

**INTERTEMPORAL DECISIONS OF FARMERS' RISK MANAGEMENT: A DYNAMIC  
OPTIMIZATION WITH GENERALIZED EXPECTED UTILITY**

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## **Abstract**

In this paper we attempt an intertemporal study of risk management decisions for wheat growers in the Pacific Northwest. We apply a generalized expected utility model (GEU) to examine the farmers' optimal choices of hedging ratios and crop insurance coverage levels in the presence of government payment programs in a multi-period production environment. A stochastic trend model is used to identify the long-term time series patterns of annual wheat yields, cash prices, and futures prices from two counties in Washington. The fitted models are then used as the base for yield and price simulation over the next five years. The stochastic dynamic optimization problem is solved numerically based on simulated data. The optimal solutions indicate that the GEU model is feasible in modeling farmers' intertemporal decisions regarding risk management. The comparison between GEU model and some commonly used expected utility models further implies the advantage of the GEU model in being flexible to specify farmers' intertemporal preferences separately and completely.

**Keywords:** intertemporal decision, generalized expected utility, dynamic optimization, risk management

# **Intertemporal Decisions of Farmers' Risk Management: A Dynamic Optimization with Generalized Expected Utility**

## **I. Introduction**

Agricultural production is a stochastic process that is greatly affected by unpredictable weather, technology, and price fluctuations in commodity markets. The risk management situation confronted by farmers is complicated with intra- and inter-temporal uncertainties when one crop cycle is taken as a period. Modeling farmers' risk management has been commonly based on a static approach, although a stochastic dynamic approach is more consistent with reality. The complexity involved in stochastic dynamic modeling requires decision making to incorporate multi-dimensional uncertainties into one entirety, which is challenging for model development in both theoretical work and empirical work.

Currently, U. S. farmers can use several risk management tools to reduce loss in bad years and save money in good years, and make long term strategic plans accordingly. Futures contracts are traditionally used by farmers to hedge price risk, and have been available for a long time. Yield-based crop insurance programs were facilitated and subsidized by the U.S. federal government dating back to the 1930s. Now crop insurance products can be used to reduce both yield and price risk, and have become by far the most popular tool used by U.S. crop producers. Besides insurance programs, government payment programs provide direct cash compensation to farmers in bad years. The 2002 Farm Bill adds a new counter-cyclical payment program to the old direct payment program and loan deficiency payment program. With increased involvement, government allocates a significant amount of tax dollars to provide and subsidize all of these programs every year. Despite that, the risk management effectiveness and farmers' participation

incentives have always been a concern (Brorsen, 1995; Ke and Wang, 2002).

Expected utility maximization, commonly used as a standard framework in many studies including agricultural risk analysis, has been shown feasible in dynamic modeling. It allows a risk averse farmer to maximize a summarized discounted von Neumann-Morgenstern expected utility function of his/her stochastic income subject to a set of policy and resource constraints. Such a specification, however, assumes utility is additively separable and therefore implies the decision maker is intertemporally risk-neutral. A generalized expected utility (GEU) maximization model, developed by Epstein and Zin (1989, 1991), provides an alternative to study intertemporal decisions with further specification of decision maker's preferences. The model utilizes a recursive utility function of constant elasticity of substitution (CES) form as the objective function. This approach incorporates the decision maker's non-neutral intertemporal substitution preference through different levels of elasticity of substitution. In this sense the recursive model disentangles intertemporal substitutability from temporal risk aversion.

The objective of this paper is to explore the feasibility of GEU as a framework in modeling farmers' intertemporal risk management decisions. The farmer's optimal risk management portfolios are examined under the GEU framework, where he/she chooses from price instruments, insurance products, and government payment programs to maximize utility.

Specifically, the paper proceeds as follows: 1) Section II reviews literature in agricultural risk management modeling; 2) Section III discusses the model structure; 3) Section IV introduces data and the simulation of yields and prices; 4) Section V discusses the optimization results based on GEU as well as some standard models; and 5) Section VI provides some conclusions.

## II. Existing Literature

Recent studies on risk management strategies have been extended from the earlier one-element models to portfolio models. They analyzed the effects of different combinations of instruments and interactions between each instrument. Among them are portfolios of crop yield insurance and futures contracts (Myers, 1988), futures market and government farm programs (Crain and Lee, 1996), crop yield insurance, futures, options and government programs (Wang, et al., 1998), and crop revenue insurance, futures and government programs (Zuniga, Coble, and Heifner, 2001; Wang, Hanson, and Black, 2003; Wang, Makus, and Chen, 2004).

Government programs have been studied either singularly (Miller, Barnett, and Coble, 2001) or in a portfolio setting together with other instruments (Wang et al., 1998; Makki and Miranda, 1998; Zuniga, Coble, and Heifner, 2001). The new counter cyclical payment program in the 2002 Farm Bill is similar to the deficiency payment program in the 1990 Farm Bill. Although having been included in a study of farmers risk management strategy for the Pacific Northwest (PNW) region (Wang, Makus, and Chen, 2004), more thorough investigation is necessary.

As a modeling framework, the expected utility (EU) maximization approach has been applied in producers' risk analysis in both static and dynamic situations since 1970s. However, unlike its counterparts in economics and finance, large amount of the existing works are still based on static scenarios, especially in agricultural economics (Nyambane et al., 2002). Examples include all the aforementioned studies, with a few exceptions such as Vukina and Anderson (1993), Myers and Hanson (1996), Atwood et al. (1996), and Nyambane et al. (2002).

In the standard specification of intertemporal EU maximization, it is common to assume an additive and homogeneous von Neumann-Morgenstern utility index. Such a specification, however, intertwines two distinct aspects of preference, intertemporal substitutability and relative

risk aversion (Epstein and Zin, 1989). Additionally, these models did not perform well in empirical examinations (Hansen and Singleton, 1983; Mehra and Prescott, 1985). A more general and flexible framework, the generalized expected utility (GEU) model was developed independently by Epstein and Zin (1989, 1991) and Weil (1990). This new specification takes the CES form for the utility function and is based on a recursive structure. The CES form adds extra flexibility in identifying intertemporal substitution along the time span, and is able to disentangle the intertemporal substitution from the risk aversion.

With the possible and testable separability of risk preference and intertemporal substitutability under the GEU framework, it is possible to estimate preference parameters separately and examine the form of objective function. Continuing on from their theoretical paper, Epstein and Zin (1991) empirically investigated the parameter estimation and the testable restrictions. Although favorable and seemingly consistent with theory, they found those estimates and test results are sensitive to consumption measures and instrumental variables. As one of the earliest and very few agricultural economists who have applied this GEU model in agricultural production, Lence (2000) used 1936-1994 U.S. farm data to study the fitness of a GEU framework and farmers' time and risk preferences. He found the estimated farmers' utility parameters satisfy the theoretical restrictions of the GEU model. The EU model is rejected in favor of the GEU model. Knapp and Olson (1996) used GEU to solve dynamic resource management problems. They found intertemporal substitution has a substantial effect while risk aversion has a very small effect on optimal solutions. Howitt et al. (2002) applied a GEU framework to stochastic water supply management. The empirical results underscore the importance of using this more general specification of intertemporal preferences.

### III. Model

#### *Theoretical Framework*

The foundation of the GEU model for intertemporal analysis is built up on independent works of Epstein and Zin (1989, 1991), and Weil (1990). Since we use Epstein and Zin's approach in this study, only Epstein and Zin's work is outlined here.

The representation of the general preference for a decision maker under risk can be identified as:

$$(1) \quad \text{Max } U_t = \left\{ (1 - \beta) C_t^\rho + \beta \left[ E_t \left( \tilde{U}_{t+1}^\alpha \right) \right]^\frac{\rho}{\alpha} \right\}^\frac{1}{\rho}$$

where  $U_t(\cdot)$  is the von-Neumann Morgenstern utility function indexed by time  $t$ ;  $E_t$  is the expectation operator at current period  $t$ ; the “ $\sim$ ” above  $U$  indicates the stochastic property of utility.  $\beta$  ( $0 < \beta < 1$ ) is the discount factor per period and implicitly defines the decision maker's time preference. By consuming at  $t + 1$ , he/she only consumes a fraction ( $\beta$ ) of the utility that would have been consumed at  $t$ .  $\alpha$  ( $0 \neq \alpha < 1$ ) denotes the risk aversion parameter, and is equal to one minus the Arrow-Pratt constant relative risk aversion (CRRA) coefficient. A smaller  $\alpha$  indicates greater risk aversion.  $\rho$  ( $0 \neq \rho < 1$ ) denotes the intertemporal substitutability, equal to  $(1 - \sigma)^{-1}$  with  $\sigma$  denoting elasticity of substitution. Early (late) resolution of risk would be preferred if  $\alpha < (>) \rho$ .  $C_t$  denotes the current consumption which is a function of the risk management choice variables. The decision maker's objective function is to maximize current utility, which comprehensively incorporates all of the lifetime expected future utilities.

The recursive GEU specification realizes a separation of risk aversion from intertemporal substitution and the non-additive intertemporal preference relations, which is not usually shared by the EU specification. However, the GEU form nests the EU form as a special case. The recursive

CES EU (CES-EU) preferences, widely used in Finance, macroeconomics and intertemporal consumption analysis, are obtained when we impose the parametric restriction  $\alpha = \rho$ .

$$(2) \quad \text{Max } U_t = \left\{ (1 - \beta) C_t^\alpha + \beta \left[ E_t \left( \tilde{U}_{t+1}^\alpha \right) \right] \right\}^{\frac{1}{\alpha}} \quad (\text{CES-EU})$$

Moreover, the standard multi-period recursive EU (MR-EU) preference is obtained when we further impose  $\alpha = \rho = 1$ . As indicated in equation (3), when the utility function is defined as a linear combination of current and future consumption levels, the optimization of MR-EU becomes a decision maker maximizing the summarized discounted expected consumption over a lifetime (finite or infinite time periods).

$$(3) \quad \text{Max } U_t = (1 - \beta) \left[ C_t + \sum_i \beta^i E_t \left( \tilde{C}_{t+i} \right) \right] \quad (\text{MR-EU})$$

Here  $C_{t+i}$  denotes the consumption for the  $i^{\text{th}}$  period in the future. The additive specification implicitly assumes preferences are homogeneous over time; each one of them carries the same weight when discounted to the current period. Such additivity is now well known to be too restrictive (Weil, 1990). Decision makers may have a clear preference for early resolution of risk compared to late resolution of risk (Kreps and Porteus, 1978).

#### *Application of GEU to Farmers' Intertemporal Decisions in the PNW*

When applying the GEU framework to our optimization problem, current consumption is further defined as a net income from the farmer's wheat production and risk management choices. The farmer uses futures contract, yield insurance, and government programs to construct risk management portfolios. Hedge ratios and insurance coverage ratios are endogenous choice variables to be determined at the optimum, based on information available at  $t$ :

$$(4) \quad C_t = NC_t + CI_t + FI_t + GI_t$$

where  $NC_t = P_t Y_t - PC_t$ ,



$$FI_t = \mathbf{x}_{t-1}[F_t - E_{t-1}(F_t)] - TC_b$$

$$CI_t = P_b \max[0, \mathbf{z}_{t-1} E_{t-1}(Y_t) - Y_t] - Pre_t$$

$$GI_t = DP_t + LDP_t + CCP_t$$

$$\text{Where } DP_t = 0.85P_D \times 0.9E_{t-1}(Y_t),$$

$$LDP_t = E_{t-1}(Y_t) \max(0, L_R - P_t),$$

$$CCP_t = 0.85 \times 0.935 E_{t-1}(Y_t) \max[0, P_T - P_D - \max(P_t, L_R)]$$

where  $NC_t$  is the net income from producing and selling the crops in the cash market;  $CI_t$  is the net income from purchasing crop insurance;  $FI_t$  is the net income from hedging in the futures market; and  $GI_t$  is the net income from government programs.

$P_t$  and  $Y_t$  represent cash prices<sup>2</sup> and yields for winter wheat at harvest time, with  $PC_t$  as the production cost.  $F_t$  is the futures price at time  $t$  and the futures market is treated as unbiased.  $\mathbf{x}_{t-1}$  is the hedging amount determined at previous period which is positive for a long position and negative for a short position.  $\mathbf{x}_{t-1}$  is in bold face to indicate its status as a choice variable.  $TC_t$  is the transaction cost of trading futures.  $P_b$  is the base price used to calculate the indemnity from crop insurance with  $Pre_t$  as the premium<sup>3</sup>, and  $\mathbf{z}_{t-1}$  is the coverage selection of the insurance and is also in bold face to indicate a choice variable.  $DP$  is direct payment program which gives a constant payment to farmers,  $LDP$  is the loan deficiency payment, and  $CCP$  is the counter cyclical payment;  $P_D$  is the direct payment rate,  $L_R$  is the loan rate, and  $P_T$  is the target price. The formulation of  $DP$ ,  $LDP$  and  $CCP$  is specified according to the 2002 Farm Bill and calibrated to PNW wheat growers, the chosen area for the empirical analysis.

Due to the nonlinearity in the objective function and complex random relationships

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<sup>2</sup> Cash price is a farm gate price after transportation cost is deducted from the spot market cash price.

<sup>3</sup> The premium of the current year's crop insurance is paid at harvest time.

among variables, closed-form optimal solutions are unavailable in stochastic dynamic programming, so the empirical solutions are obtained by numerical methods. For the dynamic optimization, we simulate yields and prices for the next five years. Optimal levels of crop insurance coverage and hedge ratios are determined simultaneously and intertemporally in the presence of government programs.

#### **IV. Data, Simulation and Model Calibration**

##### ***Data Source***

We select a representative farmer from each of the two counties in Washington State, Whitman County and Grant County. Although both are in a typical dryland farming region in the Pacific Northwest (PNW) and grow soft white wheat, these two counties have different levels of precipitation. Whitman County sits on the east central border of Washington and is part of the highest yield area for soft white wheat in the state. Whitman County is a non-irrigated area in the state with an average annual precipitation around 14 inches. In comparison, Grant County is located in the center of the state and does not border Whitman County. Grant is a much dryer county with an average annual rainfall of 5 inches in 2002. Wheat production is riskier in Grant County. However, since there is some irrigation system in Grant County, the yield is not much lower than that in Whitman County (Figure 1).

Historical data for soft white wheat yield, cash price and futures price for Whitman County and Grant County, Washington are collected and examined to identify time series patterns for simulation. The yield data for Whitman County and Grant County in Washington State are obtained from the U.S. Department of Agricultural National Agricultural Statistics Service (<http://www.usda.gov/nass/>) and Risk Management Agency (RMA) at a yearly base for 1939-2003

and 1972-2003, respectively.

Annual September wheat cash and futures prices from 1973 to 2003 are selected to represent harvest prices. September is also the time the farmer makes decisions on the following year's hedging and insurance participation and prepares for the planting year's next winter wheat crop. For cash price, we use the monthly average of daily September prices at the Portland spot market. They are taken from the USDA-ERS Wheat Yearbook (<http://www.ers.usda.gov/publications/so/view.asp?f=field/whs-bb/>). Since the PNW region grows soft white wheat which has no actively traded futures contract, the Chicago Board of Trade (CBOT) September wheat futures contract is chosen for farmers' hedging. We pick the mid-week price of the first week (Wednesday or Thursday) of September to develop our dataset.

#### ***Deterministic Trend vs. Stochastic Trend***

Because of the multi-dimensions involved in GEU specification and dynamic programming, simulation of yield data could affect the final optimization results to a large extent. Specifying a pattern that is consistent with real processes is critical in this study.

Since we have long-term annual data, the time variation is mainly reflected in the mean level due to the low-frequency feature of the data. From the time series plots of Whitman County and Grant County yield (Figure 1) for 1972 to 2003, an upward trend is visible for the last 32 years. As yield is influenced jointly by the stochastic weather and technology changes, it is important to carefully examine the yield distribution before applying any deterministic or stochastic trend models. Similarly for wheat cash and futures prices (Figure 2), the unpredictable balance of supply and demand determines the price levels and inflation associated with the macroeconomic trends further influences prices.

For multi-period analysis, we need to model the long-run inter-year randomness as well as

the short-run random effects. A stochastic trend model would be more appropriate in that it incorporates both types of randomness. Moss and Shonkwiler (1993) developed a single time-dependent stochastic trend model. Their model transforms the error term rather than the dependent variable to incorporate the possibility of both non-stationary data and non-normal errors in corn yield variation. The model is general enough to include both the standard deterministic time trend and normal errors as special cases.

Their model follows a Kalman Filter process and consists of a measurement equation;

$$(5) \quad y_t = \mu_t + \varepsilon_t$$

and two transition equations;

$$(6) \quad \begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

where  $y_t$  is the independent variable indexed by time  $t$ ;  $\begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix}$  is the state vector;  $\varepsilon_t$  is the random error describing the short run randomness with mean zero and variance  $\sigma_\varepsilon^2$ ;

and  $\begin{pmatrix} \eta_t \\ \zeta_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\zeta^2 \end{pmatrix}\right)$  is the error vector describing the long run randomness in the

transition equation that governs the evolution of the state vector. Both of the errors in the measurement equation follow normal distributions and are independent of each other.

In the basic specification,  $\mu_t$ , the mean component of the dependent variable, is shown as a random walk with a drift. Therefore the final generalization shows that the mean of the dependent variable grows at a random rate.

In the case when the dependent series contain non-normal errors,  $\varepsilon_t$  is assumed to be generated by an inverse hyperbolic sine transformation from normality. Specifically,

$$(6) \quad e_t = (\tau_t - \delta) \sim N(0,1),$$

$$\tau_t = \theta^{-1} \ln \left( \theta \varepsilon_t + \left\{ (\theta \varepsilon_t)^2 + 1 \right\}^{\frac{1}{2}} \right)$$

where  $\delta$  is the non-centrality parameter;  $\delta > 0 (< 0)$  denotes the distribution is skewed to the right (left) and if  $\delta = 0$  the distribution is symmetric.  $\theta$  is associated with the degree of kurtosis with  $\theta \neq 0$  denoting a kurtotic distribution.

$$\text{Solving for } \varepsilon_t, \text{ we get } \varepsilon_t = \frac{e^{\theta \tau_t} - e^{-\theta \tau_t}}{2\theta}.$$

The stochastic trend model reduces to a deterministic time trend model if  $\beta_0 \neq 0$  and  $\sigma_\eta^2 = \sigma_\varepsilon^2 = 0$ ; if furthermore  $\beta_0 = 0$  then it reduces to a constant mean regression model.

### ***Estimation and Simulation for Yields and Prices***

Applying the stochastic trend model to our yield and price data using maximum likelihood estimation programmed in GAUSS, we find there is no stochastic trend in the yield for Whitman County but there is one for Grant County. The stochastic trend also exists in the Portland cash prices and CBOT futures prices (Table 1).

For Grant County yield, cash price and futures price, the significance of estimated  $\sigma_\eta$  confirms the existence of a random walk in the mean component, but the insignificance of estimated  $\sigma_\varepsilon$  shows such stochastic variation doesn't exist within the trend. For Whitman County yield, however, the trend is generally a deterministic time trend and there is no randomness in the slope of the time trend. We further test for autocorrelation within the series before applying the time trend and find no evidence. The simple linear regression model with a deterministic time

trend appears to be a good model for Whitman County yield<sup>4</sup>.

The plots of predicted values versus actual values show that in general the stochastic trend models fit the data well by capturing the long-run variation in the trend for wheat yield in Grant County (Figure 3) and prices (Figure 4). The 95 percent confidence intervals include nearly all of the realizations.

For the distributions of yield and prices, we conduct normality tests first on the detrended data. Results fail to reject the null hypothesis of normality. We also estimate the stochastic model including non-normal errors. The estimates of the non-normal parameters  $\delta$  and  $\theta$  are not statistically different from zero, confirming that the data follow a normal distribution.

We use the fitted linear time trend model to simulate annual wheat yields in Whitman County for the next five years, and use the fitted stochastic trend models to simulate Grant County yield, Portland Cash price, and CBOT futures price. An empirical distribution with 2000 samples is simulated for each of the next five years and for each series. All the series are first simulated independently without autocorrelations or contemporaneous correlations. For the cash and futures prices, we then impose a correlation of 0.871 and keep yields and prices uncorrelated based on historical data. Table 2 gives the descriptive statistics of the simulated data.

### ***Parameter Calibration***

Identification of farmers' risk preferences and time preferences has been attempted in previous studies using different models (Saha, Shumway and Talpaz, 1994; Chavaz and Holt, 1996; Epstein and Zin, 1990; Lence, 2000). Among them, Lence used a similar dynamic GEU model to estimate US farmers' preference parameters based on aggregated consumption and asset return data from 1966-1994. We use these values,  $\alpha = -0.13$ ,  $\beta = 0.89$  and  $\rho = 0.9493$ , as the base for

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<sup>4</sup> Results are available upon request.

the representative farmers and assume they are constant over time.

In the determination of current consumption (or net income) level, transportation cost between the Portland spot market and the two counties is set at \$0.50 per bushel; production cost is determined as \$203 per acre for Whitman County and \$200 for Grant County (Hinman and Baldree, 2004); transaction cost associated with hedging is set at \$0.017/bushel. The price used to indemnify crop loss in the insurance programs is the CBOT September wheat futures price plus a Portland basis of \$0.45 per bushel. The insurance coverage levels are restricted to be either zero or from 50% to 85% with an increment of 5%. The insurance premium is computed as the product of the expected indemnity (actuarially fair premium level) and 1 minus the regressive subsidy rate specified in current policies.

For government programs, the direct payment rate  $P_D$  is set at \$0.52 per bushel. The base yield used to calculate a per acre payment is set at 90 percent of the expected yield. The loan rate ( $L_R$ ) for the *LDP* is \$2.86 per bushel for soft white wheat in Whitman County and \$2.91 per bushel in Grant County. The target price ( $P_T$ ) for *CCP* is \$3.86 per bushel. These parameters are based on current US farm policies.

## V. Results

We implement the stochastic dynamic optimization programming using GAUSS and numerically solve for the optimal hedge ratios and crop insurance coverage ratios for our representative farmers in two Washington State counties (Whitman and Grant). Results are shown in Table 3. Note that all the hedge ratios are negatively signed, which indicates hedging is in short position in all cases.

As we can see, the specification of the GEU model gives us extra flexibility in the

parameterization of the objective function, with which we are able to explore the feasibility of the GEU model as well as to compare the results from GEU optimization with those from other widely used expected utility optimization models. The base scenario ( $\alpha = -0.13$ ,  $\beta = 0.89$  and  $\rho = 0.9493$ ) represents the farmer who is risk averse ( $\alpha < 1$ ) and prefers an early resolution of the risk to a late resolution ( $\alpha < \rho$ ). The farmer discounts future consumption by a factor of 89% and is making a decision for the next five years based on all available information as of today.

Other scenarios of interest in our study include the two special cases of the GEU base model, CES-EU optimization with  $\alpha = \rho = -1$  and  $\beta = 0.89$ , and MR-EU optimization with  $\alpha = \rho = 1$  and  $\beta = 0.89$ . The former refers to the case where the farmer is more risk averse and has smaller intertemporal substitution preference in consumption, while the latter refers to the case when he/she is risk neutral and has perfect intertemporal substitution preference. Besides the CES-EU and MR-EU, a multi-period additive EU (MA-EU) optimization is also included. The utility function in this case is the standard constant relative risk aversion utility function  $U_t = \frac{C_t^\alpha}{\alpha}$  assuming  $\alpha = -1$ , which implied a relative risk aversion coefficient equal to 2. This utility function has been widely used in static single-period risk analyses (Mahul, 2003; Wang, Hanson, and Black, 2003; Coble, Heifner, and Zuniga, 2000). It is also easy to extend the model from single-period to multi-period as in (7), but note that this multi-period version has a static nature.

$$(7) \quad \text{Max} U_t = \left[ \sum_i \beta^i E_t \left( \frac{\tilde{C}_{t+i}^\alpha}{\alpha} \right) \right] \quad (\text{MA-EU})$$

Table 3 lists results of the Whitman County farmer's optimal choice on risk management portfolios using the four different models. In general, we see that model specification is very important in modeling farmer's risk management behavior and finding the optimal portfolios for



farmers' intertemporal decision.

For the optimal choice of crop insurance, the highest coverage of 85% is favored in all cases. This result is consistent with the model setting since the insurance is subsidized by the government and no premium loading is charged. The farmer purchases the highest available level so as to enjoy the most protection against yield risk and receives the highest subsidy. Also, the government commodity programs provide free price protection with a sizable expected income transfer. The farmer will always participate, which reduces the need for futures hedging.

From the hedge ratios, we can see the hedging levels are always below 32% due to the government program participation, but the pattern is different from the GEU base model relative to the other models, and the level of hedging is slightly higher in the GEU full optimization. With the flexibility to specify risk aversion, time preference, and intertemporal substitution separately, the GEU full model shows the farmer's optimal hedge ratios should be increasing over the first four years, which is consistent with the increasing price volatility. The generally higher level of hedging implies he/she prefers to resolve the risk earlier rather than later. Although the farmer prefers an early resolution of risk, his/her relatively high intertemporal substitutability of consumption may balance the preference in a way that hedging would be kept at a slightly increasing rate to meet the increasing price volatility. In the fifth and final year, the farmer would reduce spending on hedging and accept more risk.

In the CES-EU model when the farmer's risk aversion and intertemporal substitution of consumption is integrated as one preference, the optimal hedge ratio is higher in the first year and then becomes lower in the second through the fifth years compared to the corresponding ratios in GEU full model. The CES-EU model displays a decreasing pattern over the five years. The higher level of hedging in the first year is consistent with the farmer's higher risk aversion. The pattern

switches for the second year, however. Since the risk aversion and substitution preference are mixed together in this case, the effects of the two preferences are hard to differentiate in a cross-year setting. They may be competing against or reconciling with each other, which is not observable.

The CES-EU results are comparable to the MA-EU results in that they both share the same risk aversion. Interestingly, these two models yield nearly the same optimal hedge ratios. We have further checked with other risk aversion values including  $\alpha = -2$  and  $\alpha = 0.5$ , and get similar results. The comparison gives the impression that these two models work very similarly in modeling the optimization behavior for the decision maker's risk management. As the MA-EU model has a much simpler specification, it is probably easier to implement this model than the CES-EU in empirical work.

As a very special case of the GEU model, the MR-EU model applies to a farmer who is risk neutral and has perfect intertemporal substitutability in consumption. Consistent with these risk preferences, the optimal hedging ratio is zero for each year, reinforcing that the decision maker does not care about risks and has no specific concerns regarding consumptions across years.

Optimal choices for the representative farmer in Grant County are very similar to Whitman County. The farmer prefers slightly less hedging than the Whitman farmer but still buys the same coverage of crop insurance. Although the production is riskier in Grant County because yield is a bit more stochastic, there is no huge gap between the yield levels as shown in the historical data (Figure 1). Also we assume both farmers face the same prices, so both farmers face the same price risks. The hedge ratios are very close to those in Whitman County under the same preference set.

In summary, the comparisons between the four models for Whitman County and Grant

County in Washington State show that the GEU model is feasible in giving reasonable results on optimal risk management portfolios. For a farm planning on multi-period management, GEU shows an optimal strategy that is more consistent with reality on hedging and crop insurance for the decision maker, who wants to maximize utility over the whole time span. The GEU model framework is also flexible enough to account for separate risk, time, and substitution preferences, and is able to incorporate other commonly used EU models that have either ignored intertemporal substitution preference or integrated such substitution with risk preference.

## **VI. Summary and Conclusions**

In this study we attempt a generalized expected utility maximization framework to a risk management problem in agricultural production. A representative soft white wheat grower in Whitman County and Grant County, Washington, maximizes his/her utility by selecting an optimal portfolio of risk management tools including hedging in the futures market, purchasing crop insurance, and participating in government commodity programs. The GEU model allows the decision maker to clearly specify risk preference, time preference, and intertemporal substitution preference. It also incorporates the commonly used expected utility maximization models like MR-EU models as special cases.

We solve the maximization problem numerically by stochastic dynamic programming based on simulated yield and price data for the next five years. In simulating the data, we apply a stochastic trend model which is able to capture stochastic properties within the long-run trend in addition to those from the disturbances. It is also general enough to include the deterministic time trend regression model as a special case. Stochastic trends are found in the historical Grant County yield, Portland cash price, and CBOT futures price.

We find optimal solutions for farmers in both Whitman and Grant County under not only the GEU model, but also its special cases; the CES-EU model and the MR-EU model. This step is easy to implement due to the flexibility of GEU in parameterization. A different type of static MA-EU model is also included as one of the four cases for comparison purposes. Results vary with model specifications more so than across locations. The GEU model is feasible in modeling farmers' risk management decisions in both counties by giving more reasonable results, and the general specification form of GEU has advantages in incorporating more preference information about the decision maker.

The commonly used MA-EU model gives almost the same results when the risk aversion is specified at the same level as in the CES-EU, indicating that these two types of models probably are interchangeable if applied to empirical optimization problems. However, the results are different for the GEU model when the preferences parameters are set at different levels. The results are completely different for the risk neutral and perfect substitution setting by the MR-GEU. The optimal choice of the crop insurance purchase is always 85% under all four models and in both counties, and the hedging ratios are around 30%. These levels are in line with the existing static one period studies.

Although we have obtained favorable results concerning the feasibility and flexibility of the GEU model, further research on the GEU framework and its applicability in modeling and explaining dynamic agricultural risk management issues is still important and necessary. First, sensitivity analyses of the optimal solutions in response to the preference changes and to changes in risk management tools may provide information on farmers' preference dynamics and policy impact issues. Such sensitivity analyses will help us further explore the advantages of the GEU optimization model. Second, our results so far only focus on the two counties which are

geographically close to each other. It will be interesting to extend the research to other locations where there is more heterogeneity in farmers' preferences and yield. Third, other instruments such as revenue insurance products should be investigated to make additional contributions in policy analysis.

## References

- Atwood, J. A., J. M. Watts, and A. E. Bauquet. "An Examination of the Effects of Price Supports and Federal Crop Insurance upon the Economic Growth, Capital Structure, and Financial Survival of Wheat Growers in the Northern High Plains." *American Journal of Agricultural Economics* 78(1) (February 1996): 212-224.
- Brorsen, W.. "Optimal Hedge Ratios with Risk-neutral Producers and Nonlinear Borrowing Costs." *American Journal of Agricultural Economics* 77(1) (February 1995): 174-181.
- Chavaz, J., and M. T. Holt. "Economic Behavior Under Uncertainty: A Joint Analysis of Risk Preferences and Technology." *The Review of Economics and Statistics* 78(2) (May 1996): 329-335.
- Coble, K. H., R. G. Heifner, and M. Zuniga. "Implications of Crop Yield and Revenue Insurance for Producer Hedging." *Journal of Agricultural and Resource Economics* 25(2) (December 2000): 432-452.
- Crain, S. J., and J. H. Lee. "Volatility in Wheat Spot and Futures Markets, 1950-1993: Government Farm Programs, Seasonality, and Causality." *The Journal of Finance* 51(1) (March 1996): 325-343.
- Epstein, L. G. and S. E. Zin. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57(4) (July 1989): 937-969.
- "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis." *Journal of Political Economy* 99(2) (April 1991): 263-286.
- Hansen, L. P., and K. J. Singleton. "Stochastic Consumption, Risk Aversion, and the Temporal

- Behavior of Asset Returns.” *Journal of Political Economy* 91(2) (April 1983): 245-265.
- Howitt, R. E. et al. “Calibrated Stochastic Dynamic Models for Resource Management.” selected paper, World Congress of Environmental and Resource Economists – Monterey California, June 2002.
- Ke B., and H. H. Wang: “An Assessment of Risk Management Strategies for Grain Growers in the PNW”, *Agricultural Finance Review* 62(2) (Fall 2002): 117-133.
- Knapp K. C., and L. J. Olson: “Dynamic Resource Management: Intertemporal Substitution and Risk Aversion”, *American Journal of Agricultural Economics* 78(4) (November 1996): 1004-1014.
- Kreps, D. M., and E. L. Porteus. “Temporal Resolution of Uncertainty and Dynamic Choice of Theory.” *Econometrica* 46(1) (January 1978): 185-200.
- Lence, S. H.. “Using Consumption and Asset Return Data to Estimate Farmers’ Time Preferences and Risk Attitudes.” *American Journal of Agricultural Economics* 82(4) (November 2000): 934-947.
- Mahul, O.. “Hedging price risk in the presence of crop yield and revenue insurance.” *European Review of Agricultural Economics* 30(2) (June 2003): 217-239.
- Makki, S. S., and M. J. Miranda. “Self-insurance and the Utility of Standard Risk Management Contracts.” Selected paper, American Agricultural Economics Association Annual Meeting, Salt Lake City, Utah, August 1998.
- Mehra, R., and E. Prescott. “The Equity Premium: A Puzzle.” *Journal of Monetary Economics* 15(2) (March 1985): 145-161.
- Miller, J. C., B. J. Barnett, and K. H. Coble. “Analyzing Producer Preferences for Counter-Cyclical Government Payments.” Selected paper, American Agricultural

- Economics Association Annual Meeting, Chicago, Illinois, August 2001.
- Moss, C. B., and J. S. Shonkwiler. "Estimating Yield Distributions with a Stochastic Trend and Nonnormal Errors." *American Journal of Agricultural Economics* 75(4) (November 1993): 1056-1062.
- Myers, R. J.. "The Value of Ideal Contingency Markets in Agriculture." *American Journal of Agricultural Economics* 70(2) (May 1988): 254-267.
- Myers, R. J., and S. D. Hanson. "Optimal Dynamic Hedging in Unbiased Futures Markets." *American Journal of Agricultural Economics* 78(1) (February 1996): 13-20.
- Nyambane, G. G. et al. "Dynamic Risk Management under Credit Constraints." selected paper, NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, Missouri, April 22-23, 2002.
- Hinman, H. R., and R. Baldree. "Using Budgets for Profit Making Decisions (Winter Wheat-Dry Pea Example)." Paper presented at the Dryland Farm Management Series in Colfax, Washington, January 30, 2004.
- Saha, A., C. R. Shumway, and H. Talpaz. "Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility." *American Journal of Agricultural Economics* 76(2) (May 1994): 173-184.
- Vukina, T., and J. L. Anderson. "A State-Space Approach to Optimal Intertemporal Cross-Hedging." *American Journal of Agricultural Economics* 75(2) (May 1993): 416-424.
- Wang, H. H., S. D. Hanson, and J. R. Black. "Efficiency Costs of Subsidy Rules for Crop Insurance." *Journal of Agricultural and Resource Economics* 28(1) (April 2003): 116-137.



Wang, H. H., S. D. Hanson, R. J. Myers, and J. R. Black. "The Effects of Alternative Crop Insurance Designs on Farmer Participation and Welfare." *American Journal of Agricultural Economics* 80(4) (November 1998): 806-820.

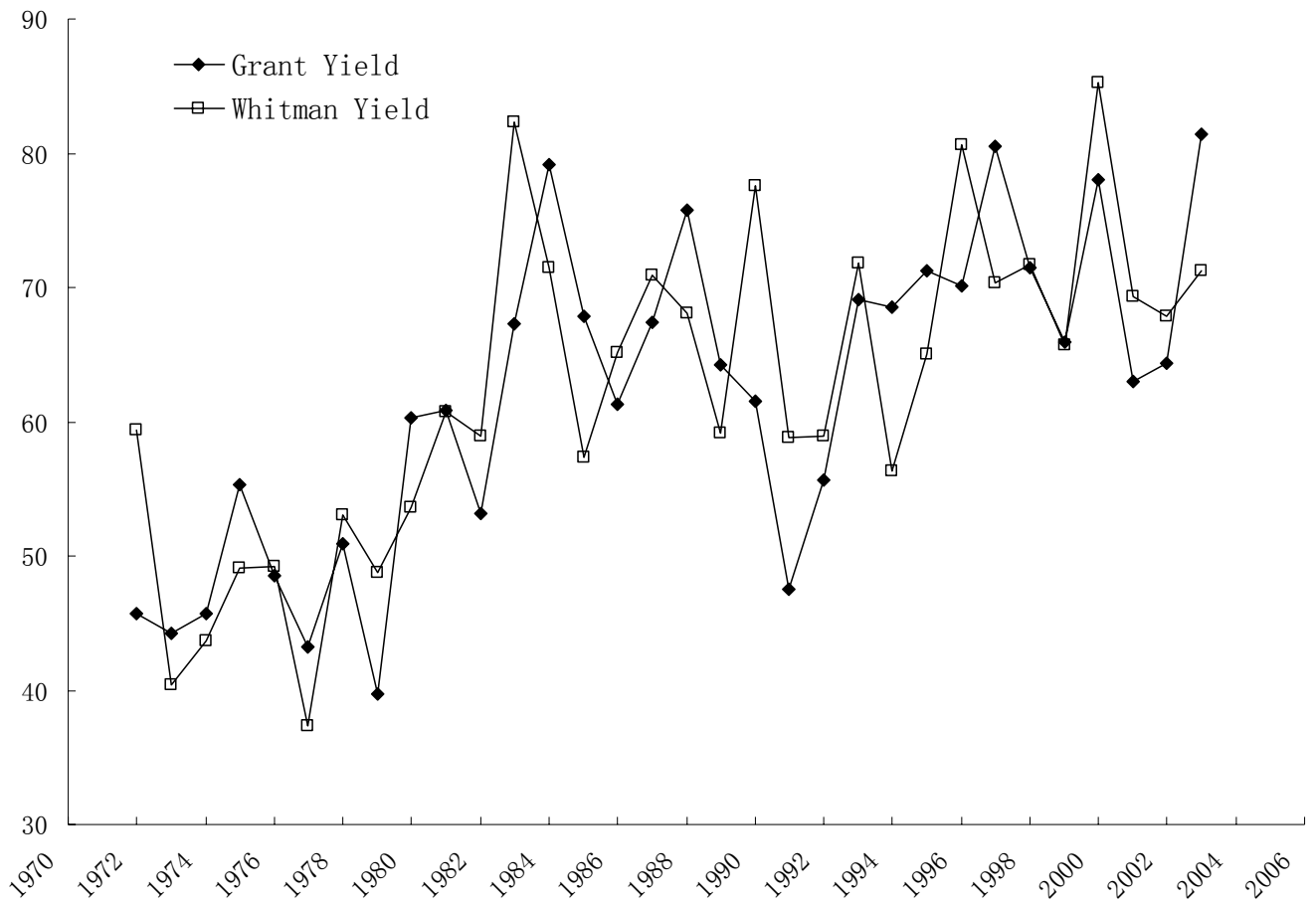
Wang, H. H., L. D. Makus, and X. Chen. "The Impact of US Commodity Programmes on Hedging in the Presence of Crop Insurance." *European Review of Agricultural Economics*, 31 (2004), in print.

Weil, P. "Nonexpected Utility in Macroeconomics." *Quarterly Journal of Economics* 105 (April 1990): 367-383.

Zuniga, M., K. H. Coble, and R. Heifner. "Evaluation of Hedging in the Presence of Crop Insurance and Government Loan Programs." Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, Missouri, April 2001.

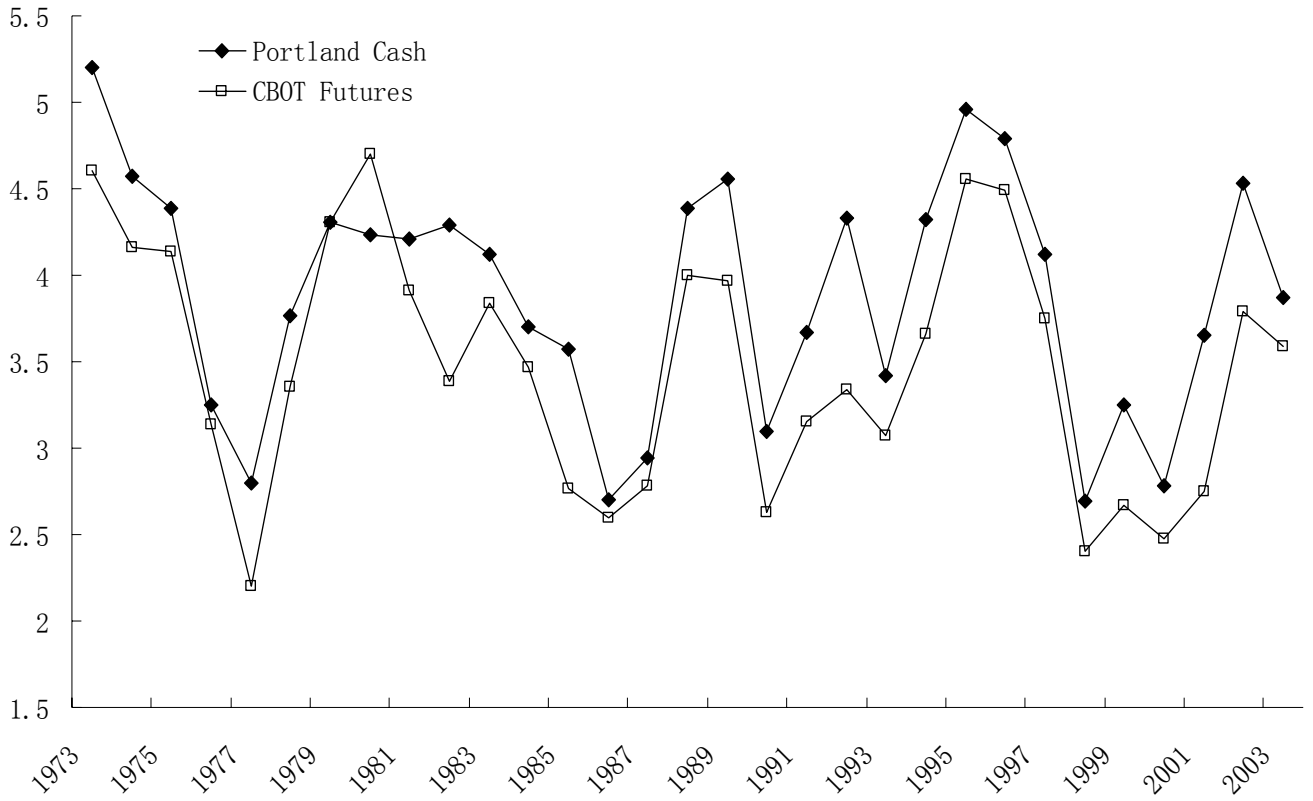
**Figure 1. Historical Soft White Wheat Yields in Whitman and Grant (1972-2003)**

Unit: Bushels/Acre



**Figure 2. Historical Wheat Cash and Futures Prices (1973-2003)**

Unit: Cents/Bushel



**Figure 3. Stochastic Trend Model Fitting for Grant Wheat Yield (1972-2003)**

**Predicted vs. Actual**

Unit: Bushel/Acre

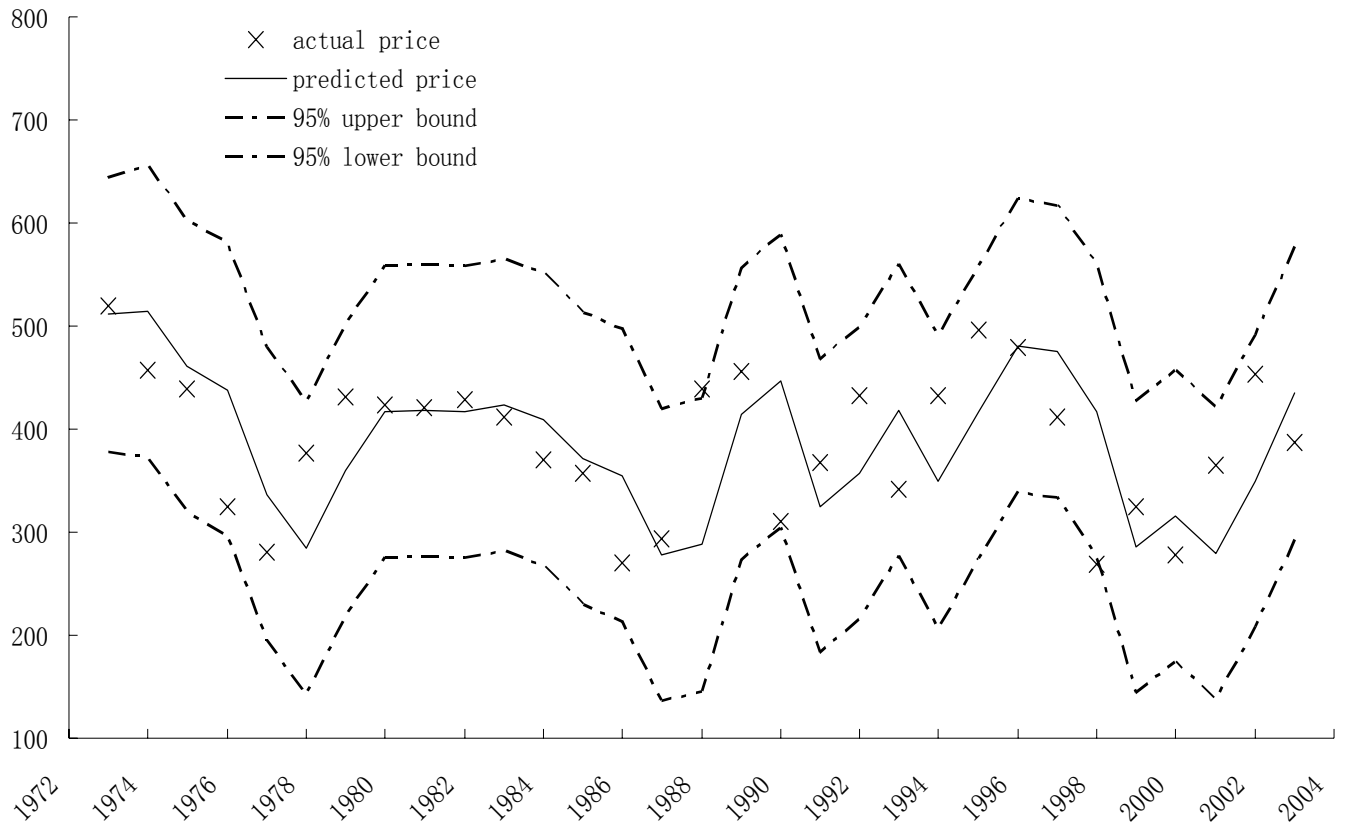


Note: The lower bound and upper bound are based on 95% confidence intervals.

**Figure 4. Stochastic Trend Model Fitting of Wheat Cash Prices**

**Predicted Vs. Actual 1973 to 2003**

Unit: Cents/Bushel



**Table 1. Stochastic Trend Estimation of Yield and Price Distributions**  
(Normal distribution)

| Parameter            | Whitman Yield  | Grant Yield    | Cash Price       | Futures Price    |
|----------------------|----------------|----------------|------------------|------------------|
| $\mu_0$              | 27.29** (3.63) | 44.22** (6.29) | 515.06** (72.91) | 463.89** (70.12) |
| $\beta_0$            | 0.73 (1.00)    | 0.94 (1.16)    | -3.92 (11.64)    | -3.40 (12.67)    |
| $\sigma_\varepsilon$ | 7.13** (0.63)  | 6.92** (1.46)  | 27.06 (33.23)    | 0.01 (0.46)      |
| $\sigma_\eta$        | 0.00 (0.15)    | 3.10* (2.04)   | 62.24** (25.56)  | 68.90** (8.75)   |
| $\sigma_\zeta$       | 0.00 (0.03)    | 0.00 (0.25)    | 0.00 (0.37)      | 0.00 (0.36)      |

Note: 1. Standard errors of the estimates are included in the parentheses.

2. “\*” denotes the estimate is statistically significant at 0.10 level, and “\*\*” denotes the significance at 0.05 level.

Table 2. Descriptive Statistics of the Simulation

| Statistics | 2004                                         | 2005   | 2006   | 2007   | 2008   | 2004                                       | 2005   | 2006   | 2007   | 2008   |
|------------|----------------------------------------------|--------|--------|--------|--------|--------------------------------------------|--------|--------|--------|--------|
|            | <i>Whitman Simulated Yield (bushel/acre)</i> |        |        |        |        | <i>Grant Simulated Yield (bushel/acre)</i> |        |        |        |        |
| Mean       | 75.28                                        | 75.93  | 76.77  | 77.36  | 78.24  | 75.19                                      | 76.27  | 76.30  | 77.34  | 78.02  |
| Std Dev.   | 7.26                                         | 7.22   | 7.28   | 7.06   | 7.23   | 7.49                                       | 8.15   | 8.36   | 9.46   | 9.65   |
| Skewness   | -0.01                                        | -0.03  | 0.02   | 0.07   | -0.04  | -0.08                                      | -0.02  | 0.03   | -0.05  | 0.02   |
| Kurtosis   | 0.24                                         | 0.14   | -0.03  | 0.07   | -0.005 | 0.08                                       | 0.26   | -0.09  | 0.16   | -0.4   |
|            | <i>Portland Cash Price (cents/bushel)</i>    |        |        |        |        | <i>CBOT Futures Price (cents/bushel)</i>   |        |        |        |        |
| Mean       | 392.68                                       | 386.16 | 382.32 | 379.39 | 376.59 | 356.02                                     | 350.67 | 349.39 | 345.95 | 343.92 |
| Std Dev.   | 66.42                                        | 91.02  | 106.55 | 121.22 | 133.68 | 68.18                                      | 95.80  | 114.89 | 128.62 | 143.83 |
| Skewness   | 0.02                                         | 0.02   | 0.06   | 0.10   | 0.06   | -0.04                                      | 0.02   | 0.10   | 0.07   | 0.05   |
| Kurtosis   | -0.05                                        | 0.06   | -0.06  | 0.20   | -0.12  | 0.03                                       | 0.01   | -0.20  | -0.26  | -0.31  |

**Table 3. Optimal Risk Management Portfolio**

| Alternative Model<br>Specifications                           | Hedge Ratio |      |      |      |      | Crop Ins.<br>Cov. Ratio |
|---------------------------------------------------------------|-------------|------|------|------|------|-------------------------|
|                                                               | $X0$        | $X1$ | $X2$ | $X3$ | $X4$ | $Z0-Z4$                 |
| <b><i>Whitman County</i></b>                                  |             |      |      |      |      |                         |
| GEU full<br>( $\alpha = -0.13, \beta = 0.89, \rho = 0.9493$ ) | 0.25        | 0.31 | 0.32 | 0.32 | 0.26 | 0.85                    |
| CES-EU<br>( $\alpha = \rho = -1, \beta = 0.89$ )              | 0.29        | 0.27 | 0.25 | 0.25 | 0.22 | 0.85                    |
| MR-EU<br>( $\alpha = \rho = 1, \beta = 0.89$ )                | 0           | 0    | 0    | 0    | 0    | 0.85                    |
| MA-EU<br>( $\alpha = -1, U(C) = -1/C, \beta = 0.89$ )         | 0.29        | 0.27 | 0.25 | 0.25 | 0.22 | 0.85                    |
| <b><i>Grant County</i></b>                                    |             |      |      |      |      |                         |
| GEU full<br>( $\alpha = -0.13, \beta = 0.89, \rho = 0.9493$ ) | 0.25        | 0.30 | 0.31 | 0.31 | 0.23 | 0.85                    |
| CES-EU<br>( $\alpha = \rho = -1, \beta = 0.89$ )              | 0.29        | 0.26 | 0.24 | 0.23 | 0.20 | 0.85                    |
| MR-EU<br>( $\alpha = \rho = 1, \beta = 0.89$ )                | 0           | 0    | 0    | 0    | 0    | 0.85                    |
| MA-EU<br>( $\alpha = -1, U(C) = -1/C, \beta = 0.89$ )         | 0.29        | 0.26 | 0.24 | 0.23 | 0.20 | 0.85                    |