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Estimation of the diffusivities and mass transfer coefficients for pears dried under different methods

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Abstract

Traditionally, in Portugal pears of the variety S. Bartolomeu are dried through an open-air sun exposure. However, this method of drying has some disadvantages, among which not being able to fully guarantee the necessary quality and sanitary standards for commercialization. Hence, the development of alternative drying methods to replace the traditional drying is of great importance nowadays, and the knowledge of the transfer phenomena happening during drying helps to understand these processes.

The present work aimed at determining the mass transfer properties of pears for air drying performed in three drying systems: solar stove with forced air convection (STFC), solar stove with natural air convection (STNC) and drying tunnel with hot air from a solar collector (DT). The methodology used to determine the mass transfer properties was a diffusion model.

The values of the diffusion and mass transfer coefficients for the drying in the three systems were estimated, and from them was possible to conclude that the values of \( D_e \) in the STNC and DT systems are similar to each other, whereas the STFC system gives a lower value of diffusivity, indicating that in this system the internal moisture transfer is less efficient. As to \( h_m \), the system that presents the higher values is DT, demonstrating a higher efficiency of surface mass transfer.

Keywords: pear drying, diffusivity, mass transfer coefficient.

1. Introduction

Open-air sun drying is the oldest drying method, being used from immemorial times to dry many agricultural products in countries where the combination of solar radiation, temperature and relative humidity is appropriate. In fact, this is the cheapest of the drying methods, but on the other hand has many important disadvantages, such as: the quality of the final product can be quite affected, due to exposure to many potentially contaminating agents; the production can be severely affected by factors like adverse weather conditions; and the production in large scale is difficult to achieve (1, 2).

In Portugal pears of the variety S. Bartolomeu (\( Pyrus communis \) L.) are dried using a traditional solar drying method, which is based on an open-air exposure (3, 2). Nevertheless, to surpass the handicaps of this method, alternative drying methods have been tested, namely using a solar stove or a drying tunnel with air heated through a solar collector, which allow conciliating cheapness with the improvement in quality by protecting the fruits against external dangers.

From the engineering point of view it is important to better understand the complex process of drying through modelling. Many mathematical models have been proposed to describe the drying process, among which the diffusion laws are quite common. During drying many changes take place inside the foods (4), and these modifications affect the product mass transfer properties such as the mass diffusion and mass transfer coefficients.

The present work aimed at determining the mass transfer properties of pears for air drying performed in three drying systems: solar stove with forced air convection, solar stove with natural air convection and drying tunnel with hot air from a solar collector.

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2. Materials and methods

Pears of the variety S. Bartolomeu were dried in three different systems: solar stove with forced convection (STFC), solar stove with natural convection (STNC) and drying tunnel (DT).

In the STFC and STNC systems, the temperature and relative humidity were variable according to the weather conditions and night/day hours. In the DT system, the temperature was kept constant at 40-42 °C and the drying air velocity was always 1.1 m/s. In the STFC system the air velocity was also kept constant at 0.4 m/s.

The drying was carried out until the pears reached a desirable moisture content of about 20 % (wet basis). The drying time was 147 hours in the STFC system, 140 hours in the STNC system and 93 hours in the DT system.

Along drying the moisture content of the pears was measured with a Halogen Moisture Analyser (model HG53, from Mettler Toledo) in the STFC and STNC systems. In the DT system the moisture was determined by weight loss, measured by weighing the whole set of pears along drying.

3. Mathematical Modelling

The non steady-state diffusion for unidirectional mass transfer, assuming that the pears can be approximated to spheres, is expressed by Fick's second law (5):

\[
\frac{\partial W}{\partial t} = \frac{1}{r} \left( D_e \frac{\partial}{\partial r} \left( r \frac{\partial W}{\partial r} \right) \right)
\]

where \( W(r,t) \) is the dry basis moisture content [kg water/kg dry solids], \( t \) is time [s], \( D_e \) is effective diffusivity [m\(^2\)/s] and \( r \) is the sphere radius [m].

Considering the initial moisture content is uniform throughout the sample and presents central symmetry, the initial and boundary conditions are:

At \( t = 0 \):

\[
W(r,0) = W_0
\]

At \( r = 0 \):

\[
\frac{\partial W(0,t)}{\partial r} = 0
\]

At \( r = R \):

\[
-D_e \frac{\partial W(R,t)}{\partial r} = h_m [W - W_a]
\]

where \( W_0 \) is the initial product moisture content, \( W_a \) is the surrounding air moisture content (all dry basis) and \( h_m \) is the convective mass transfer coefficient.

For the non steady diffusion, the solution of Fick’s Law can be approximated by an infinite series, of the form (5, 6):

\[
MR = \frac{W - W_e}{W_0 - W_e} = \sum_{n=1}^{\infty} \frac{6}{\pi^2} \exp \left[ -D_e \frac{\pi^2 t}{r^2} \right]
\]

where \( MR \) is the moisture ratio [dimensionless], and \( W \), \( W_a \) and \( W_0 \) are, respectively, the moisture content at time \( t \), the equilibrium moisture content and the initial moisture content, all expressed in dry basis [g water/g dry solids]. Considering that the second and following terms of the series can be neglected, then the solution of the Fick’s Equation is given by:

\[
MR = \left( \frac{6}{\pi^2} \right) \exp \left[ -D_e \frac{\pi^2 t}{r^2} \right]
\]

Thin-layer models admit that the moisture variations along drying are expressed in terms of some parameters, such as the drying constant, \( k \) [1/s], or the lag factor, \( k_0 \) [dimensionless], that account for combined effects of various transport phenomena during drying (7). The Henderson and Pabis model is an example of such models, and is given in expressed through the following equation (8):

\[
MR = k_0 \exp(-kt)
\]

The convective mass transfer coefficient, \( h_m \) [m/s], and the diffusivity coefficient are correlated by the dimensionless Biot number for mass transfer (9):
\[ Bi_m = \frac{h_m r}{D_i} \]  \[ \text{[8]} \]

where \( r \) is the sphere diameter [m]. Equation [8] is valid for \( Bi \) greater than 0.1 (10), and allows the estimation of \( h_m \), if the \( Bi_m \) is known.

Dincer and Hussain (2002) (10) report the equation that correlates the Biot number with the dimensionless Dincer Number:

\[ Bi_m = \frac{24.848}{Di^{0.375}} \]  \[ \text{[9]} \]

with,

\[ Di = \frac{u}{kr} \]  \[ \text{[10]} \]

where \( u \) is the flow velocity of drying air [m/s], \( k \) the drying constant and \( r \) the radius.

To determine the mass transfer properties of the pears in the three drying systems the methodology used was: 1. Estimate \( MR \) from the experimental drying data for every time \( t \); 2. From a plot \( \ln(MR) = f(t) \) estimate \( D_e \) from the slope through Eq. [6] (slope = \(-D_e \pi^2 r^2\)); 3. Estimate \( k \) and \( k_0 \) by combining Eqs. [6] and [7] \((k = D_e \pi^2 r^2, k_0 = 6/\pi^2)\); 4. Calculate \( Di, Bi_m \) and \( h_m \) from Eqs. [10], [9] and [8], respectively.

3. RESULTS AND DISCUSSION

Figure 1 shows the variations of the moisture content of pears along drying for different drying systems: solar stove with forced convection (STFC), solar stove with natural convection (STNC), drying tunnel (DT) with air heated by a solar collector. It is possible to observe a closer similarity between the two dryings performed with solar stoves, while the drying carried out in the drying tunnel presents a different kinetic profile, allowing a faster initial drying, but tending quite rapidly for stabilization in values slightly higher than those obtained in the solar stoves. The final values of moisture content were 16.5 %, 9.9 % and 23.5 % (wet basis), in the STFC, STNC and DT systems, and were achieved after 147 h, 109 h, and 93 h, respectively.

![Figure 1](image1.png)

Figure 1. Variations of the pears moisture content along drying for different drying systems. (STFC – solar stove with forced convection, STNC – solar stove with natural convection, DT – drying tunnel).

Figure 2 shows the linearization of the functions \( \ln(MR) = f(t) \) for the three drying systems, and the corresponding results are presented in Table 1. It is observed that the experimental data of the DT system follows a more regular pattern, giving better fitting results, as seen by a higher regression...
coefficient. The values of $R^2$ are thus 0.9592 for the DT system, 0.8978 for the STFC system and 0.8862 for the STNC system, being the experimental data in this last case slightly less regular.

Figure 2. Linearization of the functions $\ln(MR)=f(t)$ for the different drying systems. (Results of the fits presented in Table 1).
Table 1. Parameters for linearization of the function \( \ln(MR) = f(t) \) (method 1), in the three drying systems tested.

<table>
<thead>
<tr>
<th>Drying system*</th>
<th>Slope</th>
<th>Intercept</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>STFC</td>
<td>-5.7188x10^{-6}</td>
<td>0.2750</td>
<td>0.8978</td>
</tr>
<tr>
<td>STNC</td>
<td>-1.1037x10^{-5}</td>
<td>0.8126</td>
<td>0.8862</td>
</tr>
<tr>
<td>DT</td>
<td>-9.5825x10^{-6}</td>
<td>0.4067</td>
<td>0.9592</td>
</tr>
</tbody>
</table>

*STFC – solar stove with forced convection, STNC – solar stove with natural convection, DT – drying tunnel.

In Table 2 the values calculated using the methodology explained previously for the different mass transfer properties are presented for the three drying systems considered. The results show that the drying constant, \( k \), is higher in the STNC system and lower in the STFC system. The values of the drying constant (from 5.7188x10^{-6} s^{-1} to 1.1037x10^{-5} s^{-1}) are in the same range as those reported by Roberts et al (2008) (11) for the convective hot air drying of grape seeds in the range of temperatures from 40 to 60 ºC and with air velocities above 1.5 m/s.

Dincer and Hussain (2002) (9), for the drying of potatoes at 40 ºC, with an air velocity of 1 m/s and a characteristic dimension of 0.09 m, report a value for the lag factor of 1.0074, which is slightly higher that that in the present study.

The values of the Di number in Table 2 vary between 3.7135x10^{-5} for the STNC system and 2.3174x10^{-6} for the DT system. The value presented by Dincer and Hussain (2002) (9) is 1.2356x10^{-4} for spherical geometry for the air drying at 40 ºC of potatoes of 9 cm radius with an air velocity of 1 m/s.

Table 2. Mass transfer properties of pears calculated for the three drying systems studied.

<table>
<thead>
<tr>
<th>Drying System*</th>
<th>( D_e ) (m^2/s)</th>
<th>( k ) (s^{-1})</th>
<th>( k_0 )</th>
<th>( Di )</th>
<th>( Bi )</th>
<th>( h_m ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STFC</td>
<td>1.4218x10^{-9}</td>
<td>5.7188x10^{-6}</td>
<td>0.6079</td>
<td>1.4334x10^{-6}</td>
<td>0.1221</td>
<td>3.5040x10^{-9}</td>
</tr>
<tr>
<td>STNC</td>
<td>2.7439x10^{-9}</td>
<td>1.1037x10^{-5}</td>
<td>0.6079</td>
<td>3.7136x10^{-5}</td>
<td>0.2026</td>
<td>1.1222x10^{-8}</td>
</tr>
<tr>
<td>DT</td>
<td>2.3823x10^{-9}</td>
<td>9.5825x10^{-6}</td>
<td>0.6079</td>
<td>2.3174x10^{-6}</td>
<td>0.1020</td>
<td>4.9035x10^{-9}</td>
</tr>
</tbody>
</table>

*STFC – solar stove with forced convection, STNC – solar stove with natural convection, DT – drying tunnel.

The Biot numbers vary from 0.1020 to 0.2026, respectively in the DT and STNC systems. All the values are higher than 0.1, thus allowing the use of Equation [8] for the estimation of the mass transfer coefficients. For the convective drying of spherical potatoes at 40 ºC Dincer and Hussain (2002) (9) report a value of 0.3119.

Regarding the values of the convective mass transfer coefficient, \( h_m \), the STNC system is where the higher value was found (1.1222x10^{-8} m/s), whereas the STFC system corresponds to the lower value (3.5040x10^{-9} m/s). These values are much smaller than those found by Dincer and Hussain (2002) (9) for the convective drying of spherical potatoes (3.2665x10^{-5} m/s), but in the same range of that for the convective drying of cylindrical okra (1.6098x10^{-8} m/s) at 80 ºC, with an air velocity of 1.2 m/s and characteristic dimension of 0.003 m. Tripathy and Kumar (2009) (7) determined the convective mass transfer coefficients for potato elements in cylindrical and sliced shapes, and their results lead to \( h_m \) ranging from 1.61x10^{-7} m/s to 4.17x10^{-7} m/s in the range of temperatures from 33.74 ºC to 47.70 ºC for the cylindrical shape and from 1.70x10^{-7} m/s to 3.21x10^{-7} m/s for temperatures between 35.55 ºC and 49.88 ºC for the sliced shape.

The values of the effective diffusion coefficient, \( D_e \), vary between 1.4218x10^{-10} m^2/s in the STFC system and 2.7439x10^{-9} m^2/s in the STNC. These values are smaller if compared to that encountered by Dincer and Hussain (2002) (9) for the air drying of spherical potatoes (9.4259x10^{-9} m^2/s), but of the same magnitude of that reported in the same work for the air drying of cylindrical okra (5.6752x10^{-10} m^2/s). Tripathy and Kumar (2009) (7) refer values of \( D_e \) in the range 3.28x10^{-8} - 6.09x10^{-8} m^2/s for cylindrical potato samples for temperatures in the interval 33.74 - 47.70 ºC, and \( D_e \) values in the range 2.43x10^{-8} - 4.18x10^{-9} m^2/s for sliced potato samples at temperatures between 35.55 ºC and 49.88 ºC, being in all cases higher than the values found in the present study for all systems.
4. Conclusions

In the present work the values of the diffusion and mass transfer coefficients for the drying of pears in the three drying systems considered were estimated. It was observed that the values of diffusivity in the STNC and DT systems are similar to each other, while the STFC system gives a little lower value of diffusivity, indicating that this system is less efficient in terms of internal mass transfer. Regarding the mass transfer coefficients, the system that presents the higher value is STNC, which indicates that this drying method promotes a higher efficiency of the moisture transfer at the surface of the pears.

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References