MODELLING RISK IN FARM PLANNING

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Abstract

In this article a mathematical model is presented to assist management decisions on an integrated crop and livestock farm. Risk is incorporated into the model as the negative deviation of the actual gross income from the expected value of an activity's gross income. The model includes crop production (permitting and optimising a crop rotation system), dairy production and wool sheep production. Relevant data from a farm in the Swartland region of the Western Cape were used to test and validate the model. The results show that the adoption of crop rotation is superior in terms of gross margin to that generated from a mono-crop strategy. Empirical results also indicate that the complex interrelationships involved in a mixed crop-livestock farm operation play a major role in determining optimal farm plans. These complex interrelationships favour the introduction of crop rotation in the crop production activities of the farm under investigation. Solutions of the model with risk indicate that the crop rotation strategy and animal production levels are sensitive to different risk levels, and that the incorporation of risk greatly affects the level of land allocation to crop rotation and animal production level of the farm. Finally, the results suggest that the introduction of crop rotation is of paramount importance in improving the profitability and sustainability of the farm, thus the inclusion of forage crops such as medics into the integrated crop-livestock production is beneficial for sustained profitability.

1. INTRODUCTION

The combination of farm activities is an important factor to exploit the interrelationship between different crops and livestock. The beneficial interrelationships are better exploited if a sound management strategy is incorporated into the decision-making process. The problem of the farmer is to determine a farm planning strategy which includes the selection of cropping strategies, the number of animals and their required feed, the amount of crops to sell to the market, and the amount additional machine rent required for harvesting and baling activities on the farm. In this paper a mathematical model is developed to study the influences of the introduction of crop rotation alternatives to the decision planning of an integrated crop-livestock farm in the presence of risk. This mathematical model for farm planning incorporates

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the different activities of a farming unit. The more specific objectives of this study are: (1) to determine the optimum farm management plan which includes an optimum continuously repeatable cropping sequence; (2) to calculate optimum dairy and wool production in the presence of crop production risk; and (3) to explore the effects of risk on farm management/planning decisions.

2. SPECIFICATION OF THE MODEL

A model is developed to determine the optimal crop sequences and livestock numbers on a given farm. It is assumed that farm management is seeking to maximise the gross margin through efficient resource allocation. The model must incorporate competing criteria of gross margin maximisation and risk minimisation (risk resulting from weather unpredictability). In both criteria the model will be expected to determine an optimal crop-livestock production strategy under the given assumptions.

The concepts of risk employed in this paper focuses on the randomness or variability of outcomes. This concept of risk finds a theoretical justification in the expected utility maximisation decision model (Robinson & Barry, 1987). In this study the risk of the crop production is defined in terms of the levels of income variability associated with the different outcomes (states) of nature.

Crop production occurs in a complex system of biological, agronomical and market dynamics. Incorporating such a complex system into a decision model presents a formidable challenge. The representation of this system by means of a mathematical model is thus not a simple task and hence it is essential to include the following assumptions in the process of developing a mathematical model:

- It is assumed that the gross income, in real terms, remains constant over the period for which the problem is solved. This implies that the cost coefficients in the mathematical model remain constant.

- The year-to-year variability of the weather conditions of the farm is assumed to be categorised into three discrete states of nature. The three states considered are a normal year, a dry year and a wet year. In this study these three states of nature are used as the strategies of nature. Moreover, the cropping yield risk generated from weather variability is modelled as a deviation from the average of the three states of nature. The risk of planting crops under conditions of unpredictable weather changes is reflected in the variability of yields of crops in the three states of nature. The risk as a result of this yield variability of crops is shown by the differences in the income variation of the same cropping strategy at the three different states from the expected value. The variability of input prices is assumed to be negligible. This could, however, be a topic for future studies. Generally, the cost of different input components of the farm activity for cultivating a particular crop or managing an animal is considered as one grand cost component for each particular activity.

- The gross income from a crop is dependent on the crop itself as well as the crop that was planted on the same land in the previous year. The crop grown in the current year is dependent on the crop that was planted on the same land two years ago. A crop that grew on the same soil more three years ago was assumed to have no effect on the current crop.

- It is assumed that the optimal sequence of crops forms a cycle. Only cycles of one, two and three years will be considered (El-Nazer & McCarl, 1986; De Kock & Visagie, 1998).

- Area of arable land (A) is assumed to be divided into T unit fields (T plots). It is also assumed that the estimated yield of each crop in each field for the specified state of nature is known.

Regarding the complex interdependence between the crop and animal production activities of the farm, the following assumptions are relevant to the farm planning problem:

- The farm is assumed to be self-sufficient in forage and straw production. The implication of this assumption is that the production of forage and straw of the farm must satisfy the animals' consumption requirements for the given planning period.

- Availability in this paper is used in the sense that the animals receive the required amount of feed and roughage, which satisfies the ingredient and nutrient restrictions set by the decision maker.

- For the sake of simplicity, it is assumed that animal transactions (buying and selling) are made at the start of the planning period. For that reason animals bought are considered in the animal feed intake planning and animals sold are excluded from the animal feed considerations. Moreover, no gross income is generated from those animals sold, as they are assumed be out of the model for the planning period. The only income from these animals is the return on the capital from the sale of these animals.

- Animals are categorised into three sets, namely, adult cattle, young cattle and sheep. It is assumed that the number of young cattle is always 80% of the adult cattle. The only source of revenue from the animal production is revenue from adult cattle and sheep. The loss because of animal deaths, other natural hazards and theft is assumed to be negligible. Consequently, the cost incurred from such circumstances will not be accounted in the mathematical model.

- It is assumed that all crops produced are sold or used as animal feed in the feed mix in the period of study. This implies that no cost is incurred other than the production cost.

3. THE MATHEMATICAL MODEL

The development of the model is discussed under the headings of the objective function and then different types of constraints. After the objective function and all the constraints have been presented, the mathematical model is given. The definition of the notation used in the model is given in Appendix B.

3.1 The objective function

The large body of literature on applications of mathematical programming in agricultural decision making implies that it is critical for an agricultural firm to maximise its gross margin from its production activities, while meeting different constraints, such as availability of land, risk and cash flow. Accordingly, this paper considers management to have an objective of maximising the gross margin, which satisfies the different restrictive constraints under which the farm operates. To be more precise, the decision maker's problem is to select the optimum combination of crop production strategies and number of animals that satisfy the constraints. The objective function of the model is thus to maximise the gross margin generated from the farming activities. The gross margin is defined as the difference between the gross income derived from each enterprise (crop production and animal production activities) minus the direct attributable variable cost. The gross income is defined as:

Gross income = *gross income from crop sale* + *gross income from animal activities*

The income from animals includes the income from dairy production of the adult cattle, wool production from the sheep and animal sales. The direct attributable variable cost (DAVC) is also calculated in the following way.

Total DAVC = DAVC of crop production
+ cost of raw material used in the animal feed
+ cost of extra resources rent
+ cost of animals bought

The cost of seed, fertiliser, poisons, fuel and labour is included in the DAVC of crop production.

3.2 Constraints

3.2.1 Land, number and size of strategies

Visagie (2004) defined different cropping strategies to determine an optimal crop rotation system. Let $X = \{X_1, X_2, X_3, \ldots, X_N\}$ be all the feasible cropping strategies, where the set X includes all of the feasible one-year, two-year and three-year strategies. In an integrated crop-livestock production enterprise land is the vital factor for the different activities. The amount of crop available for the market and animal consumption and the amount of forage production is directly related to the amount of available land. The land constraint, equation (1), limits the total available area of land allocated to the different cropping strategies.

It is assumed that the integrated crop-livestock farm has a choice of n crops that can be grown on the given land. The farm has N possible crop rotation strategies from which a combination of strategies must to be implemented. No strategy can be greater than the total land available. Because of management issues it is sometimes desirable to limit the number of strategies implemented on the farm. It is, for example, not desirable to plant half a hectare with a certain strategy. In order to introduce this restriction to the model, a variable δ_i is introduced to supply a lower bound (g) and an upper bound (U) for the size of strategy X_i in the model (Williams, 1999; Winston, 1994). In reality it is impossible to implement all the feasible crop rotation strategies described in this study, as the programming model can allocate strategies which are too small for implementation.

These two constraints (upper and lower bound) as well as a restriction on how many different strategies (T) may be chosen by the model are represented by the set of equations (2). All three of these constraints may be set by the modeller at a desired level. Another indirect benefit is that it can force crop diversification. If more than one strategy is implemented, the notion of crop diversification will be introduced into the decision model. Crop diversification is one of the methods normally applied to manage risks.

If $\delta_i = 1$, then strategy i is selected for implementation in the farm planning problem on an area of X_i hectares of land.

3.2.2 Income variability as a source of risk

Farmers do not know future events with absolute certainty. In order to assess the impact of a strategy, an objective measure for a risk associated with the decision is necessary and important. To evaluate the impact of the unpredictability of the states of nature over a crop rotation strategy, selected consideration of risk is a key factor in choosing an optimal farm plan, because the introduction of risk into a production process affects the pattern of resource allocation and the level of production (Gabriel & Baker, 1980). Risk attitudes may be reflected in the farm plan analysis in different ways. In this study the assumption is made that the future income variability due to variations in weather is closely related to past variability. Thus crop income risk can be estimated by income variability over some past period (Hazell, 1971; Hazell & Norton, 1986).

There are various categories of risk in agricultural production (Anderson, Dillon & Hardaker, 1977). One such source is the biophysical environment which produces yield or production variability, which is termed the production risk (Gabriel & Baker, 1980). Production risk emanates from the unpredictable nature of weather and uncertainties in the performance of crops and livestock. Production risk will be incorporated as variability of income due to the variability of the yield of crops across the states of nature and crop rotation strategies which are assumed relevant. Hence, it is necessary to take into consideration income variability levels associated with alternative crop rotation strategies in the mathematical model.

There are different mathematical programming techniques available to introduce risk and uncertainty into an optimisation procedure. Historically game theory, mean-variance and mean-absolute deviations have been used to choose appropriate mixtures of risky agricultural production alternatives (Anderson, Dillon & Hardaker, 1977; Hardaker, Huirne & Anderson, 1997; Hazell & Norton, 1986). The approach followed here for introducing risk into the farm planning problem is a modification of Hazell's (1971) risk programming model. Hazell's variance estimator is based on the sample mean-absolute deviation instead of the more widely used sum of squares error (variance). This is a key point in Hazell's formulation that allows the incorporation of risk into a linear model. The objective function of the risk programming model formulated by Hazell is the minimisation of the total absolute deviations. The objective was thus minimising the risk level of an optimal farm plan. However, minimising risk is insufficient by itself and would result in plans with low-income levels. In this paper risk is introduced as a constraint (target level of risk as a sum of negative deviations), so that the model selects a combination of strategies that achieve a specified target level of risk with highest income. The mathematical formulation to incorporate risk as a constraint into the mathematical model which maximises income is presented in equations (3) and (4) below.

3.2.3 Animal feed activities

The integrated crop livestock production unit that is considered can produce crops for animal feeding purposes or such crops must be sold to the market at the market price. Other materials required in the feed mix which are necessary for the livestock production, including the supplies required because of insufficient farm production, can be purchased from the market or the market price. The animals on the farm are categorised in three groups. With regard to feed, each type of animal requires a certain amount of blended feed and pastures, which satisfy certain requirements, set by the manager. The quantitative modelling of the two types of food consumed by the animals is discussed below in separate sections. Feed is arguably the most important input, next to the actual animals, for a livestock operation in terms of impact on total expenses. Given the importance of feed to livestock operations, the selection of minimum-cost feed rations using linear programming has, historically, been given considerable attention in agricultural activities. The animal feeding policy and the crop production strategy influence each other as the land used to produce crops for animal consumption could also be used to grow crops to sell to the market. In formulating the feed mix, the farmer is assumed to make use of the type of raw material produced in the farm. The remaining raw material is assumed to be supplemented by purchasing from the market. The problem that should be addressed is the design of a minimum-cost feed-mix formulation that satisfies certain requirements set by the farmer. Each of the possible ingredients has a different price, and each contains different proportions of various nutrients that the animals need annually. Therefore, the problem that needs to be investigated within the feed mix context is which ingredients, and in what quantities, should be combined to meet the nutritional needs of the adult animals as cheaply as possible, taking into consideration the interdependence between the crop production and livestock production of the farm.

Let $F = \{Y_1, Y_2, Y_3, \ldots, Y_y\}$ be a set of raw materials and $N = \{N_1, N_2, N_3, \ldots, N_r\}$ be a set of nutrients. The feed mix is prepared from the set of raw materials and consists of a set of nutrients satisfying different restrictions. The mathematical model for the least-cost feed mix satisfying the nutritional and raw material requirements is formulated by a linear programming model (Klein *et al*, 1986; De Kock & Sinclair, 1987; Munford, 1989). The livestock feed mix problem for each state of nature is modelled by equations (5) through (8). Equations (5) and (6) ensure the correct restrictions on the nutrient and raw material requirements respectively. Equation (7) ensures that the total feed mix equals the consumption of the livestock.

Roughage is part of the animal feed. The young cattle and sheep require roughage feed from the farm. Since the roughage requirement of animals is totally supplied from the farm production, it is necessary that the amount of roughage produced on the farm should satisfy the demand from the animals. In order to incorporate this into the general mathematical decision-making model, the formulation of a roughage availability constraint is necessary. The roughage availability constraint is given by equation (8).

3.2.4 Availability constraints

The farm can produce all or any combination of the crops from the crops considered. The harvested crops can be used either in the feed mix or can be sold to the market, if there is an excess amount. This activity plays a pivotal role in linking both the crop and animal production enterprises. Both crop sale and preparation of the feed mix are dependent on the availability of crop yield in the farm's crop production activity. This is represented by the availability constraint (Williams, 1999), which is given by equation (9). The availability constraint ensures that the total quantity demanded (both crop sale and animal feed) does not exceed the supply from the farm production.

3.2.5 Resource renting constraints

Some of the resources available to the farm are not fixed. During the planning period the capacity of the available resources of the farm may not match the demands of some activities. If the resources available are not enough, it is assumed that the shortfalls can be supplemented through hiring or renting of additional units of the required resource.

Hazell & Norton (1986) introduced this complication into the farm planning model. The capacity of the combine harvester and baling machine on the farm is assumed to be fixed. If the demand for such machines is greater than this fixed capacity, additional units must be hired in order for the farm to cope with the demand. Equations (10) and (11) represent these constraints.

3.2.6 Animal feed storage constraint

The storage capacity of the storage area that the farm has for some feed material types is limited. That is, beyond some quantity level this constraint restricts the amount of the feed type that can be stored in the available farm storage area. The raw material storage constraint for ingredient type *m* is illustrated by equation (12).

3.2.7 Livestock buying and selling activities

The number of livestock the farm keeps depends on various factors of the farm's operation. Some of these factors include availability of space, profitability, and availability of feed and pasture. Because of such restrictive factors, the number of animals the farm keeps is constrained between a maximum and a minimum number. For animal type *a*, the upper and lower bound is given by constraints (13). These bounds could be relaxed to zero and infinity, if the modeller feels that these restrictions are not applicable.

3.3 The model

The mathematical representation of the model is given below.

$$Max\ \ G = \sum_{s=1}^{n} V_s U_s - \sum_{i=1}^{N} C_i X_i + \sum_{a=1}^{l} f_a \left[W_a + N_a - Z_a \right] + \sum_{a=1}^{l} q_a Z_a - \sum_{a=1}^{l} b_a N_a - \sum_{m=1}^{y} d_m Y_m - \sum_{w=1}^{R} h_w R_w$$

subject to

$$\sum_{i=1}^{N} X_i \leq A \tag{1}$$

$$\left.\begin{array}{l} X_i - A\delta_i \leq 0 \ \ for\ i = 1, 2, \ldots, N \\ X_i - g\delta_i \geq 0 \\ \sum_{i=1}^{N} \delta_i \leq T \\ T \geq 1, \ \ X_i \geq 0, \\ \delta_i = \begin{cases} 0 & if\ X_i = 0 \\ 1 & if\ X_i > 0 \end{cases} \end{array}\right\} \tag{2}$$

$$\sum_{i=1}^{N} \left(C_{i\tau} - C_i \right) X_i + L_\tau^- \geq 0 \ ; \tau = 1, 2, \ldots, h. \tag{3}$$

$$\sum_{\tau=1}^{h} L_\tau^- = \lambda \tag{4}$$

$$p_r \leq \frac{\sum_{m=1}^{y} a_{rm} Y_m}{\sum_{m=1}^{y} Y_m} \leq P_r \quad r = 1, 2, \ldots, x \tag{5}$$

$$e_m \leq \frac{Y_m}{\sum_{m=1}^{y} Y_m} \leq E_m \quad m = 1, 2, \ldots, y \tag{6}$$

$$\sum_{m=1}^{y} Y_m - \sum_{a=1}^{l} \pi_a \left(W_a + N_a - Z_a\right) = 0 \tag{7}$$

$$\sum_{i=1}^{N} K_i X_i - \sum_{a}^{l} \mu_a \left[\left(W_a + N_a - Z_a\right)\right] = 0 \tag{8}$$

$$\sum_{i=1}^{N} \beta_{is} X_i - U_s - Y_m = 0 \quad s = 1, 2, \ldots, n \ , \ m = 1, 2, \ldots, y \tag{9}$$

$$\sum_{i=1}^{N} \sum_{s=1}^{n} \alpha_{is} X_i - R_1 \leq \mathrm{H} \tag{10}$$

$$\sum_{silage,\ medics} U_s + \sum_{silage,\ medics} Y_m - R_2 \leq B \qquad U_S, Y_m, R_2, B \geq 0 \tag{11}$$

$$Y_m \leq \mathrm{K}_m \qquad m = 1, 2, \ldots, y \tag{12}$$

$$\theta_a \leq W_a + N_a - Z_a \leq \Theta_a, \ a = 1, 2 \ldots, l \tag{13}$$

$A,\ Y_m, p_r, P_r, e_m, E_m, K_i, \beta_{is}, X_i, U_s, Y_m, \alpha_{is}, R_1, H, R_2, B, W_a, N_a, Z_a, \theta_a, \Theta_a, \pi_a, \delta_i, \lambda \geq 0$

$\delta_i, W_a, N_a, Z_a, \theta_a, \Theta_a$ integer

λ is a parameter

4. MODEL SOLUTION (CASE STUDY)

A typical farm, which is located in the Koeberg area of Western Cape, was selected for the case study. The farm has 1800 hectares suitable for crop production. The farm's activities include crop, dairy and wool sheep production. The farm can grow wheat, canola, lupines, silage (oats) and medics. It is assumed that the farmer has 15 feasible cropping strategies (alternatives) from which the decision maker can choose, based on the decision criteria utilised. The feasible cropping alternatives include mono-cropping, two-year crop rotation and three-year crop rotation. The feasible cropping strategies are selected based on the idea that these crops are grown currently in the region where the farm is located (Hardy, 1998; Visagie, 2004). The 15 different feasible cropping alternatives (strategies) are given below. The 16th strategy (grassland) is included to incorporate the roughage-growing possibility. This strategy means that the farmer does not plant anything but uses the land as pastures.

Table 1: Feasible cropping alternative strategies

One-year strategies	Two-year strategies	Three-year strategies
Wheat	Wheat/Medics	Wheat/Wheat/Medics
Medics	Wheat/Canola	Wheat/Canola/Medics
Grassland	Wheat/Silage	Wheat /Canola/Silage
	Wheat/Lupines	Wheat/Medics/Medics
	Medics/Canola	Wheat/Wheat/Lupines
		Wheat/Silage/Medics
		Wheat/Canola/Fallow
		Wheat/Medics/Lupines

On the farm the young cattle are fed with both blended feed and roughage from the farm. Each young cattle requires 2 tons of blended feed and 2 tons of roughage per year. The different restrictions on nutrients and ingredients of the blended feed for the young cattle are the same as for the adult ones. Roughage is the only food which the sheep consume on the farm. A single sheep needs 0.5 ton of roughage per year (Perry, 1982; Subcommittee on Dairy Cattle Nutrition (USA) *et al*, 2001).

Because of farm space availability and operational business restrictions, the number of animals the farm can support is constrained. As the farm's space for animal accommodation is fixed, the maximum possible adult cattle the farm can keep is 600 and the lowest possible number of adult cattle the farm keeps because of management restrictions is 300. The maximum and minimum number of sheep kept on the farm is 2,000 and 500 respectively.

It is assumed that on average a single adult cow generates a gross income of R10,450 per year in dairy production. The gross income per sheep from wool production is assumed to be R250 per year. Another activity in the animal production is the selling and buying activities of animals. From a single cattle and sheep sold, the farm can generate an income of R600 and R50 (the income is the interest on capital) per year respectively. Furthermore, if the condition on the farm business is favourable for buying, the farm has an option of buying adult cattle and young sheep at a cost of R900 and R75 (the cost is the interest on capital) per year respectively. A combine harvester is required for harvesting wheat, canola and lupines. The capacity of the existing combine harvester is 1,200 hectares per year. The extra capacity cost of this machine is R1,000/hectare. Another machine with fixed capacity is the baling machine. Silage and medics are either sold to the market at the existing market price or can be used in the preparation of blended feed for animal consumption. In each case, both crops require baling. Baling is carried out on the farm with the existing machine capacity. The baling capacity of the machine is 2,000 tons/year. If the demand for baling is more than the existing machine capacity, the farm should employ a hired additional capacity. The cost of additional capacity for baling is 150 R/ton.

The mathematical model formulated was solved for different risk scenarios. The model was solved using the optimisation software Whats'Best! ® 7.0, Copyright © 2003, Lindo Systems, Inc. The model solution under different risk scenarios is discussed below.

4.1 Model solution in the absence of risk

Initially the mathematical model was solved in the absence of risk to compare the profitability of a wheat mono-crop system with strategies that are based on crop rotation in the integrated crop-livestock farm framework. The optimal cropping and animal production results of the model farm for mono-crop wheat production and crop rotation strategies are shown in Tables 2, 3 and 4 below.

Table 2: Optimal crop production and buy/sell plan (tons) in the absence of risk

	Wheat mono-crop strategy	Crop rotation strategy
Tons of wheat produced	4350	2490.51
Tons of medics produced	-	4220.55
Tons of silage (oats) produced	-	657.40
Tons of wheat sold to the market	4350	2490.51
Tons of medics sold to the market	-	3563.15
Extra combine harvester capacity rented (hectares)	600	-
Extra capacity of baling machine hired (tons)	-	2566.55

Table 3: Optimal animal production plan in the absence of risk

	Wheat mono-crop strategy		Crop rotation strategy	
The number of adult cattle	340	Sell 110	300	Sell 150
Number of sheep on the farm	1000	Sell 500	1696	Buy 196

Figure 3 below shows the land use specified by the mathematical model solution for the crop rotation scenario. The optimal solution indicates that approximately 91% of the farmland is allocated to the three-year crop rotation strategy (wheat-medics-medics) and the remaining 9% of the land is allocated to the two-year crop rotation strategy (wheat-silage).

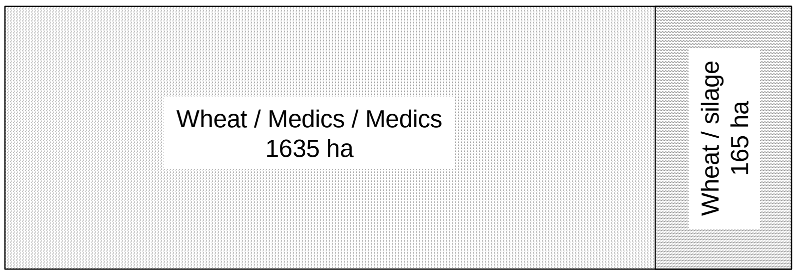

Figure 1: Optimal farmland allocation in the absence of risk

Table 4: Optimal animal feed mix production (tons) in the absence of risk

Silage	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00

Based on this solution, a farming plan which adopts crop rotation is dominant to the mono-crop counterpart – that is, if the maximisation of the gross margin is considered as the performance measure. The higher gross margin generated by crop rotation strategies in all the states of nature considered explains the benefit of crop rotation in the overall integrated farm planning decision.

The model results indicate that the number of sheep kept on the farm with the crop rotation strategy is greater than the wheat mono-crop counterpart (see Table 3)**.** This can be attributed to the better availability of roughage with crop rotation. The favourable condition for forage production created by the rotation strategy allows the increase in sheep. The gross margin generated from applying crop rotation is higher. The model solution shows that the interdependence between the crop production and the animal production of the farm favours crop rotation strategies that include roughage production, such as medics and silage.

Another interesting result is that the optimal solution suggests that there is only a slight difference in the type of ingredients used in the feed mix composition. However, the quantities utilised from the farm differ substantially. The optimal feed mix solution for the wheat mono-crop strategy indicates that from the total feed mix that the animals consume, raw materials produced on the farm amount to 7% of the feed mix. On the other hand, in the crop rotation strategy 36% of the raw materials used in the feed mix come from the crop produced on the farm. In both cases the remaining amounts were bought from the market.

4.2 Model solution in the presence of risk

This section determines which farm plans (crop production plans) maximise the gross margin for different risk scenarios. The sum of negative deviation from the expected value of the net return of a cropping sequence is considered as a measure of risk. This measure of risk is parameterised over feasible ranges, which correspond to an arbitrary chosen lower bound of R494,257 (λ_{min}) to an upper bound of R2,237,706 (λ_{max}), which is the maximum negative deviation allowed. A high enough upper bound corresponds to the maximisation problem (equivalent to the solution in the absence of risk) and the lower bound on risk corresponds to the minimum risk that can be achieved. This minimum risk value can be achieved by considering minimisation of risk as the objective function of the model. In order to investigate the effect of risk in the farm planning problem, the mathematical model was solved for three level risk scenarios.

4.2.1 Low risk

The parameter, λ, which represents the target risk level, was initially set at minimum possible value. The farmland use pattern solution of the mathematical model for the minimum risk situation of a cropping plan is given in Figure 4 below. Under this minimum-risk scenario, the model solution suggests that the decision maker allocate only 28% of the land to the crop production. Because of the conservative nature of this decision scenario, 72% of the land is not allocated to any of the 15 cropping alternatives that the decision maker has available. Based on the model solution, the remaining land is used for the purposes of roughage (grass) production. This is mainly attributed to the high-risk aversion nature of the decision maker, as the decision maker prefers production alternatives which are less risky. The maximum gross margin that can be achieved in such a case is R2,022,516. Moreover, the optimal model for this scenario specifies that the cattle production activity is carried out in the lowest minimum capacity possible. However, as can be deduced from the model solution, because of the availability of land for roughage production, sheep production is carried out at the highest possible level. The optimal solution with the low-risk scenario is given in Tables 8, 9 and 10.

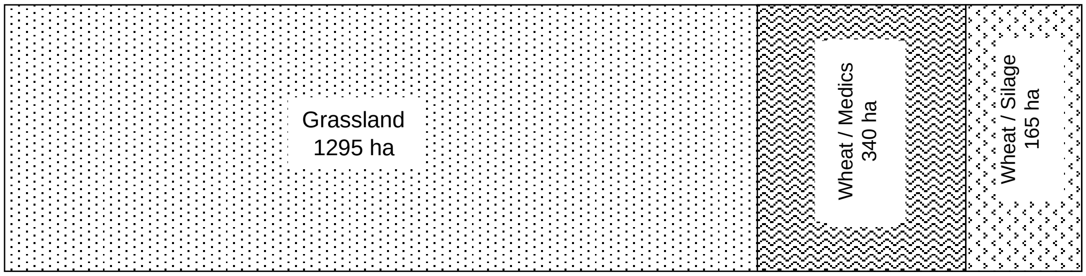

Figure 2: Farmland allocation for the minimum-risk situation (λ = R494 257)

Table 5: Amount of crops (tons) produced and sold to the market applying the minimum risk

Wheat sold to the market	716
Silage (oats) produced	346
Medics produced	657
Straw produced	242

Table 6: Optimal feed mix under minimum risk (tons)

Silage (Oats)	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00

Table 7: Optimal animal production plans for minimum-risk level

Type of animal	Number of animals kept on the farm	Number sold	Number bought
Adult cattle	300	150	-
Sheep	1955	-	455

The wheat-medics and wheat-silage strategies are profitable, if the risk level is restricted to the range R494,257 - R582,965. If the risk level is allowed to increase to the range R582,966 - R901,015, a combination of wheat-medics and wheat-medics-medics is included in the optimal solution. In this risk scenario, where the farmer is considered as highly risk averse, the land is not fully utilised by cropping strategies. A portion of land is left uncultivated. This allows for the increase of forage production, which is necessary for sheep production. With an increase of the risk level, the fraction of land used in crop production increases. The model solution also suggests that the farmer should concentrate on sheep production; while cattle production is carried out at the lowest value.

Table 8: Land allocation (ha) and gross margin (R) generated with the low-risk scenario

Risk (R)	494257	500000	582966	600000	700000	800000	900000
Gross margin (R)	2022516	2036070	2336455	2382316	2640445	288850	3091590
Wheat-medics	340	340	-	-	-	-	-
Wheat-silage	165	172	241	285	488	620	751
Wheat-medics-medics	-	-	309	297	255	255	255
Grassland	1295	1288	1249	1218	1057	925	794
Percentage of land used for crop production	28%	28%	31%	32%	41%	49%	56%

Table 9: Animal feed production plan under minimum-risk scenario

Risk(R)	Silage (oats)	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
494257	346	657	242	722	865	74	208	346
500000	346	657	242	722	865	74	208	346
582966	420	798	294	876	1050	90	252	420
600000	403	765	282	840	1007	87	242	403
700000	346	657	242	722	865	74	208	346
800000	346	657	242	722	865	74	208	346
900000	346	657	242	722	865	74	208	346

Table 10: Animal production plan for the minimum-risk scenario

Risk (R)	494257.05	500000	582966	600000	700000	800000	900000
Adult cattle	300	300	364	349	300	300	300
Sheep	1955	1946	1999	1999	1926	1764	1603

Table 11: Crop production (ton), sale and amount used for feed mix preparation

Risk (R)	Produced				Sold to the market			Used in the feed mix		
	Wheat	Silage (oats)	Medics	Straw	Wheat	Silage (oats)	Medics	Silage (oats)	Medics	Straw
494257	716	346	657	242	716	-	-	346	657	242
500000	726	361	657	242	726	15	-	346	657	242
582966	755	507	798	294	755	86	-	420	798	294
600000	797	599	766	282	797	196	1	403	765	282
700000	1014	1024	658	242	1014	678	1	346	657	242
800000	1192	1302	657	242	1192	956	-	346	657	242
900000	1370	1577	658	242	1370	1231	1	346	657	242

4.2.2 Medium risk

The mathematical model was solved for the range of risk levels R901,016 – R1,700,000. Tables 12-15 represent the model results under a medium-risk scenario. From the model results a few observations can be made. Compared to the low-risk scenario, land utilisation is increased and animal production is shifted from sheep production to cattle production. An increase of medics and silage utilisation appears in the feed mix plan. The increase in the risk level results in an increase in the gross margin.

The optimum solution involves wheat-wheat-lupines and wheat-silage-medics strategies for the risk levels R901,016 – R1,499,988, and wheat-wheat-silage and wheat-silage strategies for the risk levels R1,499,989 – R1,730,637.

Table 12: Land allocation (ha) and gross margin (R) under a medium risk scenario

Risk (R)	901016	1000000	1100000	1200000	1300000	1400000	1499988	1500000	1600000	1700000
Gross margin	3095022	3341461	3591559	3812093	4010580	4175168	4286626	4286835	4428339	4569247
Wheat-wheat-lupines	629	790	958	1005	965	925	814			
Wheat-silage-medics	511	514	514	584	704	825	986			
Wheat-silage								1208	767	321
Wheat-wheat-medics								592	1033	1479
Grass-land	660	496	328	211	131	50				

Table 13: Animal feed plan for the medium-risk scenario (tons)

Risk(R)	Silage (oats)	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
901016	346	657	242	722	865	74	208	346
1000000	346	657	242	722	865	74	208	346
1100000	346	657	242	722	865	74	208	346
1200000	394	748	275	821	984	85	236	394
1300000	476	904	333	992	1190	102	286	476
1400000	517	981	362	1077	1291	111	310	517
1499988	515	979	361	1075	1288	111	309	515
1500000	399	759	280	833	998	86	240	399
1600000	465	884	326	971	1163	100	279	465
1700000	520	988	364	1084	1300	112	312	520

Table 14: Optimal animal production plan for the medium-risk scenario

Risk(R)	901016	1000000	1100000	1200000	1300000	1400000	1499988	1500000	1600000	1700000
Adult cattle	300	300	300	341	412	447	446	346	403	450
Sheep	1783	1671	1557	1335	1034	848	791	514	502	524

Table 15: Crop production, sale and amount used for feed mix preparation (tons) for the medium-risk scenario

	Produced					**Sold to the market**				**Used in the feed mix**		
Risk (Rand)	**Wheat**	**Lupines**	**Silage (Oats)**	**Medics**	**Straw**	**Wheat**	**Lupines**	**Silage (Oats)**	**Medics**	**Silage (Oats)**	**Medics**	**Straw**
901016	1794	187	716	657	242	1794	187	370		346	657	242
1000000	2109	235	719	661	242	2109	235	373	3	346	657.4	242.2
1100000	2433	285	719	660	242	2433	285	373	3	346	657	242
1200000	2604	299	817	750	275	2604	299	423	2	394	748	275
1300000	2662	287	986	906	333	2662	287	510	1	476	904	333
1400000	2721	275	1154	1060	362	2721	275	638	78	517	981	362
1499988	2689	242	1381	1268	361	2689	242	865	289	515	979	361
1500000	2779		2537	761	280	2779		2138	2	399	759	280
1600000	3035		1610	1329	326	3035		1144	444	465	884	326
1700000	3293		675	1901	364	3293		155	913	520	988	364

At a risk level of R1,000,000, the solution from the mathematical programming model suggests that 72% of the land should be allocated for the crop production. The land allocation proposal under this risk scenario is shown in Figure 3 below. The optimal solution for the risk level λ = R1,000,000 is shown in Tables 16 to 18.

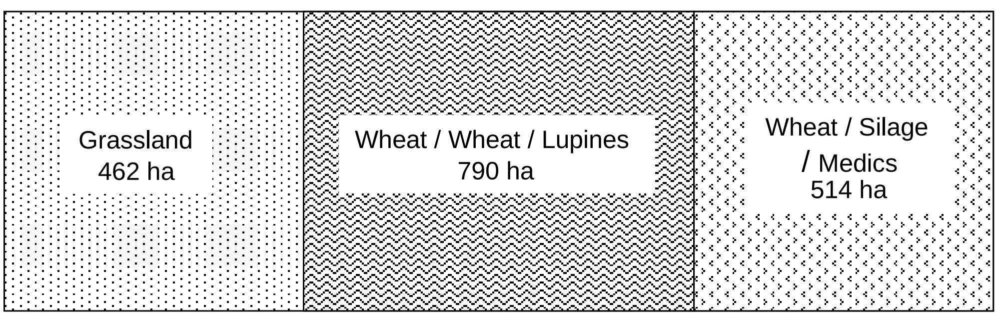

Figure 3: Land allocation for the medium-risk scenario (λ = R1,000,000)

Table 16: Quantity of crops (tons) produced and sold to the market for the constraint level λ = R1,000,000

Crop type	Produced	Sold to the market
Wheat	2109.07	2109.07
Lupines	234.73	234.73
Silage (oats)	719.28	373.28
Medics	660.54	3.14

Table 17: Optimal animal production plans at risk level λ = R1,000,000

Type of Animal	Number of animals kept on the farm	Number sold	Number bought
Adult cattle	300	150	-
Sheep	1671	-	171

Table 18: Optimal feed mix (tons) at risk level λ = R1,000,000

Silage (oats)	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
346	657.4	242.2	721.52	864.84	74.44	207.6	346

4.2.3 High risk

In this case the model solution is much like the solution without considering risk. For the high-risk values, the model has selected the combination of wheat-silage and wheat-medic-medic cropping patterns. Under this risk scenario the expected gross margin, land allocation, the number of cattle and sheep as well as the feed mix requirements are reported in Tables 19 to 22. Table 19 gives the model solution for different risk values. The optimal solution for different risk values in this case involved the production of wheat, silage and medics. Considering the farmland allocation for lower values in this category (at around R1,730,638 risk value), the model solution indicates that 62% of the land is allocated to a wheat-silage cropping sequence. The proportion of land allocated to wheat-medics-medics appears to increase gradually as the risk level increases. For example, at the risk level of R2,000,000 the model solution suggest that 66% of the land is allocated to the wheat-medics-medics strategy and the remaining land must be allocated to a wheat-silage strategy. As the amount of risk increases, the farmland allocated to the cropping sequence wheat-medic-medic increases, while the land allocated to wheat-silage decreases. As the value of the risk increases, two significant changes in the farm plan were observed. The number of sheep was increased due to the increase land allocated to forage crops, thereby increasing pasture availability for the sheep. Wheat and medics production increased and silage production decreased.

Table 19: Land allocation under high-risk values (hectares)

Risk(R)	1730638	1800000	1900000	2000000	2100000	2200000
Gross margin	4605443	4700690	4837422	4974153	5110785	5246224
Wheat-silage	1117	987	799	612	424	237
Wheat-medics-medics	683	813	1001	1188	1376	1563

Table 20: Crops used in the feed mix for high-risk values (tons)

Risk (R)	Silage (oats)	Medics	Straw	Cotton seed oil cake	Maize	Braw	Molasses	Cotton seed
1730638	513	975	359	1070	1282	110	308	513
1800000	520	988	364	1084	1300	112	312	520
1900000	520	988	364	1084	1300	112	312	520
2000000	520	988	364	1084	1300	112	312	520
2100000	520	988	364	1084	1300	112	312	520
2200000	497	944	348	1036	1242	107	298	497

Table 21: Animal production plan for the high-risk scenario

Risk level (Rand)	1730638	1800000	1900000	2000000	2100000	2200000
Adult cattle	444	450	450	450	450	430
Sheep	500	581	726	871	1016	1225

Table 22: Crop production, sale and amount used in feed production (tons)

Risk (Rand)	Produced				Sold to the market			Used in the feed mix		
	Wheat	Silage (oats)	Medics	Straw	Wheat	Silage (oats)	Medics	Silage (oats)	Medics	Straw
1730638	2458	2344	1763	359	2458	1831	789	513	975	359
1800000	2463	2072	2099	364	2463	1552	1111	520	988	364
1900000	2469	1678	2583	364	2469	1158	1595	520	988	364
2000000	2476	1284	3067	364	2476	764	2079	520	988	364
2100000	2482	891	3551	364	2482	371	2563	520	988	364
2200000	2488	497	4035	348	2488	0	3091	497	944	348

5. CONCLUDING REMARKS

An examination of the results suggests that any producer that avoids risk would prefer some combination of wheat, medic and silage. The type and size of crop rotation strategies selected change considerably, depending on the risk level allowed. The analysis of model solutions of the optimal farm plans for different levels of risk also showed that the gross margin increases considerably when the risk level increases. A graph of the expected gross margin versus the risk is shown in Figure 4.

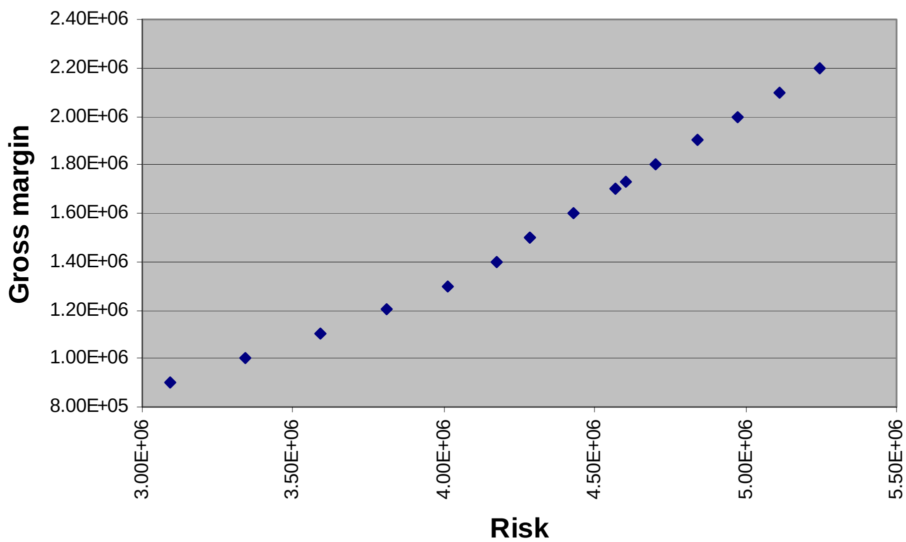

Figure 4: The expected gross margin at different risk levels (Rand)

In the highest risk scenario the model solution suggests a choice of strategies that include the crop sequences wheat-medic-medic and wheat-silage. At medium risk levels the crop sequences wheat-silage-medics, wheat-wheat-medic and wheat-wheat-lupines are in the optimal solution, depending on the particular risk level chosen. For the low-risk scenario the results of the optimal solutions suggest a combination of the cropping strategies wheat-silage, wheat-medic and wheat-medic-medic. This generally shows that with an increase in the risk level there is an increase in the diversity of the crop rotation system. The size of certain sequences (wheat-wheat-medics, wheat-medics-medics and wheat-silage) is also increased as risk level was increased in the model. As expected, the value of the objective function increased as risk become less constraining.

The choice of risk specification in the mathematical model results in significantly different crop mixes. Regardless of the decision maker's degree of risk aversion, the optimal farm plan included a diversified crop system. Crop rotation systems are generally hypothesised to be less risky compared to a mono-crop system. The model solution backs this hypothesis, as mono-crop systems failed to enter in the optimal solution. In all the optimisation done at all the different risk levels, the solution always included crop rotation strategies in the basis, which implies that a crop rotation system is always preferable above mono-crop systems. The optimal solution suggests that forage crops become evident in the low-risk scenario. This can be attributed to the interdependence of the crop and livestock activities of the farm and the lower risk associated with livestock in the model.

The question, however, remains which strategy would be optimal to follow for the planning period in question, as the differences in the alternatives offered by these different risk levels are not minor. As there is no direct theory that guarantees an explicit choice from the different solution alternatives, the decision about what strategy to adopt will depend on the behaviour of the decision maker. One evident property of the optimal solution profit-risk efficient set of the farm plan is that no one farm plan is superior to another with respect to both the performance measures, namely gross margin and risk. (This can be seen in Figure 4.) That is, farm plans with higher gross margin levels also have higher measures of risk. It follows that production plans generating a low gross margin are also associated with low-risk levels. The selection of a farm plan depends on the decision maker's preference. A risk-averse decision maker will select strategies which give him some shield against big risks. On the other hand, a decision maker who is indifferent to risk levels will prefer strategies that will give him the highest (maximum) gross margin possible.

To conclude, the model solutions point out that the choice of risk level significantly affects recommendations of crop mix and livestock production. Essentially, in the integrated crop-livestock environment diversification is the best option to maximise the gross margin at a certain risk level.

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Appendix A: Data

Three sets of data are used in this study. These are: (1) data for crop production, (2) data for animal production, and (3) data for resources hiring.

The crop production data, including cost of production, yield data and price data of crops for different strategies, are taken from the study by Visagie (2004).

The second set of data dealing with the livestock production refers to the annual animal food consumption requirements, nutrient and ingredient restrictions. Furthermore, the restriction on the number of animals the farm can keep, gross income earned and cost incurred from each type of animal per annum is required to draw up the farm plan. The Subcommittee on Dairy Cattle Nutrition (USA) *et al* (2001) and Perry (1982) are used as sources of the data for the model for animal production data requirements.

Appendix B: Indices, decision variables and parameters

In the model the following notation is used:

i. Indices

i = 1, 2, 3, …, N the crop rotation strategies
s = 1, 2, …, n the type of crop
m = 1, 2, 3,…, y the raw material type used in the feed mix
a = 1, 2 the animal type, with a = 1 adult cattle and a = 2 sheep
τ = 1, 2, …, h states of nature
w = 1, 2, …, R resource type
r = 1, 2, …, x nutrient type

ii. Decision Variables

X_i = hectares of farmland planted with strategy type i
U_s = amount of crop type s sold to the market (tons/year)
Y_m = amount of raw material (crop type) m used in the feed mix (tons/year)
W_a = the number of animal type a initially
Z_a = number of animal type a sold to the market
N_a = number of animal type a bought from the market
R_w = amount of resource w rented

iii. Parameters

C_i = direct attributable variable cost (rand/hectare) of cropping strategy *i*
V_s = selling price of crop type s (Rand/ton)
d_m = cost of food staff type m(Rand/ton)
π_a = blended feed requirement of animal type a (tons/head/year)
μ_a = roughage requirement of animal type a (tons/head/year)
β_{is} = yield of crop type s from strategy *i* (tons/hectare)
K_i = yield of roughage (tons/hectare) from strategy *i*
α_{is} = proportion of X_i cultivated by crop type *s*.
$C_{i\tau}$ = gross income from strategy *i* when state of nature τ is prevailed (Rand/hectare/year).
f_a = gross income from animal type a (Rand/head/year).
q_a = gross income from selling of animal type a (Rand/head-income from interest).
b_a = cost of buying of animal type a (Rand/head, interest cost).
θ_a = minimum number of animal type a in the farm.
Θ_a = maximum number of animal type a in the farm.
n_{rk} = percentage of nutrient r contained in raw material type k.
P_r = maximum amount (%) of nutrient *r* required in the feed mix.
p_r = minimum amount (%) of nutrient *r* required in the feed mix.
E_m = maximum amount of raw material *k* (%) desired in the feed mix.
e_m = minimum amount of raw material *k* (%) desired in the feed mix.
h_w = cost to rent resource *w*.
A = maximum available area of land in hectares the farm owns.
B = baling machine capacity (tons/year).
H = combine harvester capacity (hectares/year).