A General, Dynamic, Supply-Response Model

Robert G. Chambers and Ramon E. Lopez

Some Theoretical Issues

This paper is divided into two parts which are somewhat independent. The first part of this paper discusses certain properties of a general autonomous control model that appears promising for the analysis of general dynamic supply response models in agricultural economics, resource economics, and related fields. The second part of the paper, which can be read somewhat independently of the first, emphasizes the potential empirical applications of special cases of the general model discussed in the first part. In what follows, we always deal with continuous time and infinite horizon models because of their analytical tractability. Extension and modification of our results for discrete-time, finite-horizon problems should be fairly obvious and are left to the interested reader.

A General Model

To facilitate exposition we first concentrate our analysis upon the dynamic decision making of an agricultural or resource-based firm operating in a world of perfect certainty. The paper concludes with the discussion of a firm in an uncertain world that deals with expectation formation rationally, i.e., according to some optimization criterion.

The agricultural or resource-based firm is assumed to solve:

$$(1) \quad \underset{u}{\text{Max}} \int_0^\infty e^{-\delta t} [h(x,u) + p(x,\alpha)]\, dt$$

subject to $\dot{x} = m(x,u)$;
$x(o) = \bar{x}$.

Here $X \epsilon R^m$; $u \epsilon R^n$; $\alpha \epsilon R^k$; δ is a time discount rate, and $h(\cdot)$, $p(\cdot)$, $m(\cdot)$ are single valued, twice continuously differentiable functions; $h: R^m \times R^n \rightarrow R$; $p: R^m \times R^k \rightarrow R$; and $m: R^m \times R^n \rightarrow R^m$. In what follows it is often convenient to assume that $h(\cdot)$ and $m(\cdot)$ are concave in all their arguments while p is concave in x and convex in α. However, we shall not employ these assumptions universally.

In standard parlance, therefore, u is a vector of control variables that the firm chooses to maximize its intertemporal objective function subject to the constraints imposed by the equations of motion describing the intertemporal behavior of the state variables, x; and the initial value of the state variables. In various contexts (as will be clear from latter discussion) the vector of controls can be thought of as factors of production, outputs, levels of investment, consumption, etc. Likewise, the state vector is subject to a variety of interpretations including levels of fixed capital stock, crop or livestock inventories, etc.

The optimal value function associated with (1) shall be denoted as $J(x, \alpha)$ and since we deal with an infinite horizon, autonomous problem the associated Hamilton-Jacobi-Bellman recursion relation assumes a particularly tractable form (see e.g. Kamien and Schwartz (p. 242)):

$$(2) \quad \delta J(x,\alpha) = \underset{u}{\text{Max}}\ h(x,u) + p(x,\alpha) + \nabla_x J(x,\alpha)\, m(u,x)$$

where the notation $\nabla_x J(x,\alpha)$ denotes the gradient of $J(x,\alpha)$ with respect to x; all vectors are taken to be conformably defined for multiplication where appropriate so as to avoid unnecessary clutter in notation. In what follows $J(x,\alpha)$ is always assumed to be twice continuously differentiable.

Equations similar to (2) have been the starting point for most of the recent developments in dynamic duality theory (Cooper and McLaren; Epstein) and will provide the basis for much of what follows. The difference between the problem we pose and that posed by, say, Epstein is that we do not restrict $p(\cdot)$ to be affine in either α or x nor do we restrict m(x,u)

Associate Professors of Agricultural and Resource Economics at the University of Maryland. Scientific Article No. A-3996, Contribution No. 6980 of the Maryland Agricultural Experiment Station.

to be linear or affine in its arguments. We do assume, however, that both m(x,u) and $p(x,\alpha)$ are known and well-defined for any x, α, and u.

The starting point for the analysis is to infer the properties of $J(x,\alpha)$ implied by the maximization hypothesis. To proceed, however, it is necessary to introduce the following related problem:

$$(3)\quad h^*(x,u) = \underset{\alpha}{\text{Min}}\ \delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u,x) - p(x,\alpha).$$

If there exists a duality between h(x,u) and $J(x,\alpha)$, $h^*(x,u)$ as derived above will equal the h(x,u) that generated $J(x,\alpha)$ in (1). Contrariwise, if a particular $J^*(x,\alpha)$ generates an $h^*(x,u)$ using (3), then if a duality exists the function derived using $h^*(x,u)$ in (2) is $J^*(x,\alpha)$. In the following there is no attempt to demonstrate a formal duality between $h(\cdot)$ and $J(\cdot)$. Rather, we content ourselves with outlining properties of h(x,u) and $J(x,\alpha)$ that are consistent with a duality. This is particularly important from an empirical perspective, for as with the results of static duality theory, an ability to characterize $J(x,\alpha)$ offers a natural way to proceed in the empirical specification of dynamic response systems.

Assume that all of the curvature conditions previously mentioned for h(x,u) are in force. Then it is immediate that $J(x,\alpha)$ must be convex in α since the maximum value of any function convex in a set of parameters must inherit the convexity property (Dixit). Those of you familiar with standard results from static duality theory might suppose that this exhausts the curvature conditions in α for $J(x,\alpha)$. But as pointed out by Epstein and others, such is patently not the case here. For if one is to be able to solve problem (3) uniquely, i.e., have any hope of recapturing h(x,u), the second-order conditions for the minimization problem should be satisfied. This requires (when evaluated at the optimal controls) that

$$(4)\quad \delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u,x) - p(x,\alpha)$$

be convex in α. Of course, this implies that the Hessian matrix associated with (4) be positive semi-definite. Therefore, the conditions required in the dynamic case are a good bit more restrictive than in the static case. Now apply the envelope theorem to (3) to obtain further that

$$(5)\quad \nabla_x h^*(x,u) = \delta \nabla_x J - \nabla_{xx} J(x,\alpha) m(u,x) - \nabla_x J(x,\alpha) \nabla_x m(u,x) - \nabla_x p(x,\alpha).$$

When evaluated at the optimal controls for expression (2), expression (5) is the familiar vector-valued equation describing the trajectory of the co-state variables (the vector $\nabla_x J(x,\alpha)$) for the standard optimal control problem. If, however, one wants to impose concavity in x on h(x,u) this implies that expression (4) itself must be concave in x when evaluated at the optimal controls.

It may be somewhat difficult in general to ascertain whether $J(x,\alpha)$ actually possesses these properties. The problem is considerably simplified, therefore, when we assume

$$p(x,\alpha) = \alpha' x$$

where now k = m. In this instance, the requisite curvature properties on $J(x,\alpha)$ are that

$$\delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u,x)$$

be convex in α and concave in x.

Assuming that $\alpha' x = p(x,\alpha)$ also enables us to make some general comments about the dynamic stability of the model and how stability impinges on $J(x,\alpha)$ as well as how the state vector varies in the long-run in response to changes in the vector α (see Chambers and Lopez for a more complete discussion). When $p(x,\alpha)$ assumes this particular form applying the envelope theorem to (2) obtains

$$\delta \nabla_\alpha J(x,\alpha) = x + \nabla_{\alpha x} J(x,\alpha)\, \dot{x}^*.$$

This expression which is quite important from an empirical perspective, as latter developments demonstrate, is obviously a nonlinear differential equation in the vector x. Ascertaining its general dynamic properties is very difficult and we content ourselves with an examination of its dynamic behavior in the neighborhood of the steady state, i.e., where $\dot{x}^* = 0$. To do so, it is necessary to assume a well-defined, steady-state solution exists, and we do so without apology.

Now differentiating this expression in the neighborhood of the steady state with respect to x yields

$$\delta \nabla_{\alpha x} J(x,\alpha) = U + \nabla_{\alpha x} J(x,\alpha)\, \nabla_x \dot{x}$$

where U is the identity matrix. Approximating $\dot{x}$ in the neighborhood of the steady state linearly demonstrates that dynamic stability requires that $\delta U - \nabla_{\alpha x} J^{-1}$ be negative semidefinite and that $\nabla_{\alpha x} J$ be positive definite if $\nabla_{\alpha x} J$ is symmetric (see Chambers and Lopez for the non-symmetric case). Thus, under stability and the presumption that $p(x,\alpha) = \alpha' x$:

$$\frac{\partial^2 J}{\partial x_i \partial \alpha_i} \geq 0, \text{ and}$$

$$\frac{\partial^2 J}{\partial x_j \partial \alpha_i} = \frac{\partial^2 J}{\partial x_i \partial \alpha_j},$$

in the neighborhood of the steady state. The shadow price of the ith state variable must be always increasing in α_i and the effect of a change in α_i on the shadow price of the jth state must equal the effect of a change in α_j on the ith state variable's shadow price.

The Flexible Accelerator as an Approximation

Much of the existing empirical literature on dynamic capital adjustment, dynamic supply response, and consumer behavior relies on some version of the flexible accelerator. In its most basic form the flexible accelerator posits that optimal state-adjustment assumes the form

$$(6) \qquad \dot{x}^* = M(x - x^\infty)$$

where superscript (*) is used to denote optimality; $M \in R^m \times R^m$ the elements of which are now presumed constant; and x^∞ represents some long-run desired value of the state vector. Examples of such models are the Nerlovian partial-adjustment, supply-response model, the habit formation model of dynamic consumer behavior, and the multivariate flexible accelerator investment model of Nadiri and Rosen.

Because of its ubiquitous nature much has been made of the discovery of theoretical models that generate something like (6) as an approximation or as an exact representation of optimal state adjustment. For example, there is a rather long line of papers (Eisner and Strotz; Lucas; Treadway; Mortensen) examining the interrelationships between the flexible accelerator representation of capital stock adjustment and the adjustment cost hypothesis formulated by Edith Penrose.

Recently, Steigum has demonstrated that a simple growth model at the firm level where the firm faces constraints on the rate at which it can borrow also rationalizes the flexible accelerator as an approximation for optimal investment plans. With perfect hindsight, however, this and the other observations are rather obvious and can be easily seen in the context of the general model formulated in (1). The argument starts with the identification of x^∞ with the steady-state value of x (and the presumption x^∞ exists). Optimal state-adjustment assumes the general form

$$(7) \qquad \dot{x}^* = m(u^*,x)$$

where u^* is the vector of optimal controls expressed as a function of x and α, i.e., $u^* = u(x,\alpha)$. Expanding (7) in a Taylor-series around the steady state (recall $m(u^*,x^\infty) = 0$) yields

$$\dot{x}^* = d_x m(u^*,x^\infty)(x - x^\infty) + \tfrac{1}{2} d_{xx}{}^2 m(u^*,x^\infty)(x - x^\infty)^2 + \ldots$$

where d_x represents the total derivative with respect to x. To a first order then

$$(8) \quad \dot{x} = d_x m(u^*,x^\infty)(x - x^\infty) = [\nabla_u m(u^*,x^\infty)\nabla_x u(x,\alpha) + \nabla_x m(u^*,x^\infty)][x - x^\infty].$$

Expression (8) itself can be approximated by expression (6) where $M = \nabla_u m(u^*,x^\infty) \nabla_x u(x,\alpha) + \nabla_x m(u^*,x^\infty)$.

By (8), therefore, the ability of the general model, and therefore of the adjustment cost and other models which are special cases of (1), to rationalize the flexible accelerator lies in either the nonlinearity of $m(\cdot)$ in x or in the effect of x on the optimal control. In general, therefore, it seems obvious that a wide variety of models will be capable of generating the flexible accelerator as an approximation to the optimal, state-adjustment mechanism. Cases where the flexible accelerator is not an appropriate approximation seem to be the exception rather than the rule. Cases when the flexible accelerator is an exact representation are the subject of the next section.

Exact Flexible Accelerators

In this section we briefly examine a set of conditions under which the flexible accelerator described by (6) is an exact representation of optimal state-adjustment. In what follows, attention is only given to the case where M is a constant matrix independent of α. This greatly simplifies the problem. However, there is an ever-growing body of literature that utilizes a generalization of (6) where M is a matrix of numeric functions of δ and α. Space does not permit a detailed treatment of this issue but a related paper by Chambers and Lopez covers it in detail.

In what follows, it is convenient to assume that m = n and further that $\nabla_u m(u,x)^{-1}$ exists everywhere in the domain of $m(\cdot)$. By the implicit function theorem it is then possible to solve $\dot{x} = m(u,x)$ for u in terms of x and $\dot{x}$ to get $u = g(x,\dot{x})$. Furthermore, also assume that $m = k$ and that $p(x,\alpha) = \alpha' x$. With these assumptions in hand rewrite (1) as

$$\text{Max} \int_0^\infty e^{-\delta t}[h(g(x,\dot{x}),x) + \alpha' x]dt,$$
$$\text{s.t. } x(o) = \bar{x},$$

which is identical to the calculus of variations problem considered by Treadway in his classic treatment of the ability of the adjustment cost model to generate the flexible accelerator as an exact representation of optimal state adjustment. The problem posed by Treadway was

$$\text{(9)} \qquad \text{Max} \int_0^\infty e^{-\delta t}[f(x,\dot{x}) + \alpha' x]dt$$
$$\text{s.t. } x(o) = \bar{x}.$$

Treadway has demonstrated that for the problem described by (9) one must have

$$f(x,\dot{x}) = \phi(x - M^{-1}\dot{x}) + b\dot{x}$$

where ϕ is a strictly concave function and b is a vector of constants if the solution is to be consistent with the flexible accelerator in (6). In terms of our model it is now immediate that when $m = n = k$ and $p(x,\alpha) = \alpha' x$ that (1) is consistent with (6) when

$$\text{(10)} \quad h(x,u) = \phi(x - M^{-1}m(u,x)) + bm(u,x).$$

Therefore, the flexible accelerator is an exact representation of the optimal state adjustment of the model if the instantaneous value function $h(\cdot)$ can be written as the combination of the linear sums of the equations of motion and a concave function of the difference between the current value of the state vector and the product of M^{-1} and the equation of motion. The most important thing to realize about this result is that it means that in utilizing the flexible accelerator we are restricting ourselves to cases where the instantaneous value function is directly expressible as a function of the equation of motion. Hence, there are no exhaustive conditions for h(x,u), independent of m(x,u), that imply an exact flexible accelerator.

The fact that the flexible accelerator can only be generated by a very specific objective function suggests that the optimal value functions consistent with the flexible accelerator will assume a very special form. This is easiest to see in the case where $k = m = n = 1$; the extension to other dimensions is obvious (see Chambers and Lopez for the more general case). The Hamiltonian can be written using (10) in this instance as

$$\begin{aligned} H &= h(x,u) + \nabla_x J(x,\alpha)\, m(u,x) \\ &= \phi(x - m^{-1}m(u,x)) + bm(u,x) \\ &\qquad + \nabla_x J(x,\alpha)\, m(u,x) \end{aligned}$$

By the maximum principle, optimality requires

$$\frac{\partial H}{\partial u} = -\phi'(z)M^{-1}\frac{\partial m}{\partial u} + \frac{\partial J}{\partial x}\frac{\partial m}{\partial u} + b\frac{\partial m}{\partial u} = 0;$$

$$\frac{\partial J(x,\alpha)}{\partial x} = \left(\phi'(z)M^{-1}\frac{\partial m}{\partial u} - b\frac{\partial m}{\partial u}\right)\left(\frac{\partial m}{\partial u}\right)^{-1} = \phi'(z)M^{-1} - b$$

where $z = x - M^{-1}m(x,u)$. Recognizing that $z = x^\infty$ when evaluated at the optimum gives

$$\frac{\partial J(x,\alpha)}{\partial x} = \frac{\phi'(x^\infty)}{M} - b$$

so that $\partial^2 J(x,\alpha)/\partial x^2 = 0$. Therefore, if h(x,u) assumes the form of (10) which implies an optimal state adjustment equation of the form of (6) one must be able to write

$$\text{(11)} \qquad J(x,\alpha) = \phi(\alpha)\, x + \theta(\alpha)$$

where $\phi(\alpha)$ and $\theta(\alpha)$ are numeric functions of α. Earlier arguments suggest that they be convex in α if $J(x,\alpha)$ is to be well behaved in the sense of being able to generate the original h(x,u) via (3). Perhaps the most important thing about this result is that it implies that the shadow value of the state variable is independent of the level of the state variable.

Consistent Aggregation in Dynamic Models

The previous analysis considers the optimization decisions of a single economic entity. Most empirical studies, however, consider aggregate rather than firm decisions. Since in dynamic models the initial level of the state variables are often arbitrarily allocated across firms, we need to look at the aggregation problem. The basic problem is to determine the conditions under which there exists a consistent aggregate or industry, optimal-value function which only depends on the aggregate level of the state variables and not on their distribution across firms. It is desirable that the aggregate J(·) function satisfy the same restrictions as these of the micro functions. More formally the aggregation problem is that of elucidating the restrictions required for

$$(12)\quad \text{(i)}\ J(x,\alpha) = \Sigma_h J^h(x^h,\alpha),\ \text{and}$$
$$\text{(ii)}\quad x = \Sigma_h x^h,$$

where J^h are the micro or firm-level optimal value functions; x^h is the state vector for firm h; J is now taken to be the aggregate optimal value function; and x is the aggregate state vector. In what follows it is easiest to think of x as a scalar although the logic for the vector-valued case is identical.

Differentiating 12(i) with respect to x^h gives

$$(13)\quad \frac{\partial J}{\partial x}\frac{\partial x}{\partial x^h} = \frac{\partial J}{\partial x} = \frac{\partial J^h(x^h,\alpha)}{\partial x^h}.$$

That is, the marginal effect of the state variable on the optimal value function of each firm should be identical and equal to the marginal effect of aggregate x on the aggregate optimal value function. Since the level of x^h varies across firms (13) can only be satisfied if $\frac{\partial J^h}{\partial x^h}$ is independent of x^h for all h = 1, . . . M. That is, the firms' micro functions should be affine in x^h:

$$(14)\quad J^h(x^h,\alpha) = x^h\phi(\alpha) + \theta^h(\alpha) \qquad \forall\ h = 1, \ldots M$$

and, therefore,

$$(15)\quad J = \Sigma_h J^h = \phi(\alpha)\ x + \theta(\alpha)$$

where $\theta(\alpha) \equiv \Sigma_h\theta^h(\alpha)$ (see also Epstein and Denny). The aggregate optimal-value function should also be affine in the aggregate state variable. The structures of (14) and (15) imply that the aggregate optimal-value function is independent of the distribution of the state variable across firms. Moreover, it also implies that both the firm-level and aggregate-level, state adjustment are consistent with a generalized, flexible accelerator where the adjustment matrix depends upon α.

Blackorby and Schworm have specified slightly weaker aggregation conditions than (12). Their aggregation conditions are:

$$(16)\quad \text{(i)}\ J(x,\alpha) = \Sigma_h J^h(x^h,\alpha)$$
$$\text{(ii)}\quad x = x(x^1, x^2, \text{—}, x^M)$$

That is, instead of requiring x to be the sum of all the firm-level state variables, they only require that there exist some function x(·) of all the x^h. The aggregate x then corresponds to a representative level of the state variable rather than to the sum of the states (Muellbauer).

Differentiating (16(i)) with respect to x^h and x^k, using (16(ii)) and taking the ratios of those derivatives:

$$(17)\quad \frac{\partial J/\partial x^h}{\partial J/\partial x^k} = \frac{\frac{\partial J}{\partial x}\frac{\partial x}{\partial x^h}}{\frac{\partial J}{\partial x}\frac{\partial x}{\partial x^k}} = \frac{\partial x/\partial x^h}{\partial x/\partial x^k}$$
$$= \frac{\partial J^h(x^h,\alpha)/\partial x^h}{\partial J^k(x^k,\alpha)/\partial x^k} = \Psi(x^h,x^k) \qquad \forall h,k$$

Therefore, X(·) must be strongly separable (Chambers and Lopez);

$$(18)\quad x = F(\Sigma_j\beta_j(x^j)).$$

Now, differentiate J with respect to x^h using (18):

$$(19)\quad \frac{\partial J}{\partial x^h} = \frac{\partial J(x,\alpha)}{\partial x}\ F'(\cdot)\cdot\beta'_h(x^h)$$
$$= \frac{\partial J^h}{\partial x^h}(x^h,\alpha)\quad \forall\ h = 1, \ldots M$$

The expression in between the equality signs depends only on x^h and α and is independent of all other x^k ($k \neq h$) and of x. There exist infinite combinations of functions F′ (·) and $\frac{\partial J}{\partial x}$ which satisfy this restriction. One polar case is when F′ is required to equal one, i.e., when $x = \Sigma_j\beta_j(x_j)$. Blackorby and Schworm impose this restriction implicitly without comment or proof. In this case (19) is satisfied if and only if $\frac{\partial J}{\partial x}$ is independent of x, i.e., if J is affine in x. Using (19) in this instance gives

$$\frac{\partial J^h}{\partial x^h} = \phi(\alpha)\ \beta'_h(x_h),$$

and integration implies

$$J^h = \beta_h(x_h)\ \phi(\alpha) + \theta^h(\alpha).$$

That is, the same aggregate J functional structure as with linear aggregation but the J^h functions are more general. Note, however, that since $x = \Sigma_j\beta_j(x_j)$ one must know the $\beta_j(\cdot)$ functions in order to aggregate. Since in many empirical applications one only has aggregate data and not firm data, this aggregation rule does not seem to be very useful for empirical studies.

An interesting aspect of this type of aggregation is that the appropriate specification for J(·) hinges critically upon the specification of F(·). To see this differentiate (19) with respect to x^m to get

$$\frac{\partial^2 J(x,\alpha)}{\partial x^2} F'(\cdot)\beta'_h(x^h)F'(\cdot)\beta'_m(x^m) + \frac{\partial J}{\partial x} F''(\cdot)\beta'_h(x^h)\beta'_m(x^m) = 0$$

which implies that

$$(20)\quad \frac{\partial^2 J(x,\alpha)/\partial x^2}{\partial J(x,\alpha)/\partial x} = -\frac{F''(\cdot)}{F'(\cdot)F'(\cdot)}$$

Clearly, there will exist an infinity of J functions of the general type $J = n(x)\,\phi(\alpha) + \Theta(\alpha)$ that satisfy (20) where

$$\frac{n''(x)}{n'(x)} = -\frac{F''(\cdot)}{F'(\cdot)F'(\cdot)}$$

Unless further restrictions are placed on either F or J, this type of aggregation is, therefore, really quite empty from an empirical perspective. Of course, such results are not peculiar to the dynamic model being considered. Muellbauer finds a similar problem in the static consumer case where it is resolved by imposing homogeneity conditions that are the result of the maximization postulate. When similar restrictions are available, aggregation restrictions of the type of (16) will have much more empirical content.

Intertemporal Duality and the Estimation of Dynamic Production Decisions

This section illustrates the use of duality theory in the derivation of structural expressions for optimal decisions. As might be realized from the previous section, obtaining an explicit solution for even the simplest intertemporal optimization problem is extremely difficult. Fortunately, as in static optimization one may use duality theory in characterizing optimal value functions (OVF) and in directly deriving the optimal behavioral equations by relatively simple manipulation of a well behaved OVF. The major advantage of duality is that one can postulate a relatively complex, say, production technology and at the same time derive by simple methods the associated behavioral equations.

A second objective of this section is to illustrate the use of duality theory in the derivation of empirical models for various types of dynamic models. Modelling supply responses when there exist adjustment costs associated with investments in capital and other factors is briefly considered. Next we briefly review two models considering other sources of slow production adjustments: financial constraints on investment and biological constraints on the harvesting of a natural resource.

Generalized Envelope Relations

Consider a general problem such as (1). Assume that $p(x,\alpha)$ is linear, i.e., $p = \alpha' x$ and that $x > 0$. Under the assumed regularity conditions on $h(x,u)$ and $m(x,u)$, it follows that $J(x,\alpha)$ should satisfy the following properties:

(21)
1. $J(x,\alpha)$ is convex in α;
2. $\delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u^*,x)$ is convex in α;
3. $\delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u^*,x)$ is concave in x;
4. $\delta J(x,\alpha) - \nabla_x J(x,\alpha)\, m(u^*,x)$ is nondecreasing in α; and
5. the Hessian of $J(x,\alpha)$ is symmetric in α and in x.

Properties 1 to 3 were discussed in detail above. Property 4 follows from the nonnegativity of the state variables, and property 5 is a consequence of the assumption of twice continuous differentiability of J(·) in x and α.

Differentiating (2) with respect to α, using the envelope theorem obtains:

$$(22)\quad \delta\nabla_\alpha J(x,\alpha) = x + \nabla_{x\alpha} J(x,\alpha)\dot{x}^*$$

Equation (22), as noted above, is a system of differential equations (recall that α and x are vectors of dimension m). One can, therefore, solve:

$$(23)\quad \dot{x}^* = \nabla_{\alpha x} J(x,\alpha)^{-1}[\delta\nabla_\alpha J(x,\alpha) - x].$$

Thus, (23) is a generalized version of Shephard's lemma expressing the optimal state adjustments for the firm as a function of the exogenous variables of the system (α and x). Since each $\dot{x}^*_1$ is derived from a $J(x,\alpha)$ with known properties, one can either test or *a priori* impose the properties of $J(x,\alpha)$ summarized in (21) on the estimating system (23). If in addition, one wants to insure that the estimated system is dynamically stable in the neighborhood of the steady state, one can use the restrictions earlier derived on $\nabla_{\alpha x}J$ to that effect. Needless to say, the functions $\dot{x}^*_i$ are related to each other in a systematic manner which can be imposed in estimation. The long-run or steady-state level of the state variables x can also be derived from (22) as

$$\delta J_\alpha(x^\infty,\alpha) - x^\infty = 0. \tag{24}$$

Thus, as in the static case, by postulating an appropriate functional form for $J(x,\alpha)$ one can use (23) to derive estimating equations for the short-run optimal decisions of the firm as well as to obtain a characterization of the steady state.

The Adjustment Cost Model

In the adjustment-cost model, the firm is presumed to incur adjustment costs when it varies the level of certain inputs. These costs are typically assumed increasing and convex in the level of investment per unit of time. By impeding instantaneous adjustment, this limits the growth rate of the firm. That is, the limits to growth are entirely determined by internal properties of the firm, and there need exist no other binding constraints, such as the availability of investment funds, limiting the growth capacity of the firm.

For the most general adjustment-cost model one can write the production function as

$$Q = Q(\tilde{x},x,I)$$

where Q is output; $\tilde{x}$ is a vector of fully variable inputs; x is a vector of quasi-fixed inputs; and I is a vector of investments in quasi-fixed inputs. Instantaneous variable profits are

$$g(p,v;x,I) \equiv \max_{\tilde{x}} \{p\, Q(\tilde{x},x,I) - v\tilde{x}\}$$

where p is the output price, and v is a vector of variable input prices. The intertemporal profit maximizing problem of the firm is:

$$J(p,v,\alpha,x) \equiv \max_I \int_0^\infty \{g(p,v;x,I) - \alpha x\}e^{-\delta t}dt \tag{25}$$

$$\text{s.t.}\quad \dot{x} = I - \gamma x,\ x(o) = \bar{x}_o,$$

where α is now a vector of rental prices of quasi-fixed inputs and γ is a diagonal matrix with non-negative depreciation rates along the diagonal. Hence, the rate of depreciation is presumed exogenous and constant for each input.

Note that p, v, and α should all be time indexed. That is, their values are equal to current actual values for $t = 0$ but for $t > 0$ they correspond to *expected* values. For the time being we assume static expectations, i.e., expected prices are equal to current prices. Taylor recently pointed out some serious defects with such an assumption. We reconsider expectation problems in a later section. Clearly (25) is a special case of (1) where the function $m(\cdot)$ is linear and equal to $I - \gamma x$. We can, therefore, easily specialize our previous discussion in deriving and characterizing estimating behavioral equations of the firm. The Hamilton-Jacobi equation is (Epstein):

$$\delta J(p,v,\alpha,x) = \max_I g(p,v;x,I) - \alpha x + \nabla_x J(\cdot)\, \dot{x}. \tag{26}$$

Using the envelope theorem:

$$\text{(27)}\quad \begin{array}{ll} \text{(i)} & \dot{x} = \nabla_{\alpha x}{}^{-1} J\, [\delta\nabla_\alpha J + x]; \\ \text{(ii)} & -\tilde{x} = \delta\nabla_v J - \nabla_{vx} J\, \dot{x};\ \text{and} \\ \text{(iii)} & Q = \delta\nabla_p J - \nabla_{px} J\, \dot{x}; \end{array}$$

where $\tilde{x}$ is the vector of variable inputs, and Q is output supply. In deriving 27(ii) and 27(iii) we have used Hotelling's lemma. Using 27(i) in 27(ii) and 27(iii) one obtains the set of output supply, input demand, and investment equations in terms of the exogenous variables p,v,α and x. The properties of the $J(\cdot)$ function to be used in the empirical analysis are those outlined in earlier sections.

What are the implications of consistent aggregation for the adjustment cost model? The most important consequence of specifying a consistent, aggregate, dynamic model is that the shadow prices of the state variables (J_x) are constant throughout time. In the case of the adjustment cost model, for example,

$$\dot{J}_x = J_{xx}\, \dot{x} = 0$$

since a consistent aggregate J function is affine in x, i.e., $J_{xx} = 0$, irregardless of whether $\dot{x} \equiv 0$. Consider now the first-order conditions associated with this adjustment-cost problem. Using the maximum principle (at time zero):

$$\text{(28)}\quad \begin{array}{ll} \text{(i)} & g_I + q = 0 \\ \text{(ii)} & \dot{q} = (\delta + \gamma)\, q + \alpha - g_x \\ \text{(iii)} & \dot{x} = I - \gamma_x \\ \text{(iv)} & \lim_{t\to\infty} e^{-\delta t}qx = 0 \end{array}$$

where $q = J_x$. If the conditions for consistent aggregation are used, it can be shown (Chambers and Lopez) in the case of one state and one control that

$$g_{II}g_{xx} - g_{Ix}^2 = 0$$

which in turn implies that $g(\cdot)$ cannot be strictly concave in I and x. To estimate consistent aggregate supply, factor demand, and investment responses one must presume, therefore, that the primal profit function is not strictly concave in I and x and that it meets these restrictions. Furthermore, using 28(i):

$$\nabla_x I = -\nabla_{II}^{-1} g(\cdot) \cdot \nabla_{Ix} g(\cdot).$$

Stability requires that the matrix $\nabla_x I$ be negative. In the case of one control and one state this implies that $\partial^2 g/\partial x \partial I$ be negative.

And $J(\cdot)$ satisfying consistency in aggregation can be written as

$$J(p,v,\alpha,x) = x\, \phi(p,v,\alpha) + \theta(p,v,\alpha)$$

Therefore, using (27) the aggregate behavioral equations are:

$$\begin{aligned}
&\text{(i)} && \dot{x} = M(x - x^{\infty});\\
(29)\quad &\text{(ii)} && \tilde{x} = \phi_v(p,v,\alpha) \cdot M(x - x^{\infty})\\
& && \quad + \delta\theta_v(p,v,\alpha) - \delta\phi_v(p,v,\alpha)\, x\\
&\text{(iii)} && Q = -\phi_p(p,v,\alpha) \cdot M(x - x^{\infty})\\
& && \quad + \delta\theta_p(p,v,\alpha) + \delta\phi_p(p,v,\alpha)\, x;
\end{aligned}$$

where $[\delta u + \phi_v^{-1}]$ are the adjustment functions, and $x^{\infty} \equiv [U + \delta\phi_\alpha]^{-1}\, \theta_\alpha$ is the steady-state level of the quasi-fixed factors. Notice that the optimal investment functions are expressed in terms of a generalized flexible accelerator where the adjustment functions M are independent of x. Moreover, the variable factor demand equations ($\tilde{x}$) and the output supply response equations are affine in the state variables x.

So far we have assumed non-separable, adjustment costs. Many empirical research efforts, however, have used separable adjustment cost functions. What are the implications of imposing consistent aggregation on models assuming separable adjustment costs? Separable adjustment costs imply that $g(\cdot)$ can be written:

$$(30)\qquad g(p,v;x,I) = A(p,v;x) + C(I),$$

where $A(\cdot)$ satisfies all the properties of a variable profit function and is strictly concave in x, and C(I) is an increasing, strictly concave function. If (30) holds, the first order conditions 22(i) and 22(ii) can be written:

$$\begin{aligned}
(31)\quad &\text{(i)} && C_I(I) + q = 0; \text{ and}\\
&\text{(ii)} && \dot{q} = 0 = (\delta + \gamma)\, q + \alpha\\
& && \quad - A_x(p,v;x).
\end{aligned}$$

These equations imply that the system must always be in a steady state with $I = \gamma x^{\infty}$ where x^{∞} is the solution to 31(ii) (Chambers and Lopez). Therefore, the model is truly static since the dynamic forces vanish. This implies that consistent aggregation and separable adjustment costs are inconsistent hypotheses. There does not exist a meaningful aggregation rule when adjustment costs are separable.

Financial Constraints Models

For these models the factors limiting the growth capacity of a firm are attributed to the existence of financial constraints rather than adjustment costs (Steigum; Shalit and Schmitz; and Chambers and Lopez). Firms are presumed unable to borrow unlimited amounts of funds at constant interest rates, either because the interest rate a firm must pay increases with the debt/equity ratio or simply because there is a maximum amount of debt-per-dollar of equity that financial institutions are willing to accept. The borrowing capacity imposes a ceiling on investment. Moreover, it is assumed that this ceiling is binding. The financial constraint models are, in a sense, opposite to adjustment cost models. The former assumes that investment depends on the ability of the firm to obtain the necessary funds to finance its investment desires as well as on its own wealth or equity levels, while the latter assumes that firms' investments are only limited by the adjustments costs which a firm must accept when it expands.

Another important feature of models emphasizing financial constraints is that they require a simultaneous modelling of both the farmer production decisions and the farmer-household utility maximizing decisions (consumption, labor supply and savings). This is because the farmer's level of wealth determines his investment capacity, and the level of wealth, in turn, is closely related to the savings capacity of a farmer. Therefore, there exists a close linkage between the farmer's capacity and willingness to save and the level of farm production investment that he can afford. Farmers who have performed better in the past and, at the same time, who have been willing to consume less are now in better shape to expand their farm enterprise than those who have performed poorly in the past and/or have not been willing to save as much.

The intertemporal model of the farm-household facing financial constraints is (see Chambers and Lopez for details):

$$J(p,w,v,E,y) = \max_{u,l} \int_0^\infty u(c,l)e^{-\delta t}\,dt;$$

$$(32)\quad \text{s.t. (i) } \dot{E} = \rho(E,w,v,r) + w\,(H - l) - pc + y;$$

$$\text{(ii) } E(o) = \bar{E}_0;$$

where c is consumption; l is leisure; H is total time available for leisure and on-farm and off-farm work; w is the off-farm wage rate or opportunity cost of on-farm work; p is now a price index of consumption foods; v is a vector of output and input prices; y is fixed non-labor, non-farm income; E is the level of wealth or equity of the household; $u(\cdot)$ is a concave farm-household utility function; and $\rho(\cdot)$ is a farm-income function defined by:

$$\rho(E,w,v,r) \equiv \max_{K,L_1} \{\pi(v,K,L_1) - wL_1 - r(K - E)\colon K \leq B(E) + E\};$$

where $\pi(\cdot)$ is a farm variable profit function; K is the farm capital stock; L_1 is on-farm work by the farmer; r is the rate of interest on the farmer's debt; and B(E) is the maximum debt of a farmer as an increasing function of his/her wealth level E. Assume that $B'(E) > 0$ and $B''(E) < 0$ and that the constraint in (32) is binding, i.e., $K = B(E) + E$. Moreover, since $\pi(\cdot)$ is increasing and concave in K, $\rho(\cdot)$ is also increasing and concave in E.

Note that the only thing impeding instantaneous adjustment of the farm capital stock is the financial constraint dictating the maximum amount of indebtedness which financial institutions allow. Also, the input-demand functions and output-supply functions *conditional* on a given level of equity E can be obtained by differentiating $\rho(\cdot)$:

$$\rho_v = \pi_v = Q;\ \rho_w = -L_1;$$

where Q is a vector of net outputs conditional on E. The debt function B(E) can be recovered from ρ by differentiating with respect to r:

$$\rho_r = -B(E), \qquad \rho_{rE} = -B'$$

Intertemporal output and input adjustments are determined by the motion of E,

$$(33)\quad \begin{aligned} \dot{Q} &= \rho_{vE}\dot{E}^*;\ \text{and} \\ \dot{K} &= (1 + B'(E))\,\dot{E}^* = (1 - \rho_{rE})\,E^*. \end{aligned}$$

where $\dot{E}^*$ is the solution of (32). The solution of (32) provides the optimal short-run, farm-household consumption and leisure levels as well as the optimal equation of motion of equity. Of course, one can also obtain the steady-state solution.

Problem (32) is essentially of the same structure as the general problem (1) except that the instantaneous objective function in (32) is independent of the state variable E while the equation of motion depends on the parameter vector. Therefore, we can apply the same methodology previously described in deriving the estimating model. Moreover, there is no need to rederive the properties of $J(\cdot)$ here since they only differ slightly from those discussed in previous sections. The Hamilton-Jacobi-Bellman equation related to problem (32) is

$$(34)\quad \delta J(p,w,v,E,r,y) = \max_{c,l} u(c,l) + J_E(\cdot)[\rho(E,w,v,r) + w(H - l) - pc + y].$$

Differentiating (34) with respect to y and using the envelope theorem yields an expression for the optimal equation of motion $\dot{E}^*$:

$$(35)\qquad \dot{E}^* = J_{Ey}^{-1}[\delta J_y - J_E].$$

Next differentiate (34) with respect to w,v,p, and r to obtain:

$$(36)\quad \begin{aligned} &\text{(i)}\ \ \delta J_w = J_{Ew}\dot{E}^* + J_E L_2; \\ &\text{(ii)}\ \ \delta J_v = J_{Ev}\dot{E}^* + J_E Q; \\ &\text{(iii)}\ \ \delta J_p = J_{Ep}\dot{E}^* + J_E C;\ \text{and} \\ &\text{(iv)}\ \ \delta J_r = J_{Er}\dot{E}^* + J_E\rho_r; \end{aligned}$$

where L_2 is off-farm work supplied by the farm-household. Using (35) in (36) and recalling that $\dot{K} = (1 - \rho_{rE})\,\dot{E}$ yields structural estimating equations for the firm's decision variables:

$$(37)\quad \text{(i) } L_2 = \frac{1}{J_E}\{\delta J_w - J_{Ew}J_{Ey}^{-1}(\delta J_y - J_E)\};$$

$$\text{(ii) } Q = \frac{1}{J_E}\{\delta J_v - J_{Ev}J_{Ey}^{-1}(\delta J_y - J_E)\};$$

$$\text{(iii) } c = \frac{1}{J_E}\{-\delta J_p + J_{Ep}J_{Ey}^{-1}(\delta J_y - J_E)\};\ \text{and}$$

$$\text{(iv) } \dot{K} = \{1 - \frac{1}{J_E}\delta J_r - J_{Er}J_{Ey}^{-1}(\delta J_y - J_E)\}J_{Ey}^{-1}(\delta K_y - J_E).$$

Thus, equations (35) and (37) constitute the full system of short-run, behavioral equations of the farm-household. The approach suggests that production, consumption, savings ($\dot{E}$), and labor supply decisions are interdependent and should be estimated jointly. The theoretical properties of the estimating equations are obtained from the properties of the J(·) function from which they are derived (Chambers and Lopez).

What are the implications of consistent aggregation for the financially-constrained, farm-household problem? The first order necessary conditions of problem (32) include

$$(38)\quad \begin{aligned} &\text{(i)}\ \ u_c - J_E p = 0; \\ &\text{(ii)}\ \ u_l - J_E w = 0; \\ &\text{(iii)}\ \ \dot{J}_E = (\delta - \rho_E)J_E; \text{ and} \\ &\text{(iv)}\ \ \dot{E} = \rho(\cdot) + w(H - l) - pc + y. \end{aligned}$$

Consistent aggregation requires that $J_{EE} = 0$ and, hence, $\dot{J}_E = J_{EE}\dot{E} = 0$. Therefore, from (38(iii)):

$$\rho_E(E,w,v,r) = \delta; \text{ or}$$
$$J_E = 0$$

at all times. But this is precisely the steady-state condition (see Chambers and Lopez). That is, consistent aggregation necessarily implies that the system is in a steady state at *all* times. Again it renders the dynamic model meaningless by imposing a permanent steady state. Thus, consistent aggregation does not appear feasible in dynamic models of the financially-constrained household. The reader should note that mathematically the financial constraint model is very similar to the separable-adjustment-cost model outlined above.

Biological Models

This section uses a simple model of optimal fisheries management to illustrate the potential usefulness of the general model for natural resource economics. For simplicity, it is assumed that harvest-independent stock growth is of the logistic form:

$$rx(1 - x/k);$$

where r is now the intrinsic growth rate; x is the stock of the resource; and k is the environmental carrying capacity. r,x, and k are presumed known to the manager of the resource as a result of, say, biological sampling and survey work.

Catch is related to effort and the stock of the resource by the concave function:

$$q = Q(x,E),$$

where q is catch and E is effort. Thus, we specifically eschew the catch-per-unit-effort hypothesis in favor of a more general representation of the harvest technology. Dual to Q(·) is the short-run, stock-dependent, cost function:

$$\begin{aligned} c(q,w,x) &= \underset{E}{\text{Min}}\ \{wE{:}q = Q(E,x)\} \\ &= wE(q,x), \end{aligned}$$

where w is now the per unit cost of effort, and E(q,x) is the level of E that solves $q = Q(E,x)$ for given q and x.

Access to the fishery is strictly regulated with the manager of the resource determining optimal harvest levels according to

$$\underset{q}{\text{Max}} \left\{\int_o^\infty e^{-\delta t}[pq - c(q,w,x)]\right\}$$

subject to

$$\dot{x} = rx(1 - x/k) - q,$$
$$x(o) = \bar{x}.$$

Now this problem is somewhat unlike the general model since there is, in effect, no $p(x,\alpha)$ function from which to generate a duality in a straightforward manner. Therefore, in what follows we shall content ourselves with reasoning that can be based solely on the assumption that there exists a unique solution to this problem with a unique steady state.

The Hamilton-Jacobi-Bellman equation becomes

$$\delta J(p,w,x,r,k) = \underset{q}{\text{Max}}\ \{pq - c(q,w,x) + J_x\dot{x}\}$$

Since the solution to the above is the maximum value of limit of the sum of functions convex and linearly homogeneous in p and w J(·) inherits these same properties. Moreover, a direct application of the envelope theorem yields:

$$\begin{aligned} \delta J_p &= q^* + J_{px}\dot{x}^*; \\ \delta J_w &= -E^* + J_{wx}\dot{x}^*; \text{ and} \\ \delta J_r &= J_{rx}\dot{x}^* + J_x x(1 - x/k); \end{aligned}$$

which allows one to solve for the optimal controls and the optimal stock growth in the following manner:

$$\begin{aligned} q^* &= \delta J_p - J_{px}J_{rx}^{-1}(\delta J_r - J_x x(1 - x/k)); \\ E^* &= J_{wx}J_{rx}^{-1}(\delta J_r - J_x x(1 - x/k)) - \delta J_w; \text{ and} \\ \dot{x}^* &= J_{rx}^{-1}(\delta J_r - J_x x(1 - x/k)). \end{aligned}$$

Steady-state stock level is then given by the solution to the quadratic equation:

$$\delta J_r - J_x x + J_x x^2/k = 0$$

Unfortunately, this equation cannot be easily solved since J_r and J_x will generally depend upon x in a nonlinear fashion. However, it can be ascertained that dynamic stability requires $\delta J_{xx}(1 - x/k) + J_x(1 - \alpha x/k)$ in the neighborhood of the steady state.

Since we lack strong information on the dual relations for this problem, we continue by considering the steady-state behavior of catch and effort. In the steady state

$$q^\infty = \delta J_p; \text{ and}$$
$$E^\infty = -\delta J_w.$$

From these expressions we find that

$$\frac{\partial q^\infty}{\partial p} = \delta J_{pp} + \delta J_{px}\frac{\partial x^\infty}{\partial p}, \text{ and}$$
$$\frac{\partial E^\infty}{\partial w} = -\delta J_{ww} - \delta J_{wx}\frac{\partial x^\infty}{\partial w}.$$

Hence, the price-responsiveness of the long-run controls depends upon two effects: a pure-price effect (δJ_{pp} and $- \delta J_{ww}$) associated with a given level of the steady state; and an expansion effect associated with the adjustment in the steady-state stock level. To ascertain the sign of $\partial x^\infty/\partial p$, say, it is necessary to differentiate the equation defining x^∞ implicitly. This is left to the interested reader as an exercise (the arguments used are similar to those used in ascertaining the sign of $\nabla_{\alpha x}J$ in earlier sections). But the reader should note that the pure-price-effect in the above is consistent with the responses one would usually expect on the basis of static optimization theory since J is convex and linearly homogeneous in p and w. Letting the steady-state values of q and E be expressed as

$$q^\infty = q(x^\infty,p,w), \text{ and}$$
$$E^\infty = E(x^\infty,p,w),$$

this last result can be summarized by the expressions:

$$q(x^\infty,tp,tw) = 0,$$
$$E(x^\infty,tp,tw) = 0,$$
$$\frac{\partial q(x^\infty,p,w)}{\partial p} \geq 0$$
$$\frac{\partial E(x^\infty,p,w)}{\partial w} \leq 0, \text{ and}$$
$$\frac{\partial q(x^\infty,p,w)}{\partial w} = -\frac{\partial E(x^\infty,p,w)}{\partial p}.$$

Thus, the pure-price effects obey the standard Samuelson homogeneity and reciprocity relationships. Similar results have been obtained in a paper by Chambers and are attributable to the fact that $J(\cdot)$ is here convex and linearly homogeneous in p and w.

Since this problem is specifically aggregate in nature, it seems somewhat vacuous to consider here the question of consistent aggregation. However, we do wish to pursue the implications of the flexible accelerator for this model. By (10) the instantaneous value function is consistent with the EFA if it satisfies the following implicit equation:

$$pq - c(q,w,x) - \phi(x - M^{-1}rx(1 - xk) + q) - b(rx(1 - xk) - q) = 0.$$

Differentiating with respect to q yields:

$$p - \frac{\partial c}{\partial q} - \frac{\partial \phi}{\partial z} + b = 0;$$

and again

$$\frac{\partial^2 c}{\partial q^2} = -\frac{\partial^2 \phi}{\partial z^2}$$

which reveals a fundamental link between the curvature of c and ϕ that should be satisfied if the model is to be consistent with the flexible accelerator.

Static Expectations Without Apology

So far we have consistently assumed static expectations, i.e., that producers expect current prices to prevail in the future. How restrictive is this assumption? We argue here that if outputs and inputs are storable and if storage costs are small enough in relation to the value of commodities then there is little loss of generality in assuming static expectations. The argument here is quite similar and in places identical to Working's theory of futures-price formation.

Consider a situation where there exists a divergence between the current price of a commodity and the price that producers expect for the future. If producers or intermediaries are rational, they will participate in the commodity market whenever there is a profit potential. That is, if their discounted expected price is higher than the current price by more than the marginal storage cost, they will buy at current prices and accumulate inventories to be sold at a later date. This buying will persist until no potential profit is possible, that is, until the current price increases sufficiently. This occurs when the current price equals the discounted expected price minus the marginal storage costs. The opposite will occur if expected prices are lower than current prices. Existence of well developed commodity markets (not necessarily of futures markets) is sufficient to insure that this equilibrium between current and expected prices is rapidly achieved. A rational producer will be aware of these equilibrating forces and will consider current prices minus (plus) an appropriate premium for storage costs as an appropriate proxy for discounted future prices even if he does not participate in the commodity market. If storage costs for the relevant period are negligible as a proportion of the unit value of the commodity then the current price is a good proxy for the discounted expected price. If storage costs are not negligible but constant, i.e., independent of commodity inventories, then knowledge of the unit inventory cost (and the discount rate) is all that is required in order to determine a relationship between expected and current prices:

$$E(p_{t+1}) = (1 + \delta)p_t + a$$

where a is the unit marginal cost of inventory holding, p_t is current price and $E(p_{t+1})$ is the price expected to prevail at time $t + 1$.

The key observation is that if commodity markets exist, any new information will be absorbed by the market participants leading to incipient transactions to reestablish the above equilibrium conditions. Actually if all participants have identical expectations, there will be no transactions, current prices will be bid up immediately to levels consistent with the change in expectations. If producers have different expectations, the current price will reflect a representative expected price. Note that the previous analysis is independent of whether producers form their expectations according to the rational expectation hypothesis or not. The problem of whether producers follow rational expectation rules or not is irrelevant from the point of view of modelling dynamic supply responses. All that is needed is an estimate of the storage costs in order to derive expected prices from knowledge of current prices and the discount rate only.

Obviously these arguments seem most applicable to the agricultural examples considered above and somewhat less applicable to the fisheries example considered. But even in the absence of this argument it seems reasonable to believe that static expectations may very well be rational or more correctly optimal. The ultimate bedrock of the rational expectations hypothesis is the neoclassical postulate that economic agents optimize and that their optimization behavior carries them to the point where they discover the ultimate structural relations determining the stochastic behavior of prices. All this is fine and well, but in implementing this hypothesis it is often forgotten that the process of acquiring information is not costless (Stigler) and further that the information acquiring process and production decisions usually proceed jointly. Thus, there is a joint maximization process being carried out. But how often does one actually see the act of acquiring information enter into empirical models relying on the rational expectations hypothesis? More usually when such matters are recognized (as Lucas and Sargent clearly do in their classic overview of the rational expectations literature) it is swept under the rug in favor of the more extreme, linear-quadratic representation of the rational firm that is analytically tractable. It seems plausible that for many small to medium-sized economic agents the process of information acquisition may be extremely costly. And it may well be *rational* to rely on static expectations.

References

Blackorby, C. and W. Schworm. "Aggregate Investment and Consistent Intertemporal Technologies." *Review of Economic Studies* 49(1982):595–614.

Chambers, Robert G. "Dynamic Supply Response and the Partial-Adjustment Model"; University of Maryland, October 1983.

———. "Dynamic Stability Consistent Aggregation, and the Behavior of the Steady State." Department of Agricultural Economics, University of Maryland, June 1984.

Chambers, R. G. and R. E. Lopez. "Equity Formation and the On-Farm, Off-Farm Labor Choice." Department of Agricultural Economics, University of Maryland, October 1983.

Cooper, R. J. and K. R. McLaren. "Atemporal, Temporal, and Intertemporal Duality in Consumer Theory." *International Economic Review* 21(1980): 599–609.

Eisner, R. and R. H. Strotz. "Determinants of Business Investment." in *Impacts of Monetary Policy;* Englewood Cliffs, N.J.: Prentice-Hall, 1963.

Epstein, L. G. "Duality Theory and Functional Forms for Dynamic Factor Demands." *Review of Economic Studies* 48(1981):81–95.

Gandolfo, G. *Economic Dynamics: Methods and Models* 2d ed. New York: North-Holland, 1979.

Kamien, M. and N. Schwartz. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. New York: North-Holland, 1982.

Lucas, R. "Optimal Investment Policy and the Flexible Accelerator." *International Economic Review* 81(1967):78–85.

Lucas, R. E. and T. J. Sargent. *Rational Expectation and Econometric Practice* vol. 1—(Minneapolis; University of Minnesota, Press, 1981).

Mortensen, D. T. "Generalized Costs of Adjustment and Dynamic Factor Demand Theory." *Econometrica* 41(1973):657–65.

Muellbauer, J. "Aggregation, Income Distribution, and Consumer Demand." *Review of Economic Studies* 42(1975):525–43.

Shalit, H. and A. Schmitz. "Farmland Accumulation and Prices" *American Journal of Agricultural Economics* 64(1982):710–719.

Steigum, E. "A Financial Theory of Investment Behavior" *Econometrica* 51(1983):637–46.

Taylor, C. R. "Stochastic Dynamic Duality: Theory and Empirical Applicability" *American Journal of Agricultural Economics* (forthcoming).

Treadway, A. "The Globally Optimal Flexible Accelerator" *Journal of Economic Theory* 7(1974):17–39.