Towards Measuring Producer Welfare Under Output Price Uncertainty and Risk Non-Neutrality

by

David S. Bullock
Associate Professor, University of Illinois Department of Agricultural and Consumer Economics
326 Mumford Hall, 1301 W. Gregory Drive, Urbana, IL 61801
Phone: (157) 333-5510. Fax: (157) 333-5538. e-mail: **dsbulloc@uiuc.edu**.

Kie-Yup Shin
Senior Researcher, National Agricultural Cooperative Federation
75, 1-Ka ChungJeong-Ro
Jung-Ku SEOUL 100-707, Korea
e-mail: **kyshin@nuri.net**.

Philip Garcia
Professor, University of Illinois Department of Agricultural and Consumer Economics
326 Mumford Hall, 1301 W. Gregory Drive, Urbana, IL 61801
Phone: (157) 333-5510. Fax: (157) 333-9644. e-mail: p-garcia@uiuc.edu.

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Abstract

We combine theory with numerical integration methods to show that for any form of uncompensated supply, compensating variation of a change in higher moments of an output price distribution can be numerically derived.

Keywords: Producer welfare, price uncertainty, risk

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Introduction

There is extensive literature on evaluating welfare consequences of changes in agricultural policies, prices, and technology. Most of these studies have employed classical economic surplus measures which are based on certain prices, to identify the welfare consequences of such changes (e.g. Wallace, 1962; Cramer et al., 1990). However, production lags and weather variability cause agricultural producers typically to face uncertainty about output prices and production when they make production decisions. If producers respond to uncertain conditions (Sandmo, 1971), classical economic surplus measures are generally inappropriate (c.f. Just, 1975). The objective of this paper is to move the literature further toward a more appropriate measure.

Model, Notation, and Definitions

To facilitate communication, here we establish the notation and definition. Our basic model, notation, and definitions follow Pope, Chavas, and Just (PCJ, 1983), though our notation is slightly less sparse in order to aid in explanation and illustration of methods we present.

Begin with the expected utility model developed by Baron (1970), Sandmo (1971), Batra and Ullah (1974) and others. The firm's objective is to solve the maximization problem:

(1) $$\max_{\mathbf{x}} \mathrm{E}\left\{\mathrm{U}\left(\mathrm{W} + \mathbf{pf}(\mathbf{x}) - \mathbf{rx}\right) \middle| \gamma \right\},$$

where E is an expectations operator, U is a function showing how utility depends on wealth, **p** is an *m*-dimensional vector of output price variables, **f**(**x**) is a corresponding *m*-dimensional vector of (nonjoint) production functions, **r** is an *n*-dimensional vector of input price variables, **x** is a corresponding *n*-dimensional vector of input quantity variables, and W is a nonrandom and exogenously determined wealth variable. The price vectors **p** and **r**, and the vector of production functions **f**(**x**) may be assumed to consist of random or nonrandom variables, depending on the case at hand. All pertinent parameters are described by a vector $\boldsymbol{\gamma} \in R^n$. (We assume throughout that this parameter vector has *n* elements.) The moments of the distributions of the random variables **p**, **r**, and **f**(**x**) in (1) are included as elements of $\boldsymbol{\gamma}$, and therefore our notation explicitly denotes that expectations depend on $\boldsymbol{\gamma}$. Initial wealth W is also an element of $\boldsymbol{\gamma}$.

Firm inputs are assumed chosen optimally to solve (1), and therefore optimally chosen input quantities $\mathbf{x}^*$ are functions of the parameters of the model, $\boldsymbol{\gamma}$:

(2) $$\mathbf{x}^* = \mathbf{x}^*(\gamma).$$

The indirect risk premium function $R^*(\gamma)$ is implicitly defined by identity (3), and the indirect certainty equivalent function $L^*(\gamma)$ is defined in (4):

(3) $$U\left(W + E\left\{\mathbf{pf}\left(\mathbf{x}^*(\gamma)\right) - \mathbf{rx}^*(\gamma)\middle|\gamma\right\} - R^*(\gamma)\right) \equiv E\left\{U\left(W + \mathbf{pf}\left(\mathbf{x}^*(\gamma)\right) - \mathbf{rx}^*(\gamma)\right)\middle|\gamma\right\}.$$

(4) $$L^*(\gamma) \equiv W + E\left\{\mathbf{pf}\left(\mathbf{x}^*(\gamma)\right) - \mathbf{rx}^*(\gamma)\middle|\gamma\right\} - R^*(\gamma).$$

Substituting (4) into (3) yields,

(5) $$U\left(L^*(\gamma)\right) \equiv E\left\{U\left(W + \underbrace{\mathbf{pf}\left(\mathbf{x}^*(\gamma)\right) - \mathbf{rx}^*(\gamma)}_{\pi^*(\gamma)}\right)\middle|\gamma\right\}.$$

Note that in (5) we have provided the definition of the (uncompensated) profit function $\pi^*(\gamma)$. Assuming U to be monotonically increasing, we can invert (5) to obtain

(6) $$L^*(\gamma) \equiv U^{-1}\left(E\left\{U\left(W + \pi^*(\gamma)\right)\middle|\gamma\right\}\right).$$

The risk premium function $R^*(\gamma)$ shows the maximum amount of cash that the firm would pay to face the expected values of the random variables in $\boldsymbol{\gamma}$ instead of facing the risk inherent in these random variables. $L^*(\gamma)$ is the minimum amount of money the firm would have to receive with probability one in order to keep the firm's expected utility equal to the expected utility level provided by the (possibly risky) markets parameterized by γ.

Let γ_1 denote a vector of model parameters in an initial situation, and γ_2 denote a vector of model parameters in a subsequent situation. One measure that can be used to examine the welfare effect on the firm of a change in model parameters from γ_1 to γ_2 is simply the difference in the indirect certainty equivalent functions:

$$\Delta L^*(\gamma_1, \gamma_2) \equiv L^*(\gamma_2) - L^*(\gamma_1). \tag{7}$$

Another measure that can be used to examine the welfare effect on the firm of a change in model parameters from γ_1 to γ_2 is compensating variation, implicitly defined by identity (8):

$$\underbrace{E\left\{U\left(W + \mathbf{pf}\left(\mathbf{x}^*(\gamma_1)\right) - \mathbf{rx}^*(\gamma_1)\right) \middle| \gamma_1\right\}}_{EU^*(\gamma_1)} \equiv E\left\{U\left[W - c\left(\gamma_1, \gamma_2, EU^*(\gamma_1)\right) + \right.\right.$$

$$\left.\left.\underbrace{\mathbf{p}\overbrace{\mathbf{f}\left(\mathbf{x}^*\left(\gamma_2, W - c\left(\gamma_1, \gamma_2, EU^*(\gamma_1)\right)\right)\right)}^{\mathbf{y}_c(\gamma_1, \gamma_2, EU^*(\gamma_1))} - \mathbf{r}\overbrace{\mathbf{x}^*\left(\gamma_2, W - c\left(\gamma_1, \gamma_2, EU^*(\gamma_1)\right)\right)}^{\mathbf{x}_c(\gamma_1, \gamma_2, EU^*(\gamma_1))}}_{\pi_c(\gamma_1, \gamma_2, EU^*(\gamma_1))}\right] \middle| \gamma_2\right\}. \tag{8}$$

Also defined in (8) are the compensated profit function $\pi_c(\gamma_1, \gamma_2, EU^*(\gamma_1))$, the vector of compensated supply functions $\mathbf{y}_c(\gamma_1, \gamma_2, EU^*(\gamma_1))$, and the vector of compensated input demand functions, $\mathbf{x}_c(\gamma_1, \gamma_2, EU^*(\gamma_1))$.

A Practical Procedure of Stochastic Producer Welfare Measurement

A Change in Three Parameters

We focus on three parameters: initial wealth W, the mean of an output price distribution μ, and the standard deviation σ of that same distribution. (Procedures shown here can be generalized to examine the effects of changes in additional parameters.) We desire to measure the welfare impact of a change in these parameters from $\gamma_1 = (W_1, \mu_1, \sigma_1)$ to $\gamma_2 = (W_2, \mu_2, \sigma_2)$ (notation for all other parameters is suppressed to save space). Assume that the output price is characterized by $p = \mu + \sigma\varepsilon$, where μ and σ are constants and ε is a random variable with expected value zero. For a change in the parameters vector from $\gamma_1 = (W_1, \mu_1, \sigma_1)$ to $\gamma_2 = (W_2, \mu_2, \sigma_2)$, the compensated profit function is

$$(9)\ \pi_c\left(\overbrace{(W_1,\mu_1,\sigma_1)}^{\gamma_1};\overbrace{(W_2,\mu_2,\sigma_2)}^{\gamma_2};EU^*(\gamma_1)\right) \equiv (\mu_2 + \sigma_2\varepsilon)y_c(\gamma_1,\gamma_2,EU^*(\gamma_1)) - \mathbf{rx}_c(\gamma_1,\gamma_2,EU^*(\gamma_1)).$$

By combining the envelope theorem, the assumption that production is nonstochastic, and the assumption that ε has an expected value of zero, we have

(10a)

$$E\left\{\left.\frac{\partial\pi_c\left(\overbrace{(W_1,\mu_1,\sigma_1)}^{\gamma_1};\overbrace{(W_2,\mu_2,\sigma_2)}^{\gamma_2};EU^*(\gamma_1)\right)}{\partial\mu_2}\right|\gamma_2\right\} \equiv E\left\{y_c(\gamma_1,\gamma_2,EU^*(\gamma_1))\middle|\gamma_2\right\} \equiv y_c(\gamma_1,\gamma_2,EU^*(\gamma_1)).$$

$$(10b)\ E\left\{\left.\frac{\partial\pi_c\left(\overbrace{(W_1,\mu_1,\sigma_1)}^{\gamma_1};\overbrace{(W_2,\mu_2,\sigma_2)}^{\gamma_2};EU^*(\gamma_1)\right)}{\partial\sigma_2}\right|\gamma_2\right\} \equiv E\left\{\varepsilon y_c(\gamma_1,\gamma_2,EU^*(\gamma_1))\middle|\gamma_2\right\} \equiv 0.$$

(10c) $$E\left\{ \frac{\partial \pi_c\left(\overbrace{(W_1,\mu_1,\sigma_1)}^{\gamma_1};\overbrace{(W_2,\mu_2,\sigma_2)}^{\gamma_2};EU^*(\gamma_1)\right)}{\partial W_2} \middle| \gamma_2 \right\} \equiv E\{0|\gamma_2\} \equiv 0.$$

Substituting (10) into (8) yields compensating variation,

(11) $$c\left(\overbrace{(W_1,\mu_1,\sigma_1)}^{\gamma_1};\overbrace{(W_2,\mu_2,\sigma_2)}^{\gamma_2};EU^*(\gamma_1)\right) \equiv \int_{(W,\mu,\sigma)=(W_1,\mu_1,\sigma_1)}^{(W,\mu,\sigma)=(W_2,\mu_2,\sigma_2)} \Bigg[1\cdot dW + y_c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right)d\mu +$$

$$\left(\frac{\operatorname{cov}\left(U'\left(W - c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right) + \pi_c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right)\right),\ \varepsilon\right)}{E\left\{U'\left(W - c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right) + \pi_c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right)\right) \middle| W,\mu,\sigma\right\}} \right) y_c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right)d\sigma \Bigg].$$

The Shutdown Price Assumption and Method

Clearly, a major hurdle in using (11) for empirical work is the third term of the integrand on the right-hand side, the derivative of the compensating variation function with respect to σ, which generally cannot be derived from market data since the utility function U is not observable. To deal with this difficulty, we first follow PCJ to assume that there exists a shutdown price function, defined by

(12) $$\mu^*\left(W,\sigma,EU^*(\gamma_1)\right) = \max\left\{\mu \in \Re : y_c\left(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\right) = 0\right\}.$$

Since the line integral in (11) is path independent (see Kaplan, pp. 291 - 298), the path of integration is an arbitrary one between endpoints $\gamma_1 = (W_1, \mu_1, \sigma_1)$ and $\gamma_2 = (W_2, \mu_2, \sigma_2)$. Therefore we can elect to take a path which is the union of four subpaths. The first subpath, S_1, is a line segment between points (W_1, μ_1, σ_1) and (W_2, μ_1, σ_1). That is, along S_1, variable W changes from W_1 to W_2, while μ and σ stay constant at μ_1 and σ_1. The other three subpaths are illustrated in figure 1. In figure 1 it is assumed that the firm is risk averse, so that given W_2, as the standard deviation σ of the output price distribution rises, so does the maximum mean output price at which the firm is willing to shut down. (So, higher risk causes the vertical intercept of the compensated supply curve, when graphed against mean output price μ, to shift up.) S_2 is the line segment between point a = (W_2, μ_1, σ_1) and point b = $(W_2, \mu^*(W_2, \sigma_1, EU^*(\gamma_1)), \sigma_1)$. The third subpath, labeled $S_{shutdown}$ in figure 1, shows the locus of points $(\mu^*(W_2, \sigma, EU^*(\gamma_1)), \sigma)$ for all $\sigma \in [\sigma_1, \sigma_2]$. $S_{shutdown}$ travels between points b = $(W_2, \mu^*(W_2, \sigma_1, EU^*(\gamma_1)), \sigma_1)$ and c = $(W_2, \mu^*(W_2, \sigma_2, EU^*(\gamma_1)), \sigma_2)$. At all (μ, σ) combinations on $S_{shutdown}$, compensated supply and profits are zero since the firm is shut down. The fourth subpath, labeled S_4, is just the line segment between point c = $(W_2, \mu^*(W_2, \sigma_2, EU^*(\gamma_1)), \sigma_2)$ and point d = (W_2, μ_2, σ_2).

Letting S be the union of these four subpaths, clearly S is a path with endpoints $\gamma_1 = (W_1, \mu_1, \sigma_1)$ and $\gamma_2 = (W_2, \mu_2, \sigma_2)$. Since only parameter W changes along subpath S_1, and only parameter μ changes along subpaths S_3 and S_4, and since only parameters μ and σ change along $S_{shutdown}$ the line integral on the right-hand side of (11) implies:

$$(13)\; c\left(\overbrace{W_1,\mu_1,\sigma_1}^{\gamma_1};\overbrace{W_2,\mu_2,\sigma_2}^{\gamma_2};EU^*(\gamma_1)\right) \equiv \int_{S_1} 1\cdot dW + \int_{S_2} y_c\big(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\big)d\mu$$

$$+\int_{S_{shutdown}}\left[\underbrace{y_c\big(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\big)}_{0}d\mu + \left(\frac{\operatorname{cov}\big(U'(W-c+\pi_c),\ \varepsilon\big)}{E\big\{U'(W-c+\pi_c)\big|\mu,\sigma,W_1\big\}}\right)\underbrace{y_c\big(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\big)}_{0}d\sigma\right]$$

$$+\int_{S_4} y_c\big(\gamma_1;W,\mu,\sigma;EU^*(\gamma_1)\big)d\mu\,.$$

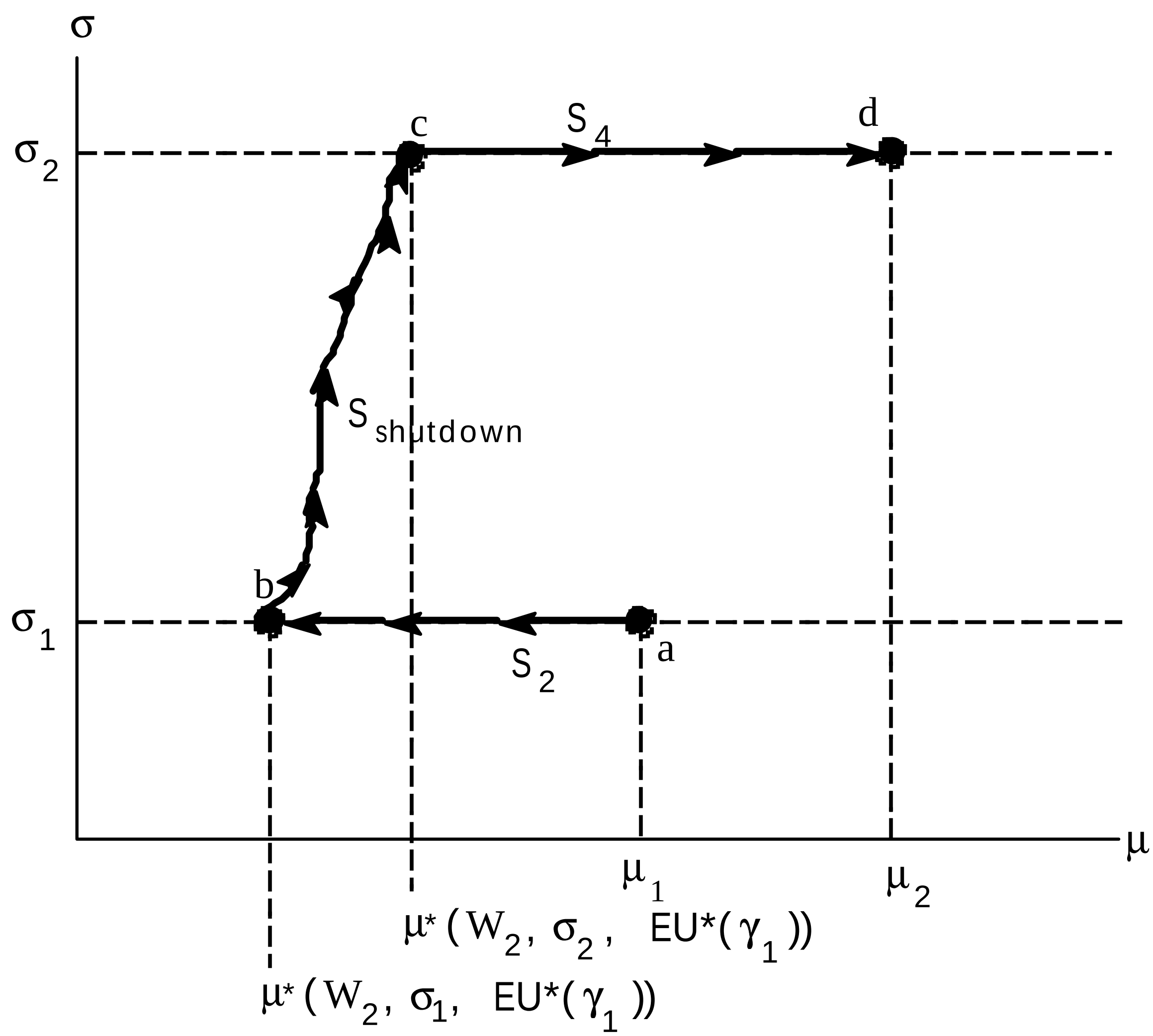

Figure 1. Three subpaths of integration

As indicated in (13), anywhere along $S_{shutdown}$, compensated supply in zero. Therefore the third integral on the right-hand side of (13) is zero, and the difficult covariance term is dispensed. The other three line integrals are easily written as definite integrals, and we obtain:

$$(14)\; c\left(\overbrace{W_1,\mu_1,\sigma_1}^{\gamma_1};\overbrace{W_2,\mu_2,\sigma_2}^{\gamma_2};EU^*(\gamma_1)\right) \equiv \underbrace{\int_{W_1}^{W_2} 1\cdot dW}_{W_2-W_1} + \int_{\mu_1}^{\mu^*(W_2,\sigma_1,EU^*(\gamma_1))} y_c\left(\gamma_1;W_2,\mu,\sigma_1;EU^*(\gamma_1)\right)d\mu$$

$$+ \int_{\mu^*(W_2,\sigma_2,EU^*(\gamma_1))}^{\mu_2} y_c\left(\gamma_1;W_2,\mu,\sigma_2;EU^*(\gamma_1)\right)d\mu\,.$$

The expression in (14) indicates that if a shutdown price subpath of integration exists, the change in stochastic producer welfare can be measured using the geometric areas behind two compensated output supply curves. The first curve must be based on risk level σ_1 and wealth W_2, and the second curve must be based on risk level σ_2 and wealth W_2. Furthermore, the researcher does not need complete knowledge of subpath $S_{shutdown}$ to apply (14), but rather only needs to know the values $\mu^*(W_2, \sigma_1, E(\gamma_1))$ and $\mu^*(W_2, \sigma_2, E(\gamma_1))$, the vertical intercepts of the compensated supply curves.

A New Procedure Based on Vartia's Method

Next we adapt Vartia's (1983) method, which he used to approximate compensated demand from the information in an uncompensated demand system, to approximate from information in an uncompensated supply function the compensating variation for changes in the first and second moments of the output price distribution. This procedure has advantages over existing procedures. Unlike Pope, Chavas, and Just (1983), it does not impose any restriction on risk preferences; nor does it place bounds around classical surplus measures as do Pope and Chavas. Unlike Larson (1988), the procedure permits complete flexibility in the specification of the functional form of the uncompensated supply function.

A Change in the Parameter Vector from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1)

First, our problem is to find compensating variation for a change in parameter vector from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1)). Let t denote an auxiliary variable such that $0 \le t \le 1$, and let be $\mu(t)$ be a differentiable function connecting $\mu_1 = \mu(0)$ to $\mu_2 = \mu(1)$. Let h(t) • $c(\gamma_1; W_1, \mu(t), \sigma_1; EU^*(\gamma_1))$. That is, h(t) is compensating variation for a change in the parameter vector from (W_1, μ_1, σ_1) to $(W_1, \mu(t), \sigma_1)$. Then, differentiating this identity with respect to t, using (12), the definition of compensated supply in (8), and the Fundamental Theorem of Calculus, we have the following relationship, which is a first order differential equation in h(t):

$$(15)\ \frac{dh(t)}{dt} \equiv \frac{\partial c\big(\gamma_1; W_1, \mu(t), \sigma_1; EU^*(\gamma_1)\big)}{\partial \mu}\frac{d\mu(t)}{dt} \equiv y_c\big(\gamma_1; W_1, \mu(t), \sigma_1; EU^*(\gamma_1)\big)\frac{d\mu(t)}{dt}$$

$$\equiv y^*\big(W_1 - h(t), \mu(t), \sigma_1\big)\frac{d\mu(t)}{dt}.$$

Note that in (15) $\mu(t)$ and $d\mu(t)/dt$ are known functions, and h(t) is an unknown function to be found. By noting that h(0) = 0 and integrating (15) with respect to t, we have:

$$(16)\qquad h(t) = h(t) - h(0) = \int_0^t \frac{dh(t)}{dt}\,dt = \int_0^t y^*\big(W_1 - h(t), \mu(t), \sigma_1\big)\frac{d\mu(t)}{dt}\,dt.$$

The compensating variation for the change from μ_1 to μ_2, denoted by h(1), is the solution to (16) when t = 1.

Next, we need to develop a practical method for calculating or approximating h(1). For any functional form of supply, equation (16) can be solved numerically. Following Vartia (1983), an algorithm that provides a numerical solution to (16) can be described intuitively as follows. By choosing numbers $t_1, t_2, \ldots, t_{N+1}$ such that $0 = t_1 < t_2 < \ldots < t_{N+1} = 1$, we derive from (16) the following equation:

$$(17)\qquad h(1) = h\big(t_{N+1}\big) = \sum_{i=2}^{N+1}\Big[h\big(t_k\big) - h\big(t_{k-1}\big)\Big] = \sum_{k=2}^{N+1}\int_{t_{k-1}}^{t_k} y^*\big(W_1 - h(t), \mu(t), \sigma_1\big)\frac{d\mu(t)}{dt}\,dt.$$

Examining the sum of integrals in (17), when $t_k - t_{k-1}$ is small (that is, when N is large) we can approximate $y^*(W_1 - h(t), \mu(t), \sigma_1)$ by the mean of its values at the limits of the integral: $y^*(W_1 - h(t), \mu(t), \sigma_1)$ ª $0.5[y^*(W_1 - h(t_{k-1}), \mu(t_{k-1}), \sigma_1) + y^*(W_1 - h(t_k), \mu(t_k), \sigma_1)]$, resulting in,

$$\begin{aligned}(18)\quad & \int_{t_{k-1}}^{t_k} y^*\left(W_1 - h(t), \mu(t), \sigma_1\right)\frac{d\mu(t)}{dt}\,dt \\ & \approx 0.5\left[y^*\left(W_1 - h(t_{k-1}), \mu\left(t_{k-1}\right), \sigma_1\right) + y^*\left(W_1 - h(t_k), \mu\left(t_k\right), \sigma_1\right)\right]\int_{t_{k-1}}^{t_k} \frac{d\mu(t)}{dt}\,dt \\ & = 0.5\left[y^*\left(W_1 - h(t_{k-1}), \mu\left(t_{k-1}\right), \sigma_1\right) + y^*\left(W_1 - h(t_k), \mu\left(t_k\right), \sigma_1\right)\right]\cdot\left[\mu\left(t_k\right) - \mu\left(t_{k-1}\right)\right].\end{aligned}$$

Next, assume that $\mu(t)$ is linear: $\mu(t) = \mu_1 + t[\mu_2 - \mu_1]$, $0 \le t \le 1$. For a given integer N, and for every $k = 1, \ldots, N + 1$, let $t_k = (k - 1)/N$. To shorten the notation, let $h_k = h(t_k)$ and let $y_k^* = y^*(W_1 - h_k, \mu(t_k), \sigma_1)$ $k = 1, \ldots, N$. First begin with starting values of $\mu(t_1) = \mu_1$ and $h_1 = 0$. Then generate a sequence $h_2, \ldots, h_{N+1}$ such that

$$(19)\qquad h_k = h_{k-1} + \frac{1}{2}\left[\underbrace{y^*\left(W_1 - h_{k-1}, \mu\left(t_{k-1}\right), \sigma_1\right)}_{y^*_{k-1}} + \underbrace{y^*\left(W_1 - h_k, \mu\left(t_k\right), \sigma_1\right)}_{y^*_k}\right]\cdot\left[\mu\left(t_k\right) - \mu\left(t_{k-1}\right)\right].$$

Since the term h_k appears on both sides of (19), it must be determined by an iterative method. To determine h_k by iteration, define $h_k^{(1)} = h_{k-1}$, and for $k = 2, \ldots, N + 1$, and $m = 2, 3, \ldots$, let

$$(20)\quad h_k^{(m)} = h_{k-1} + \frac{1}{2}\left[y^*\left(W_1 - h_{k-1}, \mu\left(t_{k-1}\right), \sigma_1\right) + y^*\left(W_1 - h_k^{(m-1)}, \mu\left(t_k\right), \sigma_1\right)\right]\cdot\left[\mu\left(t_k\right) - \mu\left(t_{k-1}\right)\right].$$

As the number *m* increases, $| h_k^{(m)} - h_k^{(m-1)} |$ will become negligibly small. When at some number M_k, $| h_k^{(M_k)} - h_k^{(M_k-1)} |$ is deemed sufficiently small, we can use (19) to say

$$(21)\qquad h_k \approx \frac{1}{2}\left[y^*\left(W_1 - h_{k-1}, \mu\left(t_{k-1}\right), \sigma_1\right) + y^*\left(W_1 - h_k^{(M_k)}, \mu\left(t_k\right), \sigma_1\right)\right]\cdot\left[\mu\left(t_k\right) - \mu\left(t_{k-1}\right)\right].$$

and start the calculation for k + 1.

Once the value of h_k is approximated by this iterative method for $k = 1, \ldots, N + 1$, the compensating variation for the change from μ_1 to μ_2 can be approximated following (17) as,

$$(22)\ c\left(W_1, \mu_1, \sigma_1; W_1, \mu_2, \sigma_1; EU^*(\gamma_1)\right) \approx$$

$$\sum_{k=2}^{N+1} 0.5\left[y^*\left(W_1 - h_{k-1}^{(M_{k-1})}, \mu(t_{k-1}), \sigma_1\right) + y^*\left(W_1 - h_k^{(M_k)}, \mu(t_k), \sigma_1\right)\right]\left[\mu(t_k) - \mu(t_{k-1})\right].$$

Letting N grow arbitrarily large allows (22) to provide an arbitrarily close approximation of compensating variation.

A Change in the Parameter Vector from (W_1, μ_1, σ_1) to (W_2, μ_2, σ_2)

Now, we need to consider the more general case in which the whole parameter vector changes from (W_1, μ_1, σ_1) to (W_2, μ_2, σ_2). By assuming the existence of appropriate shutdown mean prices, we have the formula for calculating compensating variation for the general parameter change from (14):

$$(23)\ c\left(\overbrace{\mu_1, \sigma_1, W_1}^{\gamma_1}; \overbrace{\mu_2, \sigma_2, W_2}^{\gamma_2}; EU^*(\gamma_1)\right) \equiv \underbrace{\int_{W_1}^{W_2} 1 \cdot dW}_{W_2 - W_1} + \underbrace{\int_{\mu_1}^{\mu^*(W_2, \sigma_1, EU^*(\gamma_1))} y_c\left(\gamma_1; W_2, \mu, \sigma_1; EU^*(\gamma_1)\right) d\mu}_{c\left(W_2, \mu_1, \sigma_1;\ W_2, \mu^*(W_2, \sigma_1, EU^*(\gamma_1)), \sigma_1;\ EU^*(\gamma_1)\right)}$$

$$+ \underbrace{\int_{\mu^*(W_2, \sigma_2, EU^*(\gamma_1))}^{\mu_2} y_c\left(\gamma_1; W_2, \mu, \sigma_2; EU^*(\gamma_1)\right) d\mu}_{c\left(W_2, \mu^*(W_2, \sigma_2, EU^*(\gamma_1)), \sigma_2;\ \mu_2, \sigma_2, W_2;\ EU^*(\gamma_1)\right)}.$$

We can approximate the second integral on the right-hand side of (23) in a manner very similar to (22), except that instead of finding compensating variation for a parameter change from (W_1, μ_1, σ_1) to (W_1, μ_2, σ_1), we must find compensating variation for a parameter change from (W_2, μ_1, σ_1) to (W_2, $\mu^*(W_2, \sigma_1, EU^*(\gamma_1))$, σ_1). While $\mu^*(W_2, \sigma_1, EU^*(\gamma_1))$ is initially unknown, it may be found to any desired degree of accuracy within the numerical algorithm; it is the mean output price μ at which $y_c(\gamma_1; W_2, \mu, \sigma_1; EU^*(\gamma_1)) = y^*(W_2 - c(\gamma_1; W_2, \mu, \sigma_1; EU^*(\gamma_1)), \mu, \sigma_1) = 0$. Similarly, the third integral on the right-hand side of (23) can be found numerically.

Conclusions and Limitations

Our new procedure is quite flexible in that it can be used to approximate compensating variation using any functional form of the uncompensated supply function and for changes in higher moments of the output price distribution. Similarly, with appropriate modifications, the procedure can be used to calculate equivalent variation. A limitation of our approach is that to use it shutdown prices must exist. This poses no problem when producers produce only one product, for an output price of zero will always force a shutdown. As discussed in Pope, Chavas, and Just, if a firm produces more than one good, even a zero price of one good may not cause the producer to stop producing all goods and so shut down. Additionally, it is usually the case that there are no price-quantity observations in the data in the "shutdown price" neighborhood, which causes standard deviations of estimates of vertical supply curve intercepts to be quite large. Thus, in a statistical sense, the confidence we can place in welfare measures that rely on accurate estimation of the entire supply curve may be limited.

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