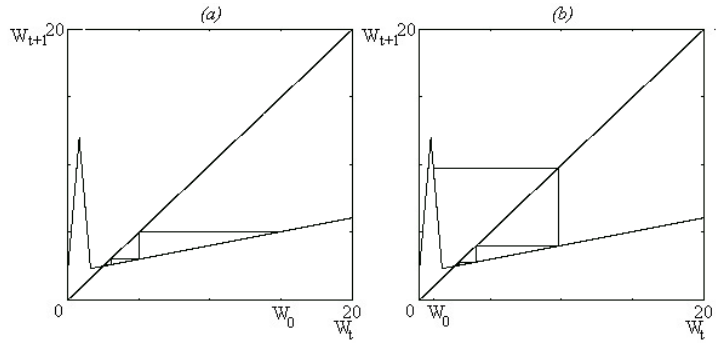

Fig. 3. Koenigs Lemerary staircase diagram for two different initial conditions: in $(a), W_0 = 15 > W_3^\star$ while in $(b), W_0 = 1 < W_3^\star$. In both cases $\mu = 60$.

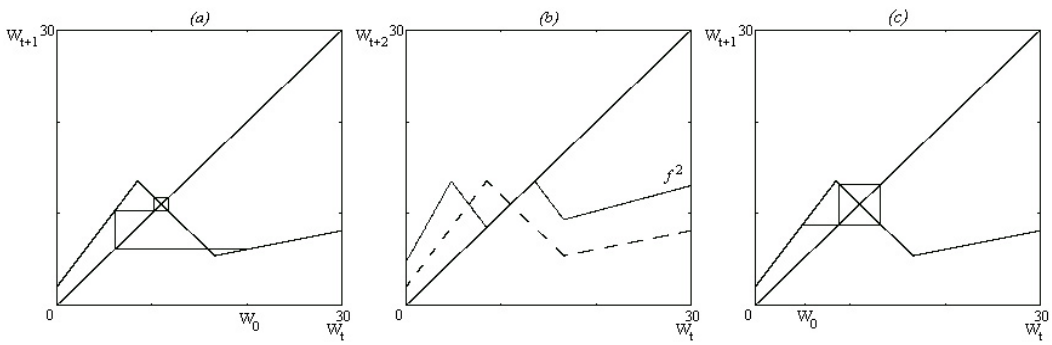

Fig. 4. Repelling period–2 orbits for $\mu \simeq 4.9$. In (a) we have a first cycle two for the initial condition $W_0 = 20$ that is different from the one in (c) for $W_0 = 5$. In (b) the fixed invariant interval of f^2 is illustrated.

$O_2 = \{W_2^\star + \gamma, W_2^\star - \gamma\}, \forall \gamma \in \left(0, \frac{f(W^M) - W^M}{2}\right]$, where $W_2^\star$ is given by (19). Two of such orbits are, for example, the ones in Figure 4 (a) and (c). The bifurcation occurring at $\mu^\star$ is not canonical: one of the repelling cycle–2 involves the maximum point W^M that is a Misiurewicz point. So such bifurcation opens a chaotic region in which the generic orbit covers two disjoint invariant sets. Figure 5 shows the trajectory for an initial condition once the bifurcation has happened and its representation versus time.

As we said, after the bifurcation at $\mu^\star$, the dynamics is suddenly chaotic so the map has the properties of density of the periodic orbits, topological transitivity and sensitively dependence on the initial condition. [19]

The following Figure 6 shows the bifurcation diagram of the map for different values of μ. The black intervals are those in which the dynamics is chaotic or

[19] We are referring to the definition of chaotic set given in [12].

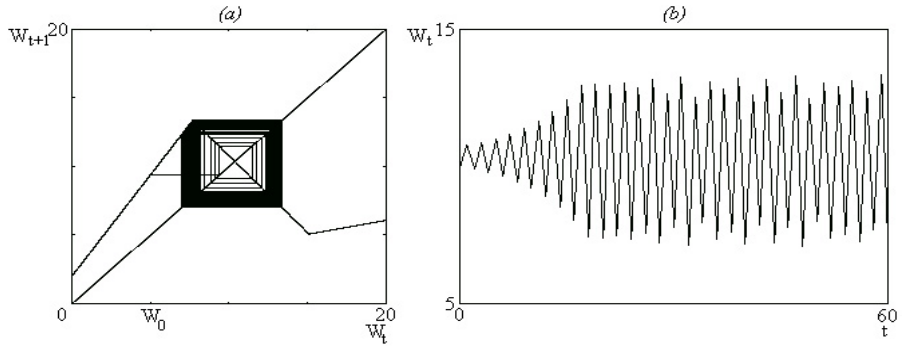

Fig. 5. (a) The generic a–periodic orbit covers two disjoint invariant sets. (b) The trajectory with respect to time. In both cases $\mu = 5.5$ and $W_0 = 5$.

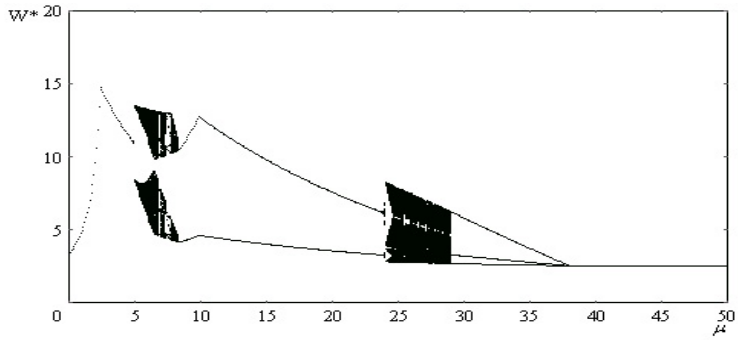

Fig. 6. Bifurcation diagram with respect to μ.

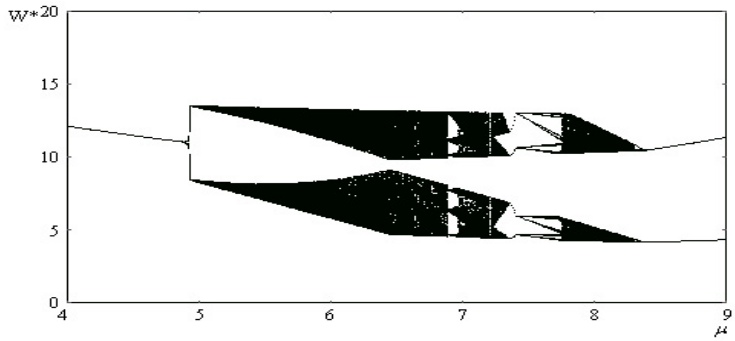

Fig. 7. Bifurcation diagram for $\mu \in (4, 9)$.

periodic with very high period. Furthermore we observe both large intervals of μ where the asymptotic behaviour is a cycle–2 (if $\mu \in (\mu_1, \mu_2)$ with $\mu_1 \simeq 8.39$ and $\mu_2 \simeq 23.5$) or a cycle–3 (if $\mu \in (\mu_1', \mu_2')$ with $\mu_1' \simeq 29.1$ and $\mu_2' \simeq 39.02$) [20]. In this case the dynamics is still predictable even in the long run.

[20] In such a case the chaotic properties could also be proven by the wellknown Li and Yorke Theorem, see [20].

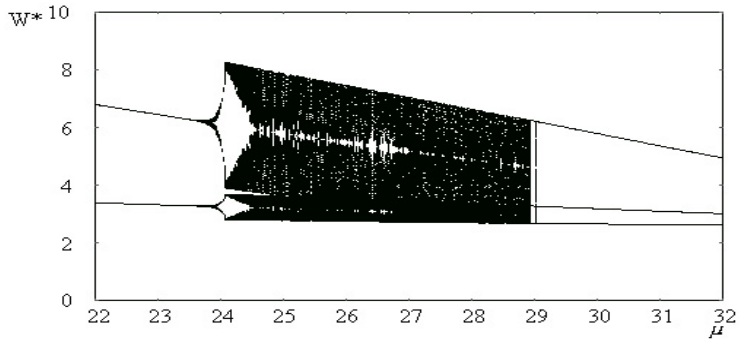

Fig. 8. Bifurcation diagram for $\mu \in (22, 32)$.

However, inside the two chaotic regions, that are visible in the following Figures 7 and 8, the dynamics is both chaotic and periodic with eventually a very high period. So, while in the first case the asymptotic behaviour is not predictable, in the second case it is still predictable even though the attractor could be non–hyperbolic and so the system would be structurally unstable. Some small intervals of μ that are periodic windows inside the chaotic region are visible in such Figures.

5 Conclusions

In this work we studied a piecewise linear dynamic system describing a small open economy where the reached level of financial development plays a central role as a source of endogenous instability.

By analyzing the qualitative dynamics we proved rigorously the global stability of economies at a low or high level of financial development. On the contrary, the economies at an intermediate level of financial development could not converge to the steady state. Consequently we assess the existence of chaotic behavior in the patterns. In this case we have been able to prove by qualitative and also quantitative study the following results.

- Economies at an intermediate level of financial development eventually converge to the fixed point by oscillations or they fluctuate indefinitely.
- They can be unstable but predictable if the attractor is a stable periodic orbit, even with high period, that can also belong to a window in the chaotic region.
- They can be unstable and unpredictable if we are in a proper chaotic region because of the sensitivity to the initial conditions.
- Economies can be structurally unstable when going trough regions governed by different asymptotic dynamics because of the lack of hyperbolicity.

- The bifurcation phenomenon is atypical because of the presence of no differentiable points.

The instability of economies that are financially developing can be understood according to the hypothesis of the model studied. In fact, during a boom, the investment expands and so does the demand for the country–specific factor. It increases its price and pushes down future profits. Less profits lead to less creditworthiness because of the presence of the credit constraint and consequently less investments. Finally, the country–specific factor will not be completely exhausted so its prices will fall down with future high profits and a possible new economic boom.

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