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Yacov Tsur and Amos Zemel

ECONOMIC DEVELOPMENT CENTER

Department of Economics, Minneapolis

Department of Agricultural and Applied Economics, St. Paul

UNIVERSITY OF MINNESOTA

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Yacov Tsur[1] and Amos Zemel[2]

Abstract

The threat on the survival of animal species due to intensive use of natural resources is incorporated within resource management models, paying special attention to uncertainty regarding the conditions that lead to extinction. The manner in which the potential benefits forgone due to the species extinction (denoted extinction penalty) induce more conservative exploitation policies is studied in detail. When the extinction penalty is ignored, the optimal policy is to drive the resource stock to a particular equilibrium level from any initial state. When the extinction penalty is considered and the conditions that lead to extinction are not fully understood (i.e., involve uncertainty), an interval of equilibrium states is identified, which depends on the penalty and the immediate extinction risk.

Correspondence address:
Yacov Tsur
Department of Agricultural and applied Economics
University of Minnesota
1994 Buford Avenue
St. Paul, MN 55108-6040
(612) 624-4746; Fax (612) 625-2729; ytsur@vm1.spcs.umn.edu

[1]Department of Agricultural and Applied Economics, University of Minnesota, 1994 Buford Avenue, St. Paul, MN 55108, USA, and Department of Agricultural Economics and Management, The Hebrew University of Jerusalem, P.O. Box 12, Rehovot 76100, Israel

[2]Center for Energy and Environmental Physics, The Jacob Blaustein Institute for Desert Research, Ben Gurion University of the Negev, Sede Boqer Campus, 84990, Israel

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I. Introduction

In August 1973, the $100 million Tellico dam was almost complete when a University of Tennessee zoologist discovered that the Little Tennessee River (soon to be turned into a reservoir) is home to the snail darter—a previously unknown species. At the same year, the Endangered Species Act (ESA) was enacted to protect endangered (under risk of extinction) and threatened (likely to become endangered in the foreseeable future) species. The snail darter was soon enlisted as endangered and a lawsuit (Hill v. Tennessee Valley Auth.) was filed against the Tennessee Valley Authority that owned the dam. Observing that "Congress...had chosen the snail darter over the dam," the Supreme Court had no choice but to rule against Tellico (Littell, 1992, p. 3), turning the incomplete dam into a giant monument of the "extinction vs. coexistence" dilemma and changing forever the landscape of wildlife protection law.

The Tellico project remained in limbo until rescued by Congress in 1978. The 1978 amendments to the ESA directed the Secretary of the Interior to designate a "critical habitat" to newly enlisted species and to consider economic impacts (Littell, 1992, p. 11). An important provision of the amendments allows for exemption of a federal project from the Act if it is determined (by a cabinet-level Endangered Species Committee) that "...the benefits of the proposed federal actions clearly outweigh the benefits of preserving the species." (US-GAO, 1992, p. 8). It is with this amendment in mind that we set to study the exploitation of a natural resource that serves a dual purpose: first, it serves human needs and is therefore exploited for beneficial use (however defined); second, it serves as a habitat for an animal population whose existence depends on adequate quality and quantity of the resource, without which the species faces an extinction risk.

The Tellico story has a happy end: The dam was exempted from the Act and completed in 1979; soon after, the snail darter was found in other rivers, where it flourishes today, and was reclassified as threatened. But a Tellico-like situation, in which resource exploitation for human needs comes at the expense of habitats essential for the existence of other species, is often encountered and will become more pervasive as the competition for the world's finite resources grows fiercer with the growing human population compounded with a rising standard of living.

• In California, water diversion from the Sacramento and San Joaquin Rivers had been restricted to protect fish and wildlife in the Delta Estuary (Fisher, Hanemann and Keeler, 1990). Coming at the expense of water entitlements for farms and cities in the Central Valley and Southern California, these restrictions lead to intense political struggles between environmentalists, farmers and city dwellers.

• In North America, the American Fisheries Society lists 364 species of fish as endangered or threatened, most are at risk due to habitat destruction. Only four adult Snake River sockeye salmon managed to reach their spawning ground in 1991, swimming from the Pacific Ocean, through eight dams in the Columbia River basin, to Idaho's Redfish Lake (Apostol, 1993).

• Reclamation of swamps and wetlands comes at the expense of habitats for migrating birds, some of which are already on the Endangered List (Weitzman, 1993).

• At the current rate, a forest area about the size of England is being cleared every year (Hartwick, 1992), leading to the extinction of incalculable number of species (Colinvaux, 1989).

• In 1988, the world population of the Chinese river dolphin, found primarily in the Yangtze River in east central China, was estimated at 300 individuals. The population decline is attributed in part to river construction (Thompson, 1988).

In the above examples the species under extinction risk may not contribute directly to human well being, yet a benefit is assigned to their very existence — a benefit often referred to as existence or nonuse value. The species may also contribute to recreational activities, so elimination entails the loss of a recreational option, and their survival contributes to biodiversity (Weitzman [1992, 1993]; Polasky, Solow and Broadus [1993]). While economists may disagree on how to measure such extra-market benefits (Hausman [1993]; Hanemann [1994]), the notion that preserving genetic diversity is desirable commands a wide consensus (about one half of medicine prescriptions originate from organisms found in the wild [Littell, 1992, p. 5, Bird, 1991]).

In this study we incorporate extinction risk within resource management models, paying special attention to uncertainty regarding the conditions that lead to extinction. As a metaphor, we consider an animal species that requires a minimal level of the resource stock to maintain its livelihood. Below this critical level the species is doomed to extinction. The critical level at which extinction occurs is incompletely known, reflecting our partial ignorance of the species ecology.

We investigate how the threat on the existence of an animal population should affect the way nature is exploited for human needs. We shall not be concerned with moral or ethical issues regarding whether or not to preserve, nor shall we deal with how to measure the benefit of preservation (this has been done by others, of which some are mentioned above). We simply assume that preservation is valuable or, alternatively, that extinction entails a penalty represented by a fine (exogenously determined) to be incurred at the time extinction occurs. It is hardly surprising that the size of this penalty has a profound effect on the optimal exploitation policy, though less obvious is how this effect is manifested. We derive below the precise manner in which the extinction risk and penalty affect optimal exploitation policies. This should help focus environmental debates on the main issues, away from groundless rhetoric.

The present work draws on the analysis of Tsur and Zemel (1994a), who extended earlier works of Cropper (1976) and Heal (1984) to study groundwater extraction under uncertainty with regard to the occurrence of an event (such as salt water intrusion) that irreversibly ruins the aquifer. The analysis was later extended to consider events that are partly reversible (i.e., their damage can be fixed at a cost) and used to incorporate global warming risks within models of fossil fuel usage (Roe, Tsur and Zemel, 1994; Tsur and Zemel, 1994b). The present effort extends the partly reversible model to situations where resource exploitation for human needs comes into conflict with other species survival. The extent of reversibility is different in the present model, since, apart from the penalty, event occurrence removes further extinction-related constraints on future plans. Similarities with the previous models allow us to refer to the above mentioned works for some of the more technical derivations.

II. The problem

To be concrete, we discuss the problem in the context of a water stream that supports a wildlife habitat if left instream, e.g., by improving spawning of some fish population, and that is also demanded as an input of production by irrigators and other manufacturers. Let S denote the state of the water stream, measured, say, by the water level at some crucial point along the river. Net natural replenishment — inflows from surface streams and springs minus outflows — is represented by R(S), and off stream diversion rate is denoted by g. These two processes determine the time evolution of S:

$$dS_t/dt \equiv \dot{S}_t = R(S_t) - g_t. \tag{2.1}$$

At the state level $\bar{S}$ net recharge is nil, i.e., $R(\bar{S}) = 0$. The cost of diverting g at the state S is C(S)g, where C(S) is the unit diversion cost, and the benefit of consuming g is Y(g). The net benefit of consuming g at the level S is Y(g)-C(S)g.

The following assumptions are made: **(i)** R(S) is decreasing and concave; **(ii)** C(S) is non-increasing and convex; and **(iii)** Y(g) is increasing and strictly concave with Y(0) = 0. The properties of C and Y are common. The properties of R are typical of a resource stock that is recharged from exogenous sources (i.e., does not reproduce itself), such as a water stream.

An exploitation policy (or plan) consists of the extraction process g_t and the associated state process S_t, $t \geq 0$. A plan is feasible if, for all t, g_t is piecewise continuous and nonnegative, and $S_t \geq 0$.

Excessive off stream diversion may lead to species extinction. To retain simplicity, we assume that only one species is at risk. When extinction occurs, a penalty of size ψ is inflicted and the exploitation process proceeds thereafter with no further extinction risk. The penalty ψ represents benefits forgone due to extinction and is treated here as an exogenous parameter.

Let $V^p(S)$ denote the post-event value function, starting from $S_0 = S$. Since no further risk is to be considered after occurrence, $V^p(S)$ is given by

$$V^p(S) = \underset{\{g_t\}}{\text{Max}} \int_0^\infty [Y(g_t)-C(S_t)g_t]e^{-\rho t}dt \tag{2.2}$$

subject to: $\dot{S}_t = R(S_t)-g_t$, $g_t \geq 0$, $S_t \geq 0$ and $S_0 = S$, where ρ is the time rate of discount. $V^p(S)$ is also the value corresponding to the pre-event problem with no extinction penalty (i.e., when $\psi = 0$).

Let X represent the minimum state level of the resource required to maintain a viable population: when S falls below X, the extinction event occurs. Suppose that X is known with certainty and the event has not yet occurred (i.e., $S_0 > X$) and let $V^c(S_0,X)$ denote the corresponding value function. The case $V^c(X,X)$, when $S_0 = X$, is of particular interest, because the planner must decide immediately whether to cross the critical level, enjoying the benefit $V^p(X)-\psi$, or to stay at or above it, avoiding the penalty. The conditions for either decision are analyzed in the next section, and the corresponding optimal benefit $\varphi(X) \equiv V^c(X,X)$ is derived.

The problem when X is known with certainty can now be formulated in terms of $\varphi(X)$. Let $T > 0$ be the first time the state process reaches the level X (if X is never encountered, $T = \infty$). Then,

$$V^c(S,X) = \underset{\{g_t,T\}}{\text{Max}} \int_0^T [Y(g_t)-C(S_t)g_t]e^{-\rho t}dt + e^{-\rho T}\varphi(X) \qquad (2.3)$$

subject to: $\dot{S}_t = R(S_t)-g_t$, $g_t \geq 0$, $S_t > X$, $S_T = X$ (if $T < \infty$), $S_0 = S > X$.

The critical state level needed to maintain a viable population is in general incompletely known and can be specified only in terms of the distribution and density functions $F(S) = \Pr\{X<S\}$ and $f(S) = dF/dS$. The distribution F is assumed to be continuously differentiable over $[\underset{\sim}{S},\bar{S}]$, where $\underset{\sim}{S} \geq 0$ is the highest instream level at which extinction is bound to occur.

Under this type of uncertainty, it is convenient to let X represent the state level at which the event occurs, so that T becomes the occurrence date. The distribution on X induces a distribution on the occurrence time T as well. Given that the event has not yet occurred, we search for the exploitation policy corresponding to

$$V(S_0) = \underset{\{g_t\}}{\text{Max}} \; E_T\left\{\int_0^T [Y(g_t)-C(S_t)g_t]e^{-\rho t}dt + e^{-\rho T}[V^p(S_T)-\psi] \mid T>0\right\} \tag{2.4}$$

subject to $\dot{S}_t = R(S_t)-g_t$, $g_t \geq 0$, $S_t \geq 0$, and S_0 given. In (2.4), E_T represents expectation with respect to the distribution of T. We assume that an optimal solution exists.

As the process evolves in time, our assessment of the distributions of X and T is modified, as X must lie below $\tilde{S}_t = \underset{\tau \in [0,t]}{\text{Min}} \{S_\tau\}$. The expected benefit (2.4), thus, involves $\tilde{S}_t$ which depends on all history to time t, unless S_t evolves monotonically in time, in which case $\tilde{S}_t = S_t$ or $\tilde{S}_t = S_0$ if S_t is non-increasing or S_t is non-decreasing, respectively. It turns out (see Appendix A for a proof) that:

Property 2.1: At least one of the *optimal* S-trajectories corresponding to (2.4) evolves monotonically in time.

We therefore restrict attention to monotonic trajectories.

For non-decreasing S trajectories, it is known with certainty that the event will never occur and the exploitation problem reduces to (2.2). For non-increasing state processes, the distribution of T is given by

$$1-F_T(t) \equiv \Pr\{T>t \mid T>0\} = \Pr\{X<S_t \mid X<S_0\} = F(S_t)/F(S_0) \tag{2.5}$$

with the density $f_T(t) \equiv dF_T(t)/dt = f(S_t)[g_t-R(S_t)]/F(S_0)$. The hazard rate associated with T is $f_T(t)/[1-F_T(t)] = \lambda(S_t)[g_t-R(S_t)]$, where

$$\lambda(S) \equiv f(S)/F(S). \tag{2.6}$$

It is assumed that $\lambda(S)$ is non-increasing.

Express the expectation in (2.4) as

$$E_T\left\{\int_0^\infty [Y(g_t)-C(S_t)g_t]I(T>t)e^{-\rho t}dt + e^{-\rho T}[V^p(S_T)-\psi] \mid T>0\right\},$$

with $I(\cdot) = 1$ or $I(\cdot) = 0$ when its argument is true or false, respectively. Since $E_T\{I(T>t)\,|\,T>0\} = 1-F_T(t) = F(S_t)/F(S_0)$, the objective function for non-increasing trajectories becomes

$$\int_0^\infty \{Y(g_t)-C(S_t)g_t+\lambda(S_t)[g_t-R(S_t)][V^p(S_t)-\psi]\}e^{-\rho t}\,\frac{F(S_t)}{F(S_0)}\,dt. \tag{2.7}$$

The allocation problem for which (2.7) is the objective is denoted the *auxiliary problem*. It is verified in Appendix A that the optimal state processes corresponding to the post-event and auxiliary problems evolve monotonically in time.

In the following section we use the optimal post-event plan to characterize the optimal exploitation policy under certainty—when the critical state level X is completely known. Using the optimal state processes of the post-event problem and of the auxiliary problem, we characterize, in Section 4, the exploitation policy under uncertainty—when X is known up to a probability distribution.

III. Certainty

The post-event value function $V^p(S)$ plays important roles in both the certainty and uncertainty problems: it affects the allocation problem under certainty in that it determines the terminal value $\varphi(X)$ at time T (see Eq. (2.3)), and it directly enters the uncertainty objective function (see Eq. (2.7)). The post event problem is similar to the certainty problem analyzed in Tsur and Zemel (1994a). We summarize below its main properties. A differential equation for the evolution of the optimal extraction rate is presented in Appendix B. The value function $V^p(S)$ is calculated given the optimal extraction trajectory.

Define $J(S) \equiv \frac{-C'(S)R(S)}{\rho-R'(S)}$ and $L(S) = [\rho-R'(S)][Y'(R(S))-C(S)-J(S)]$, and let $\hat{S}$ be the state level satisfying

$$\begin{cases} \hat{S} = 0 & \text{if } L(0) > 0 \\ \hat{S} = \bar{S} & \text{if } L(\bar{S}) < 0 \\ L(\hat{S}) = 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

Since ρ-$R'(S) > 0$, the roots of $L(S)$ are the same as the roots of $Y'(R(S))-C(S)-J(S)$. The properties of Y, R and C ensure that the latter function increases with S, hence $\hat{S}$ is unique. It follows (see Appendix B and Tsur and Zemel, 1994a), that:

Property 3.1: $\hat{S}$ is the unique steady state to which the optimal state process corresponding to the post-event problem converges from any initial state.

At equilibrium, the post-event value reduces to the equilibrium benefit $W(\hat{S})$ where $W(S) \equiv [Y(R(S))-C(S)R(S)]/\rho$. Outside equilibrium, the determination of $V^p(S)$ requires the optimal trajectory (see Appendix B).

Turning to the certainty problem (2.3), where the critical level X is known in advance, we consider initial states at or above X, for otherwise the event has already occurred and the post-event analysis applies.

Next, we assert that the optimal state process S_t^c cannot decrease when starting at $S_0 \leq \hat{S}$. For if it does, the monotonicity of S_t^c requires it to approach some value below $\hat{S}$. However, according to Property 3.1, S_t^c yields a lower benefit than the optimal post-event process S_t^p (initiated at the same level S_0) even when S_t^c carries no penalty. Moreover, the non-decreasing process S_t^p yields the same value for both the certainty and the post-event problems, hence it must be optimal for the former problem as well.

For the same reason it cannot be optimal to trigger the event when $X \leq \hat{S}$. Thus, the certainty problem may differ from the post-event problem only when $\hat{S} < X \leq S_0$, which we maintain for the reminder of this section.

Define

$$\hat{S}_c = \begin{cases} \hat{S} & \text{if } V^p(X)\text{-}W(X) > \psi \\ X & \text{if } V^p(X)\text{-}W(X) < \psi \end{cases} \tag{3.2}$$

(The singular case $V^p(X)\text{-}W(X) = \psi$ will be discussed separately.) We show that

Property 3.2: When $\hat{S} < X \leq S_0$ and $V^p(X)\text{-}W(X) \neq \psi$, $\hat{S}_c$ is the unique steady state to which the optimal state process of the certainty problem converges.

Proof: Consider any state $S > X$. We show that S cannot be an equilibrium state by constructing a plan, initiated at S, that yields a higher value than the equilibrium benefit W(S). For some arbitrary small constants $h>0$ and $\delta>0$, define the extraction plan, starting at the state S

$$g_t^{\delta h} = \begin{cases} R(S) + \delta, & 0 \leq t < h \\ R(S_h) \quad , & t \geq h \end{cases} \tag{3.3}$$

When the product $h\delta$ is small enough, the new equilibrium level S_h lies above X and the event does not occur. Under these conditions, the benefit $V^{\delta h}(S)$ associated with $g_t^{\delta h}$ is found to be (Tsur and Zemel, 1994a):

$$V^{\delta h}(S) - W(S) = L(S)\delta h/\rho + o(\delta h). \tag{3.4}$$

Since $S > \hat{S}$, $L(S) > 0$ and there exist $h > 0$ and $\delta > 0$ such that $V^{\delta h}(S) - W(S) > 0$. Thus, the steady state plan that yields the value W(S) is not optimal, ruling out the possibility that S is a steady state.

A state $S < X$ can be reached only during the post-event period, for which Property 3.1 implies that only $\hat{S}$ can be a steady state. It follows that only $\hat{S}$ or X may qualify as steady states. Now, to reach $\hat{S}$, the optimal plan must pass through X. While at X, it is always possible to enter a steady state and enjoy the benefit W(X). Proceeding towards $\hat{S}$ entails triggering the event and enjoying the benefit $V^p(X)\text{-}\psi$. Thus, it pays to proceed to $\hat{S}$ if and only if $V^p(X) - W(X) > \psi$.■

The above consideration implies that if $V^p(X)-W(X) = \psi$, *both* $\hat{S}$ and X are optimal steady states, and the choice between the processes leading to each of them is arbitrary. This singular case marks the transition from the equilibrium state $\hat{S}$, typical of the post-event (or penalty-free) problem, to the critical level X. We call the function $V^p(X)-W(X)$ the *implicit penalty*. When the actual penalty ψ exceeds the implicit penalty, extinction cannot be optimal.

Since the steady-state policy is always available, $V^p(S) \geq W(S)$, equality holding at the equilibrium state $\hat{S}$. Hence, also $V^{p\prime}(\hat{S}) = W'(\hat{S})$. Above $\hat{S}$, it is verified in Appendix B, the implicit penalty cannot decrease:

$$V^{p\prime}(S) - W'(S) \geq 0 \quad \text{for all } S > \hat{S}. \tag{3.5}$$

Thus, populations with larger critical levels X require higher penalties ψ to be saved from extinction.

In view of (2.3), The optimal state process of the certainty problem S_t^c is characterized as follows:

(i) $X \leq \hat{S}$: Here, $S_t^c = S_t^p$, which is the optimal state process of the post-event problem. In this case extinction is not desirable even with vanishing penalty and the certainty solution is the same as that of the post-event problem.

(ii) $\hat{S} < X \leq S_0$ and $V^p(X)-W(X) \leq \psi$: Here, S_t^c, $t \in [0,T]$, and T are found by solving

$$V^c(S_0,X) = \underset{\{g_t,T\}}{\text{Max}} \int_0^T [Y(g_t)-C(S_t)g_t]e^{-\rho t}dt + e^{-\rho T}W(X)$$

subject to $\dot{S}_t = R(S_t)-g_t$, $g_t \geq 0$ and $S_T = X$. For $t \geq T$, $S_t^c = X$. In this case the penalty is sufficiently high to prevent extinction, and $\varphi(X) = W(X)$.

(iii) $\hat{S} < X \leq S_0$ and $V^p(X)-W(X) > \psi$: Here, S_t^c, $t \in [0,T]$, and T are found by solving

$$V^c(S_0,X) = \underset{\{g_t,T\}}{\text{Max}} \int_0^T [Y(g_t)-C(S_t)g_t]e^{-\rho t}dt + e^{-\rho T}[V^p(X)-\psi]$$

subject to $\dot{S}_t = R(S_t)-g_t$, $g_t \geq 0$, and $S_T = X$. For $t \geq T$, S_t^c is the same as S_t^p that departs from X. In this case the penalty is not sufficiently high to prevent extinction, and $\varphi(X) = V^p(X)-\psi$.

Under certainty, the most dramatic effect of inflicting a penalty upon extinction is seen to be the shift of the equilibrium state from $\hat{S}$ to the critical level X when the extinction penalty ψ exceeds the implicit penalty $V^p(X)-W(X)$. We proceed to show, in the following section, that under event uncertainty the penalty effect assumes a very different nature.

IV. Uncertainty

In characterizing the optimal policy of the uncertainty problem (2.4) we make use of the optimal policies of the post-event and the auxiliary problems. The former has been studied above. The latter, in light of (2.7), is formulated as:

$$V^a(S_0) = \underset{\{g_t\}}{\text{Max}} \int_0^\infty \{Y(g_t)-C(S_t)g_t+\lambda(S_t)[g_t-R(S_t)][V^p(S_t)-\psi]\}e^{-\rho t}\frac{F(S_t)}{F(S_0)}dt \qquad (4.1)$$

subject to $\dot{S}_t = R(S_t)-g_t$, $S_t \geq \hat{S}$, $g_t \geq 0$, and $S_0 \geq \hat{S}$ given. As will soon become apparent, the auxiliary problem is relevant only for state levels in $[\hat{S},\bar{S}]$, hence the constraint $S_t \geq \hat{S}$.

Define

$$L_\psi(S) = L(S) + \rho\lambda(S)[V^p(S)-W(S)-\psi] \qquad (4.2)$$

where it is recalled that $L(S) = [\rho-R'(S)][Y'(R(S))-C(S)-J(S)]$. Let $\hat{S}_\psi$ be some equilibrium state of the auxiliary problem (i.e., at $S = \hat{S}_\psi$, the optimal off-stream diversion rate corresponding to (4.1) equals R(S) and it remains at this level forever). Then, following the analysis of the post-event problem (see appendix B), it is verified that:

Property 4.1: Any equilibrium state $\hat{S}_\psi$ of the auxiliary problem satisfies:

$$\begin{cases} \hat{S}_\psi = \bar{S} & \text{if } L_\psi(\bar{S}) < 0 \\ L_\psi(\hat{S}_\psi) = 0 & \text{otherwise} \end{cases} \tag{4.3}$$

According to (4.3), any equilibrium state in $[\hat{S},\bar{S})$ must be a root of L_ψ. In this respect, the function L_ψ generalizes L(S) of the post-event problem. Indeed, under certainty, when $\lambda(S)$ vanishes, the two functions coincide. (Interestingly, when ψ vanishes, i.e., when extinction entails no penalty, L_ψ does *not* reduce to L, but satisfies $L_\psi \geq L$, equality holding when $S = \hat{S}$. Therefore, both functions have the same unique root in $[\hat{S},\bar{S})$, namely $\hat{S}$, yielding the same equilibrium state.)

Can the auxiliary problem admit multiple equilibria in $[\hat{S},\bar{S}]$? The answer, in light of Property 4.1, depends on whether (4.3) has more than one solution. It turns out that $\hat{S}_\psi$ of (4.3) is unique. To see this, differentiate (4.2) to obtain

$$L_\psi'(S) = L'(S) + \rho\lambda'(S)[V^p(S)-W(S)-\psi] + \rho\lambda(S)[V^{p\prime}(S)-W'(S)]. \tag{4.4}$$

Recalling that $V^p(\hat{S})-W(\hat{S}) = L(\hat{S}) = 0$, we see that $L_\psi(\hat{S}) = -\rho\lambda(\hat{S})\psi < 0$, assuming that both ψ and $\lambda(\hat{S})$ are positive. Clearly, for states $S > \hat{S}$ satisfying $V^p(S)-W(S) \geq \psi$, we have $L_\psi(S) \geq L(S) > 0$ and such states cannot be roots of L_ψ. Thus, attention can be restricted to states $S > \hat{S}$ for which $V^p(S)-W(S) < \psi$. Now, since L(S) is increasing and $\lambda(S)$ is non-increasing, the sum of the first two terms on the right-hand side of (4.4) is positive, while (3.5) ensures that the last term is nonnegative. It follows that $L_\psi(S)$ must increase in the relevant interval, and there exists a unique state level $\hat{S}_\psi$ in $(\hat{S},\bar{S}]$, satisfying (4.3).

Property 4.1, then, implies that $\hat{S}_\psi$ is the unique equilibrium state of the auxiliary problem. Being monotonic (cf. Appendix A) and bounded, the optimal state trajectory of the auxiliary problem (henceforth denoted S_t^a) must converge to an equilibrium state. We have thus established:

Property 4.2: $\hat{S}_{\psi}$ is the unique steady state to which the optimal state process corresponding to the auxiliary problem (4.1) converges from any initial state in $[\hat{S},\bar{S}]$.

Note the similarity of this result and Property 3.1. The role of $V^p(S)-W(S)-\psi$ in shifting the equilibrium state is obvious. Unlike the certainty problem, the equilibrium state does not jump abruptly from $\hat{S}$ when the penalty ψ exceeds the implicit penalty. Rather, the auxiliary equilibrium state $\hat{S}_{\psi}$ changes continuously as ψ is increased.

The optimal state process under *uncertainty* can now be characterized using the optimal processes of the post-event and auxiliary problems in the same way it is done in Tsur and Zemel (1994a-b). We briefly sketch the main idea, avoiding technical details that can be found in these works. We denote by S_t^* the optimal state process under uncertainty and recall that S_t^p and S_t^a are the optimal state processes of the post-event and auxiliary problems.

First, note that starting at $S \leq \hat{S}$, it can never be optimal to extract above recharge, decreasing S. Such a policy is not desirable even without a penalty (S_t^p does not decrease when it lies at or below $\hat{S}$). Following the post-event path, then, involves no occurrence risk (with its associated penalty) and cannot be outperformed by some decreasing path that involves a positive risk of triggering the event and having to pay the penalty. With no extinction risk, the optimal plan coincides with the post-event (or certainty) solution and $S_t^* = S_t^p$.

Next, observe that S_t^* cannot increase when departing from an initial state at or above $\hat{S}$. This is so because for the post-event problem, the steady state policy yields a higher benefit than any increasing plan starting at $S \geq \hat{S}$. Now, uncertainty does not affect the benefit associated with nondecreasing plans, hence the steady state policy outperforms all increasing plans under uncertainty as well, and S^*_t cannot increase.

It turns out that S^*_t decreases when it departs from $S > \hat{S}_\psi$ and it is in equilibrium when initiated at $S \in [\hat{S},\hat{S}_\psi]$. To verify the former requires only showing that states for which $S > \hat{S}_\psi$ cannot be equilibrium states. This is done by comparing the value $V^{\delta h}(S)$ generated by the extraction plan $g_t^{\delta h}$ of (3.3) with the equilibrium benefit W(S) and finding that

$$V^{\delta h}(S) - W(S) = L_\psi(S)\delta h/\rho + o(\delta h).$$

(The difference with (3.4) is due, of course, to occurrence risk during $t<h$.) Since $L_\psi(S) > 0$ for $S > \hat{S}_\psi$, it is possible to obtain a value higher than the equilibrium value W(S) (when $\delta h>0$ is small enough). Thus, the equilibrium plan cannot be optimal.

To show that $[\hat{S},\hat{S}_\psi]$ consists of the equilibrium points of S^*_t, requires to show that S^*_t cannot decrease in this interval (it has already been established that it cannot increase). If S^*_t decreases, it should coincide with S^a_t. But below $\hat{S}_\psi$, the latter is increasing. Thus, S^*_t cannot decrease.

The above discussion is summarized in

Property 4.3: Let S^*_t be the optimal process corresponding to the uncertainty problem (2.4). Then: (i) S^*_t increases while passing through S levels below $\hat{S}$; (ii) S^*_t decreases while passing through S levels above $\hat{S}_\psi$; (iii) the interval $[\hat{S},\hat{S}_\psi]$ consists of the equilibrium states of S^*_t.

Indeed, parts (i) and (ii) verify our intuition that S^*_t should follow the auxiliary path when it decreases and it should coincide with the post-event trajectory when it increases. Together, parts (i) and (ii) imply that S^*_t converges to the boundaries of the equilibrium interval $[\hat{S},\hat{S}_\psi]$ from any initial state outside this interval. Entering the interval cannot be optimal because the expected loss due to event occurrence outweighs the potential gain. The optimal state process under uncertainty is now completely defined in terms of the optimal process S_t^p and S_t^a.

Clearly, the equilibrium interval depends on the difference between the implicit and actual penalties and on the extinction risk through the expected loss term $\lambda(S)[V^p(S)-W(S)-\psi]$. In fact, for a very high penalty, when $L_\psi(\bar{S}) \le 0$, every state above $\hat{S}$ must be an equilibrium state, and the policy of extracting above the recharge rate is never optimal. Comparing with the results of the certainty problem, the dramatic effect of uncertainty is apparent: rather than a mere shift of the equilibrium state, a full equilibrium interval emerges. This behavior is typical of the class of problems involving uncertain events studied by Tsur and Zemel (1994a, 1994b).

V. Closing Comments

Hardly a day goes by without one reading or hearing about some environmental disaster creeping at our doorstep or a natural catastrophe of that sort or another soon to occur. While some of these alarms turn out to be premature or even false, others pose real threats on the well-being of the living species on this Earth, and even more so, of future generations. It appears that we have been exploiting our natural environment beyond its regenerative capacity, leaving a lesser, degraded part of it from year to year. This process is irreversible to the extent that parts of the environment are lost forever. The possibility of irreversible losses requires prudent management because mistakes cannot be fixed. This is the essence of the "extinction vs. coexistence" dilemma considered in this work.

As so much is at stake, environmental debates are often loaded and tend to be long on emotion and short on economic rationale. Our aim in this work is to contribute some to the latter: We offer a framework for analyzing a situation in which natural resource exploitation may lead to the extinction of other species that do not contribute directly to human well-being. The species very existence, however, entails a biodiversity value as well as other "nonuse" benefits, hence its extinction inflicts an economic penalty.

When the extinction penalty is ignored, the optimal policy is to drive the resource stock to a particular equilibrium state ($\hat{S}$) from any initial state. When the penalty is considered and the state at which extinction occurs is known (and lies above $\hat{S}$), extinction occurs only if the benefit associate with it (penalty included) exceeds the benefit of maintaining the resource state just above the extinction level.

Extinction conditions, however, are often incompletely known and may be specified in terms of a probability distribution on the critical state level needed to maintain the species. For this case, we identify an interval of equilibrium states whose lower bound is $\hat{S}$. The upper bound $\hat{S}_{\psi}$ depends on the extinction penalty and on the immediate extinction risk. Processes initiated above the equilibrium interval converge to $\hat{S}_{\psi}$ rather than to $\hat{S}$. This behavior manifests how the economic value of preservation, as well as our partial ignorance of the associated ecology, should affect the way we deal with our natural resources.

The choice of the appropriate value of the extinction penalty allows for the great flexibility of the model under study. Its limiting values correspond to the extreme views that preservation must be guaranteed at all cost, or conversely, that only direct human benefits should determine exploitation policies. For a sound discussion of concrete situations, a rationale evaluation of the extinction penalty is clearly called for.

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Appendix A: Monotonicity of the state processes

In this appendix we consider the monotonicity properties of the optimal state processes associated with the various optimization problems introduced in Section 2. For each of these problems, we show

Proposition (Monotonicity): At least one of the optimal state trajectories evolves monotonically in time.

The proposition implies that if any of the optimization problems (2.2), (2.4) or (2.7) admits a unique solution, the corresponding state process must be monotonic. For problems with multiple solutions, at least one solution must be monotonic.

Proof: We begin with the simpler post-event and auxiliary problems. Consider first the case in which the optimal trajectory corresponding to (2.2) or (2.7) is unique. Suppose that S_t is not monotonic. For concreteness, consider three distinct time values, $t_1 < m < t_2$, such that $S_{t_1} < S_m$ and $S_{t_2} < S_m$. Since S_t is time-continuous, there must exist some $t_3 \in (t_1,m)$, at which S_t increases, and some $t_4 \in (m,t_2)$, at which S_t decreases, such that $S_{t_3} = S_{t_4}$. However, Y, C, R, F and ρ do not depend on t explicitly, hence the same decision problem is encountered at t_3 and at t_4. Thus, one cannot arrive at conflicting decisions concerning the sign of g_t-$R(S_t)$ at these times, since the optimality of both decisions violates the uniqueness of the optimal plan. This argument applies also when S_m corresponds to a minimum rather than to a maximum.

For problems with multiple optima, some optimal S trajectory may not be monotonic. We shall show, however, that it is possible to construct a monotonic plan from a non-monotonic one. Observe, first, that the optimality of the decisions at t_3 and t_4 implies that one can choose either g_{t_3} or g_{t_4} at t_3 and t_4 and obtain the same value. Furthermore, this freedom of choice prevails at any state level between S_{t_3} and S_m. Thus, the existence of a local extremum of S implies the existence of a continuum of feasible plans, all yielding the optimal value. To construct a monotonic plan, one specifies, for any state S permitting several optimal diversion rates, a particular selection rule to ensure that whenever S is encountered, the same diversion rate is adopted. For example, one can demand that among the optimal diversion rates, the minimal optimal diversion rate is selected. The ensuing plan is optimal and monotonic, because non-monotonic plans involve conflicting choices of diversion rates at the same state levels.

The uncertainty problem (2.4) differs from the post-event and auxiliary problems in that decisions may depend on history. This means that passing through the same state at different times may lead to conflicting decisions. Nevertheless, we show that monotonicity is preserved.

Consider again problems admitting a unique solution, and observe that no new information is gained (i.e., $\tilde{S}_t = \underset{\tau \in [0,t]}{\text{Min}} \{S_\tau\}$ does not change) while passing through a local maximum. Thus, the argument used above applies, implying that a local maximum conflicts with the assumption of a unique solution, and once S_t starts increasing, it cannot decrease at later times.

The analysis of a possible local minimum is more involved. Suppose that diversion exceeds recharge at $t_3 < m$ but falls short of recharge at $t_4 > m$, yet $S_{t_3} = S_{t_4}$ and S_m is the minimum level obtained during $[t_3, t_4]$. Although the state level is the same, the decision problems at t_3 and t_4 may differ, since $\tilde{S}_{t_4} = S_m < \tilde{S}_{t_3}$. To rule out such situations, observe first that since local maxima are excluded, the local minimum S_m is, in fact, a global minimum. We can also restrict attention to minima satisfying $S_m > \underline{S}$, where $\underline{S}$ is the state level at which the event is bound to occur. This is so because monotonicity refers only to the pre-event component of the uncertainty plan and by the time $\underset{-}{S}$ is reached, the event will have occurred with probability one. Thus if $S_m \leq \underset{-}{S}$, the pre-event trajectory is trivially monotonic.

Let S_t^* and g_t^* represent the optimal processes corresponding to the uncertainty problem (2.4), and $\varphi(S) = V^p(S)-\psi$ be the value at the occurrence date T if the critical level turns out to equal S. The pre-event value is

$$V(S) = E_T\left\{\int_0^T [Y(g_t^*)-C(S_t^*)g_t^*]e^{-\rho t}dt + e^{-\rho T}\varphi(S_T^*)\,|\,T>0\right\},$$

while $U(S) = \int_0^\infty [Y(g_t^*)-C(S_t^*)g_t^*]e^{-\rho t}dt$ is the benefit associated with the uninterrupted plan. In the expressions for V(S) and U(S), time is measured relative to the passage time through the state level S. For notational convenience we suppress the possible explicit time dependence of U and V representing differences in histories and future plans as S is encountered at different times. We now show that if the process S_t^* is not designed to reach $\underset{-}{S}$, then unintentional occurrence cannot be advantageous, that is

Lemma A1: If $\text{Inf}\{S_t^*\} > \underset{-}{S}$, then $U(S) \geq V(S)$.

proof: The relation among the benefit measures is expressed as

$$V(S)=E_T\left\{\int_0^\infty [Y(g_t^*)-C(S_t^*)g_t^*]e^{-\rho t}dt+e^{-\rho T}\{\varphi(S_T^*)-\int_0^\infty [Y(g_{t+T}^*)-C(S_{t+T}^*)g_t^*]e^{-\rho t}dt\}\,|\,T>0\right\},$$

or

$$V(S) = U(S) + E_T\left\{e^{-\rho T}[\varphi(S_T^*) - U(S_T^*)]\,|\,T>0\right\}. \tag{A1}$$

Since the plan corresponding to $V^p(S)$ is feasible, $V(S) \geq \varphi(S)$, which together with (A1) implies $U(S) + E_T\left\{e^{-\rho T}[\varphi(S_T^*) - U(S_T^*)]\,|\,T>0\right\} \geq \varphi(S)$. In terms of $\theta(S) \equiv U(S) - \varphi(S)$, this result is written as

$$\theta(S) \geq E_T\left\{e^{-\rho T}\theta(S_T^*)\,|\,T>0\right\}. \tag{A2}$$

Define $\underline{\theta} \equiv \operatorname*{Inf}_{t\in[0,\infty]} \{\theta(S_t^*)\}$. Clearly, the lemma follows from (A1) if we show that $\underline{\theta} \geq 0$. For every $\varepsilon > 0$ there exists some time q for which $\theta(S_q^*) < \underline{\theta} + \varepsilon$. Inequality (A2) applies for every S along the optimal plan. In particular, for S_q^* it implies that $\theta(S_q^*) \geq E_T\left\{e^{-\rho T}\theta(S_{q+T}^*)\,|\,T>0\right\} \geq \underline{\theta}E_T\left\{e^{-\rho T}\,|\,T>0\right\}$. In terms of $d_q \equiv E_T\left\{e^{-\rho T}\,|\,T>0\right\} < 1$, this results reduces to $\underline{\theta}(1-d_q) > -\varepsilon$. Since the state process never comes too close to $\underline{S}$, d_q does not approach unity as $\varepsilon \to 0$. Moreover, since ε can be made arbitrarily small, it follows that $\underline{\theta} \geq 0$.■

This result can be used to eliminate the possibility of a local minimum by comparing the benefit expected from the four feasible plans:

a) S_t^{33}, starting at t_3 and following the path $S^*_{t+t_3}$; (optimal).

b) S_t^{34}, starting at t_3 and following the path $S^*_{t+t_4}$; (suboptimal).

c) S_t^{43}, starting at t_4 and following the path $S^*_{t+t_3}$; (suboptimal).

d) S_t^{44}, starting at t_4 and following the path $S^*_{t+t_4}$; (optimal).

Note that the time index t of S_t^{ij} measures the time elapsed from the corresponding start time t_i. In fact, $S_t^{33} = S_t^{43}$ for all t, and the two plans differ only with respect to the prior information involved: $\tilde{S}_t^{43} = S_m$ and S_t^{43} is carried out knowing that the event will never occur, whereas $\tilde{S}_0^{33} > S_m$ and S_t^{33} is planned under the risk that it will be interrupted by an event before the minimum level S_m is arrived at.

Let $V(S^{ij})$ denote the benefit expected from each path, evaluated at its start time t_i. Judging by the decisions taken, $V(S^{33}) > V(S^{34})$ and $V(S^{44}) > V(S^{43})$. We also know that $S_t^{34} = S_t^{44}$ and these paths are increasing, hence $V(S^{34}) = V(S^{44})$. (For increasing plans the probability of non-occurrence reduces to unity and does not affect the value.) It follows that $V(S^{33}) > V(S^{43})$. However, $V(S^{33}) = V(S_{t_3})$

while $V(S^{43}) = U(S_{t_3})$, hence the latter inequality contradicts Lemma A1.

For problems admitting multiple optima, the strong inequalities of the previous paragraph may be replaced by equalities, and the non-monotonic plan cannot be ruled out. Yet, the construction of a monotonic optimal path from this non-monotonic plan follows the discussion of the post-event and the auxiliary problems: One chooses a selection rule according to which, for each state level, a particular diversion rate is chosen among all optimal rates. The resulting optimal plan is monotonic, because conflicting decision at the same state levels are not allowed.■

Appendix B: Properties of the Post-Event Plan

The current-value Hamiltonian and Lagrangian functions corresponding to the post event problem (2.2) are

$$H(S_t,g_t,p_t,t) = Y(g_t) - C(S_t)g_t + p_t[R(S_t)-g_t]$$

and

$$\mathcal{L}(S_t,g_t,p_t,\alpha_t,\gamma_t,t) = H(S_t,g_t,p_t,t) + \gamma_t g_t + \alpha_t S_t$$

where p_t is the current value costate variable, and γ_t and α_t are the current value Lagrange multipliers associated with the constraints $g_t \geq 0$ and $S_t \geq 0$. Denoting optimal quantities by the superscript p, the necessary conditions include (Arrow and Kurz, 1970, pp. 48-49): $\partial\mathcal{L}/\partial g = 0$, giving

$$Y'(g_t^p) - C(S_t^p) = p_t - \gamma_t, \tag{B1}$$

and $\dot{p}_t-\rho p_t = -\partial\mathcal{L}_t/\partial S_t$, yielding

$$\dot{p}_t = p_t[\rho-R'(S_t^p)] + C'(S_t^p)g_t^p - \alpha_t. \tag{B2}$$

The slackness conditions read:

$$\gamma_t \geq 0, \quad \alpha_t \geq 0, \quad \gamma_t g_t^p = 0, \quad \alpha_t S_t^p = 0. \tag{B3}$$

Using (B1) to eliminate p_t from (B2), we obtain

$$\dot{p}_t = [Y'(g_t^p)-C(S_t^p)+\gamma_t][\rho-R'(S_t^p)] + g_t^p C'(S_t^p) - \alpha_t, \tag{B4}$$

which reduces, since $g_t^p = R(S_t^p) - \dot{S}_t^p$, to

$$\dot{p}_t = \{-C'(S_t^p)-[\rho-R'(S_t^p)]Y''(g)\}\dot{S}_t^p + L(S_t^p) + \gamma_t[\rho-R'(S_t^p)] - \alpha_t. \quad \text{(B5)}$$

In (B5) g is some value between g_t^p and $R(S_t^p)$, $L(S) = [\rho-R'(S)][Y'(R(S))-C(S)-J(S)]$ and $J(S) = -C'(S)R(S)/[\rho-R'(S)]$, as defined in Section 3. For an equilibrium state $\hat{S}$, $\dot{p}$ and $\dot{S}$ must vanish and (B5) implies

$$L(\hat{S}) + \gamma[\rho-R'(\hat{S})] - \alpha = 0, \quad \text{(B6)}$$

from which, using (B3), Property 3.1 is deduced.

Next, we derive a first-order differential equation for the optimal process whose solution permits the evaluation of the value function $V^p(S)$. For simplicity, we consider the case of an interior equilibrium point, so that $L(\hat{S}) = \alpha = \gamma = 0$. Taking the time derivative of (B1), we obtain

$$Y''(g_t^p)\dot{g}_t^p - C'(S_t^p)\dot{S}_t^p = \dot{p}_t. \quad \text{(B7)}$$

Comparing with (B4) we find

$$Y''(g_t^p)\dot{g}_t^p = [Y'(g_t^p)-C(S_t^p)][\rho-R'(S_t^p)] + R(S_t^p)C'(S_t^p). \quad \text{(B8)}$$

As the problem is autonomous, g^p can be expressed as a function of S only: $g^p = g^p(S)$. With $g^{p\prime}(S) \equiv dg^p/dS$ and $\dot{g}^p = g^{p\prime}(S)[R(S)-g^p]$, (B8) is rewritten as

$$g^{p\prime}(S) = \frac{L(S) + [Y'(g^p)-Y'(R(S))][\rho-R'(S)]}{Y''(g^p)[R(S)-g^p]}. \quad \text{(B9)}$$

The boundary condition associated with (B9) is $g^p(\hat{S}) = R(\hat{S})$. Note that (B9) is not singular at $\hat{S}$, because the numerator also vanishes at this state. Indeed, taking the limit $S \to \hat{S}$, the right-hand side of (B9) reduces to $L'(\hat{S})/\{Y''(R(\hat{S}))[R'(\hat{S})-g^{p\prime}(\hat{S})]\} - [\rho-R'(\hat{S})]$. Solving for $g^{p\prime}(\hat{S})-R'(\hat{S})$, noting that this quantity must be positive to allow the equilibrium level $\hat{S}$ to attract the optimal state process, we find

$$g^{p\prime}(\hat{S}) = R'(\hat{S}) + \frac{1}{2}\left\{\sqrt{\rho^2 - 4L'(\hat{S})/Y''(R(\hat{S}))} - \rho\right\}. \quad \text{(B10)}$$

With (B10) providing the starting step, (B9) is conveniently treated numerically.

Once (B9) is solved for $g^p(S)$, S_t^p can be determined via

$$t = \int_{S_0}^{S_t^p} \frac{dS}{R(S)-g^p(S)} . \tag{B11}$$

Given $g^p(S)$ and using $V^{p\prime}(S) = p(S) =$ the costate variable at the passage time through S (see, e.g., Arrow and Kurz, 1970, p. 35), the post-event value function $V^p(S)$ can be evaluated from the Dynamic Programming relation

$$\begin{aligned}\rho V^p(S) &= Y(g^p(S))-C(S)g^p(S) + V^{p\prime}(S)[R(S)-g^p(S)] \\ &= Y(g^p(S))-C(S)g^p(S) + [Y'(g^p(S))-C(S)][R(S)-g^p(S)].\end{aligned} \tag{B12}$$

Our next task is to establish the monotonicity of the implicit penalty $V^p(S)-W(S)$ for $S > \hat{S}$, as stated in relation (3.5). Since the steady-state policy is always available, $V^p(S) \geq W(S)$, equality holding at the equilibrium state $\hat{S}$. Thus, $V^{p\prime}(\hat{S}) = W'(\hat{S})$. With $\rho W(S) = Y(R(S)) - C(S)R(S)$, we find

$$\rho W'(S) = [Y'(R(S))-C(S)]R'(S) - R(S)C'(S) = L(S)R'(S)/[\rho-R'(S)] + \rho J(S).$$

Since $L(S) > 0$ for $S > \hat{S}$ and $R'(S) \leq 0$,

$$W'(S) \leq J(S) \quad \text{for all } S > \hat{S}. \tag{B13}$$

We now show that

$$p(S) \geq J(S) \quad \text{for all } S > \hat{S}, \tag{B14}$$

where it is recalled that $p(S)$ is the costate value at the passage time through S. From (B2)-(B3) we find

$$\dot{p}_t = p_t[\rho-R'(S_t^p)] + C'(S_t^p)g_t^p = [\rho-R'(S_t^p)][p_t-J(S_t^p)] - C'(S_t^p)[R(S_t^p)-g_t^p]$$

or

$$d[p_t+C(S_t^p)]/dt = [\rho-R'(S_t^p)][p_t-J(S_t^p)]. \tag{B15}$$

The optimal state process that departs from $S > \hat{S}$ must decrease (cf. Property 3.1), hence $C(S_t^p)$ and $J(S_t^p)$ cannot decrease. Suppose that $p_0 \equiv p(S) < J(S)$. According to (B15), $p_t+C(S_t^p)$ decreases with time, hence p_t must decrease. It follows that the (initially positive) difference $J(S_t^p)-p_t$ cannot shrink as S_t^p

approaches $\hat{S}$. In particular, $J(\hat{S}) > p_\infty \equiv p(\hat{S})$. But according to (B1), at $\hat{S}$ we must have $p_\infty \geq J(\hat{S})$, equality holding if $\hat{S} > 0$. Hence, $p(S) \geq J(S)$ as stated by (B14). Recalling that $V^{p\prime}(S) = p(S)$, (B13) and (B14) imply (3.5)■

Another interesting consequence of (B15) is the monotonicity of the control variable g_t^p (to be distinguished from the monotonicity of the state S_t^p). As $g_t^p > 0$ at $S > \hat{S}$, (B1) and (B15) give $dY'(g_t^p)/dt = [\rho - R'(S_t^p)][p_t - J(S_t^p)] \geq 0$, implying (since Y is strictly concave) that g_t^p cannot increase with time. One can establish, in a similar manner, that g_t^p cannot decrease along processes initiated at $S < \hat{S}$.

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