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Estimating Efficiency Measures in North Dakota Farms

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Abstract

A normalized Quadratic Cost function is estimated using data from 1998-2003 to analyze the cost structure of North Dakota farms. Results indicate that there is overall evidence of increasing returns to scale. However, we do not find differences in efficiency across the different farm sizes. Marginal costs scale economies do not differ significantly across the different size categories, indicating that small and medium farms are just as efficient as the larger farms.

Keywords: normalized quadratic cost function, economies of scale

Highlights

Declining net farm income resulting from low commodity prices and adverse weather conditions has been a problem faced by North Dakota farmers in recent years. However, some farms continue to be profitable and prosper even in adverse conditions.

The objective of this study is to examine the profitability of differently sized farms in North Dakota. The study especially focuses on: (1) whether scale efficiency exists in North Dakota farms and (2) if large-size farms have a cost advantage over smaller farms.

Results from our empirical analysis indicate that, overall, there is evidence of increasing returns to scale but there is little difference in production efficiency across the farm sizes. Marginal costs and economies of scale are similar between small, medium, and large-size farms. Thus, small and medium sized farms are just as efficient as the large ones. These findings do not explain the loss in farm numbers from the middle-size farm for the last several decades in North Dakota. The number of middle-size farms has declined, mainly because the farms may not be able to generate enough income to live as full-time farmers. On the other hand, large-size farms may not be more efficient in their operation, yet may generate enough income for their living. This may be a main reason for the increasing number of large-size farms in North Dakota.

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INTRODUCTION

Low commodity prices have been a problem faced by U.S., and, particularly, North Dakota farmers, since the Federal Improvement and Reform Act (FAIR) Act was passed in 1996. By this Act, farmers received government payments independent of farm prices (Taylor, Koo, and Swenson, 2002). Without government subsidies, net farm incomes in North Dakota farms would have been negative in most years since the late 1980s. Government spending in North Dakota has increased substantially during this period: $353 million in 1996 to $1170 million in 2000 (North Dakota Agricultural Statistical Service, 2001). U.S. exports, on the other hand, have been relatively stagnant. Although exports rose to $62.4 billion in 2004, they are forecasted to decline to $56 billion in 2005 (U.S. Department of Agriculture (USDA), 2004).

These factors have contributed to lower commodity prices, increased government spending, and declining net farm incomes. However, farm profitability varies greatly. Some farms continue to be profitable in spite of low commodity prices and unfavorable weather conditions (Taylor, Koo, and Swenson, 2002). Every year, it is typical for the top 25 percent of the farms to be extremely profitable and the lower 25 percent to show very little profit (Edwards and Kay, 1994).

In North Dakota in 2002, there were over 15,000 farms with sales less than $50,000 (small farms), about 8,000 farms with sales categories greater than $100,000 (large farms), and less than 5,000 farms with sales between $50,000-$100,000 (medium farms). While the number of large farms has increased since 1987 and has remained stable recently, the number of medium farms has fallen from over 5,000 in 1987 to about 3,000 in 2002. The number of small farms also decreased from over 20,000 in 1987 to just over 15,000 in 1992, but the numbers have increased steadily thereafter (Figure 1).

The net farm income for the medium and large farms has been increasing steadily, except for a slight drop in 1997. But for the smaller farms, net farm income has been declining into the negative since 1997 (Figure 2).

Of the total production in 2002, large farms accounted for over 80 percent, while medium and small farms accounted for a little over 10 percent each. Over the period 1987-2002, the percentage of total production increased for large farms and decreased for the medium and smaller farm income categories (Figure 3).

Of the total number of farms in 2002, almost 60 percent were small farms, a little over 10 percent were in the category of medium, while almost 30 percent were large farms. In general, there has been an increase in the percentage of large farms, while the percentage of small and medium farms has been relatively stable (Figure 4).

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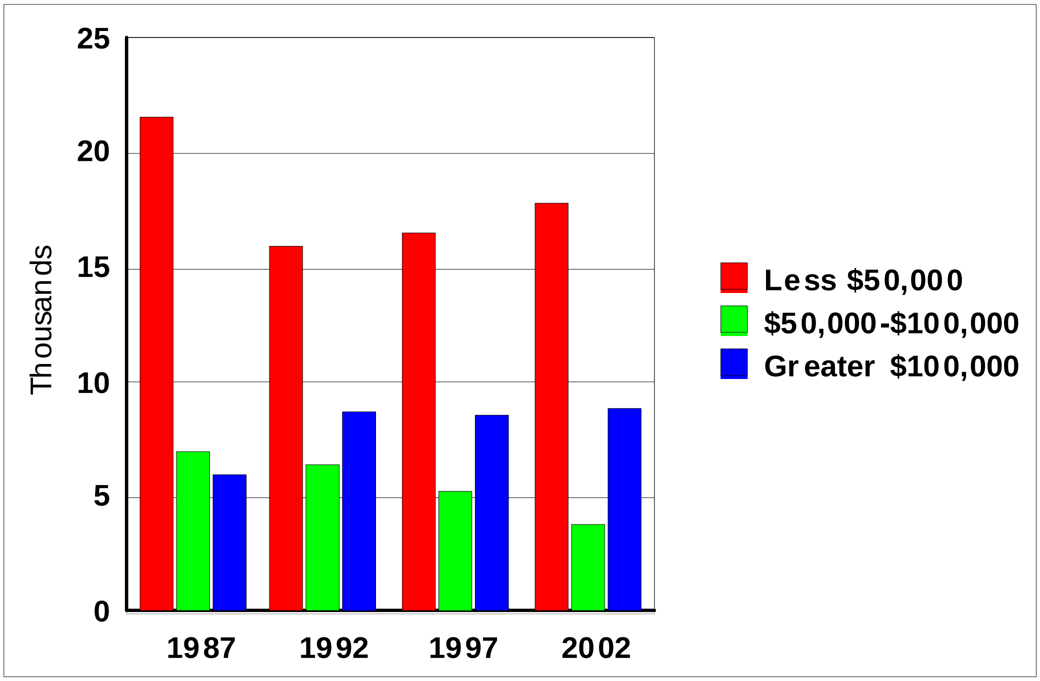

Figure 1. Number of Small, Medium, and Large Farms by Sales Categories in North Dakota, 1987-2002

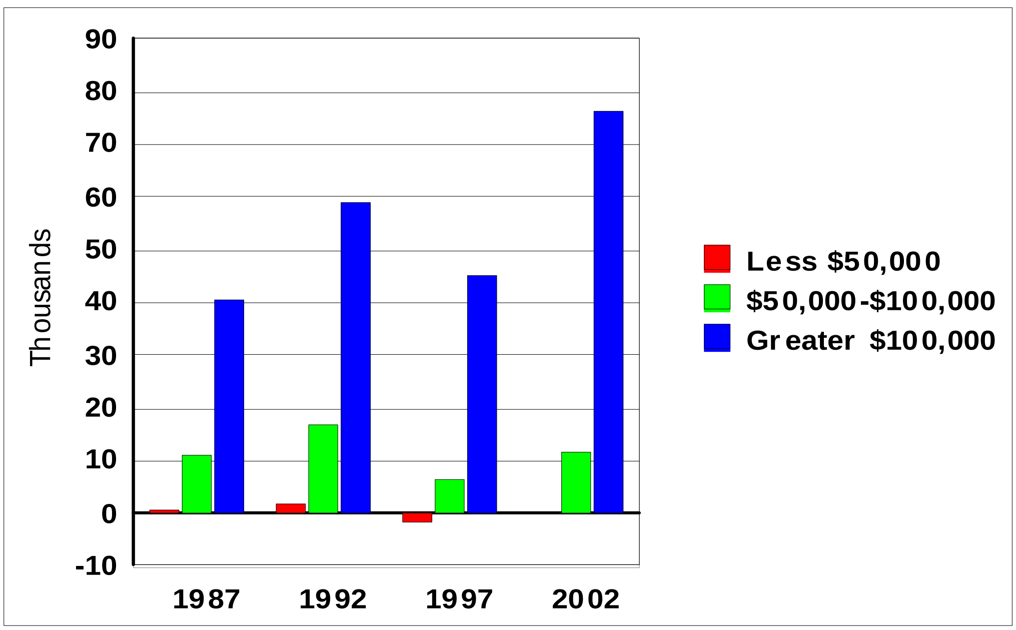

Figure 2. Net Farm Income of North Dakota Farms by Sales Categories, 1987-2002

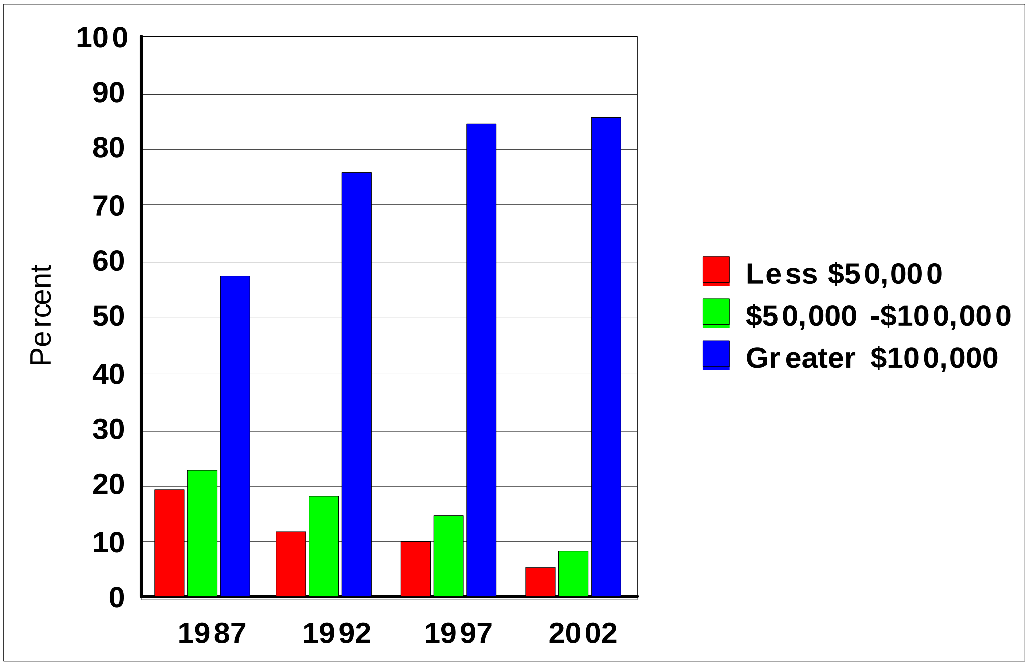

Figure 3. Percentage of Contribution to Total Production in North Dakota by Small, Medium, and Large Farms, 1987-2002

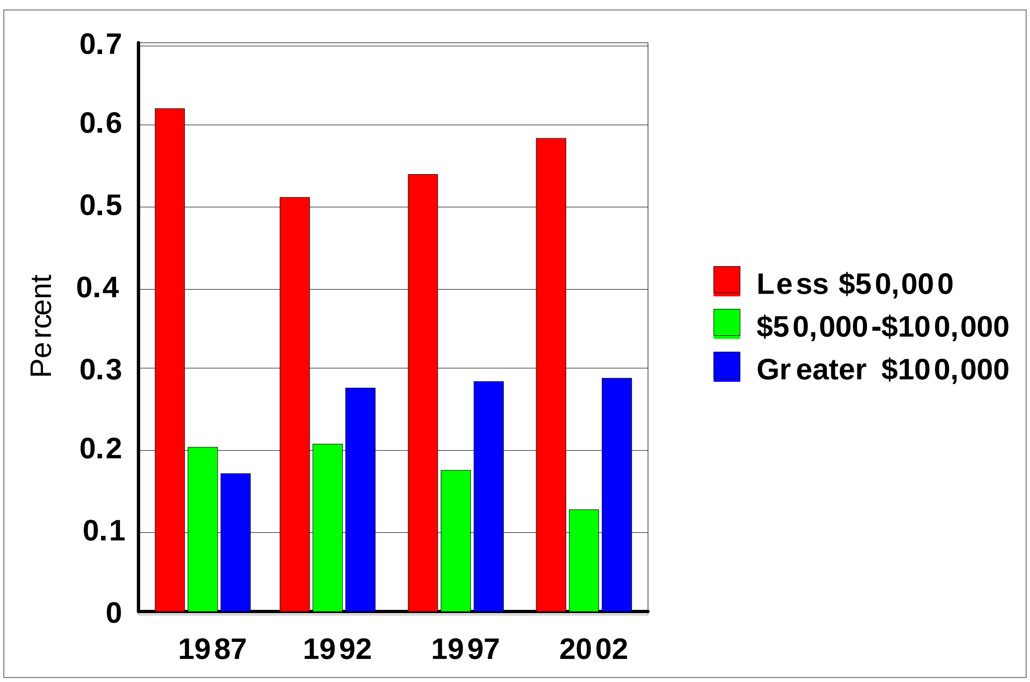

Figure 4. Percentage of Total Number of Farms in North Dakota Classified as Small, Medium, and Large Farms by Sales Categories, 1987-2002

However, total factor productivity (TFP) for 2003 does not increase as the farm size grows (Figure 5). TFP is total output value/ total input cost. If TFP is greater than 1, the farm shows positive net farm income. If TFP is less than 1, the farm has a negative net farm income.

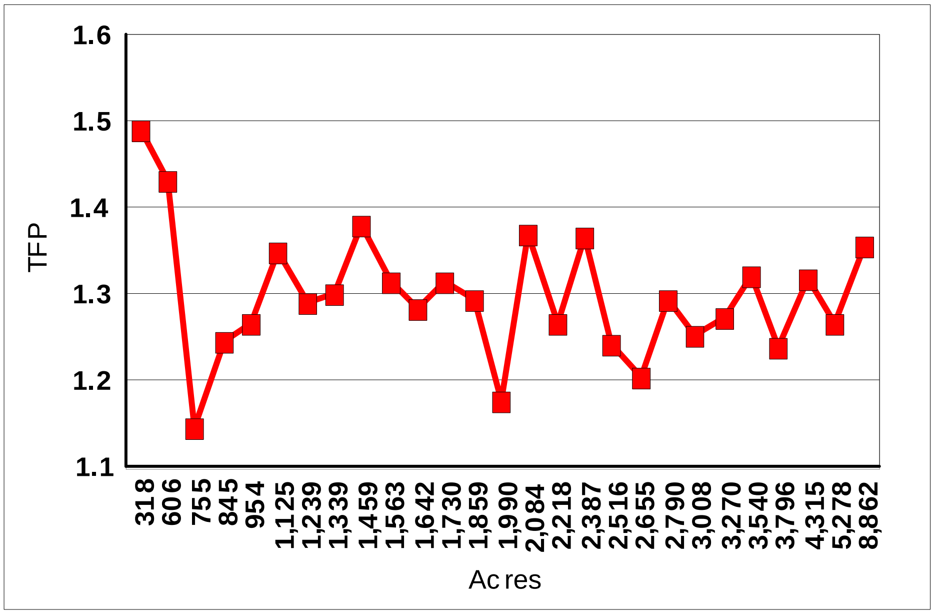

Figure 5. Total Factor Profitability for North Dakota Farms in 2003

Therefore, the question that arises is what drives the changes in farm numbers, particularly, in medium-size farms? Are large farms more cost efficient and able to take advantage of economies of scale, or are there other factors?

For these reasons, it is imperative to study the cost structure of North Dakota farms. Our paper has two main objectives:

1. To investigate the cost structure of North Dakota farms in different size categories.

2. To estimate economies of scale and input elasticities for North Dakota farms and to determine whether cost efficiencies exist in the different size categories.

The results from this study will help us understand what drives profitability and the changing farm numbers in North Dakota. The existence of scale economies would suggest that larger farms have a cost advantage over small farms in producing the same level of output. Thus, in the future we may expect the average farm size in North Dakota to grow as farmers take advantage of economies of scale.

METHODOLOGY

In this paper, we utilize the normalized quadratic cost function, a widely used flexible functional form which in its simplest form is a Tayler series expansion of order two (See Appendix A for details of the estimated cost function). The normalized quadratic cost function has the usual advantages of other flexible functional forms in terms of reducing specification errors, increasing deduction, and obtaining price elasticities at a point without imposing stringent restrictions on input elasticities (Gallant and Golub, 1984; Terrell,1996). A cost function should satisfy homogeneity, symmetry, and curvature conditions as required by economic theory. One of the limitations of the normalized quadratic cost functions is that, unlike the translog cost function, homogeneity cannot be directly imposed on the parameters without destroying the flexibility of the functional form (Caves, Christensen, and Tretheway, 1980). Thus, normalization (dividing the cost function and all factor prices by one common factor price) is used to overcome this problem and impose homogeneity.

The normalized quadratic function estimated in this paper takes the following general form:

$$C^{*\prime} = b_0 + \sum_{i=1}^{m-1} b_i W_i' + \sum_{i=m+1}^{n} b_i Y_i + 1/2\left(\sum_{i=1}^{m-1}\sum_{j=1}^{m-1} b_{ij} W_i' W_j' + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} b_{ij} Y_i Y_j \right) + \sum_{i=1}^{m-1}\sum_{j=m+1}^{n} b_{ij} W_i' Y_j \tag{1}$$

where $C^{*\prime} = C^{*}/W_m$ is the normalized cost (cost divided by the mth normalized price), $W_i' = {W_i}/{W_m}$ are the ith normalized input prices while Y_i is the ith output quantity. The cost function is assumed to be twice continuously differentiable and linearly homogeneous in input prices. In order to conform to economic theory, the cost function must satisfy homogeneity, concavity in input prices, and convexity in outputs.

Using Shephard's lemma, we can obtain the compensated input demand functions which are the first derivatives of the cost function as

$$\partial C/\partial W_i = X_i \quad \text{for } i=1\ldots\ldots,m-1 \tag{2}$$

Cross-equation symmetry restrictions are imposed by setting

$$b_{ij}=b_{ji} \text{ for all } i, j$$

and homogeneity, as mentioned before, is imposed by normalization (Featherstone and Moss 1994).

Curvature restrictions on the input side are satisfied if the Hessian matrix of prices is negative semi-definite; on the output side, curvature restrictions hold if the Hessian matrix of quantities is positive semi-definite. Curvature restrictions are first checked by calculating the eigen values for the Hessian matrices of input prices and output. Eigenvalues need to be negative for the matrix of prices to satisfy concavity and positive for the matrix of output to satisfy convexity (Featherstone and Moss 1994).

If curvature restrictions do not hold, curvature is imposed using the Cholesky decomposition method. A negative semi-definite Hessian matrix ensures that appropriate curvature restrictions are met on the input side. We can ensure negative semi-definiteness of the Hessian matrix by letting

$$B \equiv -AA^T \quad (3)$$

where B represents matrices of the parameters of the system we wish to estimate and A is a n x n lower triangular matrix. Using Cholesky decomposition, we can then reparameterize the model and estimate the parameters in A instead of the parameters in B (See Appendix A for details). This ensures that the Hessian matrix, $B \equiv -AA^T$, is negative semi-definite (Featherstone and Moss 1994). A similar approach is used to ensure positive semi-definiteness on the output side. Following Featherstone and Moss (1994) our own- price elasticities are calculated as follows:

$$Z_{ii} = (\partial X_i / \partial Wi)\ (W_i / X_i), \quad (4)$$

while cross-price elasticities are calculated as follows:

$$Z_{ij} = (\partial X_i / W_j)\ (W_j / X_i). \quad (5)$$

Economies of scale for output Y expressed as S(Y) are estimated as follows:

$$S(Y) = C(Y)/Y^*C(Y), \quad (6)$$

where Y is the output and $C(Y) = \partial C(Y)/\partial Y$ is the cost of producing output Y.

DATA

The estimated normalized quadratic cost function consists of eight inputs and one output. The data for this study were obtained from the North Dakota Farm Business Management Education Program for the period 1998-2003. A total of 106 farms were available for the six-year period, amounting to 630 observations. Each year, new farms may enroll in the farm management program, while other farms leave or may not finish the records by the deadline. The chosen farms existed over the entire period of six years and produced only crops. The output was represented by Y_1 (an aggregation of soybean, corn and wheat). The inputs consisted of seed (X_1), fertilizer (X_2), chemicals (X_3), repairs (X_4), fuel and oil (X_5), wages (X_6), land (X_7), and machinery (X_8). Input and output quantities were calculated by dividing the revenue by the price indices. Input and output price indices were used to represent input and output prices and were obtained from the USDA, National Agricultural Statistical Service (NASS), Agricultural Statistics (2004) (Chapter 9-Farm Resources, Income and Expenses).

All input prices, the output price, and total cost were normalized using the price of machinery. Summary statistics for the data are reported in Table 1.

Table 1. Summary Statistics of Sample Farms Observations (630)

Variable	Minimum	Maximum	Average	Standard Deviation
Seed	0.0000	629.68	108.26	107.84
Fertilizer	0.0000	726.11	150.04	123.63
Chemicals	0.0000	754.28	126.75	107.00
Repairs	0.0000	0.3606	5818.6	0.14
Fuel	0.0000	347.33	60.17	47.89
Wages	0.0000	413.24	32.99	62.02
Rent	6.8590	4147.0	990.0	756.45
Machinery	0.0000	685.00	77.29	74.829
Output-Crop	0.0000	175.29	36.70	30.12
Price of Seed	0.41990	1.0336	0.93	0.05
Price of Fertilizer	0.72973	2.2373	0.80	0.07
Price of Chemicals	0.80405	2.0539	0.85	0.06
Price of Repairs	0.87050	2.2373	0.89	0.05
Price of Fuel	0.63636	2.1823	0.80	0.13
Price of Wage	0.97122	1.5404	1.0152	0.03
Price of Rent	0.15924	2.3657	0.26825	0.11
Price of Machinery	1.0000	1.0000	1.0000	1.00
Price of Crop	0.68750	2.4207	0.73060	0.07
Cost	6.8795	0.80708	13667	0.31

EMPIRICAL RESULTS

The cost function and the eight input demand equations were estimated using iterative, seemingly unrelated regression. The usual procedure of deleting one equation and recovering the parameters of the deleted equation by homogeneity was followed. The equation for the input machinery was deleted in our analysis. Estimation was completed with and without curvature imposed (we report results only after curvature was imposed). The cost function was first estimated for the entire data set. The data set was later divided into large, medium, and small farms based on the average net farm income. Separate cost functions and economies of scale measures were estimated for the different farm sizes. All the models were estimated using Shazam 10.0 statistical software.

The parameter estimates for the system of equations are reported in Table 2. Of the 55 parameters estimated, 25 were statistically significant. Eighteen parameters were significant at the one percent level of significance, two at the five percent level of significance, and five at the ten percent level.

Table 2. Parameter Estimates of the Normalized Quadratic Cost Function with Curvature Imposed

	COEFFICIENT	STD. ERROR	T-RATIO
Constant (b0)	18.246	10.355	1.7621**
Seed (w1)	103.80	39.194	2.6483***
Fertilizer (w2)	-16.107	31.628	-0.50926
Chemicals (w3)	215.18	83.139	2.5882***
Fuel (w4)	7.0788	43.094	0.16426
Wage(w5)	24.964	18.446	1.3534*
Rent (w6)	36.608	19.407	1.8863**
Machinery (w7)	-290.54	71.019	-4.0910***
Crop (y1)	22.086	18.102	1.2201
A11(w1*w1)	14.255	1.0302	13.837***
A12 (w1*w2)	-7.0676	1.9313	-3.6594***
A13 (w1*w3)	20.633	2.2102	9.3357***
A14 (w1*w4)	-17.862	4.7779	-3.7384***
A15 (w1*w5	2.7638	0.76778	3.5997***
A16 (w1*w6)	-1.2014	2.5389	-0.47321
A17 (w1*w7)	-13.267	1.5770	-8.4129***
B19 (w1*y1)	2.7954	0.070794	39.486***
A22 (w2*w2)	-0.98372	1.6185	-0.60779
A23 (w2*w3)	-2.8171	1.7271	-1.6311*
A24 (w2*w4)	3.2682	4.0160	0.81380
A25 (w2*w5)	-1.0701	0.88914	-1.2035
A26 (w2*w6)	0.75564	2.4721	0.30567
A27 (w2*w7)	-25.056	1.8957	-13.217***
B29 (w2*y1)	3.3896	0.099341	34.121**
A33 (w3*w3)	-5.1306	3.1490	-1.6293*
A34 (w3*w4)	-4.8302	16.003	-0.30183
A35 (w3*w5)	3.5737	1.0398	3.4368***
A36 (w3*w6)	1.1080	11.466	0.096637
A37 (w3*w7	0.36788	3.5956	0.10232
B39 (w3*y1)	2.6414	0.082552	31.997***
A44 (w4*w4)	-16.021	11.664	-1.3736*
A45 (w4*w5)	0.44184	3.2145	0.13745
A46 (w4*w6)	8.3372	7.5316	1.1070
A47 (w4*w7)	1.0563	3.7378	0.28260
B49 (w4*y1)	16.913	13.473	1.2553
A55 (w5*w5)	0.16173	1.8143	0.89143
A56 (w5*w6)	10.362	1.8559	5.5835***
A57 (w5*w7)	0.24268	3.9716	0.061104
B59 (w5*y1)	1.2161	0.040349	30.139***
A66 (w6*w6)	2.5446	1.9732	1.2896*
A67 (w6*w7)	0.046462	3.9394	0.11794
B69 (w6*y1)	1.4046	0.064166	21.890***
A77 (w7*w7)	0.000703	3.9003	0.0001803
B79 (w7*y1)	25.223	0.23500	107.33***
A99 (y1*y1)	-0.00009700	0.10177	-0.0009532

***indicates significance at the 1percent level; **indicates significance at the 5 percent level; ***indicates significance at the 10 percent level**

Note: The parameters A11 through A99 are the curvature parameters transformed using Cholesky decomposition, while the parameters B19 through B79 are interaction parameters of the seven inputs prices (seed through rent) with the output price (crop). Please see Appendix A for details.

Table 3 shows the price elasticity estimates for the eight inputs (seed, fertilizer, chemicals, repairs, fuel, wages, land, and machinery) with curvature imposed. The elasticities were estimated at the mean of input and output price. Before imposing curvature, the own-price elasticities of chemicals, fuel, wages, land, and machinery were negative and inelastic except for machinery. The own-price elasticities for seed, fertilizer, and repairs were positive. This indicates that curvature properties were not satisfied (Featherstone and Moss, 2004). After curvature was imposed, all the input curves were downward sloping. In general own-price elasticities are a little larger after curvature imposition. The own-price elasticties are elastic for seed, chemicals, and wages and inelastic for the other inputs.

The input seed is substitutable with fertilizer, feed, wage, rent, and machinery and complementary with other inputs. The fertilizer input is a net substitute with chemical and fuel, while the inputs feed, wage, rent, and machinery are net substitutes with chemical. The input pairs, feed and fuel, feed and wage, fuel and rent, fuel and machinery, wage and machinery, and rent and machinery, are also net substitutes. While the high elasticity estimates for some of the inputs, particularly wages, are unexpected, they can be attributed to the fact that we had zero values for the wage variable for some observations.

Table 3. Price Elasticities at Mean for the Normalized Quadratic Cost Function with Curvature Imposed

	SEED	FERT	CHEM	FEED	FUEL	WAGE	RENT	MACH
SEED	-1.744714	0.7453973	-2.312625	2.089409	-0.291891	0.1592399	0.4646696	0.8905137
FERT	0.6315990	-0.2750668	0.8212725	-0.737118	0.0999722	-0.5259963	-0.052599	-0.275634
CHEM	-2.159181	0.9049405	-3.092313	2.476486	-0.264185	0.2591828	0.4307446	1.444326
FEED	0.3580997	-0.1490953	0.4546033	-0.819600	0.0936935	0.1751417	-0.054890	-0.057951
FUEL	-0.613120	0.2478282	-0.594361	1.148297	-0.292419	-0.0874647	0.0357836	0.1554580
WAGE	0.4867737	-0.1897598	0.8485896	3.123808	-0.127286	-5.743251	-0.072012	1.673139
RENT	0.3209924	-0.1731858	0.3187033	-0.221242	0.0117682	-0.01627356	-0.391025	0.1502629
MACH	0.0946773	-0.3458481	0.1644701	-0.035949	0.0786853	0.5819188	0.0231263	-0.277800

Table 4 presents the marginal cost and economies of scale estimates after curvature was imposed for the entire set and for the different farm categories: small, medium, and large. If economies of scale are greater than one, there is evidence of increasing returns to scale; a value less than one indicates decreasing returns to scale. Before imposing curvature, the estimate for economies of scale was 0.86, indicating decreasing returns to scale. After curvature was imposed, the scale measure was 1.07, showing increasing returns to scale. Within the different farm categories, we also found evidence of efficiency (all the three categories showed increasing returns to scale).

Table 4. Marginal Costs and Economies of Scale with Curvature Imposed

Farms	Marginal Cost	Economies of Scale
All	54.04145	1.07225
Small	17.70475	1.19265
Medium	18.61417	1.21634
Large	19.54811	1.20774

Data selection bias may be one of the reasons behind our surprising results (we selected only those farms that existed over a six-year period) which are against the common belief that large farms have an advantage in terms of fixed cost (small farms cannot make optimal use of expensive equipment). However, previous research on North Dakota farms indicates that large farms have advantage over small farms only in terms of borrowing: operating interest was found to be a significant variable in Taylor, Koo, and Swenson (2002). Smaller farms are also more efficient in terms of management and labor (Rosset, 1999).

Our results also indicate that marginal costs are about the same for small, medium, and large farms. Thus, we do not find a significant decrease in marginal costs as the average farm size increases.

CONCLUSIONS

We estimated a normalized quadratic cost function to determine if cost efficiencies exist for North Dakota farms. Results from our empirical analysis indicate that, overall, there is evidence of increasing returns to scale, but across the farm sizes there is little difference in production efficiency. Marginal costs and economies of scale are similar across small, medium, and large-size farms. Thus, small and medium sized farms are just as efficient as the large ones.

If per acre profitability does not explain the loss of the middle-size farms in North Dakota, something else must be behind the falling numbers. Several possible reasons include: (1) the middle-size farm does not have the resources for new technology or modern machinery; (2) the middle-size farm may have difficulty transferring assets to the next generation (i.e., it cannot generate family living expenses for more than one family during the transition); and (3) unlike the small-size farm, the middle-size farm has to generate enough income for family living expenses because the producer does not have time available for non-farm work. Any or all of these could be the reason behind the loss of the middle-size farm; however, further research is needed before a determination can be made.

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Appendix

The cost and factor demand equations estimated in this paper are shown below:

Cost Equation:

Cost = b0+b1*w1+b2*w2+b3*w3+b4*w4+b5*w5+b6*w6+b7*w7+b9*y1
+.5*b11*w11+b12*w12+b13*w1*w3+b14*w1*w4+b15*w1*w5+b16*w1*w6+b17*w1*w7+b1
9*w1*y1+0.5*b22*w2+b23*w2*w3+b24*w2*w4+b25*w2*w5+b26*w2*w6+b27*w2*w7+b29
*w2*y1+0.5*b33*w3*w3+b34*w3*w4+b35*w3*w5+b36*w3*w6+b37*w3*w7+b39*w3*y1+0
.5*b44*w4*w4+b45*w4*w5+b46*w4*w6+b47*w4*w7+b49*w4*y1+0.5*b55*w5*w5+b56*w
56+b57*w57+b59*w5*y1+0.5*b66*w6*w6+b67*w6*w7+b69*w6*y1+0.5*b77*w7*w7+b79*
w7*y1+0.5*b99*y1*y1

Where w1 through w7 are the seven inputs (seed, fertilizer, chemicals, repairs, fuel and oil, wages, and land) normalized by machinery (w8), y1 is the output (crop), and cost is the total cost normalized by machinery.

Factor demand Equations:

x1=b1+b11*w1+b12*w2+b13*w3+b14*w4+b15*w5+b16*w6+b17*w7+b19*y1
x2=b2+b12*w1+b22*w2+b23*w3+b24*w4+b25*w5+b26*w6+b27*w7+b29*y1
x3=b3+b13*w1+b23*w2+b33*w3+b34*w4+b35*w5+b36*w6+b37*w7+b39*y1
x4=b4+b14*w1+b24*w2+b34*w3+b44*w4+b45*w5+b46*w6+b47*w7+b49*y1
x5=b5+b15*w1+b25*w2+b35*w3+b45*w4+b55*w5+b56*w6+b57*w7+b59*y1
x6=b6+b16*w1+b26*w2+b36*w3+b46*w4+b56*w5+b66*w6+b67*w7+b69*y1
x7=b7+b17*w1+b27*w2+b37*w3+b47*w4+b57*w5+b67*w6+b77*w7+b79*y1

Where x1 to x8 represent the eight inputs (seed, fertilizer, chemicals, repairs, fuel and oil, wages, land, and machinery, respectively).

In order to impose curvature, we have to ensure that the Hessian matrix of input prices is negative-semidefinite for concavity.

We first calculate the eigenvalues for our B matrix.

$$B=\begin{bmatrix} b11 & b12 & b13 & b14 & b15 & b16 & b17 & b18 \\ b12 & b22 & b23 & b24 & b25 & b26 & b27 & b28 \\ b13 & b23 & b33 & b34 & b35 & b36 & b37 & b38 \\ b14 & b24 & b34 & b44 & b45 & b46 & b47 & b48 \\ b15 & b25 & b35 & b45 & b55 & b56 & b57 & b58 \\ b16 & b26 & b36 & b46 & b56 & b66 & b67 & b68 \\ b17 & b27 & b37 & b47 & b57 & b67 & b77 & b78 \\ b18 & b28 & b38 & b48 & b58 & b68 & b78 & b88 \end{bmatrix} \text{ All eigenvalues} \leq 0$$

Next, we impose curvature restrictions using the Cholesky decomposition, which will transform our B matrix above into a lower triangular matrix.

$$P=\begin{bmatrix} a11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a12 & a22 & 0 & 0 & 0 & 0 & 0 & 0 \\ a13 & a23 & a33 & 0 & 0 & 0 & 0 & 0 \\ a14 & a24 & a34 & a44 & 0 & 0 & 0 & 0 \\ a15 & a25 & a35 & a45 & a55 & 0 & 0 & 0 \\ a16 & a26 & a36 & a46 & a56 & a66 & 0 & 0 \\ a17 & a27 & a37 & a47 & a57 & a67 & a77 & 0 \\ a18 & a28 & a38 & a48 & a58 & a68 & a78 & 0 \end{bmatrix}$$

$$B= PP'= -$$

$$\begin{bmatrix} a11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a12 & a22 & 0 & 0 & 0 & 0 & 0 & 0 \\ a13 & a23 & a33 & 0 & 0 & 0 & 0 & 0 \\ a14 & a24 & a34 & a44 & 0 & 0 & 0 & 0 \\ a15 & a25 & a35 & a45 & a55 & 0 & 0 & 0 \\ a16 & a26 & a36 & a46 & a56 & a66 & 0 & 0 \\ a17 & a27 & a37 & a47 & a57 & a67 & a77 & 0 \\ a18 & a28 & a38 & a48 & a58 & a68 & a78 & 0 \end{bmatrix} \begin{bmatrix} a11 & a12 & a13 & a14 & a15 & a16 & a17 & a18 \\ 0 & a22 & a23 & a24 & a25 & a26 & a27 & a28 \\ 0 & 0 & a33 & a34 & a35 & a36 & a37 & a38 \\ 0 & 0 & 0 & a44 & a45 & a46 & a47 & a48 \\ 0 & 0 & 0 & 0 & 0 & a56 & a57 & a58 \\ 0 & 0 & 0 & 0 & 0 & a66 & a67 & a68 \\ 0 & 0 & 0 & 0 & 0 & 0 & a77 & a78 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a88 \end{bmatrix}$$

$$B = -$$

$$\begin{bmatrix}
(a11*a11)\ (a11*a12)\ (a11*a13)\ (a11*a14)\ (a11*a15)\ (a11*a16)\ (a11*a17)\ (a11*a18) \\
(a12*a11)\ (a12*a12+a22*a22)\ (a12*a13+a22*a23)\ (a12*a14+a22*a24)(a12*a15+a22*a25)(a12*a16+a22*a26) \\
(a12*a17+a22*a27)(a12*a18+a22*a28) \\
(a13*a11)\ (a13*a12+a23*a22)\ (a13*a13+a23*a23+a33*a33)(a13*a14+a23*a24+a33*a34)(a13*a15+a23*a25+a33*a35) \\
(a13*a16+a23*a26+a33*a36)(a13*a17+a23*a27+a33*a37)(a13*a18+a23*a28+a33*a38) \\
(a14*a11)\ (a14*a12+a24*a22)\ (a14*a13+a24*a23+a34*a33)(a14*a14+a24*a24+a34*a34+a44*a44) \\
(a14*a15+a24*a25+a34*a35+a44*a45)(a14*a16+a24*a26+a34*a36+a44*a46)(a14*a17+a24*a27+a34*a37+a44*a47) \\
(a14*a18+a24*a28+a34*a38+a44*a48) \\
(a15*a11)\ (a15*a12+a25*a22)\ (a15*a13+a25*a23+a35*a33)(a15*a14+a25*a24+a35*a34+a45*a44) \\
(a15*a15+a25*a25+a35*a35+a45*a45+a55*a55)(a15*a16+a25*a26+a35*a36+a45*a46+a55*a56) \\
(a15*a17+a25*a27+a35*a37+a45*a47+a55*a57)(a15*a18+a25*a28+a35*a38+a45*a48+a55*a58) \\
(a16*a11)\ (a16*a12+a26*a22)\ (a16*a13+a26*a23+a36*a33)(a16*a14+a26*a24+a36*a34+a46*a44) \\
(a16*a15+a26*a25+a36*a35+a46*a45+a56*a55)(a16*a16+a26*a26+a36*a36+a46*a46+a56*a56+a66*a66) \\
(a16*a17+a26*a27+a36*a37+a46*a47+a56*a57+a66*a67)(a16*a18+a26*a28+a36*a38+a46*a48+a56*a58+a66*a68) \\
(a17*a11)\ (a17*a12+a27*a22)\ (a17*a13+a27*a23+a37*a33)(a17*a14+a27*a24+a37*a34+a47*a44) \\
(a17*a15+a27*a25+a37*a35+a47*a45+a57*a55)(a17*a16+a27*a26+a37*a36+a47*a46+a57*a56+a67*a66) \\
(a17*a17+a27*a27+a37*a37+a47*a47+a57*a57+a67*a67+a77*a77) \\
(a17*a18+a27*a28+a37*a38+a47*a48+a57*a58+a67*a68+a77*a78) \\
(a18*a11)\ (a18*a12+a28*a22)\ (a18*a13+a28*a23+a38*a33)(a18*a14+a28*a24+a38*a34+a48*a44) \\
(a18*a15+a28*a25+a38*a35+a48*a45+a58*a55)(a18*a16+a28*a26+a38*a36+a48*a46+a58*a56+a68*a66) \\
(a18*a17+a28*a27+a38*a37+a48*a47+a58*a57+a68*a67+a78*a77) \\
(a18*a18+a28*a28+a38*a38+a48*a48+a58*a58+a68*a68+a78*a78+a88*a88)
\end{bmatrix}$$

We then substitute our B parameters using the above transformation and estimate the maximum likelihood with curvature imposed.