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Forward Discount Puzzle and Liquidity Effects: Some Evidence from Exchange Rates among US, Canada, and Japan

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Abstract. This paper empirically examines whether the interaction between foreign exchange markets and monetary markets can help to resolve the forward discount puzzle. Following the monetary models of Lucas [1990] and Fuerst [1992], we define as liquidity effects (the negative impact of monetary injection on nominal interest rates), temporary deviations from the standard Euler equation. The liquidity effect identified by these models weakens the linkage between current forward rates and expected future spot rates, and improves on the standard rational expectations model that predicts a one-to-one correspondence between the two. Using time-series of exchange rates among US, Canada, and Japan, this paper shows that the liquidity measure identified above has an impact on forward premiums, and that once the liquidity effect is taken into consideration, the unbiased prediction of the forward discount rate is recovered to some extent in a theoretically consistent manner.


Keywords: forward discount anomaly, liquidity effects
1. **Introduction** One of the most puzzling phenomena of foreign exchange markets is that forward discount rates (the difference between forward rates and spot rates) cannot serve as an unbiased predictor for expected changes in spot rates. Furthermore, empirical studies frequently document that the currency of a country with higher nominal interest rates tends to appreciate at later times. Such an empirical finding is dramatically contrary to the standard prediction that forward rates and future spot rates will be jointly depreciated under high nominal interest rates. In the literature on international finance, this puzzling phenomenon is called the forward discount anomaly or the forward discount puzzle.

Serious efforts to resolve the forward discount puzzle have been made in past decades. Hodrick [1987], Engel [1996], and others offer systematic surveys on recent developments of theoretical and empirical research in this field. In one direction, researchers introduce risk averse behavior into the standard rational expectations model. In another direction, they adopt several alternatives to rational expectations models, including peso problems, irrational expectations, and speculative bubbles. Our reading of the literature is that neither direction has yet completely resolved the puzzle.\(^1\)

This paper empirically investigates the extent to which the explicit consideration of monetary markets can help to resolve the forward discount puzzle. In particular, we examine whether introducing liquidity effects (the negative impact of monetary shocks on nominal interest rates) is able to contribute to solving the puzzle. Intuitively, it is easy to understand that the consideration of liquidity effects is potentially useful for the resolution of the puzzle. Suppose that money supply grows unexpectedly. As a consequence of this, current nominal interest rates decrease immediately due to liquidity effects, whereas inflation rates are accelerated in time to come due to the quantity theory of money.

The first consequence makes current forward rates appreciate by the covered interest parity. In contrast, the second will cause future spot rates to depreciate by the purchasing power parity. Thus, liquidity effects are able to weaken the one-to-one linkage between current forward rates and future spot rates, thereby offering a reasonable explanation for
the action of current forward rates against the movement of future spot rates.

Several empirical studies based on vector autoregression models (hereafter, VAR models) indeed find that monetary policy shocks induce the forward discount puzzle phenomenon. For example, Eichenbaum and Evans [1995] show that a contractionary shock to US monetary policy leads to an immediate increase in US interest rates together with a contemporaneous depreciation of forward rates, while it induces a sharp and persistent appreciation of spot rates. That is, a monetary contraction yields both depreciated current forward rates and appreciated future spot rates. Grilli and Roubini [1996] also find such a monetary-policy-induced forward discount puzzle using VAR models.

These findings suggest that liquidity effects have a chance to play an important role in resolving the puzzle; however, they cannot identify any theoretical mechanism of liquidity effects. It is therefore necessary to examine empirical implications based on structural monetary models in order to investigate thoroughly the relationship between monetary shocks and the forward discount puzzle.2

The literature on monetary economics has paid serious attention to the monetary model built by Lucas [1990], Fuerst [1992], and others, as a theoretical mechanism of liquidity effects.3 In their models, households adjust cash positions more slowly than firms, and such an asymmetric cash adjustment generates liquidity impacts on nominal interest rates in money markets. Grilli and Roubini [1992] present a two-country open economy version of Lucas’ model. In their model, monetary shocks which yield liquidity effects have impacts on nominal and real exchange rates.

In the context more directly relevant to the forward discount puzzle, Yaron [1995] introduces into an open economy model, not only the Lucas-Fuerst type liquidity effect, but also portfolio adjustment costs for household sectors.4 Choosing a set of parameters based on a simulated moments estimation, his simulation generates some results in favor of the resolution of the forward discount puzzle. In particular, his model succeeds in weakening substantially the linkage between future depreciation (appreciation) and forward discount.

As a complement to Yaron’s simulation study, this paper examines more directly by
data, empirical implications available from the Lucas-Fuerst type liquidity effect. Following the formulation of Fuerst, we identify as the liquidity effect, a temporary deviation from the standard Euler equation available from the optimization behavior of households. Then, for the estimation purpose we modify the standard Euler equation in consideration of such a deviation.

A relevant investigation here is accordingly to examine statistically the empirical implications based on the Lucas-Fuerst model of liquidity effect using actual data from foreign exchange markets. In addition, we quantitatively evaluate the extent to which the above liquidity effect is able to resolve the forward discount puzzle. Although, unlike in Yaron, portfolio adjustment costs are not introduced explicitly in our specification, we pay attention to the persistent liquidity effect which is possibly caused by such a cost.

This paper is organized as follows. In Section 2, we first derive tractable empirical specifications based on the above-mentioned liquidity effect. We then apply these specifications to time-series of Yen/US dollar rates and Canadian/US dollar rates in Section 3. Section 4 presents our conclusions.

2. Empirical Specifications

2.1. Sketch of Lucas-Fuerst model In this section, we first review the Lucas-Fuerst type of liquidity effect in a closed economy, and then explore the impact of their formulated liquidity effect on foreign exchange markets. There is a single goods usable for consumption and investment in the Lucas-Fuerst model. Both households and firms are subject to cash-in-advance constraints, but households adjust cash positions more slowly than firms. Their model therefore differs from the conventional monetary model with cash-in-advance constraints in which households and firms adjust cash positions simultaneously.

More concretely, households do not immediately change cash positions in response to monetary shocks, whereas firms do. One economic consequence of this setup is that unexpected monetary injection must be absorbed at least temporarily by firms because households have already maintained fixed cash positions when monetary shocks are realized.
Immediately after the monetary injection, therefore, real interest rates must decrease in order to make firms enhance monetary demand for purchasing investment goods, thereby absorbing the unexpected monetary injection. As Lucas [1990] and Fuerst [1992] demonstrate, such a negative impact on real interest rates may lower nominal interest rates by offsetting the Fisher effect or the positive impact of monetary injection on nominal rates.

One footnote to this class of monetary models is that monetary injection does not generate one-sided effects on investment demand. As a result of price increases due to unexpected monetary growth, purchasing power deteriorates on the household side, and households are forced to reduce consumption demand. That is, monetary shocks generate investment demand at the sacrifice of consumption demand. In this regard, the Lucas-Fuerst type liquidity effect may be interpreted as the economic consequence of the effect of monetary injection on the distribution between households and firms.

Fuerst [1992] identifies the above liquidity effect as a temporary deviation from the standard Euler equation derived from the optimization behavior of households. Because households do not adjust cash positions immediately in response to the current realization of monetary shocks, the Euler equation holds for the household sector in terms of not the current information, but the one-period lagged information. Restating this, when households determine the intertemporal allocation between time $t$ consumption and time $t + 1$ consumption, the Euler equation holds in terms of the time $t - 1$ information instead of the time $t$ information. Thus, we obtain

$$E_{t-1} \left[ \frac{1 + i_t U'(C_{t+1})/P_{t+1}}{1 + \rho U'(C_t)/P_t} \right] = 1, \quad (1)$$

where $E_{t-1}$ is the expectation operator conditional on information at time $t - 1$, $\rho$ is the time preference rate, $C_t$ is the consumption level at time $t$, $U'(C)$ is the marginal utility of consumption, $P_t$ is the nominal price level at time $t$, and $i_t$ is the one period nominal
interest rate at time $t$. Using equation (1), we define

$$\frac{1 + i_t}{1 + \rho} E_t \left[ \frac{U''(C_{t+1})/P_{t+1}}{U''(C_t)/P_t} \right] = 1 + \Lambda_t,$$

(2)

where

$$\Lambda_t = \frac{1 + i_t}{1 + \rho} E_t \left[ \frac{U''(C_{t+1})/P_{t+1}}{U''(C_t)/P_t} \right] - E_{t-1} \left[ \frac{1 + i_t}{1 + \rho} \frac{U''(C_{t+1})/P_{t+1}}{U''(C_t)/P_t} \right].$$

(3)

By construction, the unconditional mean of $\Lambda_t$ is equal to zero.

Fuerst [1992] interprets $\Lambda_t$ defined by equation (3) as the measure of the allocation of liquidity between consumption and investment. When $\Lambda_t$ is negative, households would want to increase current consumption if more cash were immediately available. To put it differently, the consumption goods market is less liquid than the investment goods market. In this case, a decrease in real interest rates shifts resources from consumption to investment. Conversely, when $\Lambda_t$ is positive, the consumption goods market is more liquid than the investment goods market. Thus, negative $\Lambda_t$ implies an increase in investment demand at the sacrifice of consumption demand under lower real interest rates. In addition, negative $\Lambda_t$ may lead not only to decreases in real interest rates, but also in nominal interest rates by offsetting the Fisher effect of monetary injection.

Assuming that the period utility function is characterized by the following preference with constant relative risk aversion

$$U(C) = \frac{C^{1-\gamma} - 1}{1 - \gamma},$$

where $\gamma$ is the degree of relative risk aversion, and expanding equation (3) up to second moments for random variables and up to first moments for deterministic variables, we obtain

$$i_t = E_t \pi_{t+1} + \Lambda_t - \gamma Cov_t (\pi_{t+1}, c_{t+1} - c_t) + MRS_{t+1},$$

(4)

where

$$MRS_{t+1} = \rho + \gamma E_t (c_{t+1} - c_t) - \frac{\gamma(\gamma + 1)}{2} Var_t (c_{t+1} - c_t),$$

(5)
\( p \) and \( c \) are the logarithms of \( P \) and \( C \). \( \pi_{t+1} \) is the inflation rate. \( Cov_t \) is the conditional covariance operator, while \( Var_t \) is the conditional variance operator. In accordance with usual practice, several adjustment terms (the Jensen’s inequality terms) are omitted when deriving both equations (4) and (5). The first term of equation (4) is the inflation effect, the second is the liquidity effect, and the third is the inflation-hedge effect. Finally, the fourth term \( MRS_{t+1} \) implies a real interest rate or the intertemporal marginal rate of substitution. The second and third terms make additions to the conventional factors appearing in the Fisher equation \( (i_t = E_t \pi_{t+1} + MRS_{t+1}) \).

2.2. Introduction of foreign exchange markets We next introduce the foreign exchange market into the above monetary model. Throughout this paper, \( e_t \) denotes the logarithm of spot rates of one unit of the foreign currency converted into the domestic currency, while \( f_t \) is the logarithm of forward rates. Hence, higher \( e_t \) or \( f_t \) implies the depreciation of the domestic currency or the appreciation of the foreign currency. Adding asterisks implies the variables belonging to the foreign country in the two-country setup.

Foreign exchange markets are characterized for this model by two simple parity conditions. First, spot rates are determined by the purchasing power parity (PPP):

\[
e_t = p_t - p_t^*. \tag{6}
\]

Under the assumption of PPP, a change in spot rates reflects the difference in the inflation rate between the two countries as follows:

\[
e_{t+1} - e_t = \pi_{t+1} - \pi_t^*. \tag{7}
\]

Second, forward rates are determined by the covered interest parity as below:

\[
f_t - e_t = i_t - i_t^*. \tag{8}
\]

On the other hand, domestic and foreign bond markets are influenced by the liquidity
effect as formulated above. That is, equation (4) holds for the domestic bond market, while the following equation holds for the foreign bond market when the preference of the foreign consumers is identical to that of the domestic consumers:

\[ i_t^* = E_t \pi_{t+1}^* + \Lambda_t^* - \gamma \text{Cov}_t (\pi_{t+1}^*, c_{t+1}^* - c_t^*) + MRS_{t+1}^*. \] (9)

Gathering equations (4), (7), (8) and (9), we obtain

\[ E_t e_{t+1} - e_t = (f_t - e_t) + (-\Lambda_t + \Lambda_t^*) + \left[ \gamma \text{Cov}_t (\pi_{t+1}, c_{t+1} - c_t) - \gamma \text{Cov}_t (\pi_{t+1}^*, c_{t+1}^* - c_t^*) \right] + \left( -MRS_{t+1} + MRS_t^* \right). \] (10)

Equation (10) indicates that changes in spot rates are predicted by not only forward discount rates \((f_t - e_t)\), but also three additional factors: the cross-country difference in the liquidity effect \((-\Lambda_t + \Lambda_t^*)\), the difference in the inflation-hedge effect \((\gamma \text{Cov}_t (\pi_{t+1}, c_{t+1} - c_t) - \gamma \text{Cov}_t (\pi_{t+1}^*, c_{t+1}^* - c_t^*)\)), and the difference in the intertemporal marginal rate of substitution \((-MRS_{t+1} + MRS_t^*)\). In other words, the current model generalizes the unbiasedness hypothesis of the standard forward exchange rate in two directions, the risk averse behavior and the liquidity effect.

To see the extent to which the current model is generalized, we consider one special case where consumers are risk averse \((\gamma > 0)\), but the liquidity effect is not present in either country \((\Lambda_t = \Lambda_t^* = 0)\). When households in both countries allocate resources based on current information, and make complete arbitrage between domestic and foreign bond markets, intertemporal marginal rates of substitution (real interest) become equal in the two countries \((MRS_{t+1} = MRS_t^*)\). Furthermore, the consumption profile is parallel in the two \((c_{t+1} - c_t = c_{t+1}^* - c_t^*)\). Together with the purchasing power parity (equation (7)), equation (10) is simplified as

\[ E_t e_{t+1} - e_t = f_t - e_t + \gamma \text{Cov}_t (e_{t+1} - e_t, c_{t+1} - c_t). \] (11)

In equation (11), the conditional variance \(\text{Cov}_t (e_{t+1} - e_t, c_{t+1} - c_t)\) is expected to resolve
the forward discount anomaly.

Previous empirical studies (e.g. Mark [1985] and Hodrick [1989]) suggest, however, that adopting equation (11), or the model with only risk averse behavior, does not contribute to the resolution of the forward discount anomaly. The principal reason for this is that the estimated risk premium term \(-\gamma \operatorname{Cov}_t(e_{t+1} - e_t, c_{t+1} - c_t)\) is rather small in magnitude, and the risk premium term and future spot rates do not move in opposite directions.

Based on the argument presented by Fama [1984], the above empirical failure of the theoretical prediction is described as follows. As discussed in the introduction, empirical studies frequently find that expected changes in spot rates \((E_t e_{t+1} - e_t)\) and forward discount rates \((f_t - e_t)\) move in opposite directions. As Fama points out, to have such a negative correlation between the two requires that the risk premium term \(-\gamma \operatorname{Cov}_t(e_{t+1} - e_t, c_{t+1} - c_t)\) should be negatively correlated with the expected change in spot rates, and that the variance of the risk premium term should be larger than that of expected changes in spot rates. That is,

\[
\operatorname{Cov}(E_t e_{t+1} - e_t, -\gamma \operatorname{Cov}_t(e_{t+1} - e_t, c_{t+1} - c_t)) < 0, \tag{12}
\]

and

\[
\operatorname{Var}(E_t e_{t+1} - e_t) < \operatorname{Var}(-\gamma \operatorname{Cov}_t(e_{t+1} - e_t, c_{t+1} - c_t)). \tag{13}
\]

The actual data, nevertheless, often fail to satisfy either of these two necessary conditions, thereby rejecting the empirical specifications based on risk averse behavior.

2.3. **Simplified empirical specifications** For the derivation of empirical specifications, we simplify equation (10) by dropping the third and fourth terms on the right hand side:

\[
E_t e_{t+1} - e_t \approx (f_t - e_t) + (-\Lambda_t + \Lambda_t^*) + \text{constant term}. \tag{14}
\]

The principal reason for this omission is that these terms are often small in magnitude; also, as shown in existing empirical studies, they do not contribute to the resolution of the puzzle. Abstracting risk aversion behavior and the intertemporal substitution motive,
accordingly, we focus our attention on the effect of liquidity on foreign exchange rates.

For the same reasons as above the measures of liquidity effects $\Lambda_t$ and $\Lambda_t^*$ are simplified as

$$\tilde{\Lambda}_t \approx (i_t - E_{t-1}i_t) - (E_t \pi_{t+1} - E_{t-1} \pi_{t+1}) - \gamma \left[ E_t \left( c_{t+1} - c_t \right) - E_{t-1} \left( c_{t+1} - c_t \right) \right],$$  \hspace{1cm} (15)

for the home country and

$$\tilde{\Lambda}_t^* \approx (i_t^* - E_{t-1}i_t^*) - \left( E_t \pi_{t+1}^* - E_{t-1} \pi_{t+1}^* \right) - \gamma \left[ E_t \left( c_{t+1}^* - c_t^* \right) - E_{t-1} \left( c_{t+1}^* - c_t^* \right) \right],$$  \hspace{1cm} (16)

for the foreign country. In these specifications, the liquidity effect is measured by the revision in expectations with respect to the level of real interest rates in relation to consumption growth. That is, the liquidity term $\tilde{\Lambda}_t$ becomes negative when the revision in real interest rates is smaller than the revision in consumption growth in the home country.

Another benefit of the above exclusion of second moment terms from explanatory variables is that we can avoid one serious econometric problem caused by generated regressors. As discussed later, we use as generated regressors, the moment terms which are estimated by the VAR system. As Pagan and Ullah [1988] demonstrate, the OLS method leads to inconsistent estimators on generated second moments such as estimated covariance terms, and it is then necessary to adopt the instrumental variables approach (hereafter, the IV approach). Pagan [1984], however, shows that the OLS method still guarantees the consistency of estimated coefficients on generated first moment terms. Therefore, we do not have to resort to the IV approach in this case, as far as a proper correction is made for the estimation of standard errors.

Substituting equation (15) and (16) into equation (14) leads to

$$E_t \pi_{t+1} - f_t = (-\tilde{\Lambda}_t + \tilde{\Lambda}_t^*) + \text{constant term.}$$  \hspace{1cm} (17)

The above equation implies that the difference in liquidity effects between the two countries
is able to describe those systematic changes in spot rates that cannot be explained by
forward discount rates. More concretely, when liquidity effects do matter, the forecasting
error of the anticipated real interest rate adjusted by the consumption growth term can
offer an additional explanation for the forward premium (the expected spot rate relative
to the current forward rate) of foreign exchange markets.

One thing to emphasize about equation (17) is that although the conditional expectation
of \( \widetilde{\Lambda}_t \) and \( \widetilde{\Lambda}_t^* \) given the time \( t-1 \) information is exactly equal to zero (see equation (3)), both
\( \widetilde{\Lambda}_t \) and \( \widetilde{\Lambda}_t^* \) are realized variables at time \( t \). Therefore, these liquidity measures are able to
work as state variables for the prediction of future spot rates given the time \( t \) information.
There would be no explicit reason for \( \widetilde{\Lambda}_t \) and \( \widetilde{\Lambda}_t^* \) to be able to predict the forward premium
without considering the liquidity effect.

This paper explores the empirical implications of equation (17) in several respects. First,
we investigate whether the two inequalities (12) and (13), pointed out by Fama [1984], are
satisfied when the risk premium term \(-\gamma \text{Cov}_t(e_{t+1} - e_t, c_{t+1} - c_t)\) is replaced by the relative
liquidity term \(-(-\widetilde{\Lambda}_t + \widetilde{\Lambda}_t^*)\). Second, by evaluating equation (18)

\[
E_t e_{t+1} - f_t = \beta(-\widetilde{\Lambda}_t) + \beta^* (\widetilde{\Lambda}_t^*) + \text{constant term},
\]

we examine how the liquidity term of each country helps to predict the forward premium
(the future exchange rate relative to the current forward rate). Third, more strict restric-
tions such as \( \beta = 1 \), \( \beta^* = 1 \), and \( \beta = \beta^* \) are also tested statistically. Fourth, we are
interested in the extent to which forward discount rates \( f_t - e_t \) can recover predictive power
for future spot rates once the liquidity effect is taken into consideration.

In addition to these tests, we explore the impact of the lagged liquidity measure, as well
as the current one using the specification:

\[
E_t e_{t+1} - f_t = \beta(-\widetilde{\Lambda}_t) + \beta_{-1} (-\widetilde{\Lambda}_{t-1}) + \beta^* (\widetilde{\Lambda}_t^*) + \beta^*_{-1} (\widetilde{\Lambda}_t^{*-1}) + \text{constant term}.
\]
While the liquidity effect persists for only one period in the current model, the lagged liquidity measure may have an impact on forward rates for the following reasons. First, the decision interval of consumers may be lengthier than the frequency of data used for estimation. Second, liquidity effects are rather persistent as documented for the US monetary market by several empirical papers including Christiano and Eichenbaum [1992] and Strongin [1995].

For the second possibility, persistent liquidity effects may be generated by either transaction costs that delay the adjustment of cash positions for household sectors\textsuperscript{12} or habit formation that keeps consumers from quickly adjusting their consumption plans. As mentioned before, following Christiano and Eichenbaum [1992], Yaron [1995] introduces costly adjustment of cash positions in order to generate persistent liquidity effects on forward premiums. In allowing for persistent liquidity effects, our empirical specification is not structural, but rather reduced.

3. Estimation Results

3.1. Data In this section we apply the empirical implications derived in the previous section to our two datasets of foreign exchange markets, that is, Canadian/US dollar rates and Yen/US dollar rates. We define forward discount rates using three-month forward rates. Three-month Treasury bills rates are used as nominal interest rates for both the US and Canada, while three-month GENSAKI rates (rates on repurchase contracts, one of the most typical money market rates in Japan) are adopted for Japan.\textsuperscript{13} For all three countries, inflation rates are calculated based on the seasonally-adjusted consumer price index (CPI).

In the case of Canadian/US dollar rates, quarterly consumption growth is defined by the quarterly per-capita seasonally-adjusted consumption (service and non-durable goods) reported in the System of National Account (SNA). Consistent with the quarterly consumption growth, we construct the March-June-September-December panel for both inflation rates and financial returns, thereby eliminating the potential information overlapping between time $t$ and time $t + 1$ as much as possible. That is, spot rates, forward rates, and
nominal interest rates are evaluated at the end of the last month of each quarter, while the quarterly inflation rate is the growth rate of the monthly CPI between the last month of each quarter and that of the next quarter. The sample period is from March 1980 to December 1995.

In the case of Yen/US dollar rates, we exploit the availability of monthly consumption data. That is, the US monthly per-capita seasonally-adjusted consumption (service and non-durable goods) is obtainable from the SNA, while the Japanese monthly per-capita seasonally-adjusted consumption (total consumption) is available from the Family Income and Expenditure Survey. The quarterly consumption growth is calculated from these monthly datasets. While corresponding to the consumption data, the panel of inflation rates and financial returns can begin with either January, February, or March, this paper reports only the results based on the January-April-July-October panel. The estimation results using this panel do not differ substantially from either the results from the other two panels or those based on the quarterly consumption data. The sample period is from January 1980 to December 1995.

3.2. **Construction of liquidity measures** To construct the liquidity measure defined in the previous section ($\tilde{\Lambda}_t$ or $\tilde{\Lambda}_t^*$), we estimate the expectation revision for nominal interest rates, inflation rates, and consumption growth as follows. First, we estimate the VAR model of the nominal interest rates, inflation rates, and consumption growth rates of Japan and US in the case of Yen/US dollar rates, and Canada and US in the case of Canadian/US dollar rates. The order of lags is determined according to the Schwarz’s Bayesian Information Criterion. Second, we calculate the expectation of both these rates conditional on the time $t$ information set and those conditional on the time $t-1$ information set using the estimated VAR model.

Finally, from these estimated expectations we obtain the forecast error of the nominal interest rate, the anticipated inflation rate, and the expected consumption growth. Assuming a particular value for the degree of relative risk aversion ($\gamma$), we define the sum of $(i_t - E_{t-1}i_t), - (E_t\pi_{t+1} - E_{t-1}\pi_{t+1}),$ and $-\gamma [E_t(c_{t+1} - c_t) - E_{t-1}(c_{t+1} - c_t)]$ as the liquidity
measure of each country, using the calculations given above. In the estimation below, we plausibly assume that \( \gamma \) is either 0.5, 1.0, 1.5, or 2.0. In addition, we estimate the cases of \( \gamma = 5.0 \) and \( \gamma = 10.0 \) for the main equations (18) and (19) in order to examine how the estimation results are robust.

3.3. **Data features**  Our quarterly datasets share a feature of foreign exchange markets that occurs frequently. When equation (20) is estimated

\[
E_t e_{t+1} - e_t = b(f_t - e_t) + \text{constant term,}
\]

(20)

the unbiasedness prediction of forward rates for future spot rates \((b = 1)\) is strongly rejected for both Canadian/US dollar rates and Yen/US dollar rates (see Panels 1 and 2 of Table 1). Furthermore, the estimated \( b \) is significantly negative, thereby suggesting that the actual data yields results radically opposite to the theoretical prediction.

Among the three countries, liquidity effects are empirically investigated most intensively for the US monetary market, and our constructed liquidity measure of the US \((\tilde{A}_t)\) shows consistency with such documented results. Several empirical studies including Christiano and Eichenbaum [1992], Strongin [1995] and Beaudry and Saito [1998], find that the liquidity effect of the US monetary market can be captured by the innovation in the non-borrowed reserves; these innovations have negative impacts on nominal interest rates. As Figure 1 shows, the innovation in the growth of the non-borrowed reserves\(^{15}\) is indeed negatively correlated with our constructed liquidity measure, and this negative correlation is statistically significant (see Table 2). That is, the unexpected monetary growth makes \(\tilde{A}_t\) negative as the Lucas-Fuerst model predicts.

3.4. **Estimation results (Canada-US rates)**  This subsection reports the case of Canadian/US dollar rates regarding Canada as the home country. We first examine whether the two inequalities (12) and (13) pointed out by Fama [1984] are satisfied when the risk premium part is replaced by the relative liquidity measure \((-(-\tilde{A}_t + \tilde{A}_t^r))\). As Panel 1 of Table 3 shows, the correlation between the estimated changes in spot rates and the
relative liquidity measure is positive and opposite to the theoretical prediction, although
the comparison of variance between the two is consistent with the prediction for \( \gamma = 1.5 \)
and \( \gamma = 2.0 \). The inclusion of the lagged liquidity measure \((-\bar{\Lambda}_{t-1} + \bar{\Lambda}_{t-1}^*)\), however,
makes the two inequalities hold in a theoretically consistent manner (see Panel 2 of Table
3). These results suggest that the lagged liquidity measure may work to predict future spot
rates.

Panel 1 of Table 4-1 reports the result of the estimation of equation (18). Throughout
this section, Newey-West corrected standard errors (Newey and West [1987]) are adopted
to remove the two-step estimation bias of the standard errors. While the estimates of \( \beta \)
and \( \beta^* \) are imprecise, the theoretical restriction \( \beta = \beta^* = 1 \) is not rejected statistically
except when the degree of relative risk aversion (\( \gamma \)) is high. The point estimates of \( \beta \)
and \( \beta^* \), however, become smaller as \( \gamma \) increases, in particular when \( \gamma \) is more than 1.5, \( \beta^* \) is
negative and \( \beta^* = 1 \) is rejected statistically; an increase in \( \gamma \) does not necessarily improve
the fitness of this model. In summary, the specification (18) is not rejected, but statistical
support for the model is weak.

Panel 2 of Table 4-1 reports the result of the case with the lagged liquidity measures
for the estimation of equation (19).\(^{16}\) One noticeable result is that the lagged liquidity
measures of both countries (\( \bar{\Lambda}_{t-1} \) and \( \bar{\Lambda}_{t-1}^* \)) help to predict future spot rates statistically
significantly for each value of \( \gamma \) (except for the US case of \( \gamma = 0.5 \)). As suggested before,
this finding may indicate persistent liquidity effects on forward premiums.\(^{17}\)

3.5. Estimation results (Japan-US rates) This subsection reports the case of Yen/US
dollar rates regarding Japan as the home country. With the inclusion of the relative liquidity
measure of Japan and the US as the risk premium term, the two inequalities suggested
by Fama [1984] are strongly violated (the calculation result is not reported here); the corre-
lation between the expected change in spot rates and the relative liquidity term is positive,
while the variance of the former is far larger than that of the latter. The inclusion of the
lagged liquidity measures does not alter the obvious violation.

Panel 1 of Table 4-2 reports the result of the estimation of equation (18). The coef-
cient of the Japanese liquidity measure ($\beta$) is imprecise, and becomes negative when $\gamma$ is larger than one. On the other hand, the result for the US liquidity measure is mixed. The estimated coefficient ($\beta^*$) is significantly positive and useful for the prediction of the forward premium, but it is much greater than one and the difference is statistically significant. Consequently, $\beta = \beta^* = 1$ is strongly rejected. In brief, except for the finding that $\hat{\Lambda}_t^*$ helps to predict the forward premium, there is little evidence for the model in the case of Japan-US rates.

Panel 2 of Table 4-2 reports the result of the estimation of equation (19). Except for the coefficient on the lagged US liquidity measure with $\gamma = 0.5$, the inclusion of the lagged liquidity measure does not help to predict future spot rates. This result is in contrast with that of the Canada-US case.

3.6. **Prediction of forward rates for future rates** In this final subsection, we explore the extent to which the unbiased prediction of forward rate is fixed by considering liquidity effects using the following specification:

$$E_t e_{t+1} - e_t = b(f_t - e_t) + \beta(-\hat{\Lambda}_t) + \beta_{-1}(-\hat{\Lambda}_{t-1}) + \beta^*(\hat{\Lambda}_t^*) + \beta^*_{-1}(\hat{\Lambda}_{t-1}^*) + \text{constant term.} \quad (21)$$

That is, we examine how the inclusion of liquidity measures makes the coefficient on forward discount rates ($b$) more appropriate, or closer to one.

While we initially estimate equation (21) in full, we cannot find any reasonable estimator for $b$ mainly because that the term $(f_t - e_t)$ is correlated with the term $(-\hat{\Lambda}_t - \hat{\Lambda}_{t-1} + \hat{\Lambda}_t^* + \hat{\Lambda}_{t-1}^*)$.\(^{18}\) We therefore estimate equation (21) with $b$ fixed. Given the range of $b$ between -1.50 and 1.50, $b$ is chosen such that the sum of squared residuals (SSR) is minimized. Table 5 reports the result of this investigation (Panel 1 for the Canada-US case and Panel 2 for the Japan-US case). Without considering the liquidity effect (the second column of each panel of Table 5), the critical value of $b$ is -1.35 for the Canada-US quarterly case, and -1.15 for the Japan-US quarterly case.
In the case of Canadian/US dollar rates, once the liquidity effect is considered, the value of \( b \) calculated as above approaches one. More concretely, when \( \gamma \) is assumed to be 0.5, 1.0, 1.5, or 2.0, the critical value of \( b \) becomes -0.89, -0.39, -0.15, or -0.03, respectively. \( b = 1 \) is not rejected at the conservative significance level when \( \gamma \) is 1.0, 1.5, or 2.0 (see the last row of Panel 1, Table 5). These results suggest that with consideration for the liquidity effect, the unbiased prediction of forward rates for future spot rates is fixed to some extent in a theoretically consistent manner.

The results are comparable to what Yaron [1995] finds for the remedy for the unbiasedness hypothesis by considering liquidity effects; for \( b \) in equation (20), his simulated open economy fails to yield negative estimates, but it succeeds in generating estimates far below one. Both Yaron’s simulation and our estimation suggest that liquidity effects offer a partial remedy for the forward discount puzzle.

In the case of Yen/US dollar rates, however, considering the liquidity effect does not help to recover the unbiased prediction of forward rates at all (see Panel 2, Table 5). The critical value of \( b \), which minimizes the SSR, is not very different from that without the liquidity effect \( (b = -1.15) \), and \( b = 1 \) is rejected strongly. In contrast to the Canada-US case, there is little room for consideration of the liquidity effect to recover the unbiased prediction in this case.

4. Conclusion This paper has presented tractable empirical specifications in order to identify the liquidity effect on forward rates of foreign exchange markets, and to explore the extent to which considering liquidity effects can help to resolve the forward discount puzzle. Using time-series of Canadian/US dollar rates and Yen/US dollar rates, we have examined whether liquidity effects are present in determining the forward premium of foreign exchange.

The estimation results are summarized as follows. On the one hand, in the Canada-US quarterly case, the empirical specification is not rejected, although statistical support for the model is weak. The lagged liquidity measure helps to predict future spot rates, thereby suggesting several theoretical possibilities. Furthermore, once the liquidity effect is taken
into consideration, the unbiased prediction of forward rates for future spot rates is recovered to some extent in a theoretically consistent manner. As mentioned before, this finding is comparable to what Yaron [1995] finds for the remedy for the unbiasedness hypothesis with consideration of liquidity effects. On the other hand, in the Japan-US case, there is little evidence for the model with liquidity effects. Even the explicit consideration of liquidity effects does not help to fix the unbiased prediction of forward rates for the latter case.

The above results call for two important extensions for future research. First, there are many empirical evidence against PPP among the three countries (for example, see Frankel and Rose [1996], Lothian [1997], and Wu [1996]); therefore, it is highly desirable that models should be constructed without the explicit assumption of PPP. Second, as mentioned before, underlying models should be formulated such that the forward premium of foreign exchange rates explicitly depends not only on the current liquidity measure, but also on the lagged one. For this purpose, either the adjustment cost of cash positions or habit formation may be a promising candidate in extending the structural model.
REFERENCES


Table 1: Tests of Forward Exchange Rate Unbiasedness, Three-month Rates, from 1980 to 1995

**Panel 1: Exchange Rates between Canada and US**

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-1.350</td>
</tr>
<tr>
<td></td>
<td>(0.435)</td>
</tr>
<tr>
<td>constant term</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>R square</td>
<td>0.056</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.116</td>
</tr>
</tbody>
</table>

**Panel 2: Exchange Rates between Japan and US**

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-1.153</td>
</tr>
<tr>
<td></td>
<td>(0.276)</td>
</tr>
<tr>
<td>constant term</td>
<td>-0.029</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
</tr>
<tr>
<td>R square</td>
<td>0.081</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.287</td>
</tr>
</tbody>
</table>

(i) The numbers in parentheses are the Newey-West corrected standard errors.

(ii) The degree of freedom of $Q$-statistics is equal to 15.

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimated regression coefficient</td>
<td>-0.040 (0.009)</td>
<td>-0.063 (0.021)</td>
<td>-0.085 (0.034)</td>
<td>-0.107 (0.047)</td>
</tr>
</tbody>
</table>

(i) The surprise in the growth of the US non-borrowed reserves plus extended credit is estimated based on the AR(3) specification.

(ii) Standardizing as a unit the variances of the surprise and the liquidity measure, the coefficient is estimated by regressing the constructed liquidity measure $\Lambda_t$ on the contemporaneous surprise in non-borrowed reserves.

(iii) The numbers in parentheses are the Newey-West corrected standard errors.

Panel 1: With Only Current Liquidity Effects

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Cov}(E_t e_{t+1} - e_t, -(\tilde{\Lambda}_t + \tilde{\Lambda}_t^*) ) )</td>
<td>+1.31x10^{-6}</td>
<td>+1.78x10^{-6}</td>
<td>+2.25x10^{-6}</td>
<td>+2.72x10^{-6}</td>
</tr>
<tr>
<td>( \text{Var}(E_t e_{t+1} - e_t) )</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
</tr>
<tr>
<td>( \text{Var}(-(\tilde{\Lambda}_t + \tilde{\Lambda}_t^*)) )</td>
<td>3.26x10^{-5}</td>
<td>6.24x10^{-5}</td>
<td>10.8x10^{-5}</td>
<td>17.0x10^{-5}</td>
</tr>
</tbody>
</table>

Panel 2: With One Period Lagged Liquidity Effects

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Cov}(E_t e_{t+1} - e_t, -(\tilde{\Lambda}<em>t - \tilde{\Lambda}</em>{t-1} + \tilde{\Lambda}<em>t^* + \tilde{\Lambda}</em>{t-1}^*)) )</td>
<td>-1.39x10^{-5}</td>
<td>-2.43x10^{-5}</td>
<td>-3.47x10^{-5}</td>
<td>-4.50x10^{-5}</td>
</tr>
<tr>
<td>( \text{Var}(E_t e_{t+1} - e_t) )</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
<td>6.66x10^{-5}</td>
</tr>
<tr>
<td>( \text{Var}(-(\tilde{\Lambda}<em>t - \tilde{\Lambda}</em>{t-1} + \tilde{\Lambda}<em>t^* + \tilde{\Lambda}</em>{t-1}^*)) )</td>
<td>7.34x10^{-5}</td>
<td>14.5x10^{-5}</td>
<td>25.3x10^{-5}</td>
<td>39.5x10^{-5}</td>
</tr>
</tbody>
</table>

(i) The estimation of \( E_t e_{t+1} - e_t \) is based on the VAR system consisting of forward discount rates, changes in spot rates, inflation rates, nominal interest rates, and consumption growth rates.

Panel 1: With Only Current Liquidity Effects (Quarterly)

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) on current ( \Lambda_t )</td>
<td>1.618</td>
<td>0.840</td>
<td>0.572</td>
<td>0.450</td>
<td>0.206</td>
<td>0.108</td>
</tr>
<tr>
<td>(0.978)</td>
<td>(0.682)</td>
<td>(0.494)</td>
<td>(0.388)</td>
<td>(0.168)</td>
<td>(0.086)</td>
<td></td>
</tr>
<tr>
<td>( \beta^* ) on current ( \Lambda_t^* )</td>
<td>1.988</td>
<td>0.096</td>
<td>-0.234</td>
<td>-0.271</td>
<td>-0.161</td>
<td>-0.088</td>
</tr>
<tr>
<td>(1.370)</td>
<td>(0.946)</td>
<td>(0.613)</td>
<td>(0.441)</td>
<td>(0.159)</td>
<td>(0.076)</td>
<td></td>
</tr>
<tr>
<td>Constant term</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>P-value for testing ( \beta = \beta^* )</td>
<td>0.712</td>
<td>0.337</td>
<td>0.143</td>
<td>0.088</td>
<td>0.038</td>
<td>0.030</td>
</tr>
<tr>
<td>P-value for testing ( \beta = 1 ) and ( \beta^* = 1 )</td>
<td>0.757</td>
<td>0.583</td>
<td>0.129</td>
<td>0.016</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>R square</td>
<td>0.045</td>
<td>0.037</td>
<td>0.045</td>
<td>0.052</td>
<td>0.065</td>
<td>0.070</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.102</td>
<td>0.177</td>
<td>0.186</td>
<td>0.188</td>
<td>0.191</td>
<td>0.192</td>
</tr>
</tbody>
</table>

Panel 2: With One Period Lagged Liquidity Effects (Quarterly)

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) on current ( \Lambda_t )</td>
<td>1.699</td>
<td>1.062</td>
<td>0.751</td>
<td>0.575</td>
<td>0.231</td>
<td>0.114</td>
</tr>
<tr>
<td>(0.977)</td>
<td>(0.638)</td>
<td>(0.447)</td>
<td>(0.347)</td>
<td>(0.152)</td>
<td>(0.079)</td>
<td></td>
</tr>
<tr>
<td>( \beta ) on lagged ( \Lambda_t )</td>
<td>1.986</td>
<td>2.419</td>
<td>1.968</td>
<td>1.603</td>
<td>0.723</td>
<td>0.372</td>
</tr>
<tr>
<td>(0.937)</td>
<td>(0.627)</td>
<td>(0.441)</td>
<td>(0.347)</td>
<td>(0.163)</td>
<td>(0.087)</td>
<td></td>
</tr>
<tr>
<td>( \beta^* ) on current ( \Lambda_t^* )</td>
<td>1.897</td>
<td>0.017</td>
<td>-0.150</td>
<td>-0.160</td>
<td>-0.097</td>
<td>-0.054</td>
</tr>
<tr>
<td>(1.538)</td>
<td>(0.819)</td>
<td>(0.526)</td>
<td>(0.389)</td>
<td>(0.151)</td>
<td>(0.075)</td>
<td></td>
</tr>
<tr>
<td>( \beta^* ) on lagged ( \Lambda_t^* )</td>
<td>2.207</td>
<td>2.783</td>
<td>1.906</td>
<td>1.373</td>
<td>0.471</td>
<td>0.219</td>
</tr>
<tr>
<td>(1.938)</td>
<td>(0.937)</td>
<td>(0.543)</td>
<td>(0.378)</td>
<td>(0.134)</td>
<td>(0.065)</td>
<td></td>
</tr>
<tr>
<td>Constant term</td>
<td>-0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>R square</td>
<td>0.114</td>
<td>0.215</td>
<td>0.259</td>
<td>0.279</td>
<td>0.305</td>
<td>0.311</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.048</td>
<td>0.133</td>
<td>0.231</td>
<td>0.302</td>
<td>0.443</td>
<td>0.484</td>
</tr>
</tbody>
</table>

(i) The numbers in parentheses are the Newey-West corrected standard errors.
(ii) The degree of freedom of Q-statistics is equal to 15.

### Panel 1: With Only Current Liquidity Effects

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ on current $\Lambda_t$</td>
<td>1.431</td>
<td>-1.678</td>
<td>-3.743</td>
<td>-4.648</td>
<td>-3.151</td>
<td>-1.582</td>
</tr>
<tr>
<td></td>
<td>(3.082)</td>
<td>(3.724)</td>
<td>(3.601)</td>
<td>(3.058)</td>
<td>(1.382)</td>
<td>(0.745)</td>
</tr>
<tr>
<td>$\beta^*$ on current $\tilde{\Lambda}_t$</td>
<td>11.280</td>
<td>9.002</td>
<td>6.496</td>
<td>4.461</td>
<td>0.538</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(4.084)</td>
<td>(2.989)</td>
<td>(1.728)</td>
<td>(1.081)</td>
<td>(0.588)</td>
<td>(0.362)</td>
</tr>
<tr>
<td>Constant term</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>P-value for testing $\beta = \beta^*$</td>
<td>0.118</td>
<td>0.082</td>
<td>0.031</td>
<td>0.008</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>P-value for testing $\beta = 1$ and $\beta^* = 1$</td>
<td>0.009</td>
<td>0.010</td>
<td>0.006</td>
<td>0.003</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>R square</td>
<td>0.069</td>
<td>0.073</td>
<td>0.079</td>
<td>0.085</td>
<td>0.092</td>
<td>0.089</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.390</td>
<td>0.383</td>
<td>0.352</td>
<td>0.332</td>
<td>0.359</td>
<td>0.384</td>
</tr>
</tbody>
</table>

### Panel 2: With One Period Lagged Liquidity Effects

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ on current $\Lambda_t$</td>
<td>-0.110</td>
<td>-2.027</td>
<td>-4.090</td>
<td>-5.260</td>
<td>-3.669</td>
<td>-1.822</td>
</tr>
<tr>
<td></td>
<td>(2.291)</td>
<td>(3.417)</td>
<td>(3.510)</td>
<td>(2.831)</td>
<td>(1.284)</td>
<td>(0.744)</td>
</tr>
<tr>
<td>$\beta$ on lagged $\tilde{\Lambda}_t$</td>
<td>-0.050</td>
<td>-0.648</td>
<td>1.056</td>
<td>2.442</td>
<td>1.943</td>
<td>0.812</td>
</tr>
<tr>
<td></td>
<td>(2.795)</td>
<td>(3.644)</td>
<td>(3.476)</td>
<td>(2.987)</td>
<td>(1.436)</td>
<td>(0.695)</td>
</tr>
<tr>
<td>$\beta^*$ on current $\tilde{\Lambda}_t$</td>
<td>7.953</td>
<td>7.492</td>
<td>5.996</td>
<td>4.271</td>
<td>0.443</td>
<td>-0.040</td>
</tr>
<tr>
<td></td>
<td>(4.142)</td>
<td>(3.759)</td>
<td>(2.256)</td>
<td>(1.403)</td>
<td>(0.577)</td>
<td>(0.359)</td>
</tr>
<tr>
<td>$\beta^*$ on lagged $\tilde{\Lambda}_t$</td>
<td>15.649</td>
<td>6.651</td>
<td>2.322</td>
<td>0.927</td>
<td>0.454</td>
<td>0.220</td>
</tr>
<tr>
<td></td>
<td>(3.923)</td>
<td>(3.679)</td>
<td>(3.233)</td>
<td>(2.797)</td>
<td>(1.137)</td>
<td>(0.554)</td>
</tr>
<tr>
<td>Constant term</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>R square</td>
<td>0.174</td>
<td>0.109</td>
<td>0.097</td>
<td>0.109</td>
<td>0.136</td>
<td>0.128</td>
</tr>
<tr>
<td>P-value of Q-stat.</td>
<td>0.227</td>
<td>0.239</td>
<td>0.266</td>
<td>0.266</td>
<td>0.318</td>
<td>0.347</td>
</tr>
</tbody>
</table>

(i) The numbers in parentheses are the Newey-West corrected standard errors.
(ii) The degree of freedom of Q-statistics is equal to 15.
Table 5: Grid Search for $b$ that minimizes the SSR of the Forward Discount Equation, Three-month Rates, from 1980 to 1995


<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>no liquidity effects</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value $b$ that minimizes SSR</td>
<td>-1.35</td>
<td>-0.89</td>
<td>-0.39</td>
<td>-0.15</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\beta$ on current</td>
<td>0.816 (0.977)</td>
<td>0.608 (0.645)</td>
<td>0.488 (0.457)</td>
<td>0.396 (0.355)</td>
<td></td>
</tr>
<tr>
<td>$\beta$ on lagged</td>
<td>0.785 (0.941)</td>
<td>1.691 (0.616)</td>
<td>1.500 (0.433)</td>
<td>1.266 (0.340)</td>
<td></td>
</tr>
<tr>
<td>$\beta^*$ on current</td>
<td>0.644 (1.479)</td>
<td>-0.415 (0.815)</td>
<td>-0.345 (0.522)</td>
<td>-0.273 (0.385)</td>
<td></td>
</tr>
<tr>
<td>$\beta^*$ on lagged</td>
<td>1.356 (1.762)</td>
<td>2.436 (0.887)</td>
<td>1.755 (0.532)</td>
<td>1.292 (0.377)</td>
<td></td>
</tr>
<tr>
<td>P-value for testing $b = 1$</td>
<td>0.000</td>
<td>0.003</td>
<td>0.067</td>
<td>0.131</td>
<td>0.171</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>no liquidity effects</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value $b$ that minimizes SSR</td>
<td>-1.15</td>
<td>-0.64</td>
<td>-0.89</td>
<td>-1.03</td>
<td>-1.10</td>
</tr>
<tr>
<td>$\beta$ on current</td>
<td>-0.306 (2.213)</td>
<td>-2.235 (3.020)</td>
<td>-4.464 (3.080)</td>
<td>-5.753 (2.524)</td>
<td></td>
</tr>
<tr>
<td>$\beta$ on lagged</td>
<td>-1.137 (2.701)</td>
<td>-1.100 (3.657)</td>
<td>0.640 (3.542)</td>
<td>1.877 (3.026)</td>
<td></td>
</tr>
<tr>
<td>$\beta^*$ on current</td>
<td>5.416 (4.023)</td>
<td>5.477 (3.955)</td>
<td>4.491 (2.710)</td>
<td>3.088 (1.871)</td>
<td></td>
</tr>
<tr>
<td>$\beta^*$ on lagged</td>
<td>9.886 (3.975)</td>
<td>2.299 (3.252)</td>
<td>-0.677 (2.426)</td>
<td>-1.256 (1.967)</td>
<td></td>
</tr>
<tr>
<td>P-value for testing $b = 1$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(i) The numbers in parentheses are the Newey-West corrected standard errors.
Footnotes

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1. Mark and Wu [1998] show that a model of noise trading receives more support from empirical evidence than the standard intertemporal asset pricing model.

2. Kumah [1996] demonstrates that the emergence of the forward discount puzzle phenomenon is not statistically robust, and it depends on how monetary shocks are identified within statistical VAR models. His finding suggests that it is rather difficult to give evidence for the impact of liquidity effects on exchange rates using only a reduced-form-type statistical model. Even in this regard, it is necessary to examine empirical implications directly based on structural monetary models in order to determine whether liquidity effects are responsible for the resolution of the forward discount puzzle.


4. The portfolio adjustment cost introduced by Yaron [1995] is similar to the one assumed by the closed economy model of Christiano and Eichenbaum [1992].

6. As Engel [1996] points out, when only risk averse behavior is taken into consideration, the forward discount puzzle for the foreign exchange market is unsolved as much as the risk premium puzzle for the stock market (Mehra and Prescott [1985]). On the analogy of the risk premium puzzle, Backus, Gregory, and Telmer [1993] generalize models with risk aversion adopting more flexible utility functions in order to resolve the forward discount puzzle.

7. Following the tradition of the literature, the risk premium term is defined as \( f_t - E_t e_{t+1} \).

8. See Engel [1996] for the detailed argument on this empirical failure.

9. Backus, Forcsì, and Telmer [1998] find several conditions under which stochastic discount factors (state prices) are consistent with the two inequalities presented by Fama [1984].

10. At the preliminary stage of estimation, we added the inflation-hedge effect represented by the covariance term, to the same empirical setup reported in Section 3. We however could not find any reasonable coefficient on the covariance term. That is, the estimated coefficients on the inflation-hedge term were insignificant in most cases. In addition, the inclusion of the hedge term did not have impacts on the estimation of the coefficient on the liquidity measure. For example, in the case of Canada and US with the lagged liquidity effect, the estimated coefficient on the inflation-hedge term ranged from -120.8 with the standard deviation 116.7 to 13.9 with the standard deviation 34.3, while the other estimated parameters remained intact even in the presence of the inflation-hedge term. Not only the existing literature which finds no significant evidence for the risk aversion factor in explaining forward premiums, but also the above examination led us to the exclusion of covariance terms in our
empirical specification.

11. With respect to the omission of \((MRS_{t+1} + MRS^*_{t+1})\), the failure of the equalization of the intertemporal marginal rate of substitution between the two countries is caused by the presence of liquidity effects in this open economy model. It is therefore difficult to distinguish between the liquidity effect and the difference in \(MRS\) in this setup. In our empirical specification, we focus on the term of liquidity effects.


13. Because we are particularly interested in liquidity effects in domestic money markets, we use onshore money market rates instead of Eurocurrency rates for the three countries. Through a detailed examination of our dataset, we find that covered interest parity still holds reasonably for domestic rates to an extent that it holds based on Eurocurrency rates quoted as LIBOR, as far as our sample period between 1980 and 1995 is concerned. Ito [1992] demonstrates that covered interest parity almost held for the Japanese onshore rate after 1980, because most of the governmental restrictions on the Japanese domestic money market had been lifted until 1980 and her domestic money market had been integrated into the Euroyen market.

14. There is clear evidence based on the Dickey-Fuller test (D-F test) for the rejection of a unit root in both inflation and consumption growth. The D-F test, on the other hand, cannot reject the presence of a unit root in nominal interest rates in either country. Nevertheless, we treat these three variables as stationary, partly because it is known that the power of unit root tests is rather low, and partly because both inflation and nominal interest should be of the same order of integration on a priori theoretical grounds.

15. The innovation is estimated by the AR(3) process.
16. Another possibility is that as Brown and Gibbons [1985], Ogaki [1993] and others suggest, the estimation based on quarterly consumption data may be subject to a time aggregation problem. While we use monthly consumption data for the three countries in order to control time aggregation, we cannot find any significant estimators on the liquidity measures.

17. Controlling the endogeneity of monetary aggregates, Beaudry and Saito [1998] find even more persistence for the liquidity effect than the previous literature suggests. That is, liquidity effects persist beyond two quarters. Their finding is broadly consistent with our result that one quarter-lagged liquidity measures help to explain forward premiums.

18. As mentioned in Section 3.3, the forward discount is negatively correlated with the expected change in spot rates. As found in Section 3.4, on the other hand, current and lagged liquidity measures can predict the future movement in spot rates. Accordingly, both factors are correlated with each other.
Figure 1: Surprises in Non-borrowed Reserves and US Liquidity Measure