IS INVERSE DEMAND PERVERSE?

PRESENTED AT WAEA ON JULY 8, 2005 IN SAN FRANCISCO

CARLO RUSSO, NAVIN YAVAPOLKUL, DAVID ZETLAND

Abstract. Our non-representative sample of 245 undergraduates had significantly lower scores on questions presented in the standard heterogeneous form (i.e., Direct Demand equation and Inverse Demand graph) than on questions presented in non-standard homogenous forms. This result, which holds for advanced students, highlights one reason why 95 percent of students in economics principles classes do not enter the major—economics can be *gratuitously* mathematical. We argue that the Inverse Demand standard hurts rather than helps economics when it is used in early courses, but that professors have no incentive to change their methods. We recommend that early classes use either no graphs or a homogenous combination of graph and equation. The "standard" should be introduced later, when benefits outweigh costs.

Most instructors teach the principles of economics (e.g., Economics 1) to undergraduates with a combination of intuition, algebra and graphs. Although the weight given to each component varies by instructor, some elements are universally accepted as "the standard." Inverse Demand, the inverted graphical presentation of a Direct Demand function, with the independent variable, price, on the vertical axis (an inversion of mathematical convention) is one of them.[1] See Figure 1.

Key words and phrases. Economics teaching and education. JEL: A22, B1. 2,400 words.
PhD Students, Department of Agricultural & Resource Economics, University of California, Davis. One Shields Avenue, Davis, CA 95616-8512. Carlo Russo is also affiliated with the Universitá degli Studi di Cassino, Dipartimento Economia e Territorio. Corresponding Author: David Zetland (`david@primal.ucdavis.edu`).
We thank Aslıhan Arslan, Derek Berwald, Chris Dawes, Jennifer Lee, Brendan Livingston, Matthew Pearson, and Teddy Wong for surveying students; the 283 students who answered surveys; and Byeong-il Ahn, Jennifer Keeling, Gorm Kipperberg, Joaquim Silvestre, Rich Sexton, Ed Taylor, an anonymous JEE referee, and session participants at the 2005 WAEA for their helpful comments. All errors are ours.
 This paper is based on an earlier working paper (Russo et al., 2004) that is both thorough and long-winded. Readers interested in the details of surveys, history of Inverse Demand, or the formal model of switching standards can download that paper from `http://www.agecon.ucdavis.edu/uploads/grad_papers/inversedemand.pdf`.

[1]We say a question-answer pair are *homogenous* if the question and answer have the same form, i.e., $P(Q)$ **or** $Q(P)$. Thus, a Direct (Inverse) graph mapped to/from a Direct (Inverse) equation is homogenous. (We also consider equation-equation pairs.) If the question is *heterogeneous*, the graph maps to/from an equation in another form, i.e. Direct (Inverse) graph to Inverse (Direct) equation.

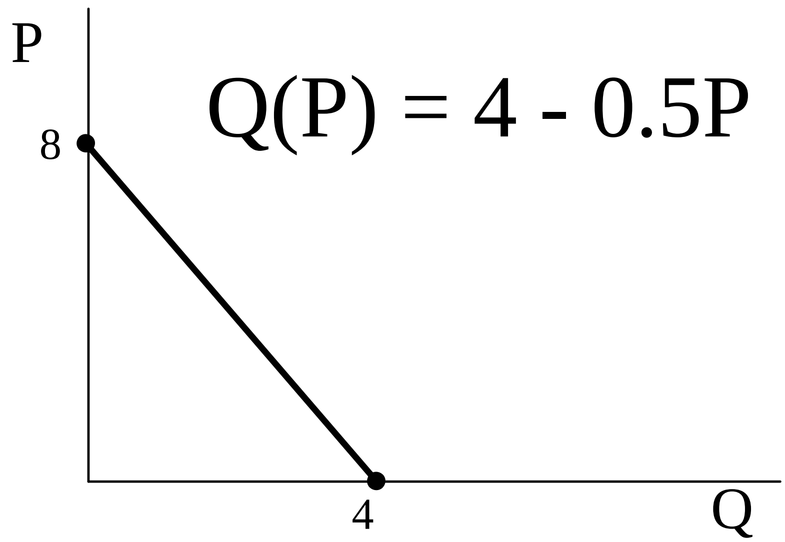


FIGURE 1. The typical Inverse Demand graph and Direct Demand equation. Price appears on the vertical axis in all economic figures to maintain consistency across various analytical goals. Since these applications are uncommon in early economics classes, students do not benefit from the costs of using this form. The algebraic presentation of quantity as a function of price is intuitive to the beginning student, who then expects to see the same relationship in the graph—and does not.

Inverse Demand may be one reason why 95 percent of the students who take economics principles graduate in another major (Hansen et al., 2001). We claim that Inverse Demand is the result of historical inertia (not of theoretical necessity or pedagogical efficiency) and question the effectiveness of this standard. In this paper, we evaluate if Inverse Demand makes it more difficult for students to understand and apply demand theory. We find that it does and suggest ways to keep Inverse Demand while improving economic teaching by putting Inverse Demand in context.

1. The Backstory on Marshall and Walras

When Marshall wrote *Principles of Economics*, he used the contemporary analytical presentation of $P(Q)$ and graphically represented the market with price on the vertical axis.[2] He kept to this notation, even in situations he knew to be

[2]Marshall (1920, p 90): "There may be even more violent changes than this in the price of thing which is not necessary, if it is perishable and the demand for it is inelastic: thus fish may be very dear one day, and sold for manure two or three days later."

$Q(P)$, trading logic for consistency, while solidifying the standard. Léon Walras developed an alternative standard, following on Cournot, of $Q(P)$. These "standards" reflected a deeper theoretical division. "The basic difference between Walras and Marshall, with regard to the market adjustment mechanism, is that Walras regarded price as the adjusting variable when markets are in disequilibrium whereas Marshall focused on quantity as the adjusting variable in the same circumstances" (Ekelund Jr. and Hebert, 1975, p 314). Mainstream economics intertwines both traditions, allowing for both $P(Q)$ and $Q(P)$. Graphically, however, economists have settled on the Marshallian presentation.

2. Empirics—Do Students Understand Inverse Demand?

Economists use Inverse Demand often and are comfortable with its idiosyncracies. We suspected that Inverse Demand was not so easy for beginners, in the same way that idiomatic English can be confusing to non-native speakers. We set out to see how professors and students perceive and use Inverse Demand.

2.1. **Survey of Professors.** Most of the 13 professors surveyed in a non-representative sample said that Inverse Demand is an unusual standard, that students ("especially the smart ones") have no problems with it, and that they (almost) always use the Inverse form for graphing—for the "usual network externality."

2.2. **Survey of Students.** We surveyed 283 undergraduate students taking economics classes at UC Davis.[3] We collected descriptive data on the students and asked eight multiple choice questions in three parts. (Appendix A is the survey.[4]) Part I asked them to look at a demand graph and choose the demand equation which matched it (graph → equation). Part II asked them to add two individual demand equations and choose the equation for aggregate demand (equation → equation). Part III asked them to add two individual demand equations and then choose the aggregate demand graph which matched the sum (equation → graph).

[3] We had 245 observations after eliminating surveys with unanswered test questions. Our data are not representative; we prefer to think of our sample as a case study to provoke further thought.
[4] The answers are 1A, 2C, 3B, 4B, 5B, 6C, 7A, 8B.

Number of Economics Classes	1	2 or 3	4+	Total
Characteristics				
Number of Students	64	79	102	245
Average GPA	3.02	3.10	3.19	3.12
% Students in Econ Major[8]	53	70	88	73
Percentage Correct Answers				
Part I (graph → equation)				
1: Inverse → Direct	50	58	73	62
2: Inverse → Inverse	64	79	94	82
3: Direct → Direct	64	91	84	81
Part II (equation → equation)				
4: (Direct → Direct)	91	95	99	96
5: (Inverse → Direct)	91	95	95	94
Part III (equation → graph)				
6: Direct → Inverse	41	58	61	55
7: Direct → Direct	59	60	74	66
8: Inverse → Inverse	47	55	69	59

TABLE 1. Survey Results. Sub-samples in three middle columns are cohorts who have taken the same number of economics classes.

Each part had questions and answers in Direct and/or Inverse form.[5] If the forms were the same (e.g., Direct (Inverse) graph to Direct (Inverse) equation), we considered the question to be "homogenous" in form. If the form was Inverse graph and Direct equation, we considered the question to be in "heterogeneous" form.[6] We hypothesized that students would perform worse with the heterogenous form.[7]

2.2.1. *Survey Results.* We found two major results in the data:[9]

(1) Our sample students do significantly worse when answering a question with graph and equation in heterogenous form compared to a question in homogenous form. Thus, the *status quo* presentation of Inverse Demand may be sub-optimal for students. See Table 2.

[5]We designed the survey by randomly using the inverse or direct form for a set of simple demand equations. The answer choices were randomly sorted from a set which included the correct answer, a random answer, and an answer which came from an "upside down" method—i.e., as if the student did not invert when they should have or did invert when they should not have.

[6]We skipped Inverse equation and Direct graph since this is rarely seen in the classroom.

[7]Since Inverse Demand is the standard, students should do better on questions in Inverse Demand form—unless it's a *really bad* standard.

[8]This self-reported statistic may not be representative and is conditional on enrollment in an economics class where the survey was administered. The numbers in Table 1 are not directly comparable to the 5 percent statistic reported by (Hansen et al., 2001); at our university, 7.1 percent of all undergraduates are in the economics or managerial economics major.

[9]We used asymptotic z-statistics to test the difference in the rate of success between pairs of questions within parts. See Appendix B for more detail.

Heterogeneous vs. Homogeneous (Direct)				
		% Inverse → Direct	% Direct → Direct	Test Stat
Part I	gph → eqn	62	81	4.82***
Part II	eqn → eqn	94	96	0.81
Part III	eqn → gph	55	66	2.51**
Heterogeneous vs. Homogeneous (Inverse)				
		% Inverse → Direct	% Inverse → Inverse	Test Stat
Part I	gph → eqn	62	82	4.94***
Part III	eqn → gph	55	59	0.91
Homogeneous (Direct) vs. Homogeneous (Inverse)				
		% Direct → Direct	% Inverse → Inverse	Test Stat
Part I	gph → eqn	81	82	0.12
Part III	eqn → gph	66	59	1.59

* p < 0.1, ** p < 0.05, *** p < 0.01

TABLE 2. **Total sample ($n = 245$).** Students have consistently and significantly higher success rates answering graph-equation questions in homogenous forms (either Direct-Direct or Inverse-Inverse) compared to heterogenous forms. There is no difference between the homogenous forms across the whole sample.

(2) There is a Learning Curve, but not the type we want. Students who have taken more economic classes generally perform better then those who have taken fewer classes, but Advanced students still perform significantly worse on questions in heterogenous form. See Figure 2, Table 3 and Table 4.

2.3. Interpretation.

- Students do not have difficulty with inverse form *per se* but the heterogeneous mix of graph and equation.
- A homogenous presentation of the material increases student success. The type of homogeneity does not usually matter.
- There is no inherent "math bias" of Direct-Direct over Inverse-Inverse for students used to seeing independent variables on the vertical axis.
- Advanced students work better with Inverse-Inverse than Direct-Direct, the likely result of learning that presentation form.

Part I (graph → equation)	Beginners	Advanced	Test Statistic
Inverse → Direct	50	73	3.01***
Direct → Direct	64	94	4.66***
Inverse → Inverse	64	84	2.85***
Part II (equation → equation)			
Direct → Direct	96	99	1.14
Inverse → Inverse	94	95	0.27
Part III (equation → graph)			
Inverse → Direct	41	61	2.56**
Direct → Direct	59	74	1.99**
Inverse → Inverse	47	69	2.84***

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3. **Learning Curve:** Advanced students performed significantly better than beginning students in graph → equation and equation → graph questions but not in the equation → equation questions. They still do poorly on heterogeneous forms. (See Table 4.)

Heterogeneous vs. Homogeneous (Direct)				
		% Inverse → Direct	% Direct → Direct	Test Stat
Part I	gph → eqn	73	84	2.06**
Part II	eqn → eqn	95	99	1.67*
Part III	eqn → gph	61	74	1.96**
Heterogeneous vs. Homogeneous (Inverse)				
		% Inverse → Direct	% Inverse → Inverse	Test Stat
Part I	gph → eqn	73	94	4.32***
Part III	eqn → gph	61	69	1.18
Homogeneous (Direct) vs. Homogeneous (Inverse)				
		% Direct → Direct	% Inverse → Inverse	Test Stat
Part I	gph → eqn	84	94	2.29**
Part III	eqn → gph	69	74	0.77

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4. **Advanced Students Still Don't Get It:** Students with more than four economics classes ($n = 102$) are still challenged by heterogenous forms. They do better with the Inverse-Inverse over the Direct-Direct homogenous form, probably because of familiarity.

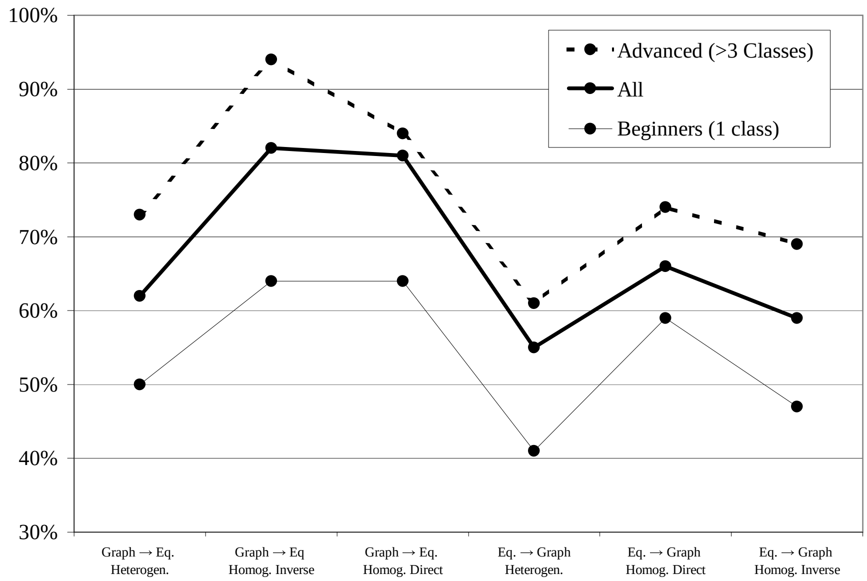


Figure 2. Performance by cohorts (defined by the number of economic classes taken) on selected questions.

3. Explaining the Status Quo

So—if our results are not specific to the data and indicate a larger pattern in economics instruction, why has this sub-optimal situation persisted? This was the exact question of our *Journal of Economic Education* reviewer:[10]

> If there was such an interest in change because of some significant pedagogical problem then some enterprizing textbook author would have exploited the problem and developed a book using direct demand graphs. That event has not happened.

Unfortunately, this critique misses our point—that students are the ones who lose from a confusing standard. Since professors use Inverse Demand often, when they (naturally) use it for teaching, they may not see that students have a harder time with Inverse Demand. Professors who do not compare performance under homogeneous vs. heterogeneous presentations (as we did) may only see that student performance with Inverse Demand improves over time (which we also found), not that it is always worse than the alternative.

[10]The paper was rejected.

Students are helpless. They know no alternative to Inverse Demand—only that it is the standard upon which their grade depends.[11] Many quit economics when they fail to see the "intuition."

3.1. **A Model, in Brief.** In our earlier paper (Russo et al., 2004), we created an elaborate model to show why a professor has no incentive to switch to Direct Demand from Inverse Demand. A Professor who switches receives students who care more about grades than economics.[12] The quality of students falls, the number of students rises and the workload increases—three bads. Since the system does not award popularity (for good reason), the Professor gets little compensation for these bads. Thus, there is no incentive to switch to Direct Demand. We can summarize this result as:

	Professor	Student
Benefit	More students (?)	Better understanding (or grades)
Cost	Effort to form new curriculum. Cognative dissonance in departing from professional norms.	Knowledge incompatible with standards for further progress.

On further thought, we discovered that a smaller adjustment can help both students and professors, i.e., putting Inverse Demand in context.

4. Suggestions for Change

To work with, not against, Inverse Demand, we suggest that professors:

(1) Start without graphs so that $Q(P)$ algebra is clear.
(2) Use Direct Demand graphs to show how supply and demand equate. The gains from trade are now measured from the horizontal axis but easy to understand.
(3) Now introduce cost and utility functions that are optimized to give marginal cost (supply) and marginal benefit (demand) functions and curves that have quantity as independent variables. Map them to the vertical axis.

[11]Our anecdotal (i.e., without statistical significance) impression from our research on this topic is that a number of students only memorize the Inverse Demand pattern; they do not learn the mathematical or economic principles or retain the concept beyond their intermediate microeconomics class.

[12]We assume that students who care about economics stay with the standard because their future depends on it. Grade-seekers don't care since they are leaving anyway.

(4) Use both algebraic forms (Direct Demand and Inverse Demand) as appropriate. This can help students understand the pros and cons of the inconsistency.

When they have mastered the basics of market equilibrium and are ready to derive profit- and utility-maximizing supply and demand curves from utility and cost functions, the place of the Inverse Demand graph within the whole will make sense.

5. Conclusion

Inverse Demand is a sub-optimal standard for students if test performance is correlated with student comprehension. This does not seem to be the case with Direct Demand. The problem arises when the graphical form is inconsistent with the equation form (heterogeneous). Rather than throw out the baby with the bathwater—Inverse Demand is very useful in other contexts—we suggest that Inverse Demand be introduced later in the student's instruction and that either no graphs or homogenous graphs be used for early instruction—especially in principles classes.

We hope that our results provoke thought and discussion among economics teachers. If notation hinders students from understanding principles, economics loses.

References

Ekelund Jr., R. B. and Hebert, R. F. (1975). *A History of Economic Theory and Method*. McGraw-Hill Book Company.

Frey, B. S. (2000). Does Economics Have an Effect? Towards an Economics of Economics. *Working Paper N^o. 36*. Retrieved Feb. 2004, from `www.iew.unizh.ch/wp/iewwp036.pdf`.

Hansen, W. L., Salemi, M. K., and Siegfried, J. J. (2001). Creating a Standards-Based Economics Principles Course. *Working Paper 01-W05*.

Marshall, A. (1920). *Principles of Economics*. MacMillan and Company, Ltd., London (UK), 8^{th}, 1952 reprint edition.

Russo, C., Yavapolkul, N., and Zetland, D. (2004). Is Inverse Demand Perverse? *Working Paper*. Available at `http://www.agecon.ucdavis.edu/uploads/grad_papers/inversedemand.pdf`.

Appendix A: Economic Teaching Methods Evaluation

Instructions

This evaluation is voluntary and confidential. Do NOT write any identifying information on this page! Please try to answer all questions in the time allowed.

About You

Major: Are you now declared (or intending to declare) economics or managerial economics as your major? Please circle one . . . Yes No

Classes: Please circle all classes that you have taken or are currently taking:

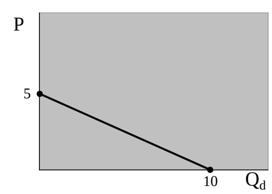
ARE100A	ARE100B	ARE155	ARE156	ECN 1A	ECN 1B
ECN 100	ECN 101	Math16A	Math16B	Math21A	Math21B

GPA: What's your cumulative GPA? ________

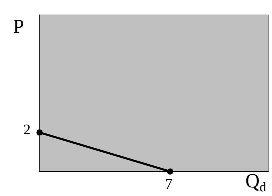
Questions on Demand

Part I: Choose the multiple choice answer corresponding to the given graph (for the given axis combination)

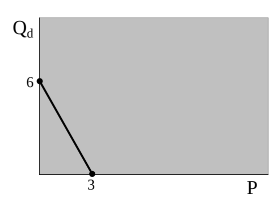
(1) Choose the answer which matches the graph to the right (Q_d on x-axis).
  (a) $Q_d = 10 - 2p$
  (b) $Q_d = 5 - \frac{1}{2}p$
  (c) $Q_d = 5 - 2p$



(2) Choose the answer which matches the graph to the right (Q_d on x-axis).
  (a) $p = 7 - \frac{7}{2}Q_d$
  (b) $p = 7 - \frac{2}{7}Q_d$
  (c) $p = 2 - \frac{2}{7}Q_d$



(3) Choose the answer which matches the graph to the right (P on x-axis).
  (a) $Q_d = 3 - \frac{1}{2}p$
  (b) $Q_d = 6 - 2p$
  (c) $Q_d = 6 - \frac{1}{2}p$



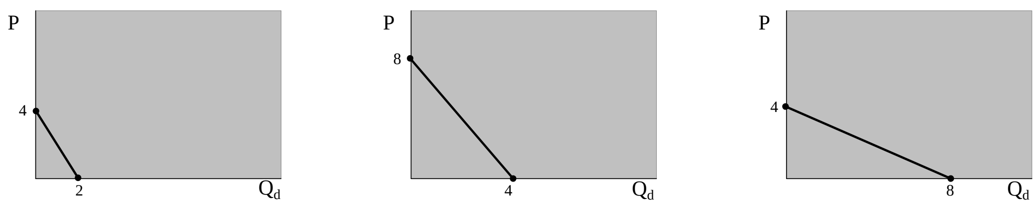
Part II Given the following individual demand equations (q_1, q_2) and market price (p), circle the correct value of aggregate demand, Q_d (where $Q_d = q_1 + q_2$).

(4) If $q_1 = 3 - 2p$, $q_2 = 3 - 2p$ and $p = 1$, then:
 (a) $Q_d = 4$
 (b) $Q_d = 2$
 (c) $Q_d = 3$

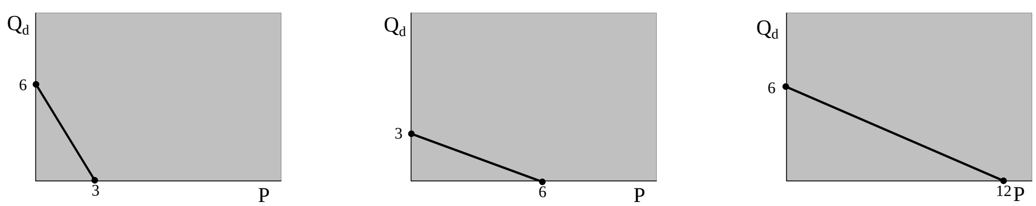
(5) If $p = 4 - 3q_1$, $p = 4 - 3q_2$ and $p = 1$, then:
 (a) $Q_d = 4\frac{1}{2}$
 (b) $Q_d = 2$
 (c) $Q_d = 7\frac{1}{3}$

Part III Circle the graph representing the aggregate demand function (for the given axis combination)

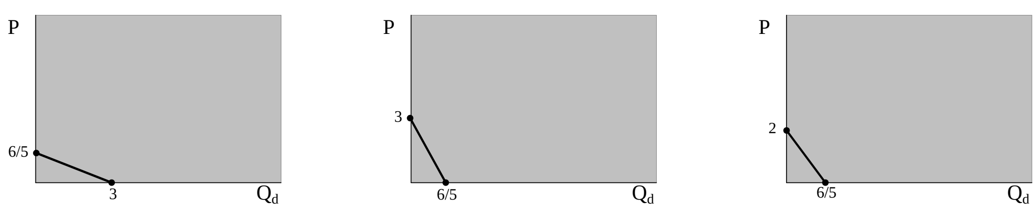
(6) If $q_1 = 4 - p$ and $q_2 = 4 - p$, then aggregate demand looks like (Q_d on x-axis):



(7) If $q_1 = 3 - p$ and $q_2 = 3 - p$, then aggregate demand looks like (P on x-axis):



(8) If $p = 3 - 5q_1$ and $p = 3 - 5q_2$, then aggregate demand looks like (Q_d on x-axis):



Appendix B: Empirics

Methodology. We assume that the answer for each question is distributed as multinomial with three possible outcomes per trial (student). (y_i, p_i) describes the outcomes and probabilities for $i = 1, 2, 3$, where 1 is the correct answer. A maximum likelihood estimator for p_k, given observed (y_1, y_2, y_3) is simply the number of students who answer the kth choice divided by the total number of students. Thus, we are implicitly assuming the same *a priori* p_i's across all students.

We constructed an asymptotic z-statistic to test the difference in the rate of success between pairs of questions within parts for significance. We found the z-statistic with

$$Z_{\frac{a}{2}} = \frac{\hat{p}_1^s - \hat{p}_1^t}{\sqrt{\left(\frac{\hat{p}_1^s(1-\hat{p}_1^s)}{n_s}\right) + \left(\frac{\hat{p}_1^t(1-\hat{p}_1^t)}{n_t}\right)}} \text{ which is } \overset{a}{\sim} N(0,1)$$

$$\text{and where } \hat{p}_1^s = \frac{y_1^s}{n_s} \text{ and } \hat{p}_1^t = \frac{y_1^t}{n_t}, \text{ for } s \neq t.$$

Note that $\hat{p}_1^s$ and $\hat{p}_1^t$ indicate an estimated parameter for the probability of choosing the right answer (choice 1) in two different questions (e.g. Direct-Direct and Direct-Inverse). This allows us to test hypotheses on student performance on various types of questions.