# INVERSE DEMAND RELATIONSHIPS FOR WHEAT FOOD USE BY CLASS

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**Abstract:** A normalized quadratic input distance system is applied to estimate inverse demand relationships for wheat by class. Semi-nonparametric and Bayesian estimators are used to impose curvature on inputs and outputs. Price flexibilities are estimated for hard red winter, hard red spring, soft red wheat, soft white winter, and durum wheat. Durum wheat is found to be the most price flexible. Economically and statistically important differences in price formation across classes of wheat are found and are supportive of government programs differentiating wheat by class.

**Introduction**

Policymakers in the U.S. have been recently altered and introduced farm programs that recognize differences in demand and supply responses for wheat classes. For example, the Commodity Credit Corporation (CCC) released market loan rates by class "to establish loan rates that are in line with market forces in order to avoid over-production of wheat in a county in response to the benefits that are available under the marketing loan program" (U.S. Department of Agriculture 2002). To better understand price formation and market response for wheat food use, we conceptualize and specify an industry distance function with the different wheat classes as an input into flour production. A normalized quadratic distance function is used from which a factor demand system and flexibilities are derived and then jointly estimated with the distance function itself. Moreover, an interesting empirical digression on comparing alternative approaches to imposing curvature is provided.

Previous research on wheat by class is limited. Chai (1972) estimated domestic demand for wheat by class over the period from 1929 to 1963. Chai concluded that price elasticities were more elastic for hard classes than soft classes of wheat. Barnes and Shields (1998) estimated a double-log demand system for wheat by class. Annual data from 1981 to 1998 were used in a demand system analysis with regional prices at the farm level. Inelastic own-price elasticities were reported for each of the five wheat classes, but different from Chai, soft white wheat was reported as being the most elastic and durum being the least elastic. Wilson and Gallagher (1990) examined price responsiveness for wheat classes using a Case function approach and found important quality differentials in international markets. Marsh (2003) reported cost, price, and substitution elasticities for hard red winter, hard red spring, soft red wheat, soft white winter, and durum wheat over the period 1974-1999. In general, hard red winter and spring wheat varieties were much more responsive to their own price than were soft wheat varieties and durum wheat.

Previous research on normalized quadratic distance functions is also limited. On the consumer demand side, Holt and Bishop (2002) recently specified a normalized quadratic distance function and used it to estimate inverse demand relationships for fish. In contrast, our focus is on the production side where we apply an alternative functional form of the normalized quadratic function for an input distance function (Marsh, Featherstone, and Garrett 2003). The normalized quadratic input distance function specified in the current study accommodates both single and multiple output production processes and allows direct testing or imposition of input and output curvature conditions. Even for the case of a single input where the properties of the consumer and input distance function are equivalent (Cornes 1992), the functional specification is different.

Several approaches are compared that estimate the distance function jointly with the inverse demand functions and impose curvature restrictions. To do this we exploit the stochastic frontier approach (Stevenson 1980; Greene 1980; Battese and Coelli 1988), which effectively estimates the objective function itself, and extend the approach to include inverse demand relationships. This framework is sufficiently flexible to impose curvature on both inputs and outputs, as well as allow estimation of a complete system of equations. For this input distance system, we first explore a semi-nonparametric estimator with curvature conditions imposed following Lau (1978). Next, we explore a parametric estimator that uses a maximum likelihood function for a complete system of equations to construct a Bayesian model with curvature restrictions imposed following Geweke (1986). This research compliments recent studies by Atkinson and Primont (2002) and Atkinson, Färe, and Primont (2003), who estimated complete systems of inverse demand relationships jointly with the distance function using a GMM estimator. However, neither study specified a likelihood function for the complete system of equations nor did they consider curvature restrictions.

The paper proceeds in the following manner. First, a normalized quadratic input distance function is specified. Second, the data for the empirical analysis are discussed. Third, the empirical model and key econometric issues are presented, including curvature restrictions and the maximum likelihood function for a complete system of equations with extensions to a Bayesian estimator. Fourth, results are reported and interpreted. This includes empirical inverse demand relationships for wheat by class and price flexibilities. Finally, implications and concluding comments are provided.

**Normalized Quadratic Input Distance Function**

*Input-Distance Function*

The direct input distance function is defined by

(1)
$$D(\mathbf{x}, \mathbf{y}) = \sup_{\delta} \left\{ \delta > 0 \,|\, (\mathbf{x}/\delta) \in S(\mathbf{y}), \forall y \in \mathbf{R}_+^M \right\}$$

where $\delta \geq 1$. In (1), $\mathbf{y}$ is a (m$\times$ 1) vector of outputs, $\mathbf{x} = (x_1, \ldots, x_k)'$ is a ($n \times 1$) vector of inputs and $S(\mathbf{y})$ is the set of all input vectors $\mathbf{x} \in \mathbf{R}_+^n$ that can produce the output vector $\mathbf{y} \in \mathbf{R}_+^m$. The underlying behavioral assumption is that the distance function represents a rescaling of all the input levels consistent with a target output level. Intuitively, $\delta$ is the maximum value by which one could divide $\mathbf{x}$ and still produce $\mathbf{y}$. The value $\delta$ places $\mathbf{x}/\delta$ on the boundary of $S(\mathbf{y})$ and on the ray through $\mathbf{x}$. Investigating the distance function is interesting because it is a dual representation of the cost function and both are valid representations of multiple output technologies. The input distance function measures the extent to which the firm is input inefficient in producing a fixed set of output. Moreover, it provides direct estimates of input inefficiency and price flexibilities that are informative economic measures of price formation.

The standard properties of a distance function are that it is homogenous of degree one, nondecreasing, and concave in input quantities $\mathbf{x}$, as well as nonincreasing and quasi-concave in outputs

**y** (Shephard 1970; Färe and Primont 1995). From this framework, inverse factor demand equations may be obtained by applying Gorman's Lemma

(3)
$$\frac{\partial D(\mathbf{x},\mathbf{y})}{\partial \mathbf{x}} = \mathbf{p}^*(\mathbf{x},\mathbf{y})$$

where $\mathbf{p}^*=(p_1,\ldots,p_n)'$ is a $(n \times 1)$ vector of cost normalized input prices or $p_i^* = p_i / \sum_{j=1}^{n} p_j x_j$. The Hessian matrix is given by the second order derivatives of the distance function (Antonelli matrix)

(4)
$$A = \begin{bmatrix} \dfrac{\partial^2 D(\mathbf{x},\mathbf{y})}{\partial \mathbf{x} \partial \mathbf{x}'} & \dfrac{\partial^2 D(\mathbf{x},\mathbf{y})}{\partial \mathbf{x} \partial \mathbf{y}'} \\ \dfrac{\partial^2 D(\mathbf{x},\mathbf{y})}{\partial \mathbf{y} \partial \mathbf{x}'} & \dfrac{\partial^2 D(\mathbf{x},\mathbf{y})}{\partial \mathbf{y} \partial \mathbf{y}'} \end{bmatrix}$$

The input distance function is often used as a measure of technical efficiency (Farrell 1957; Debreu 1951). Inefficiencies arise if firms do not use cost minimizing amounts of input for several reasons, including regulated production, production quotas, or shortages (Atkinson and Primont 2002; Atkinson, Färe, and Primont 2003). The input-oriented measures of technical efficiency is given by

(5)
$$TE = 1/D = \inf_{\delta} \{\delta : \delta x \in S(\mathbf{y})\}$$

where *TE* lies between zero and one. This efficiency measure can be equivalently specified as

(6)
$$\ln D + \ln TE = \ln D - u = 0$$

where the term $u = -\ln TE$ can be expressed as $TE = \exp(-u)$. Hence, *u* is nonnegative being bounded below by zero and unbounded from above.

*Normalized Quadratic Distance Function*

To complete the model specification, the inverse demand equations in (3) are derived from a normalized quadratic distance function (Marsh, Featherstone, and Garrett 2003). The normalized

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quadratic allows estimation of flexibilities, as well as the explicit investigation of the interactions

between inputs and outputs. The proposed normalized quadratic distance function is given by

$$(7) \quad D(\mathbf{x},y) = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2}\left( \left( \sum_{k=1}^{n} \alpha_k x_k \right)^{-1} \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j + \sum_{i=n+1}^{n+m}\sum_{j=n+1}^{n+m} b_{ij} y_i y_j \right) + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m} b_{ij} x_i y_j$$

with $n$ inputs and $m$ outputs. The $b_i's$ and $b_{ij}'s$ are parameters to be estimated, while the $\alpha_i$ are

predetermined positive constants that dictate the form of normalization. Symmetry is imposed by

restricting $b_{ij} = b_{ji}$. The normalized quadratic distance function in (7) is semiflexible at a reference

vector $x^*$ (Diewert and Wales 1988).

Using Gorman's Lemma, the input demand equations are given by

$$(8) \quad p_i^* = b_i + \left( \sum_{k=1}^{n} \alpha_k x_k \right)^{-1} \sum_{j=1}^{n} b_{ij} x_j + \alpha_i \left( \sum_{k=1}^{n} \alpha_k x_k \right)^{-2} \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j + \sum_{j=n+1}^{n+m} b_{ij} y_j$$

where the input prices are normalized as $p_i^* = p_i / \sum_{j=1}^{n} p_j x_j$ such that cost of producing the target level

of output is unity. Homogeneity of degree zero in inputs in the input demand equations implies that

$\sum_{j=1}^{n} b_{ij} = 0$, while the normalization restriction requires that $\sum_{k=1}^{n} \alpha_k x_k = 1$ at a reference vector. The

equivalent share equation is given by

$$(9) \quad w_i = b_i x_i + \left( \sum_{k=1}^{n} \alpha_k x_k \right)^{-1} \sum_{j=1}^{n} b_{ij} x_j x_i + \alpha_i x_i \left( \sum_{k=1}^{n} \alpha_k x_k \right)^{-2} \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} x_i x_j + \sum_{j=n+1}^{n+m} b_{ij} y_j x_i$$

Normalizing quantities by their mean values yields unit means, or $x^* = (1,...,1)' = l_n$, which can be used

as a reference bundle. At a reference vector $x^*$, the demand restrictions become

$$(10) \quad \sum_{k=1}^{n} \alpha_k x_k^* = \sum_{k=1}^{n} \alpha_k = 1, \ \alpha_k \geq 0, \forall k, \text{ and } \sum_{j=1}^{n} x_j^* b_{ij} = \sum_{j=1}^{n} b_{ij} = 0$$

Given the distance function is homogeneous of degree one quantities, then it is possible to normalize by some $\lambda$ (e.g., an input or output or convex combinations),

$$(11) \qquad \frac{1}{\lambda}D(\mathbf{x},\mathbf{y}) = D^*\left(\frac{\mathbf{x}}{\lambda},\mathbf{y}\right) \quad \Leftrightarrow \quad \ln D(\mathbf{x},\mathbf{y}) - \ln \lambda = \ln D^*\left(\frac{\mathbf{x}}{\lambda},\mathbf{y}\right)$$

From (6) the relationship can be rewritten as

$$(12) \qquad -\ln \lambda = \ln D^*\left(\frac{\mathbf{x}}{\lambda},\mathbf{y}\right) - u$$

In empirical applications, the term $u = -\ln TE$ has been exploited to form an estimable equation of the distance function itself that provides a direct measure of input inefficiency (Stevenson 1980; Greene 1980; Battese and Coelli 1988; Morrison Paul, Johnston, and Frengley 2000; Brümmer, Glauben, and Thussen 2002). Alternatively, Atkinson and Primont (2002) and Atkinson, Färe, and Primont (2003) directly estimate equations (6) with generalized methods of moments.

Compensated price flexibilities at $x^* = (1,...,1)' = l_n$ are given by the equation

$$(13) \qquad f_{ij}^* = \frac{\partial \ln p_i}{\partial \ln x_j} = \frac{b_{ij} x_j}{p_i} \quad \text{for } i,j=1,\ldots,n$$

using the estimated $b_{ij}$ and the predicted $p_i$.

*Stochastic Input-Normalized Distance System*

To define a distance function normalized by the *kth* input let $x_s^* = x_s / x_k \ \forall s = 1,...,n$. Define the predetermined constants as $\alpha = (0,...,0,\alpha_k,0...,0) \ni \alpha_k = 1$, then $\sum_{s=1}^{n} \alpha_s x_s^* = 1$. Using the homogeneity property of the distance function, it can be written as

$$(14) \quad D^*(\mathbf{x},y) = \frac{D(\mathbf{x}/x_k, y)}{x_k} = b_0^* + \sum_{i=1}^{n-1} b_i^* x_i^* + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2}\left(\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} b_{ij}^* x_i^* x_j^* + \sum_{i=n+1}^{n+m}\sum_{j=n+1}^{n+m} b_{ij} y_i y_j\right) + \sum_{i=1}^{n-1}\sum_{j=n+1}^{n+m} b_{ij} x_i^* y_j$$

Hence, the distance function in (14) is a special case of that in (7). The input demand functions for (14) become

(15)
$$p_i^* = b_i^* + \sum_{j=1}^{n-1} b_{ij}^* x_j^* + \sum_{j=n+1}^{n+m} b_{ij}^* y_j + \varepsilon_i \quad \text{for } i = 1,\ldots,n-1$$

with stochastic error terms $\varepsilon_i$. Flexibilities follow those specified in (13).

From (11) the *kth* input-normalized distance function can be represented by

(16)
$$-\ln x_k = b_0^* + \sum_{i=1}^{n-1} b_i^* x_i^* + \sum_{i=n+1}^{n+m} b_i y_i + \frac{1}{2}\left( \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} b_{ij}^* x_i^* x_j^* + \sum_{i=n+1}^{n+m}\sum_{j=n+1}^{n+m} b_{ij} y_i y_j \right) + \sum_{i=1}^{n-1}\sum_{j=n+1}^{n+m} b_{ij} x_i^* y_j - u + \varepsilon_0$$

where $\varepsilon_0$ is assumed to be an identically distributed stochastic error term and independent of $u$. This representation is important because estimation of the demand equations [equation (15)] without the distance functions in (16) limits curvature testing and imposition to inputs and not outputs. Alternatively, estimating a system including (15) and (16) offers opportunity to impose the complete set of curvature restrictions defined in (4) and potentially increase econometric efficiency. We assume cross correlation of the $\varepsilon_i$'s $i = 0,1,\ldots,n-1$ with covariance $\mathbf{S}_*$, but independence among the $u$ and $\varepsilon_i$'s $i = 0,1,\ldots,n-1$. Estimation issues concerning (16) are complicated by that fact that $u$ is unobserved, but have been addressed in several ways in the stochastic frontier production literature, which we discuss in more detail below.

**Data**

Annual prices and quantities for the empirical analysis for each of the five wheat classes are based on June to May marketing years, from 1974/1975 to 1999/2000. Descriptive statistics are provided in Table 1. Wheat quantity and price data were collected from U.S. Department of Agriculture's Economic Research Service, *Wheat Year Book*, annually from 1974 to 2001. Total flour production increased from 251 million cwt in 1974 to 412 million cwt in 1999, averaging 332 million cwt over the period. Total

wheat food use (the sum of *HRW*, *HRS*, *SRW*, *SWW*, and *DUR* food use) has increased from 545 million

bushels in 1974 to 925 million bushels in 1999. Figure 1 presents food use by wheat class, showing

food use has been trending upwards over time. From 1974 to 1999 the average proportion of total food

use was 0.42, 0.25, 0.19, 0.07, and 0.07 for *HRW*, *HRS*, *SRW*, *SWW*, and *DUR*, respectively.

Given the importance of protein content for hard wheat varieties in flour production, we estimate

the empirical model with wheat cash prices from major markets. This is because *HRW* and *HRS* prices

are sensitive to protein content across regions (Parcell and Stiegert 1998) and that these quality impacts

from protein may be averaged out in the regional price data (Marsh 2003). In particular, the *HRW* price

is represented by Kansas City, No.1 (13% protein); *HRS* price by Minneapolis, dark No.1 spring (13%

protein); *SRW* price by Chicago, No. 2; *SWW* price by Portland No.1; and *DUR* by Minneapolis, No.1

hard amber durum. Figure 2 shows cost normalized prices for wheat by class.

**Econometric Estimation**

Following Wholgenant (1989) and Marsh (2003), the raw product is considered as an input into food

production. Hence, we specify an industry distance function for the flour milling industry and derive

inverse factor demand equations. In specification of the distance function, we do not differentiate

between types of flour produced, but rather assume flour output is a homogeneous product. Although

this is a simplification, the assumption is empirically practical because of limited quantity data for flour.

Finally, millfeed output is not considered in the conceptual model specification. This is because millfeed

is a by-product of flour milling that is used as feed input in the livestock industry and prices typically

follow other feed stuffs such as corn prices (Harwood et al., 1989).

The econometric system consists of the four inverse factor demand equations in (15) and the

transformed distance function in (16), including *HRW*, *HRS*, *SRW*, and *SWW*. *DUR* quantity was used to

normalize the distance function. Flexibilities were recovered for the *DUR* equation using standard properties of general demand restrictions.

*Curvature*

In this analysis we consider two approaches to imposing curvature, including Choleskey decomposition and Bayesian estimation. The Choleskey decomposition approach for the normalized quadratic only requires reparameterization of the Hessian matrix. For example, to impose concavity the Antonelli matrix can be reparameterized into a negative semidefinite matrix by $\mathbf{A} = -\mathbf{BB}'$ where $\mathbf{B}$ is a lower triangular matrix (Lau 1978). Under the Bayesian framework, demand restrictions are imposed following Geweke (1986) and imposing uniform priors on parameters of interest. Griffith, O'Donnell, and Cruz (2000) also use Geweke's approach to imposing restrictions using the Metropolis-Hastings algorithm.

*Semi-Nonparametric Estimation*

Consider the error term $v = \varepsilon_0 - u$ from (16). Because the term $u = -\ln TE$ is an unobservable independent variable, specification of equation (16) requires further assumptions to achieve econometric identification and subsequent estimation. However, standard procedures in the stochastic frontier literature are to assume the unobservable variable $u$ is represented by a distribution with nonegative support and with mean $\mu$. Most often the choice has been the truncated normal or a gamma distribution (see Stevenson 1980; Greene 1980; Battese and Coelli 1988). Following Morrison Paul, Johnston, and Frengley (2000), Brümmer, Glauben, and Thussen (2002), the mean $\mu$ can be specified as a function of observable predetermined variables $\mathbf{Z}$, or $\mu = f(\mathbf{Z}, \gamma)$, with unknown parameters $\gamma$. The $\gamma$ parameters are then estimated by directly substituting $\mu = f(\mathbf{Z}, \gamma)$ into (16). This approach is similar to Zellner's (1970) instrumental variable approach to unobservable independent variables that provides consistent parameter estimates.

Initially, we follow a fixed effect approach and specify **Z** as discrete shift variables representing technical efficiency over the periods from 1974-1980, 1981-1990, and 1991-1999.  The system of equations represented by (15) and (16) with fixed effects is estimated using a method of moments estimator. Atkinson and Primont (2002) point out that neither the fixed nor random effect dominate one another.  For a fixed effect specification, identification can be difficult.  While for a random effects specification, strong distributional assumptions are made about the distributions of the error.  The random effect specification is taken up when we apply a maximum likelihood estimator of the system of equations in (15) and (16).

To measure the significance of price and substitution flexibilities, bootstrapped confidence intervals are constructed.  Bootstrap procedures are convenient for intractable inference problems and are often equivalent or superior to first-order asymptotic results (Mittelhammer, Judge, and Miller 2000). Bootstrap estimates are obtained by (a) resampling the residuals of the model, (b) predicting cost and prices of wheat, (c) reestimating the five-equation system with predicted values, and (d) then recalculating the flexibilities. This process was repeated 500 times to generate distributions of cost, price, and substitution flexibilities.  Then 90% confidence intervals for each flexibility were constructed based on the percentile method, which requires ordering the estimated flexibilities and then selecting outcome 25 (0.05*500) for the lower critical value and outcome 475 (0.95*500) for the upper critical value.  For hypothesis testing, if the bootstrapped confidence interval for the flexibility contains zero, then the flexibility value is not considered significantly different from zero at the 0.10 level.

*Parametric Estimation*

To derive a likelihood function of (15) and (16) with a random effects component, the error term $v = \varepsilon_0 - u$ is specified as the sum of a truncated normal with mean $\mu$ and variance $\sigma_u^2$ and the $\varepsilon_i$'s are distributed $N(\mathbf{0}, \mathbf{S}_*)$.  Further, the $u$ are distributed independently of the $\varepsilon_i$'s.  Using a

change of variable technique (Mittelhammer 1996), the likelihood function for (15) and (16)

becomes

(17)
$$L(\boldsymbol{\beta},\Sigma,\boldsymbol{m},\boldsymbol{s}_u \mid \mathbf{Y}, \mathbf{X}) = \prod_{t=1}^{T} (2\boldsymbol{p})^{-T/2} \det(\Sigma)^{1/2} \exp\left(-.5\left\{\boldsymbol{e}_t'\Sigma\boldsymbol{e}_t + 2\boldsymbol{e}_t'\left(\Sigma[.,1]\right)u_t + u_t^2 \boldsymbol{s}_{11}\right\}\right)$$
$$\left[1 - F\left(-\boldsymbol{m}/\boldsymbol{s}_u\right)\right]^{-1} \exp\left(-.5\left\{(u_t - \boldsymbol{m})/\boldsymbol{s}_u\right\}\right) I_{[0,\infty)}(u_t)$$

where for convenience we denote $\Sigma = \Sigma_*^{-1}$ and $\sigma_{ij}$ is the $(i,j)$ element of $\Sigma$. Here, $F$ is the

standard normal cdf and $I_{[0,\infty)}(u_i)$ is the standard indicator function taking the value 1 when

$u_i \in [0,\infty)$ and 0 otherwise. This represents a generalization of the likelihood function presented

by Stevenson (1980) by including not only the stochastic distance function in (16) but also the

system of inverse demand relationships in (15).

To specify a posterior pdf for the system of equations in (15) and (16), we assume prior

information on the $\theta = \left(\beta', \Sigma_*', \mu, \sigma_u\right)'$ with prior pdf $\pi(\theta) = \pi(\beta)\pi(\Sigma_*)\pi(\mu)\pi(\sigma_u^2)$. The

parameters $\beta$ and $\mu$ are assumed to have uniform distributions that bound the parameter space.

The prior on $\Sigma_*$ is an inverted Wishart distribution, while the inverted gamma is used for a prior

on $\sigma_u^2$. These priors have been used in numerous Bayesian studies (e.g., Zellner, Bauwnes, and

Van Dijk 1988). The posterior pdf is then defined as

$$p(\boldsymbol{\beta},\Sigma,\boldsymbol{m},\boldsymbol{s}_u) = L(\boldsymbol{\beta},\Sigma,\boldsymbol{m},\boldsymbol{s}_u \mid \mathbf{Y},\mathbf{X})\boldsymbol{p}(\boldsymbol{b})\boldsymbol{p}(\Sigma)\boldsymbol{p}(\boldsymbol{m})\boldsymbol{p}(\boldsymbol{s}_u^2)$$

A Monte Carlo method based on importance sampling is used to estimate moments of the

posterior distribution (Mittelhammer, Judge, and Miller 2000; Van Dijk, Hop and Louter 1987).

Further details about the derivation of the likelihood function and specification of the Bayesian

estimator are provided in Marsh, Featherstone, and Garrett (2003).

**Results and Discussion**

*Semi-Nonparametric Estimation*

Parameter estimates, asymptotic standard errors, and 90% bootstrapped confidence intervals are presented in Table 2 for the model with symmetry and curvature imposed using Cholesky decomposition. Based on the bootstrapped confidence intervals, sixteen of the twenty-five estimated coefficients are statistically significant at the 0.10 level. The output coefficients are negative and significant at the 0.10 level for each demand equation. R-square values, which explain variation in quantity of wheat for food use, were 0.997, 0.966, 0.969, 0.920, and 0.894 for the distance function, *HRW*, *HRS*, *SRW*, and *SWW*, respectively.

Linear ($b_{1t}$) and quadratic ($b_{1t}$) time trend and efficiency ($b_{1970}$ and $b_{1990}$) parameters were also significant. The time trend coefficients indicate a quadratic upwards trend over time, representing potential technical change and other factors. The parameters representing technical efficiency from the periods 1974-1980 and 1991-1999 yielded nearly identical technical efficiency values of 0.983 and 0.985, respectively. These results complement those reported by Hossain and Bhuyan (2000) who estimated an output distance function and found that productivity growth in the flour sector (SIC 2041) from 1960-1994 came primarily from technical change rather than change in efficiency.

Table 3 contains price flexibilities at the mean for each demand equation. Signs of the own-flexibilities were negative as required with the imposition of concavity and are inflexible for each wheat class. Durum wheat exhibits the own-flexibility with the largest magnitude (-0.74), while the own-flexibility of the remaining wheat classes range from -0.03 to -0.22. Cross-price effects are inflexible.

*Bayesian Estimation*

Bayesian parameter estimates and 90% confidence intervals are presented in Table 4. Seventeen of the twenty-four coefficients are statistically significant at the 0.10 level. The output coefficients are

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negative and significant at the 0.10 level for each demand equation. The trend coefficients indicate an increasing upwards trend, but at a slower rate than that of the semi-nonparametric model. The parameter *m* representing mean technical efficiency yielded an estimate for technical efficiency with a value of 0.896.

Table 5 contains the mean price flexibilities for each demand equation. Signs of the own-flexibilities were negative as required with the imposition of concavity and are inflexible for each wheat class. Durum wheat exhibits the own-flexibility with the largest magnitude (-0.94) followed by hard red winter wheat (-0.52), while the own-flexibility of the remaining wheat classes range from -0.13 to -0.37. Cross-price effects are also inflexible. Only the cross-effects between *HRW* and *DUR* are statistically significant.

*Discussion*

Comparing across the estimators, the price flexibilities were all inelastic. However, the own-price flexibilities were larger (especially for *HRW* and *HRS*) from the Bayesian estimation relative to those from the semi-nonparametric estimation. In contrast, the trend variables were less prominent in the Bayesian estimator. Interestingly, the measures of input inefficiency were relatively similar.

Overall, durum wheat is the most price flexible over the sample. Revisiting Figure 2, which presents the cost normalized prices by wheat class, clearly exhibits that durum wheat has the most price volatility over the sample period. This is not surprising given its limited geographical production and stringent quality requirements. Moreover, it has limited substitutability with the exception of high quality hard red wheat (Barnes and Shields 1998). Soft white wheat exhibits the least flexibility. Mean price flexibilities across the two estimators for soft white wheat range from -0.10 to -0.13. Meanwhile soft red winter wheat ranges from -0.22 to -0.35. The difference in the mean price flexibilities across the estimators is surprising for hard red spring and winter wheat. Intuitively, the higher price

flexibilities from the Bayesian estimator seem more reasonable because they are higher quality wheat and prices depend on protein content (Parcell and Stiegert 1998; Bale and Ryan 1977).

**Conclusions**

We conceptualized and specified a normalized quadratic input distance function from which to derive inverse demand functions for the different wheat classes as an input into flour production. A semi-nonparametric estimator with fixed effects for input inefficiency and a Bayesian estimator with random effects for input inefficiency (both imposing curvature restrictions) were used to estimate a complete system of equations (distance function and inverse demand relationships) and to calculate price flexibilities for wheat food use by class.

Empirical findings of this study are important to policymakers. Overall, results were relatively robust across estimators in that the own-price flexibilities were all inelastic. Durum wheat exhibited the own-flexibility with the largest magnitude (-0.74 to -0.94), while the own-flexibilities of soft red winter (-0.22 to -0.35) and soft white (-0.10 to -0.13) wheat were relatively consistent across the two estimators. In contrast, hard red winter and hard red spring wheat exhibited larger changes in own-price flexibilities across the two estimators. Hard red winter wheat ranged from -0.03 to -0.52, while hard red spring wheat ranged from -0.10 to -0.37. Nevertheless these results indicate important differences in price formation across wheat classes and are supportive of government programs that no longer assume wheat to be a homogenous product. Programs that differentiate wheat by class address concerns of surplus (and other) problems arising from government policies that distort price spreads between different types and qualities of wheat (e.g., Farnsworth 1961).

**Table 1.** Descriptive statistics for nominal price and quantity data from 1974 to 1999.

| Variable | Mean | St. Dev. | Min | Max |
|---|---|---|---|---|
| Quantity of Flour (1000 cwt) | 332090.00 | 51405.00 | 251100.00 | 411970.00 |
| Price of Hard Red Winter ($US/bu) | 3.93 | 0.66 | 2.81 | 5.69 |
| Price of Hard Red Spring ($US /bu) | 3.94 | 0.67 | 2.83 | 5.64 |
| Price of Soft Red Wheat ($US /bu) | 3.43 | 0.63 | 2.19 | 4.83 |
| Price of Soft White Wheat ($US /bu) | 3.86 | 0.61 | 2.90 | 5.27 |
| Price of Durum ($US /bu) | 4.74 | 1.11 | 3.30 | 7.03 |
| Quantity of Hard Red Winter (million bu) | 305.35 | 45.95 | 251.00 | 387.00 |
| Quantity of Hard Red Spring (million bu) | 178.46 | 39.52 | 128.00 | 260.00 |
| Quantity of Soft Red Wheat (million bu) | 133.65 | 17.10 | 94.00 | 155.00 |
| Quantity of Soft White Wheat (million bu) | 54.23 | 14.50 | 31.00 | 85.00 |
| Quantity of Durum (million bu) | 53.15 | 17.63 | 32.00 | 80.00 |

**Table 2.** Semi-Nonparametric parameter estimates
from the normalized quadratic system. Study period from 1974 to 1999.[a]

| Coefficient | Coefficient Estimate | t-value | p-value |
|---|---|---|---|
| $b_0$ | -4.25930* | -3.37861 | 0.00073 |
| $b_1$ | 3.07246* | 23.41018 | 0.00000 |
| $b_2$ | 2.66231* | 19.06190 | 0.00000 |
| $b_3$ | 3.81680* | 13.26632 | 0.00000 |
| $b_4$ | 3.87449* | 12.19213 | 0.00000 |
| $b_5$ | 2.41177* | 5.73059 | 0.00000 |
| $b_{11}$ | 0.08385* | 2.27983 | 0.02262 |
| $b_{12}$ | -0.19495* | -2.75455 | 0.00588 |
| $b_{13}$ | 0.17115 | 0.65944 | 0.50961 |
| $b_{14}$ | 0.26480 | 0.85603 | 0.39198 |
| $b_{22}$ | 0.04855 | 0.16040 | 0.87257 |
| $b_{23}$ | 0.08073 | 0.04513 | 0.96400 |
| $b_{24}$ | 0.26063 | 0.59963 | 0.54875 |
| $b_{33}$ | 0.26194 | 0.31598 | 0.75202 |
| $b_{34}$ | -0.00327 | -0.00172 | 0.99862 |
| $b_{44}$ | -0.00003 | 0.00000 | 1.00000 |
| $b_{15}$ | -0.47943* | -18.37210 | 0.00000 |
| $b_{25}$ | -0.39437* | -14.60137 | 0.00000 |
| $b_{35}$ | -0.66601* | -11.61678 | 0.00000 |
| $b_{45}$ | -0.64058* | -10.59324 | 0.00000 |
| $b_{55}$ | -0.00006 | 0.00000 | 1.00000 |
| $b_{1t}$ | -2.64166* | -5.07957 | 0.00000 |
| $b_{2t}$ | 0.73554* | 2.93002 | 0.00339 |
| $b_{1970}$ | 0.01725* | -2.17105 | 0.02993 |
| $b_{1990}$ | 0.01549* | -2.58818 | 0.00965 |

[a] Quantity of flour was scaled by 100,000 in estimation.

* Significant at 10% level.

**Table 3.** Semi-Nonparametric price flexibility estimates from the normalized quadratic system with bootstrapped 90% percentile confidence intervals.

| Price | Equation Cholesky Decomposition Price Flexibilities | | | | |
|-------|----------|-----------|-----------|-----------|-----------|
|       | HRW | HRS | SRW | SWW | DUR |
| HRW | -0.03000* | 0.06969 | -0.06946 | -0.09552 | 0.17064 |
| HRS | 0.04021 | -0.09920* | 0.08216 | 0.09665 | -0.16227 |
| SRW | -0.02719 | 0.05574 | -0.22441* | -0.12513 | 0.53143* |
| SWW | -0.01663 | 0.02915 | -0.05563 | -0.10422* | 0.20169* |
| DUR | 0.03361 | -0.05538 | 0.26733* | 0.22822* | -0.74150* |

*90% confidence interval does not contain zero.

**Table 4.** Bayesian Regression estimates and confidence intervals from the normalized quadratic system. Study period from 1974 to 1999.[a]

| Coefficient | Coefficient Estimate | 90% Confidence Interval (Lower) | 90% Confidence Interval (Upper) |
|---|---|---|---|
| $b_0$ | -4.26837* | -4.35244 | -4.17123 |
| $b_1$ | 3.16721* | 2.98338 | 3.16467 |
| $b_2$ | 2.63845* | 2.57417 | 2.75 |
| $b_3$ | 3.91642* | 3.72912 | 3.90753 |
| $b_4$ | 3.84199* | 3.78666 | 3.95921 |
| $b_5$ | 2.4766* | 2.32197 | 2.50249 |
| $b_{11}$ | -0.08632* | -0.10409 | -0.03313 |
| $b_{12}$ | -0.01357 | -0.05099 | 0.06599 |
| $b_{13}$ | 0.01381 | -0.07089 | 0.06454 |
| $b_{14}$ | -0.0173 | -0.08321 | 0.06447 |
| $b_{22}$ | -0.02378* | -0.13679 | -0.03541 |
| $b_{23}$ | -0.02073 | -0.05745 | 0.0818 |
| $b_{24}$ | 0.02364 | -0.05059 | 0.09746 |
| $b_{33}$ | -0.11639* | -0.19665 | -0.04967 |
| $b_{34}$ | -0.07959 | -0.12494 | 0.02557 |
| $b_{44}$ | -0.17926* | -0.23322 | -0.06696 |
| $b_{15}$ | -0.40074* | -0.56946 | -0.38792 |
| $b_{25}$ | -0.39817* | -0.48715 | -0.30291 |
| $b_{35}$ | -0.74633* | -0.75767 | -0.5788 |
| $b_{45}$ | -0.59522* | -0.72857 | -0.54998 |
| $b_{55}$ | -0.01033* | -0.09562 | -0.00439 |
| $b_{1t}$ | -0.26847* | -0.35138 | -0.17638 |
| $b_{2t}$ | 0.07288 | -0.02305 | 0.10031 |
| $\mu$ | 0.10956* | 0.04192 | 0.95994 |

[a] Quantity of flour was scaled by 100,000 in estimation.
* Significant at 10% level.

**Table 5.** Bayesian Regression price flexibility estimates from the normalized quadratic system with 90% percentile confidence intervals.

| Price | Equation Bayesian Price Flexibilities | | | | |
|---|---|---|---|---|---|
| | HRW | HRS | SRW | SWW | DUR |
| HRW | -0.52289* | -0.02616 | -0.06202 | -0.07445 | 0.54002* |
| HRS | 0.00472 | -0.36521* | 0.02397 | 0.03032 | 0.16272 |
| SRW | -0.01793 | 0.01688 | -0.34736* | -0.08047 | 0.18212 |
| SWW | -0.01143 | 0.01454 | -0.0368 | -0.13307* | 0.05211 |
| DUR | 0.54753* | 0.35994 | 0.42222 | 0.25768 | -0.93697* |

*90% confidence interval does not contain zero.

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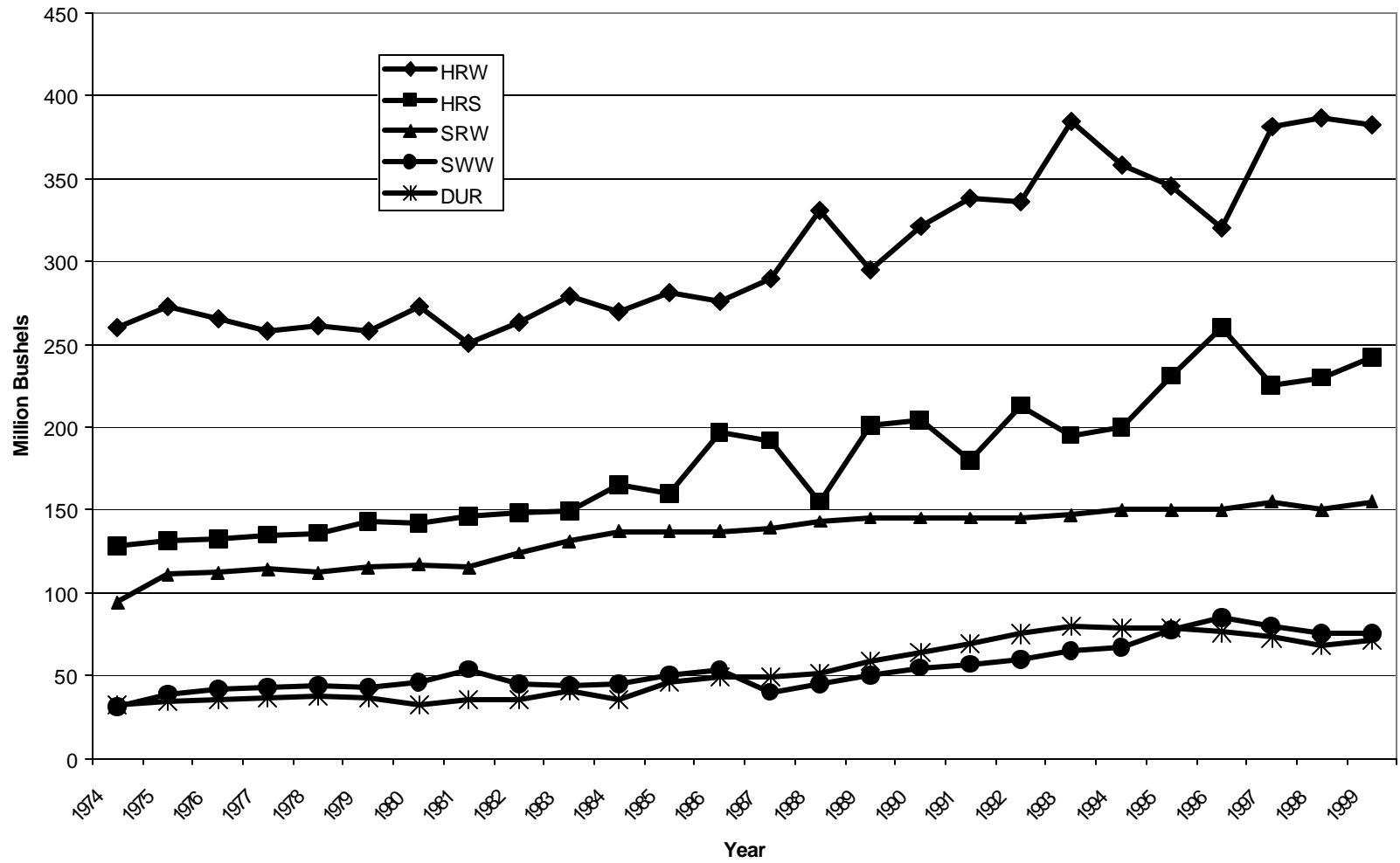
**Figure 1**. Domestic food use in the US by wheat class from 1974 to 1999.

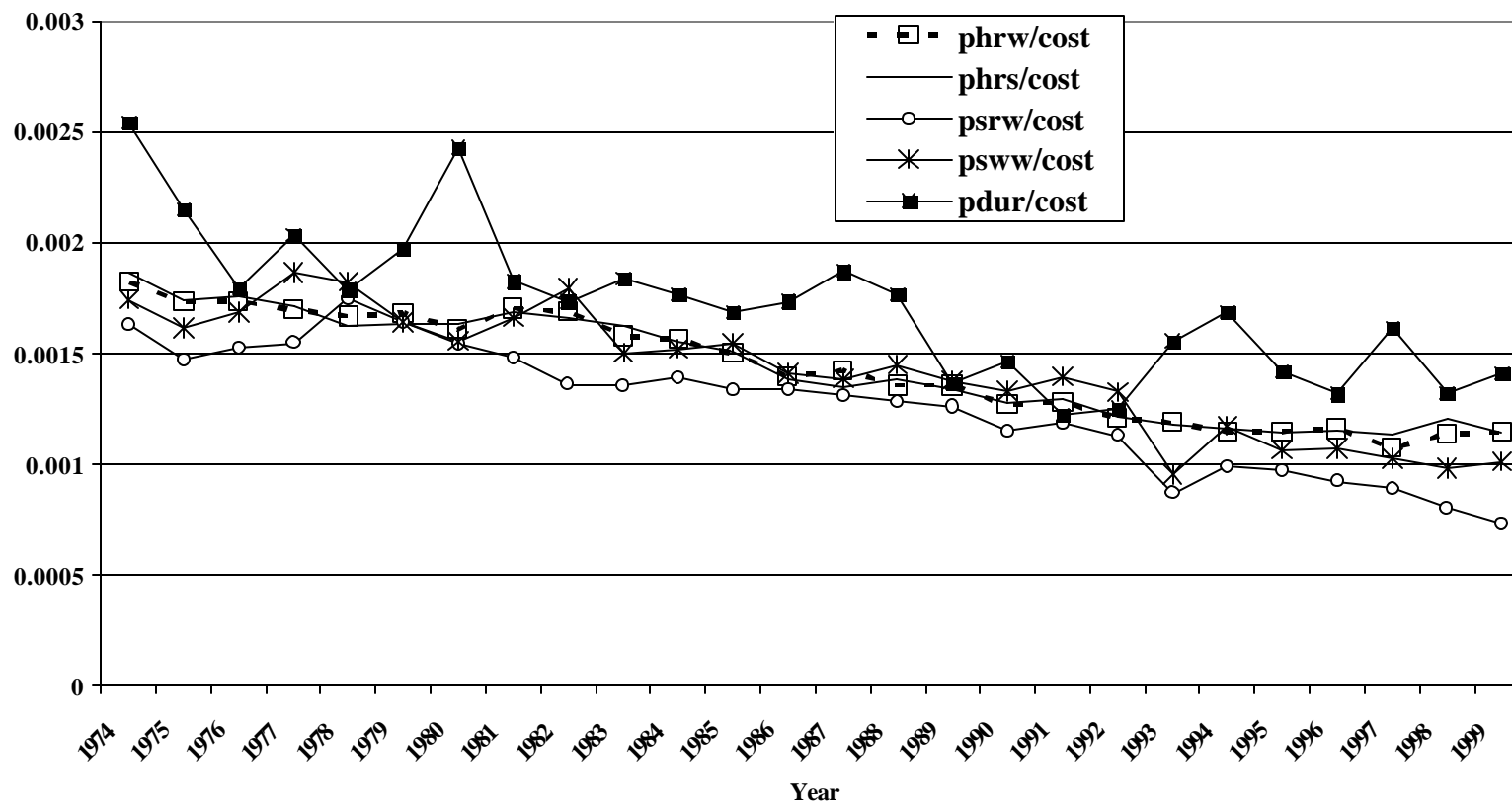**Cost Normalized Prices by Wheat Class**



**Figure 2. Cost Normalized Prices for domestic food use by wheat class from 1974 to 1999.**