# Environmental Regulation with Innovation and Learning: Rules versus Discretion

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# Environmental Regulation with Innovation and Learning: Rules versus Discretion \*

#### Abstract

We analyze a model of environmental regulation with learning about environmental damages and endogenous choice of emissions abatement technology by a polluting firm. We compare environmental policy under discretion, in which policy is updated upon learning new information, versus under rules, in which policy is not updated. When investment in abatement technology is made prior to the resolution of uncertainty, neither discretion nor rules with either taxes or standards achieve an efficient solution. When there is little uncertainty, rules are superior to discretion because discretionary policy gives the firm an incentive to distort investment in order to influence future regulation. However, when uncertainty is large, discretion is superior to rules because it allows regulation to incorporate new information. Under discretionary policy, taxes are superior to standards regardless of the relative slopes of marginal costs and marginal damages.

JEL Code: H23, Q2.

Key words: environmental regulation, emissions taxes and standards, rules versus discretion, technology adoption and innovation.

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## 1 Introduction

Virtually without exception, decisions about environmental policy are made without complete information about either the benefits or the costs of environmental improvement. Setting environmental policy under uncertainty in a static setting has been the subject of a fairly large literature in economics beginning with Weitzman (1974). (See Lewis 1996 for an excellent summary.) Setting environmental policy, however, is not a static proposition. Results of new scientific studies can lead to updated beliefs on how much damage is caused by emissions. New technologies or investment in new plant and equipment may make pollution abatement easier and cheaper to accomplish. Such new information should lead a welfare-maximizing regulator to adjust environmental policy. In fact, environmental regulations are periodically adjusted based on updated understanding or new circumstances. For example, the National Ambient Air Quality Standards for ozone and particulate matter were tightened in 1997. EPA stated that "... many important new studies have been published which show that breathing particulate matter at concentrations allowed by the current primary standard can likely cause significant health effects —including premature death and an increase in respiratory illness."<sup>1</sup> More recently, the standards on arsenic in drinking water were lowered from 50 parts per billion to 10 parts per billion. Yet knowing that regulations may be adjusted may give regulated firms scope to adjust their behavior in order to try to influence future regulation.

In this paper, we analyze environment regulation with learning about environmental damages and endogenous choice of emissions abatement technology by a polluting firm. We model the order of moves in a game between the regulator and the firm based on the ease or speed with which a variable or decision can change. We assume that the most difficult decision to change in a short period of time is the form of the regulatory regime. The regulatory

<sup>&</sup>lt;sup>1</sup>Cited from http://www.epa.gov/oar/oaqps/ozpmbro/partmat.htm.

regime is typically based on environmental statutes or administrative procedures that require concerted effort to change. In our model, the regulatory regime determines whether regulation occurs via emissions taxes or emissions standards. The regulatory regime also specifies whether regulation is fixed (rules), or may change based on new information (discretion). Given the regulatory regime, the firm chooses investment in technology where greater investment results in lower (expected) abatement costs. After the investment decision, uncertainty about abatement costs and environmental damages is resolved. With discretion, the particular level of the tax or standard is then chosen. However, under rules, regulation is fixed in the first stage. Finally the firm chooses its abatement level and payoffs are realized.

We consider two variants of the model: one with technology adoption and the other with technology innovation by a polluting firm. In section 2, we analyze an adoption model in which the firm chooses a technology from a menu of available existing technologies. Larger investment leads to lower abatement costs with certainty. In section 3, the firm chooses its expenditure on research and development for technology innovation where greater investment results in a larger probability of finding a new technology with lower abatement costs. The major difference between the innovation model and the adoption model is the stochastic response of cost to investment.

The main question we investigate in this paper is whether it is better for a regulator to commit to an emissions policy prior to learning about environmental damages and technology choice by firms (rules), or whether it is better to adjust policy after learning about environmental damages and technology choice (discretion). In both variants (adoption and innovation), we show that the regulator cannot achieve the first-best solution with taxes or standards under either rules or discretion. Rules are not first best because regulation may not reflect actual benefits or costs of abatement after technology choice and uncertainty is resolved. As in Kydland and Prescott (1977), discretion is not first best because of the strategic nature of the game. The investment decision of the firm will be distorted in order to influence regulation. Ideally, a regulator would like to make regulation conditional on the resolution of uncertainty but not have regulation conditional on investment. This outcome is not possible because investment occurs prior to resolution of uncertainty.

We also analyze whether taxes or standards are preferable. With rules, taxes and standards yield exactly the same solution. The regulator sets regulation such that expected marginal benefits of abatement equal expected marginal cost (post investment). This can be accomplished with either taxes or standards. On the other hand, with a discretionary policy, taxes and standards yield different outcomes. Under an emissions standard, discretionary policy results in a lower incentive to invest because lower marginal emissions costs cause the regulator to tighten the standard. Under an emissions tax, discretionary policy increases the incentive to invest because lower marginal emissions costs cause the regulator to set a lower tax rate (Kennedy and Laplante 1999, Karp and Zhang 2001, Moledina et al. 2003). In section 4, we compare expected social costs (abatement costs plus pollution damages) under rules, discretionary taxes and discretionary standards. When uncertainty about damages is relatively small, rules are preferred to discretion because avoiding distortion of investment incentives is more important than adjusting policy in light of new information. On the other hand, with relatively large uncertainty about damages, discretion is preferred to rules. Under discretion, we find that taxes are preferred to standards for a model with quadratic costs and benefits regardless of the slopes of marginal cost and marginal benefit. This result contrasts with Weitzman (1974) where taxes are preferred to standards if and only if the absolute value of the slope of marginal benefits is smaller than the absolute value of the slope of marginal cost.

These results have direct implications for the policy debate on the effectiveness of technology forcing standards. Technology forcing standards are set at levels that cannot be met by the regulated firms with current technology. The idea behind setting strict standards is to stimulate research and development and force technological innovation. Technology forcing standards have been used in North America and Europe to regulate emissions of air pollutants. For example, the U.S. Clean Air Act required a 90 percent reduction in emissions when there were few means available to achieve the emissions reduction goal (Leone 1998). Our results show that committing to standards (rules) when there is large uncertainty about costs can lead to large expected losses. In such cases, discretionary policy is preferable, and with discretion, taxes are preferred to standards (at least for quadratic costs and benefits).

There is a growing literature on dynamic environmental regulation, much of it inspired by interest in climate change policy. Kolstad (1996) characterized optimal regulation with learning about damages for a stock pollutant. Learning can delay the timing of irreversible investment in abatement technology. Other papers that analyze learning and irreversible investment in dynamic environmental contexts include Ulph and Ulph (1997), Kelly and Kolstad (1999), Pindyck (2000) and Saphores (2002). These papers characterize optimal regulation; they do not consider the strategic aspects of the game between the regulator and the firm. Several other papers consider the problem of asymmetric information about abatement cost in a dynamic setting (Benford 1998, Hoel and Karp 2001, Karp and Zhang 2001, 2002, Moledina et al. 2003, Newell and Pizer 1998). Of these papers, the three most similar to ours are Karp and Zhang (2001, 2002) and Moledina et al. (2003). The two papers by Karp and Zhang (2001, 2002) analyze a model with stock pollution and nonstrategic firms. Karp and Zhang (2001) analyze a model with investment while Karp and Zhang (2002) analyze a model with learning about damages. In the model with learning, they find that the relative efficiency of taxes over standards increases as the regulator has more opportunities for learning. Moledina et al. (2003) analyze a model with strategic firms that adjust their behavior to alter future regulation, but assume a naïve regulator. None of these papers analyze a game in which both regulators and regulated firm(s) are strategic.

The other strand of relevant literature analyzes technological change and environmental regulation (see Jaffe et al. for a recent survey). Much of this literature analyzes incentives to adopt new technology when regulation is fixed, as in our model with rules, and there is no uncertainty (see, for example, Milliman and Prince 1989). In a paper closer in spirit to our paper, Kennedy and Laplante (1999) analyze a model in which regulation changes in response to technology adoption decisions of firms. They consider a case with strategic firms and show that firms over-invest when regulated via taxes and under-invest when regulated via tradable emissions permits. Unlike our model, there is no uncertainty about either damages or costs, and they only consider discretionary policy rather than compare rules versus discretion.

In Section 2, we describe a game with endogenous technology adoption. We define the alternative policy schemes that we consider—rules and discretion—and define the corresponding subgame perfect equilibria. Then we characterize the welfare consequence of each scheme when the policy instrument consists of emissions taxes and emissions standards. Section 3 introduces an alternative model with endogenous technology innovation. Section 4 compares the expected total cost under rules and discretion for taxes and standards using numerical simulations. We analyze both the adoption model and the innovation model in the simulations. Section 5 contains concluding remarks and comments on potential future research.

# 2 A model with technology adoption

## 2.1 Model Environment

This subsection describes a game-theoretic model of pollution regulation with endogenous technology adoption involving a regulator and a single polluting firm. At the outset of the game, the regulator chooses the policy scheme to be employed. We consider two types of policy schemes: 1) discretion, in which the regulator may update policy based on new conditions or information, and 2) rules, in which policy chosen at the outset is fixed and

cannot be updated. For each scheme we consider two alternative policy instruments: simple (linear) emissions taxes and emissions standards.

In the next stage of the game, the firm chooses investment in adoption of emissions abatement capital. Let e represent the level of emissions by the firm and let k represent investment. Let r represent the unit cost of investment. The firm's emissions abatement cost is given by C(e, k). We assume that the emissions abatement cost function is decreasing in emissions and abatement investment ( $C_e < 0, C_k < 0$ ), convex ( $C_{ee} > 0, C_{kk} > 0, C_{ee}C_{kk} - C_{ek}^2 \ge 0$ ) and twice continuously differentiable. We also assume that marginal abatement cost,  $-C_e$ , is decreasing in investment,  $C_{ek} > 0$ .<sup>2</sup>

Emissions of pollution by the firm cause damages, which are external to the firm. Initially there is uncertainty about the damage function. Let S represent the set of possible states and let D(e; s) represent damages caused by emissions in state  $s \in S$ . Let  $\pi(s)$  be the probability that state s occurs. Uncertainty about which state will occur is resolved after the firm has chosen investment. We assume  $D_e(\cdot; s) > 0$ ,  $D_{ee}(\cdot; s) > 0$  for all  $s \in S$ . We also assume that  $D_e(e; s) > D_e(e; s')$  for all  $e \ge 0$  for some states  $s, s' \in S$  with  $\pi(s), \pi(s') > 0$ .

After uncertainty about damages is resolved, the regulator sets the tax or standard if they are in a discretionary policy regime (otherwise taxes and standards are fixed and cannot be changed). The firm then chooses emissions. Finally payoffs to the firm and the regulator are realized. Payoffs to the firm are:

$$-rk - \sum_{s \in S} \pi(s)C(e(s), k)$$
 under standards, and  
 $-rk - \sum_{s \in S} \pi(s)[C(e(s), k) + \tau(s)e(s)]$  under taxes

where  $\tau(s)$  is the per unit tax on emissions in state s. (Under rules, the tax rate  $\tau$  is the same across the states.) The regulator is assumed to care about minimizing the total cost

 $<sup>{}^{2}</sup>C_{i}$  is the first-order partial derivative of C with respect to  $i \in \{e, k\}$ ).  $C_{ij}$  is the second-order derivative of C with respect to  $i, j \in \{e, k\}$ ).

of pollution (abatement cost plus damages):

$$-rk - \sum_{s \in S} \pi(s) [D(e(s);s) + C(e(s),k)].$$

The complete order of moves of the games is summarized in figure 1. In figure 1a, we show the sequence of moves for the discretionary policy game. Figure 1b shows the sequence of moves for the rules game. The difference between discretion and rules is that the tax or standard is selected in the initial move in the rules game, but is chosen after investment and uncertainty is resolved in the discretionary game.

### 2.2 Emissions taxes

### 2.2.1 Equilibrium of the tax subgame under rules

Given a tax  $\tau$  on emissions, the firm solves

$$\min_{e,k\ge 0} rk + C(e,k) + \tau e.$$

The necessary and sufficient conditions for an interior solution are

$$r + C_k(e, k) = 0,$$
$$C_e(e, k) + \tau = 0.$$

Denote the solution by  $e(\tau), k(\tau)$ . Given  $e(\tau), k(\tau)$ , the regulator solves

$$\min_{\tau \ge 0} rk(\tau) + \sum_{s \in S} \pi(s) [C(e(\tau), k(\tau)) + D(e(\tau); s)]$$

A subgame perfect equilibrium of tax subgame under rules is given by a strategy profile  $(\tau^{RT}, (k^{RT}(\tau), (e^{RT}(\tau))_{\tau \ge 0}))$  that solves the above optimization problems by the firm and the regulator. (Superscript *RT* denotes the rules tax scheme.)

#### 2.2.2 Equilibrium of the discretionary-tax subgame

With discretionary taxes, the regulator chooses a state- and investment-dependent tax plan. Given investment  $k \ge 0$ , state  $s \in S$  and tax  $\tau$ , the firm chooses the level of emissions to solve

$$\min_{e>0} C(e,k) + \tau e$$

The necessary and sufficient condition for an interior solution is

$$C_e(e,k) + \tau = 0.$$

Denote the solution by  $e(k, \tau)$ . Given the firm's emissions plan (as functions of taxes and investment)  $\{e(k, \tau)\}_{s \in S}$ , the regulator solves

$$\min_{\tau \ge 0} C(e(k,\tau),k) + D(e(k,\tau);s)$$

for all  $s \in S$  given investment k. Denote the solution by  $\{\tau(k,s)\}_{k\geq 0,s\in S}$ .

Given the state- and investment-contingent tax schedule  $\{\tau(k,s)\}_{k\geq 0,s\in S}$ , the firm solves

$$\min_{k\geq 0} rk + \sum_{s\in S} \pi(s) [C(e(\tau(k,s)),k) + \tau(k,s)e(\tau(k,s))].$$

A subgame perfect equilibrium of a discretionary-tax subgame is given by a strategy profile  $(\{(\tau^{DT}(k,s))_{k\geq 0}\}_{s\in S}, (k^{DT}, \{(e^{DT}(\tau,s))_{\tau\geq 0}\}_{s\in S}))$  that solves the above optimization problems by the firm and the regulator. (Superscript DT denotes the discretionary tax scheme.)

### 2.3 Welfare properties of emissions taxes

Now we compare the alternative policy schemes—discretion and rules—in terms of efficiency. First we characterize the optimal (socially least cost) investment/emissions plan. Then we examine whether taxes, under either rules or discretion, can achieve efficiency. The optimal investment/emissions plan  $(k^*, \{e^*(s)\}_{s \in S})$  is given by a solution to

$$\begin{split} \min_{\substack{k,\{e(s)\}_{s\in S}\\\text{ s.t. }}} & rk + \sum_{s\in S} \pi(s) [C(e(s),k) + D(e(s);s)]\\ \text{ s.t. } & k \geq 0 \text{ and } e(s) \geq 0 \text{ for all } s \in S. \end{split}$$

Given the convexity of the functions, the following first order conditions are necessary and sufficient for an interior solution:

$$C_e(e^*(s), k^*) + D_e(e^*(s)) = 0 \text{ for all } s \in S,$$
 (1)

$$r + \sum_{s \in S} \pi(s) C_k(e^*(s), k^*) = 0.$$
(2)

Throughout the paper, we assume that the optimal solution is interior.

Proposition 1 states that the regulator can implement the optimal investment/emissions plan if the regulator can choose taxes that are contingent only on the realized state.

**Proposition 1** A state-contingent and investment-independent tax plan  $\{\tau(s)\}_{s\in S}$  achieves the socially minimum cost if  $\tau(s) \equiv D_e(e^*(s); s)$  for all  $s \in S$  where  $e^*(s)$  denotes the optimal emissions in state s.

(See Appendix A for the proof.) A state-contingent, investment-independent tax scheme allows taxes to be adjusted to reflect actual conditions allowing marginal abatement costs to equal marginal damages. However, since taxes are not a function of investment, there is no scope for the firm to manipulate the tax through its investment decision. The optimal policy is not available to the regulator in the game because the firm chooses investment prior to the realization of the state. Hence, if the regulator minimizes the total cost after resolution of damage uncertainty, then the regulator needs to specify the tax rate depending on the firm's technology choice. Once the regulator makes the taxes state-dependent, a time-consistent regulator cannot make the taxes investment-independent. We now show that the two policy schemes—discretion and rules—cannot achieve efficiency. As a first step to prove that discretionary policy will be inefficient, we use the following lemma.

#### **Lemma 1** The equilibrium discretionary tax rates are decreasing functions of k.

(See Appendix A for the proof.) The lemma shows that the regulator will choose a lower tax rate if higher investment is observed, Therefore, the firm will have an incentive to invest in order to manipulate the regulator into setting a lower tax. This is the source of inefficiency in discretionary policies. Using this lemma, we show that discretionary tax policies lead to suboptimal technology adoption and emissions choices. In addition, under the assumption that the firm's cost function, given how the regulator adjusts taxes, is convex in investment, a discretionary tax scheme will result in over-investment relative to the optimal investment (Proposition 2).

**Proposition 2** In equilibrium, the discretionary tax scheme does not achieve the efficient solution (socially minimum cost). Furthermore, if the firm's objective function is convex in investment given that taxes depend on investment, then the equilibrium investment in a discretionary-tax subgame  $k^{DT}$  is larger than the optimal investment  $k^*$ .

(See Appendix A for the proof.) The firm's investment optimization problem under discretionary tax policies, where the firm takes into account the effect of its investment on the tax, is not necessarily convex in investment even if the functions C and  $\{D(\cdot; s)\}_{s \in S}$  are convex. As discussed in Appendix B, the convexity assumption holds if the functions C and D are second-order polynomials of emissions and investment.

The following proposition states that a tax rule also fails to achieve the efficient solution as long as S is non-degenerate.

**Proposition 3** The equilibrium tax rate under rules does not achieve the socially minimum cost.

(See Appendix A for the proof.)

### 2.4 Emissions standards

Here we describe the subgame perfect equilibria for the emissions-standard subgames. With emissions standard  $q(s) \ge 0$  in state s, the firm is restricted to choose emissions e(s) so that  $e(s) \le q(s)$ . Alternatively, one could assume that e(s) can exceed q(s) but that this would invoke a large fine such that the firm would never find it optimal to choose e(s) > q(s).

#### 2.4.1 Equilibrium of the standard subgame under rules

Given a standard q on emissions, the firm solves

$$\min_{e,k \ge 0} \quad rk + C(e,k)$$
  
s.t.  $0 \le e \le q.$ 

Given the emissions abatement cost is decreasing in emissions, the firm will choose e = q. Then the necessary conditions for an interior solution are

$$r + C_k(q, k) = 0,$$
$$e = q.$$

Denote the solution by q, k(q). Given that the firm's choice q and k(q), the regulator solves

$$\min_{q \ge 0} rk(q) + \sum_{s \in S} \pi(s) [C(q, k(q)) + D(q; s)]$$

A subgame perfect equilibrium of a standard subgame under rules is given by a strategy profile  $(q^{RS}, (k^{RS}(q), e^{RS}(q))_{q\geq 0})$  that solves the above optimization problems by the regulator and the firm. (Superscript RS denotes the rules standard scheme.)

#### 2.4.2 Equilibrium of the discretionary-standard subgame

Given investment  $k \ge 0$  and state  $s \in S$ , under standard q(k, s) the firm chooses the level of emissions e(k, s) = q(k, s). Given the firm's emissions plan, the regulator solves

$$\min_{q>0} C(q,k) + D(q;s)$$

for all  $s \in S$  given investment k. Denote the solution by  $\{q(k,s)\}_{k\geq 0,s\in S}$ .

Given a state- and investment-contingent standard plan  $\{q(k,s)\}_{k\geq 0,s\in S}$ , the firm solves

$$\min_{k \ge 0} rk + \sum_{s \in S} \pi(s) [C(q(k,s),k)]$$

A subgame perfect equilibrium of a discretionary-standard subgame is given by a strategy profile  $(\{(q^{DS}(k,s))_{k\geq 0}\}_{s\in S}, (k^{DS}, \{(e^{DS}(q,s))_{q\geq 0}\}_{s\in S}))$  that solves the above optimization problems by the regulator and the firm. (Superscript *DS* denotes the discretionary standard scheme.)

### 2.5 Welfare properties of emissions standards

Here we consider the welfare properties of emissions standards under discretion and rules.

Fact 1 A state-contingent standard scheme achieves the socially minimum cost if, for all  $s \in S$ , the emissions standard q(s) in state s is equal to  $e^*(s)$ , the optimal emissions in state s.

Fact 1 immediately follows from the assumption that the abatement cost function C is strictly decreasing in emissions (and hence the constraint on emissions induced by a standard is binding). As in the proof for Proposition 1, given that the emissions level in each state is optimal, the firm's investment choice is equal to  $k^*$ , the optimal investment level. If the standard is contingent on both the state and the investment by the firm, then the regulator cannot achieve efficiency. To show this, first we prove that the regulator has an incentive to strengthen the standard if a higher level of investment is observed (Lemma 2).

**Lemma 2** The equilibrium discretionary emissions standards  $\{q(s)\}_{s\in S}$  are decreasing functions of k.

(See Appendix A for the proof.) The firm, therefore, has an incentive to reduce investment to get a more lenient standard. This result leads to the following proposition.

**Proposition 4** The equilibrium discretionary standards do not achieve the socially minimum cost. Furthermore, if the firm's objective function is convex in investment given that standards depend on investment, then the equilibrium investment in a discretionary-standard subgame  $k^{DS}$  is smaller than the optimal investment  $k^*$ .

(See Appendix A for the proof.) It is worthwhile noting that discretionary taxes and discretionary standards are both suboptimal, but they are suboptimal in different ways. The discretionary emissions tax results in over-investment whereas the discretionary emissions standard causes the firm to under-invest.

As with taxes, emissions standards under rules are suboptimal. When standards are set prior to the resolution of uncertainty, the standard set may not achieve an efficient result given the realized damage function.

### 2.6 Comparison of Taxes versus Standards

Here we compare the relative efficiency of taxes versus standards under both rules and discretion. We begin by comparing taxes and standards under rules.

**Proposition 5** Equilibrium standard rules and equilibrium tax rules result in the same expected social cost in equilibrium.

(See Appendix A for the proof.) Because the regulator has committed to a policy (tax or standard), the firm faces no uncertainty when it makes its choice of investment and emissions level. Therefore, the regulator can induce the firm to choose a given investment and emissions choice via either a standard or a tax. The regulator will then choose policy such that it attains minimum ex-ante expected social cost from the set of possible induced investment and emissions responses of the firm. Note that this result is not optimal because in fact the firm's emissions choice should reflect the true state of damages.

Next, we compare the relative performance of taxes versus standards under discretionary policy. Because investment levels differ under taxes and standards, causing differences in resulting regulatory policy and emissions, comparing performance is complicated. To simplify the task, we restrict attention for the following proposition to a case with quadratic costs and benefits. Suppose the emissions abatement cost function is given by

$$C(e,k) = \frac{1}{2}c(\bar{e} - e - ak)^2 \quad \text{for } e, k \text{ such that } 0 \le e \le \bar{e} - ak, k \ge 0$$
(3)

where  $c, \bar{e}$  and a are positive scalars. Suppose the damage function is given by

$$D(e;s) = \frac{de^2}{2} + f(s)e \quad \text{for } e \ge 0$$
(4)

where d > 0 and  $f(s) \ge 0$  for all  $s \in S$ , and the mean of f(s) is given by  $\sum_{s \in S} \pi(s)f(s) = f$ for some f > 0. These functions satisfy all of the properties assumed for cost and benefit functions. With this specification, we have the following proposition.

**Proposition 6** Under a discretionary policy regime and quadratic cost and benefit functions (given by equations 3 and 4), the expected total costs are lower in equilibrium with emissions taxes compared to the expected total costs in equilibrium with emissions standards.

(See Appendix A for the proof). Proposition 6 states that emissions taxes are more efficient than emissions standards, at least under assumptions of quadratic costs and benefits. This result contrasts with the results of Weitzman (1974) in which taxes are preferred to standards if an only if the marginal benefits curve is flatter than the marginal cost curve. Proposition 6 holds regardless of the slopes of marginal benefit and marginal cost. Under discretionary policy, equilibrium results under standards and taxes would both be optimal conditional on investment being set optimally. Inefficiency occurs because investment is distorted: overinvestment with taxes and under-investment with standards. The degree to which investment is distorted away from the optimal level is greater under standards than under taxes, which generates the result that taxes are preferred to standards.

One comparison that cannot be made unambiguously is the comparison between rules and discretion. Whether rules are preferred to discretion, or vice-versa, depends upon the degree of uncertainty about damages. We will illustrate this point and the magnitudes of the inefficiency of various policy schemes with a numerical example in section 4.

# 3 A model with technology innovation

In this section, we modify the model of section 2 from one of adoption of existing technology to one of innovation to discover new technology. The key modelling difference between innovation and adoption is that the results of innovation are stochastic while those of adoption are deterministic. We assume that greater investment results in a larger probability of finding a new technology with lower abatement costs. Otherwise, we retain the model structure of section 2.

### 3.1 Model Environment

Denote the probability of innovation success by  $\lambda$ . Obtaining a higher probability of success can be achieved through increased investment. Let  $G(\lambda)$  be the cost of innovation with success probability  $\lambda$ . The function G is strictly increasing, convex and twice continuously differentiable in success probability:  $G'(\lambda) > 0$  and  $G''(\lambda) > 0$  for  $\lambda \in [0, 1)$ . Further assume that the cost of innovation is zero when the success probability is zero and the cost tends to infinity as the success probability goes to one: G(0) = 0 and  $\lim_{\lambda \to 1} G(\lambda) = +\infty$ . Because *G* is monotonic, we can think of  $\lambda$  as representing the level of investment as well as the probability of success.

If the firm is unsuccessful in innovating, then it retains the status-quo emissions reduction cost function C(e; H). (*H* stands for 'high' marginal abatement costs.) If the firm is successful, then the cost function is C(e; L). (*L* stands for 'low' marginal abatement costs.) We assume  $C_e(e; \cdot) < 0$ ,  $C_{ee}(e; \cdot) > 0$  for all  $e \ge 0$  for both *H* and *L*. Further,  $-C_e(e; H) > -C_e(e, L)$  for all  $e \ge 0$ , i.e., marginal abatement costs are higher when innovation is not successful

The ex ante expected total cost of pollution is given by

$$G(\lambda) + \lambda \sum_{s \in S} \pi(s) [C(e(s,L);L) + D(e(s,L);s)] + (1-\lambda) \sum_{s \in S} \pi(s) [C(e(s,H);H) + D(e(s,H);H)]$$

where e(s, L) (e(s, H)) is the amount of emissions in state s when innovation was successful (failed).

## 3.2 Welfare properties of emissions taxes with technology innovation

The optimal investment/emissions plan  $(\lambda^*, \{e^*(s, H), e^*(s, L)\}_{s \in S})$  is given by a solution to

$$\begin{split} \min_{\lambda,\{e(s,H),e(s,L)\}_{s\in S}} & G(\lambda) + \lambda \sum_{s\in S} \pi(s)[C(e(s,L);L) + D(e(s,L);s)] \\ & + (1-\lambda) \sum_{s\in S} \pi(s)[C(e(s,H);H) + D(e(s,H);s)] \\ \text{s.t.} & \lambda \in [0,1], \ e(s,H) \geq 0 \text{ and } e(s,L) \geq 0 \text{ for all } s \in S. \end{split}$$

Given the convexity of functions C and D, the following first order conditions are necessary and sufficient for an interior solution:

$$C_e(e^*(s,L);L) + D_e(e^*(s,L);s) = 0 \text{ for all } s \in S,$$
(5)

$$C_e(e^*(s,H);H) + D_e(e^*(s,H);s) = 0 \text{ for all } s \in S$$
 (6)

and

$$G'(\lambda^*) + \sum_{s \in S} \pi(s) [C(e^*(s,L);L) + D(e^*(s,L);s)] - \sum_{s \in S} \pi(s) [C(e^*(s,H);H) + D(e^*(s,H);s)] = 0.$$
(7)

The next two propositions (6 and 7) show that emissions taxes under both discretion and rules fail to achieve an efficient outcome.

**Proposition 7** Equilibrium discretionary taxes do not achieve the socially minimum cost. Furthermore, the equilibrium success probability  $\lambda^{DT}$  is larger than the optimal level probability  $\lambda^*$ .

(See Appendix A for the proof.)

Proposition 8 The equilibrium tax rule does not achieve socially minimum cost.

Proposition 8 follows from the fact that the tax rule does not induce the firm to choose different emissions levels for different realizations of states nor does the tax rate change for different results of innovation.

## 3.3 Welfare properties of emissions standards with technology innovation

If the standard is contingent on both the state and the investment by the firm, the regulator cannot achieve efficiency because the firm's innovation effort is lower than optimal.

**Proposition 9** The equilibrium discretionary standards do not achieve the socially minimum cost. Furthermore, the equilibrium success probability  $\lambda^{DS}$  is lower than the optimal probability  $\lambda^*$ . (See Appendix A for the proof.) As in the case with technology adoption, the standard under rules and the taxes under rules do equally well in terms of efficiency (Proposition 10).

**Proposition 10** Equilibrium standard rules and equilibrium tax rules result in the same expected social cost in equilibrium.

The proof is similar to the proof for Proposition 5 and 3.

In sum, the results of the innovation model are qualitatively similar to the results in the adoption model of section 2. Under rules, taxes and standards yield the same result. This result is inefficient because it does not reflect actual conditions of damages. Under discretion, taxes result in over-investment while standards result in under-investment. Hence, discretionary emissions taxes result in over-investment whereas discretionary emissions standards causes the firm to under-invest in technology innovation.

# 4 Numerical Examples

In the previous two sections we showed that neither discretion or rules, taxes or standards, achieves an efficient result. In this section we use numerical simulation to investigate the relative efficiency of these alternative regulatory schemes. Since taxes and standards yield the same outcome under rules, we compare rules with discretionary taxes and discretionary standards. We begin by analyzing the adoption model (4.1) and then analyze the innovation model (4.2).

### 4.1 Simulation with technology adoption

In what follows we use simple quadratic cost and damage functions to illustrate the relative efficiency of rules versus discretionary taxes versus discretionary standards. The emissions abatement cost function and the damage function are given by equations (3) and (4) introduced in subsection 2.6. For the random variable f(s), we assume  $P(f(s) = f + \varepsilon) =$   $P(f(s) = f - \varepsilon) = \frac{1}{2}$  for some  $\varepsilon > 0$ . These functions satisfy the properties assumed in section 2. With this specification, the firm's objective function is concave under discretionary schemes. Parameters  $c, \bar{e}, a, d, f, \varepsilon$  and the unit price of investment r are chosen so that all the equilibrium solutions are interior. Hence, Propositions 1- 6 apply to this example.

In figure 2 we show the effect of increased uncertainty on the relative efficiency of alternative policy schemes. With no uncertainty ( $\varepsilon = 0$ ), rules result in an efficient solution. The regulator can set standards or taxes to induce the firm to choose the correct levels of investment and emissions. Discretionary policy, however, does not result in an efficient solution even with no uncertainty. This result occurs because of the distortion in investment incentives. With increasing uncertainty, rules become relatively less efficient. Rules may be set in ways that are far from optimal given actual conditions. Inefficiency of rules increases in a quadratic fashion with increases in  $\varepsilon$ . On the other hand, the relative inefficiency of discretionary policy is hardly affected by increased uncertainty because policy will be set to reflect actual conditions. Inefficiency arises because of distortion of investment, which is affected little by changes in uncertainty. As shown in figure 2, discretionary policy is preferred to rules for high levels of uncertainty.

As shown in figure 2, discretionary taxes are more efficient than standards for the complete range of uncertainty (see Proposition 6). Note that, in Weitzman (1974), taxes are preferred to standards if and only if the marginal benefits curve is flatter than the marginal cost curve. In figures 3 and 4 we show the effect of changes in the slopes of marginal damages and marginal abatement cost on the relative superiority of discretionary taxes compared to standards. In figure 3, we fix the slope of marginal damages equal to 1 and vary the slope of the marginal abatement cost function (parameter c). As shown in figure 3, taxes become increasingly favorable as c increases. In figure 4, we fix the slope of the marginal abatement cost equal to 1 and vary the slope of marginal damages (parameter d). Taxes become increasingly superior to standards as d decreases. The advantage of taxes over standards

under discretionary policy increases as the marginal benefits curve becomes flatter compared to the marginal cost curve, consistent with Weitzman (1974). However, in our model with quadratic costs and benefit functions, taxes are superior to standards regardless of the slopes of the marginal cost and marginal benefit functions.

In figure 5, we show the effect of varying the cost of investment (parameter r). As the cost of investment becomes larger, the ratio of deadweight loss to the first-best level of cost under each scheme becomes larger.

### 4.2 Simulation with technology innovation

For the innovation cost function G, assume  $G(\lambda) = A(\frac{1}{1-\lambda} - 1 - \lambda)$  where A is a positive constant. We assume the abatement cost function C is given by

$$C(e,H) = \frac{c(\bar{e}^H - e)^2}{2} \text{ for } 0 \le e \le \bar{e}^H, \quad C(e,L) = \frac{c(\bar{e}^L - e)^2}{2} \text{ for } 0 \le e \le \bar{e}^L$$

where  $\bar{e}^L = \alpha \bar{e}^H$  with  $0 < \alpha < 1$ . (The smaller  $\alpha$ , the larger improvement in abatement technology when innovation is successful.) The damage function is identical with that given in section 4.1. Parameters  $A, c, \bar{e}^H, \bar{e}^L, d, f$  and  $\varepsilon$  are positive and chosen so that all the equilibrium solutions are interior. These functions satisfy the properties assumed in section 3 and hence Propositions 7 - 10 apply to this example.

In figure 6, we show results for a case where there is little difference in costs with and without successful innovation. When the difference between the two abatement cost functions,  $C(\cdot, H)$  and  $C(\cdot, L)$ , is small ( $\alpha = 0.95$ ), there is little uncertainty about abatement costs, making this model quite similar to the adoption model. In fact, we observe much the same pattern in the rankings between rules, discretionary taxes and discretionary standards as shown in figure 2 with the adoption model. When uncertainty about damages is low, rules are preferable to discretion. With high uncertainty about damages, discretion is preferable to rules. In figure 7 we show results for a case where there is a substantial cost reduction (50 percent, or  $\alpha = 0.5$ ) with successful innovation. In this case, discretionary policy is preferable to rules even when there is little to no uncertainty about damages. In cases where there is great uncertainty about the state of future technology, there is large risk in committing to technology forcing standards. If it turns out that innovation is unsuccessful, the emissions standard will be far more stringent ex post than conditions warrant. On the other hand, if innovation is successful, then standards should be tightened further. Even factoring in the distortion to innovation incentives, discretionary policies yield far lower expected costs than do rule when there is large uncertainty about the state of future technology.

# 5 Discussion

In this paper we compared environmental policy under discretion, in which policy is updated upon learning new information, versus under rules, in which policy is not updated. When investment in abatement technology is made prior to the resolution of uncertainty, neither discretion nor rules with either linear taxes or standards achieve an efficient solution. When uncertainty about damages or the results of investment are small, rules are superior discretion, because discretionary policy schemes give the firm an incentive to distort investment in order to influence future regulation. However, when uncertainty about either damages or the results of investment is large, discretion is superior to rules because it allows regulation to incorporate new information. We found that with discretionary policy, taxes are superior to standards even in cases where marginal costs are flatter than marginal damages, in contrast to Weitzman (1974).

The inefficiency of environmental policy under both rules and discretion is caused by the fact that investment occurs prior to the resolution of uncertainty. If it were possible to reverse the order so that all uncertainty were resolved prior to investment, the regulator could make policy dependent on actual conditions but not dependent upon investment. This would avoid distorting investment incentives while still allowing regulation to reflect actual conditions. That this order cannot be reversed is clearest in the innovation model where investment in R&D must take place prior to realizing the results of such activity. Even with adoption, investments tend to be long-lived while new information is learned on a fairly frequent basis.

Even with timing fixed as in this model, the inefficiency of environmental policy with learning and innovation under both rules and discretion could be overcome with sufficiently sophisticated regulatory policy. One way to achieve an efficient result is for the regulator to set non-linear taxes. An efficient result will occur if the regulator sets a tax schedule equal to realized marginal damages. In this case, the firm always faces the social costs of its actions and it will choose efficient levels of emissions abatement and investment. By definition, fully internalizing all external costs will correct externalities, but such solutions cannot typically be implemented in practice.

Another possible route to overcome inefficiency in cases with learning and innovation is to consider the introduction of an environmental investment policy in addition to traditional environmental policy targeted to emissions. Innovation policies would need to be coordinated with emissions policy. Under discretionary emissions standards, the firm will tend to investment too little. This distortion could be corrected by subsidizing investment in emissions abatement equipment. On the other hand, under discretionary emissions taxes, the firm will tend to invest too much. Therefore, somewhat paradoxically, with a discretionary emission tax scheme, investment in emissions abatement equipment should also be taxed.

We assumed there is only one polluting firm in our model to highlight the strategic aspects of the regulator-regulated firm interaction. At the other extreme, a large number of small firms might each believe that their own actions have no influence on future regulation. In this case, there would be no distortion of investment incentives and discretionary policy would be the optimal approach. In the more interesting intermediate case with a small number of strategic firms, each firm must consider the effect of their investment on rival firms as well as on the regulator opening up numerous possible results. In addition, having more than one firm raises the issue of appropriability of rents from successful innovation among firms (see Fischer et al. 2003 for analysis of this issue).

In this model we focused on symmetric uncertainty about damages in the adoption model and symmetric uncertainty about the result of R&D in the innovation model. An alternative formulation of the innovation model would be to assume that the results of innovation are private information to the firm, which would then make the model one of regulation under asymmetric information. In addition to firms' private information about costs, the regulator's type may be another source of asymmetric information. For example, perhaps the commitment to rules is somewhat less than categorical. The firm may be uncertain whether a regulator really can or cannot commit to rules. The firm will form a belief on the regulator's type (the ability of the regulator to commit rules) and the firm's response will depend on such beliefs. We leave analysis of asymmetric information models to future research.

# Appendix A

## **Proof of Proposition 1**

The socially optimal (i.e. least cost) emissions levels and investment are given by the solution to

$$\min_{\substack{k, \{e(s)\}_{s \in S}}} \quad rk + \sum_{s \in S} \pi(s) [C(e(s), k) + D(e(s); s)]$$
s.t. 
$$k \ge 0, \ e(s) \ge 0 \text{ for all } s \in S.$$

The optimal plan  $(k^*, \{e^*(s)\}_{s \in S})$  is characterized in the text by equations (1) and (2). On the other hand, the firm chooses state-*s* emissions given investment *k* and tax rate  $\tau(s)$  to solve

$$\min_{e(s)} C(e(s), k) + \tau(s)e(s)$$

for all  $s \in S$ . The necessary and sufficient conditions are

$$C_e(e(s), k) + \tau(s) = 0$$
 for all  $s \in S$ .

Denote the solutions by  $\{e(k,s)\}_{s\in S}$ . At the investment stage, the firm solves

$$\min_{k\geq 0} rk + \sum_{s\in S} \pi(s) [C(e(k,s),k) + \tau(s)e(k,s)].$$

The necessary and sufficient condition for an interior solution is

$$r + \sum_{s \in S} \pi(s) C_k(e(k,s),k) = 0.$$

With  $\tau(s) \equiv D_e(e^*(k,s);s)$  for all  $s \in S$ , the equilibrium emissions and investment are the same as the unique optimal solution.

### Proof of Lemma 1

Denote the optimal emissions given investment k and state  $s \in S$  by  $e^*(k, s)$ . The optimal emissions satisfy, for all  $s \in S$ ,

$$C_e(e^*(k,s),k) + D_e(e^*(k,s);s) = 0.$$

Totally differentiating with respect to investment and emissions, we have

$$\frac{\partial e^*(k,s)}{\partial k} = -\frac{C_{ek}(\cdot,\cdot)}{C_{ee}(\cdot,\cdot) + D_{ee}(\cdot;s)} < 0.$$

The equilibrium tax rate  $\tau(k, s)$  given investment k and state  $s \in S$  must satisfy

$$\tau(k,s) = D_e(e^*(k,s);s).$$

Differentiating both sides with respect to k, we have

$$\frac{\partial \tau(k,s)}{\partial k} = D_{ee}(e^*(k,s);s) \cdot \frac{\partial e^*(k,s)}{\partial k} < 0$$

for all  $k \ge 0$  since  $D(\cdot; s)$  is strictly convex in emissions. Hence, the equilibrium discretionary tax rate is strictly decreasing in investment k.

## **Proof of Proposition 2**

Given investment k, a realized state  $s \in S$  and a tax  $\tau$ , the firm chooses emissions to solve

$$\min_{e \ge 0} C(e,k) + \tau e.$$

The necessary and sufficient condition for an interior solution is

$$C_e(e,k) + \tau = 0. \tag{8}$$

Let  $e(k, \tau)$  represent the emissions level that solves this problem. Holding  $\tau$  constant, the partial derivative of e with respect to k is given by

$$\frac{\partial e(k,\tau)}{\partial k} = -\frac{C_{ee}}{C_{ek}}.$$
(9)

Holding k constant, the partial derivative of e with respect to  $\tau$  is given by

$$\frac{\partial e(k,\tau)}{\partial \tau} = -\frac{1}{C_{ee}}.$$
(10)

As in Lemma 1, the regulator will set taxes such that  $\tau(k, s) = D_e(e^*(k, s); s)$ . From Lemma 1, we have  $\frac{\partial \tau(\cdot, s)}{\partial k} < 0$ . At the investment stage, the firm's objective function is

$$V_{DT}(k) = rk + \sum_{s \in S} \pi(s) [C(e(k, \tau(k, s)), k) + \tau(k, s)e(k, \tau(k, s))]$$

The subscript DT stands for discretionary taxes. The derivative of  $V_{DT}$  evaluated at the optimal investment  $k^*$  is

$$V'_{DT}(k^*) = r + \sum_{s \in S} \pi(s) [C_e(e(k^*, \tau(k^*, s)), k^*) \{ \frac{\partial e}{\partial k} + \frac{\partial e}{\partial \tau} \cdot \frac{\partial \tau}{\partial k} \} + C_k(e(k^*, \tau(k^*, s)), k^*)$$
$$+ \frac{\partial \tau(k^*, s)}{\partial k} e(k^*, \tau(k^*, s)) + \tau(k^*, s) \{ \frac{\partial e}{\partial k} + \frac{\partial e}{\partial \tau} \cdot \frac{\partial \tau}{\partial k} \} ]$$
$$= r + \sum_{s \in S} \pi(s) [C_k(e(k^*, \tau(k^*, s)), k^*) + \frac{\partial \tau(k^*, s)}{\partial k} e(k^*, \tau(k^*, s))]$$

where the second equality follows from condition (8), and  $e(k^*, \tau(k^*, s)) = e^*(s)$ , the optimal state-*s* emissions. Note that  $r + \sum_{s \in S} \pi(s)C_k(e^*_s, k^*) = 0$  by equation (2). Since  $\frac{\partial \tau(k^*,s)}{\partial k}e(\tau(k^*,s)) < 0$ , it follows that  $V'_{DT}(k^*) < 0$ . Assuming that  $V_{DT}$  is convex in investment *k*, this implies that the equilibrium investment by the firm is larger than the optimal investment. Therefore, the discretionary tax scheme fails to achieve the social cost minimum outcome characterized by (1) and (2). (We discuss the assumption of convexity of  $V_{DT}$  further in Appendix B.)

### **Proof of Proposition 3**

A state-independent tax induces the firm to choose the same amount of emissions across different states. As long as the marginal cost function varies across states, the optimal emissions will differ across states. Hence, a tax scheme where the tax rate is uniform across states does not achieve the optimal outcome.

## Proof of Lemma 2

Denote the optimal standard given investment k and state  $s \in S$  by q(k, s). It must satisfy

$$C_e(q(k,s),k) + D_e(q(k,s)) = 0.$$

Totally differentiating with respect to investment and emissions yields:

$$\frac{\partial q(k,s)}{\partial k} = -\frac{C_{ek}(\cdot,\cdot)}{C_{ee}(\cdot,\cdot) + D_{ee}(\cdot;s)} < 0$$

for all  $k \ge 0$  and  $s \in S$ . Hence, the equilibrium discretionary standard level is strictly decreasing in investment k.

### **Proof of Proposition 4**

Given investment k and an emissions standard plan  $\{(q(k,s))_{k\geq 0}\}_{s\in S}$ , the firm chooses emissions to minimize cost. From Lemma 2, we know that the optimal discretionary standard level is decreasing in investment. At the investment stage, the firm solves

$$\min_{k \ge 0} V_{DS}(k) = rk + \sum_{s \in S} \pi(s) [C(q(k, s), k)].$$

The subscript DS stands for discretionary standards. The first-order derivative is

$$V_{DS}'(k) = r + \sum_{s \in S} \pi(s) [C_e(q(k,s),k) \cdot \frac{\partial q(k,s)}{\partial k} + C_k].$$

Evaluating this expression at the optimal solution, we have:

$$V_{DS}'(k^*) = r + \sum_{s \in S} \pi(s) [C_e(q(k^*, s), k^*) \cdot \frac{\partial q(k^*, s)}{\partial k} + C_k(e(k^*, s), k^*)]$$

where  $q(k^*, s) = e^*(s)$ , the optimal state-*s* emissions. Note that  $r + \sum_{s \in S} \pi(s)C_k(e^*(s), k^*) = 0$  by equation (2). Since  $C_e(q(k^*, s), k^*) \cdot \frac{\partial q(k^*, s)}{\partial k} > 0$ , it follows that  $V'(k^*) > 0$ . Given the convexity of  $V_{DS}$ , this implies that the equilibrium investment by the firm is less than the optimal investment. Therefore, the discretionary standard scheme fails to achieve the social cost minimum outcome characterized by equations (1) and (2).

### **Proof of Proposition 5**

With rules, the regulator sets a single tax or a single standard so that the firm faces the same regulation no matter which state  $s \in S$  occurs. In the tax case, the firm facing emissions tax  $\tau$  will choose emissions level and investment given by the following equations:

$$r + C_k(e, k) = 0,$$

$$C_e(e, k) + \tau = 0.$$
(11)

Denote the solution by  $e(\tau)$ ,  $k(\tau)$ . The regulator will choose the tax rate in order to minimize social cost knowing the firm's emissions and investment choices as a function of  $\tau$ :

$$\min_{\tau \ge 0} rk(\tau) + C(e(\tau), k(\tau)) + \sum_{s \in S} \pi(s)D(e(\tau); s)$$

The necessary and sufficient condition for solving this minimization problem is

$$rk'(\tau) + C_e e'(\tau) + C_k k'(\tau) + e'(\tau) \sum_{s \in S} \pi(s) D_e(e(\tau); s) = 0.$$

Using equation (11) and the fact that  $e'(t) \neq 0$ , we have

$$C_e(e,k) + \sum_{s \in S} \pi(s) D_e(e;s) = 0.$$

In the case of standards, the firm will set emissions equal to the standard: e = q. The firm will choose investment k(q) to satisfy

$$r + C_k(q, k(q)) = 0.$$
 (12)

The regulator will choose the standard in order to minimize social cost knowing the firm' emissions and investment choices as a function of standard:

$$\min_{q\geq 0} rk + C(q, k(q)) + \sum_{s\in S} \pi(s)D(q; s).$$

The necessary and sufficient condition for solving this minimization problem is

$$rk'(q) + C_e + C_k k'(q) + \sum_{s \in S} \pi(s) D_e(q; s) = 0$$

Using (12) we can simply this equation:

$$C_e(q, k(q)) + \sum_{s \in S} \pi(s) D_e(q; s) = 0.$$

Noting that e = q, we therefore have the same equilibrium emissions and investment under the tax rule and the standard rule.

### **Proof of Proposition 6**

Let  $\Delta$  be the expected total costs in the equilibrium under discretionary standards minus the expected total costs in the equilibrium under discretionary taxes. We want to show that  $\Delta > 0$ . First, for discretionary standards, we derive the equilibrium standards and emissions as functions of investment. Then we derive the equilibrium investment under discretionary standards. We follow the same steps for deriving equilibrium tax rates, emissions and investment under discretionary taxes. Then we show that  $\Delta > 0$ .

### i) Discretionary standards

Given state s and investment k, the regulator sets the standard q(k, s) to solve

$$\min_{q \ge 0} \frac{c(\bar{e} - ak - q)^2}{2} + \frac{de^2}{2} + f(s)e.$$

Solving this problem, we obtain  $q(k,s) = \frac{c(\bar{e}-ak)-f(s)}{c+d}$ . Given standards  $\{(q(k,s))_{k\geq 0}\}_{s\in S}$ , in the investment stage the firm solves

$$\min_{k \ge 0} rk + \sum_{s \in S} \pi(s) \frac{c(\bar{e} - ak - q(k, s))^2}{2}.$$

The solution  $k^{DS}$  is given by

$$k^{DS} = \frac{-r(c+d)^2 + acd(d\bar{e}+f)}{a^2cd^2}.$$

(Note that  $f \equiv \sum_{s \in S} \pi(s) f(s)$ .)

### *ii)* Discretionary taxes

In the emissions abatement stage, given investment k and a tax  $\tau$ , the firm chooses emissions  $e(k, \tau)$  to solve

$$\min_{e \ge 0} \frac{c(\bar{e} - ak - e)^2}{2} + \tau e.$$

The solution is given by  $e(k,\tau) = \frac{c(\bar{e}-ak)-\tau}{c}$ . Hence, in state s, given investment k the regulator sets the tax rate  $\tau(k,s)$  to solve

$$\min_{\tau \ge 0} \frac{c(\bar{e} - ak - e(k, \tau))^2}{2} + \frac{de(k, \tau)^2}{2} + f(s)e(k, \tau).$$

The solution is given by  $\tau(k,s) = \frac{cd(\bar{e}-ak)+cf}{c+d}$ , and hence  $e(k,\tau(k,s)) = \frac{c(\bar{e}-ak)-f(s)}{c+d}$  (note that  $e(k,\tau(k,s)) = q(k,s)$ ). In the investment stage, the firm chooses investment  $k^{DT}$  to solve

$$\min_{k \ge 0} rk + \sum_{s \in S} \pi(s) \left[ \frac{c(\bar{e} - ak - e(k, \tau(k, s)))^2}{2} + \tau(k, s)e(k, \tau(k, s)) \right].$$

The solution  $k^{DT}$  is given by

$$k^{DS} = \frac{-r(c+d)^2 + 2ac^2d\bar{e} + acd^2\bar{e} + ac^2f}{a^2cd(2c+d)}$$

#### iii) Comparison of equilibrium costs

Given the equilibrium quantities found above, we have

$$\begin{split} \Delta &= rk^{DS} + \sum_{s \in S} \pi(s) [\frac{c}{2} \{\bar{e} - ak^{DS} - q(k^{DS}, s)\}^2 + \frac{d}{2} \{q(k^{DS}, s)\}^2 + f(s)q(k^{DS}, s)] \\ &- \left[ rk^{DT} + \sum_{s \in S} \pi(s) [\frac{c}{2} \{\bar{e} - ak^{DT} - e(k^{DT}, \tau(k^{DT}, s))\}^2 \right. \\ &+ \frac{d}{2} \{e(k^{DT}, \tau(k^{DT}, s))\}^2 + f(s)e(k^{DT}, \tau(k^{DT}, s))] \right] \\ &= \left[ -r + \frac{ac(d\bar{e} + f)}{c + d} - \frac{a^2cd(k^{DT} + k^{DS})}{2(c + d)} \right] (k^{DT} - k^{DS}). \end{split}$$

In the last equation, the expression inside the square bracket is

$$-r + \frac{ac(d\bar{e} + f)}{c+d} - \frac{a^2cd(k^{DT} + k^{DS})}{2(c+d)} = \frac{c(2cr + adf)}{2d(2c+d)} > 0.$$

We have  $k^{DT} - k^{DS} > 0$  by Propositions 2 and 4. Hence, we have  $\Delta > 0$ . We conclude that the expected total cost under taxes is less than the expected total cost under standards.

### **Proof of Proposition 7**

Denote the equilibrium success probability under discretionary taxes by  $\lambda^{DT}$ . We will show that  $\lambda^{DT} > \lambda^*$ .

Given technology (H or L) and tax rate  $\tau$ , the firm sets emissions to minimize cost, which occurs where marginal abatement cost equals the tax rate. Denote  $e(\tau, H)$  and  $e(\tau, L)$ as the emissions level chosen by the firm given tax rate  $\tau$  and technology H and L, respectively.

Given state s and technology H or L, the regulator chooses a tax rate  $\tau(s, H)$  or  $\tau(s, L)$ ). Denote  $e^*(s, H)$  and  $e^*(s, L)$  as the optimal emissions in state (s, H) and (s, L), respectively. The equilibrium discretionary tax rates are given by

$$\tau(s,H) = D_e(e^*(s,H);s), \quad \tau(s,L) = D_e(e^*(s,L);s) \quad \text{for all } s \in S.$$

Given  $\{\tau(s, H), \tau(s, L)\}_{s \in S}$ , the firm chooses emissions  $\{e^*(s, H), e^*(s, L)\}_{s \in S}$ .

At the investment stage, the firm's objective function given the regulator's optimal discretionary tax rates is

$$V_{DT}(\lambda) = G(\lambda) + \lambda \sum_{s \in S} \pi(s) [C(e^*(s, L); L) + \tau(s, L)e^*(s, L)] + (1 - \lambda) \sum_{s \in S} \pi(s) [C(e^*(s, H); H) + \tau(s, H)e^*(s, H)].$$

We have  $V''_{DT}(\lambda) = G''(\lambda) < 0$ , so  $V_{DT}$  is strictly concave in  $\lambda$ . Hence,  $V'_{DT}(\lambda) = 0$  is the necessary and sufficient condition for the cost minimization. The first-order derivative of

 $V_{DT}$  evaluated at  $\lambda^*$  is

$$V'_{DT}(\lambda^*) = G'(\lambda^*) + \sum_{s \in S} \pi(s) [C(e^*(s,L);L) + \tau(s,L)e^*(s,L)] - \sum_{s \in S} \pi(s) [C(e^*(s,H);H) + \tau(s,H)e^*(s,H)].$$

Adding and subtracting  $\sum_{s \in S} \pi(s) \tau(s, H) e^*(s, L)$ , we have

$$V'_{DT}(\lambda^*) = G'(\lambda^*) + \sum_{s \in S} \pi(s) [C(e^*(s,L);L) + \tau(s,L)e^*(s,L) - \tau(s,H)e^*(s,L)] - \sum_{s \in S} \pi(s) [C(e^*(s,H);H) + \tau(s,H)e^*(s,H) - \tau(s,H)e^*(s,L)].$$

Note that  $\tau(s, L) < \tau(s, H)$ , and it follows from convexity of  $D(\cdot; s)$  that  $\tau(s, H)e^*(s, H) - \tau(s, H)e^*(s, L) > D(e^*(s, H); s) - D(e^*(s, L); s)$ . Therefore

$$V'_{DT}(\lambda^*) < G'(\lambda^*) + \sum_{s \in S} \pi(s) [C(e^*(s, L); L)] - \sum_{s \in S} \pi(s) [C(e^*(s, H); H) + D(e^*(s, H); s) - D(e^*(s, L); s)] = 0$$
(13)

where the equality in the last line of (13) follows from the first order condition for optimality shown in equation (7) in section 3. This result along with the concavity of  $V_{DT}$  implies that the equilibrium effort  $\lambda^{DT}$  is larger than the optimal effort  $\lambda^*$ . Therefore, the discretionary tax scheme fails to achieve the social cost minimum outcome characterized by (5), (6) and (7).

### **Proof of Proposition 9**

Denote the equilibrium success probability under discretionary standards by  $\lambda^{DS}$ . We will show that  $\lambda^{DS} < \lambda^*$ .

The firm facing emissions standard q will set emissions equal to q. Therefore, given state s and technology H or L, the regulator chooses an emissions standard  $q(s, H) = e^*(s, H)$  and  $q(s, L) = e^*(s, L)$ , where \*(s, H) and  $e^*(s, L)$  are the optimal emissions in state (s, H) and (s, L), respectively. Then the firm's problem given the regulator's equilibrium discretionary standards is

$$\min_{\lambda \ge 0} V_{DS}(\lambda) = G(\lambda) + \lambda \sum_{s \in S} \pi(s) [C(e^*(s, L); L)] + (1 - \lambda) \sum_{s \in S} \pi(s) [C(e^*(s, H); H)].$$

We have  $V_{DS}'(\lambda) = G''(\lambda) < 0$ , so  $V_{DS}$  is strictly concave in  $\lambda$ . Hence,  $V_{DS}'(\lambda) = 0$  is the necessary and sufficient condition for the cost minimization. The first-order derivative of  $V_{DS}$  evaluated at  $\lambda^*$  is

$$V'_{DS}(\lambda^{*}) = G'(\lambda^{*}) + \sum_{s \in S} \pi(s)[C(e^{*}(s,L);L)] - \sum_{s \in S} \pi(s)[C(e^{*}(s,H);H)]$$

$$> G'(\lambda^{*}) + \sum_{s \in S} \pi(s)[C(e^{*}(s,L);L) - C(e^{*}(s,H);H)]$$

$$+ \sum_{s \in S} \pi(s)[D(e^{*}(s,L);s) - D(e^{*}(s,H);s)]$$
(14)
$$= 0$$
(15)

where the inequality in (14) follows from  $D(e^*(s, L); s) < D(e^*(s, H); s)$ , and the equality in (15) is from the first order condition for optimality shown in equation (7) in section 3. This implies that the equilibrium effort  $\lambda^{DS}$  is smaller than the optimal effort  $\lambda^*$ . Therefore, the discretionary standard scheme fails to achieve the social cost minimum outcome characterized by (5), (6) and (7).

# Appendix B: Convexity of the firm's objective function

Here we discuss the conditions under which the firm's objective, as a function of investment, is convex under discretionary policies in the technology adoption model.

### Firm's objective function under discretionary taxes

As in proposition 2, let  $e(k, \tau)$  be the cost-minimizing choice of emissions by the firm given investment k and tax rate  $\tau$ . Let  $\{(\tau(k, s))_{k\geq 0}\}_{s\in S}$  be the equilibrium discretionary tax rates by the regulator. Given  $\{(\tau(k, s))_{k\geq 0}\}_{s\in S}$ , the firm solves, in the investment stage,

$$\min_{k \ge 0} V_{DT}(k) = rk + \sum_{s \in S} \pi(s) [C(e(k,s),k) + \tau(k,s)e(k,s)].$$

The first-order derivative is

$$\begin{split} V'_{DT}(k) &= r + \sum_{s \in S} \pi(s) [C_e \cdot \{ \frac{\partial e(k, \tau(k, s))}{\partial k} + \frac{\partial e(k, \tau(k, s))}{\partial \tau} \cdot \frac{\partial \tau(k, s)}{\partial k} \} + C_k \\ &+ \frac{\partial \tau(k, s)}{\partial k} \cdot e(k, \tau(k, s)) + \tau(k, s) \cdot \{ \frac{\partial e(k, \tau(k, s))}{\partial k} + \frac{\partial e(k, \tau(k, s))}{\partial \tau} \cdot \frac{\partial \tau(k, s)}{\partial k} \} ] \\ &= r + \sum_{s \in S} \pi(s) [C_k + \frac{\partial \tau(k, s)}{\partial k} \cdot e(k, s)] \end{split}$$

where the second equality follows from equation (8) in the proof of proposition 2. The second-order derivative is

$$V_{DT}''(k) = \sum_{s \in S} \pi(s) [C_{ke} \cdot \{\frac{\partial e(k, \tau(k, s))}{\partial k} + \frac{\partial e(k, \tau(k, s))}{\partial \tau} \cdot \frac{\partial \tau(k, s)}{\partial k}\} + C_{kk} \\ + \frac{\partial^2 \tau(k, s)}{\partial k^2} \cdot e(k, \tau(k, s)) + \frac{\partial \tau(k, s)}{\partial k} \cdot \{\frac{\partial e(k, \tau(k, s))}{\partial k} + \frac{\partial e(k, \tau(k, s))}{\partial \tau} \cdot \frac{\partial \tau(k, s)}{\partial k}\}\}] \\ = \sum_{s \in S} \pi(s) [\{C_{ke} + \frac{\partial \tau(k, s)}{\partial k}\} \cdot \{\frac{\partial e(k, \tau(k, s))}{\partial k} + \frac{\partial e(k, \tau(k, s))}{\partial \tau} \cdot \frac{\partial \tau(k, s)}{\partial k}\} + C_{kk} \\ + \frac{\partial^2 \tau(k, s)}{\partial k^2} \cdot e(k, \tau(k, s))].$$

From lemma 1, we have

$$\frac{\partial \tau(k,s)}{\partial k} = -\frac{D_{ee}C_{ek}}{C_{ee} + D_{ee}}.$$

From equations (9) and (10) in the proof of Proposition 2, we have

$$\frac{\partial e(k,\tau(k,s))}{\partial k} + \frac{\partial e(k,\tau(k,s))}{\partial \tau} \cdot \frac{\partial \tau(k,s)}{\partial k} = -\frac{C_{ek}}{C_{ee}} + \frac{1}{C_{ee}} \cdot \frac{D_{ee}C_{ek}}{C_{ee} + D_{ee}} = \frac{-C_{ek}}{C_{ee} + D_{ee}}.$$

Hence,

$$V_{DT}''(k) = \sum_{s \in S} \pi(s) \left[ \{ C_{ke} - \frac{D_{ee} C_{ek}}{C_{ee} + D_{ee}} \} \cdot \frac{-C_{ek}}{C_{ee} + D_{ee}} + C_{kk} + \frac{\partial^2 \tau(k, s)}{\partial k^2} \cdot e(k, \tau(k, s)) \right]$$

where the terms inside the square brackets are, for all  $s \in S$ ,

$$\{C_{ke} - \frac{D_{ee}C_{ek}}{C_{ee} + D_{ee}}\} \cdot \frac{-C_{ek}}{C_{ee} + D_{ee}} + C_{kk} + \frac{\partial^2 \tau(k,s)}{\partial k^2} \cdot e(k,\tau(k,s))$$

$$= \left[\frac{C_{ee}C_{ek}}{C_{ee} + D_{ee}}\right] \left[-\frac{C_{ek}}{C_{ee} + D_{ee}}\right] + \frac{C_{kk}(C_{ee} + D_{ee})^2}{(C_{ee} + D_{ee})^2} + \frac{\partial^2 \tau}{\partial k^2} e(k,\tau(k,s))$$

$$= \frac{-C_{ee}C_{ek}^2 + C_{kk}(C_{ee}^2 + 2C_{ee}D_{ee} + (D_{ee})^2)}{(C_{ee} + D_{ee})^2} + \frac{\partial^2 \tau}{\partial k^2} e(k,\tau(k,s))$$

$$= \frac{C_{ee}[C_{ee}C_{kk} - C_{ek}^2] + C_{kk}(2C_{ee}D_{ee} + (D_{ee})^2)}{(C_{ee} + D_{ee})^2} + \frac{\partial^2 \tau}{\partial k^2} e(k,\tau(k,s))$$

where  $C_{ee}C_{kk} - C_{ek}^2 \ge 0$  since C is convex. Therefore, the first term is positive. In the second term, we have

$$\frac{\partial^2 \tau(k,s)}{\partial k^2} = \frac{\partial}{\partial k} [D_{ee}(e^*(k,s);s) \cdot \frac{\partial e^*(k,s)}{\partial k}] = D_{eee}(e^*(k,s);s) \cdot [\frac{\partial e^*(k,s)}{\partial k}]^2 + D_{ee}(e;s) \cdot \frac{\partial^2 e^*(k,s)}{\partial k^2},$$

which involves the third-order derivatives of C and D. Therefore, the sign of  $V''_{DT}$  is indeterminate. However, if we assume that the absolute values of these third-order derivatives are zero or small enough, then  $V''_{DT}$  is positive and hence  $V_{DT}$  is strictly convex.

### Firm's objective function under discretionary standards

By Lemma 2, the optimal discretionary standard is decreasing in investment k:

$$\frac{\partial q(k,s)}{\partial k} = -\frac{C_{ek}(\cdot,\cdot)}{C_{ee}(\cdot,\cdot) + D_{ee}(\cdot)} < 0.$$

Given  $\{(q(k,s))_{k\geq 0}\}_{s\in S}$ , in the investment stage the firm solves

$$\min_{k\geq 0}V(k)=rk+\sum_{s\in S}\pi(s)[C(q(k,s),k)].$$

The first-order derivative is

$$V_{DS}'(k) = r + \sum_{s \in S} \pi(s) [C_e \cdot \frac{\partial q}{\partial k} + C_k].$$

The second-order derivative is

$$V_{DS}''(k) = \sum_{s \in S} \pi(s) \left[ (C_{ee} \frac{\partial q}{\partial k} + C_{ek}) \frac{\partial q}{\partial k} + C_e \frac{\partial^2 q}{\partial k^2} + C_{ke} \frac{\partial q}{\partial k} + C_{kk} \right].$$

Arrange the terms inside the square brackets to have, for all  $s \in S$ ,

$$\begin{aligned} & (C_{ee}\frac{\partial q}{\partial k} + C_{ek})\frac{\partial q}{\partial k} + C_e\frac{\partial^2 q}{\partial k^2} + C_{ke}\frac{\partial q}{\partial k} + C_{kk}) \\ = & (-\frac{C_{ee}C_{ek}}{C_{ee} + D_{ee}} + C_{ek})(\frac{-C_{ek}}{C_{ee} + D_{ee}}) - \frac{C_{ek}^2}{C_{ee} + D_{ee}} + C_{kk} + C_e\frac{\partial^2 q}{\partial k^2} \\ = & \frac{C_{ee}C_{ek}^2 - 2C_{ek}^2(C_{ee} + D_{ee}) + C_{kk}(C_{ee} + D_{ee})^2}{(C_{ee} + D_{ee})^2} + C_e\frac{\partial^2 q}{\partial k^2} \\ = & \frac{(C_{ee} + 2D_{ee})(C_{ee}C_{kk} - C_{ek}^2) + C_{kk}D_{ee}^2}{(C_{ee} + D_{ee})^2} + C_e\frac{\partial^2 q}{\partial k^2} \end{aligned}$$

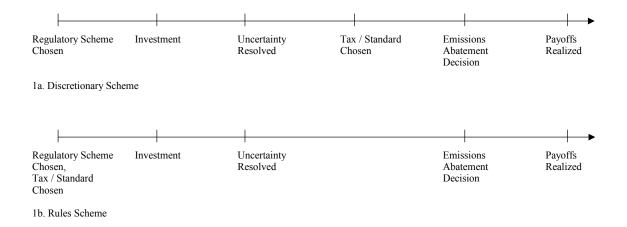
where  $C_{ee}C_{kk} - C_{ek}^2 \ge 0$  since C is convex, and hence the first term is positive. The second term includes the second-order derivative of q, which involves the third-order derivatives of C and D. Therefore, the sign of  $V_{DS}''$  is indeterminate. However, if we assume that the absolute values of these third-order derivatives are zero or small enough, then  $V_{DS}''$  is positive and hence  $V_{DS}$  is strictly convex.

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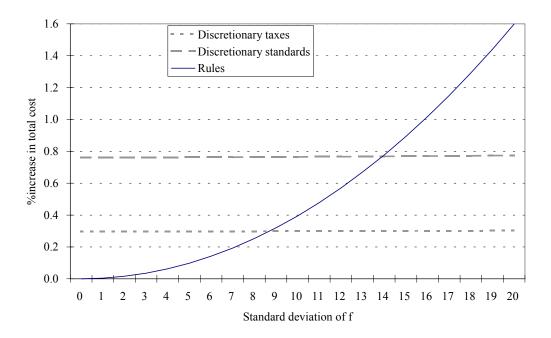
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#### Order of moves under discretionary schemes and rules schemes



# Figure 1

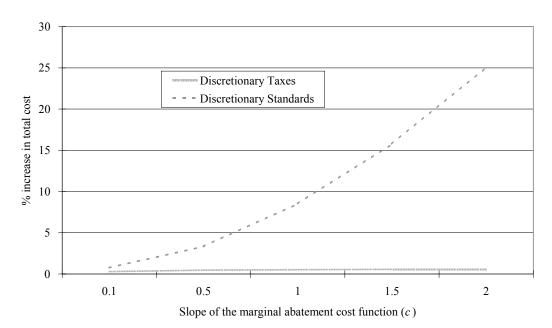
Rules versus Discretion in Adoption Model



$$\overline{e} = 500, a = 0.5, c = 0.1, d = 1, f = 10, r = 20$$

Figure 2

#### Price versus Quantity in Adoption Model (1)

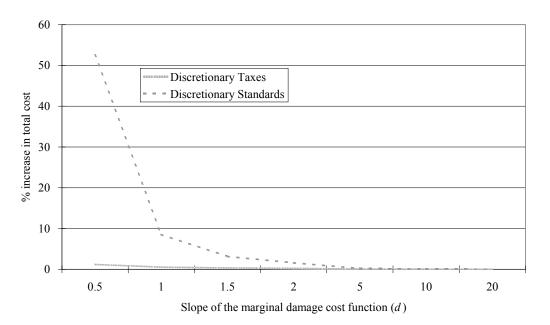


Note: Slope fo the marginal damage cost function (d) is fixed at 1.

 $\overline{e} = 500, a = 0.5, d = 1, f = 10, \varepsilon = 10, r = 20.$ 

Figure 3

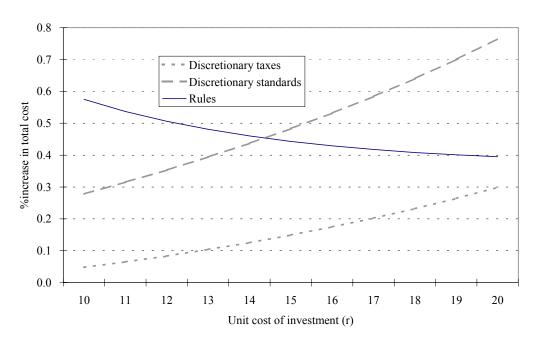
#### Price versus Quantity in Adoption Model (2)



Note: Slope fo the marginal abatement cost function (c) is fixed at 1.

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Figure 4



Effect of Changes in Investment Cost in Adoption Model

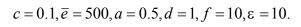
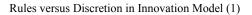
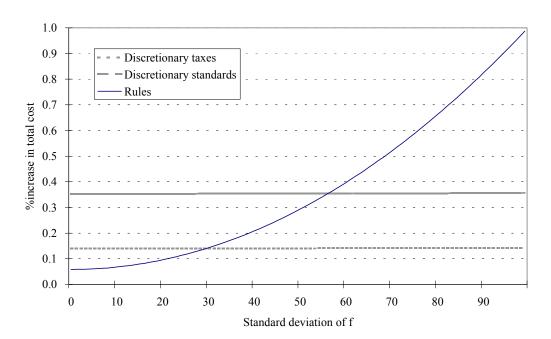


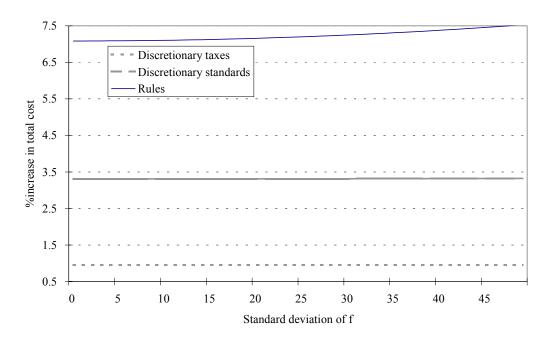
Figure 5





$$c = 100, e^{H} = 10, d = 100, f = 50, A = 100, \alpha = 0.95.$$
  
Figure 6

Rules versus Discretion in Innovation Model (2)



$$c = 100, e^{H} = 10, d = 100, f = 50, A = 100, \alpha = 0.5.$$

Figure 7