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A COMPUTABLE GENERAL EQUILIBRIUM MODEL FOR DEVELOPMENT POLICY ANALYSIS

The purpose of this paper is to present an example of the so-called Computable General Equilibrium (CGE) models which were introduced into the Applied Economics literature just a little more than a decade ago and proved to be very useful for both medium/long-term planning and development policy analysis exercises. The underlying motivation for such modelling has arisen in response to the well-known short-comings and limitations of the partial equilibrium constructs.

Due to the complexities and the interwoven structure of the real world economies, applied policy analysts have always been skeptical about the analytical powers of partial equilibrium models. Thus, the growing need for increasingly general models has led to the explicit aim of converting the Walrasian general equilibrium system from an abstract mathematical apparatus (as was formalized by Kenneth Arrow, Gerard Debreu and others in the 1950's) into a realistic and applicable model of actual economies.

The CGE Models have, until now, been successfully applied to many countries, addressing different policy questions.[1] To cite some examples, the existing models include, but not limited to: Taylor & Black (1974, Chile); Adelman & Robinson (1978, S. Korea); Dervis & Robinson (1978, Turkey); Ahluwalia & Lysy (1979, Malaysia); Cardoso & Taylor (1979, Brazil); de Melo (1980, Sri Lanka); Feltenstein (1980, Argentina); de Melo & Robinson (1980, Colombia); Lewis & Urata (1983, Turkey); Lundborg (1984, Malaysia); Gupta & Togan (1984, India, Kenya and Turkey).

The paper first introduces the broad class of general equilibrium, macro models that are antecedent to the contemporary CGE formulations. There, I try to provide an interpretative essay on such early multi-sector constructs and follow the path to the evolution of the idea of building price-endogenous, non-linear macro models that can capture both the market-optimization behavior of individual agents and the commanding nature of the exogenously specified government policies.

The second and third sections in turn, build the different segments of the CGE model. The paper concludes with a compact presentation of the core CGE equations and two Appendices, one on the Linear Expenditure System: and another on the endogenous derivation of the price elasticity of demand, both of which will be used in the modelling process.

1. INTRODUCTION: A PRELUDE TO THE CGE MODELS

The earliest multisector planning models were based on the simple input-output linkages among various sectors of the economy. With these models, assuming a fixed-cofficients -- Leontieff -- Technology for each sector, and given the estimates of input-output coefficients across sectors, the planner was in a position to calculate the necessary output level at each sector in order to satisfy a targeted final consumption bundle.

To be more concise, letting A be the matrix of fixed input-output coefficients a_{ij}, with a_{ij} being the amount of input i necessary to produce 1 unit of output j; and denoting gross output vector by X, and the final consumption vector by C, the material-balance equation can be written as:

$$X = AX + C \qquad (I-1)$$

Now, suppose the planner has some targeted consumption bundle, indicated by the vector $\hat{C}$. Then the "solution" to the problem can be found by solving the material-balance equation for X:

$$X^{*} = (I - A)^{-1}\hat{C} \qquad (I-2)$$

Equation (I-2) gives gross production requirements in order to satisfy the targeted consumption demand.

Leaving aside the rather simplistic and cumbersome nature of the static, fixed input-output modelling, the most important drawback of such models has been the lack of an optimization criteria for setting the targeted consumption bundle, $\hat{C}$. Yet, in the absence of such criteria, the determination of $\hat{C}$, and hence of the gross output vector X^{*}, becomes an ad hoc exercise in matrix algebra.

Later, another class of multisector models, known as Linear Programming Models succeeded in overcoming this, and many other shortcomings of the early input-output exercises.[2] Here an explicit objective function was introduced and the problem involved optimizing this function subject to certain (linear) constraints. Again, retaining the same notation for X and A, the static linear programming model can be formulated as follows:

$$\text{Max } RX \text{ s.t. } Ax \leq B$$

$$X \geq 0$$

where R is a vector of objective function weights and B is a vector of resource constraints. Given data on R, A and B, if the feasible set $F = \{X \| AX \leq B; X \geq 0\}$ is bounded and non-empty, then a solution vector X^* can be found to the above problem.[3]

Linear Programming Models coupled with the so-called Duality Theorems have provided interesting implications for economic problems. In particular, given the above linear programming problem (primal), the dual problem could be written as:

$$\text{Min } WB \text{ s.t. } WA \geq R$$
$$W \geq 0$$

where W is a vector of constants. Now, the Duality Theorem states that a feasible vector, X^*, for the primal problem is optimal if and only if a feasible vector W exists for the dual problem; such that,

$$W^*(AX^*-B) = 0 \qquad \text{(I-3)}$$

$$(WA - R)X^* = 0 \qquad \text{(I-4)}$$

Equations (I-3) and (I-4) together constitute the complementary slackness conditions. Decomposing (I-3) and (I-4) we can get the following relations:

$$W^*_i > 0 \text{ implies } \sum_j a_{ij} X^*_j - B_i = 0 \qquad \text{(I-5)}$$

$$\sum_j a_{ij}X^*_j - B_j > 0 \text{ implies } W^*_i = 0 \qquad \text{(I-6)}$$

$$X^* > 0 \text{ implies } \sum_i W^*_i a_{ij} - R_j = 0 \qquad \text{(I-7)}$$

$$\sum_i W^*_i a_{ij} - R_j > 0 \text{ implies } X^*_j = 0 \qquad \text{(I-8)}$$

Thus, in general if, at optimum, any one of the constraints in either problems is not binding (i.e. is either strictly negative or strictly positive) then the corresponding dual variable carries a zero value.

To carry the analysis a little further, if we interpret the B vector as fixed supplies of inputs, the R vector as objective function weights, and the elements of matrix A as input requirements of per unit output levels, the complementary slackness conditions naturally allows us to interpret the dual multipliers, vector W, as "input prices". In this context, equations (I-5) and (I-6) state that only fully used inputs have positive prices and for those inputs where the fixed supply exceeds demand, the associated price must be zero. Further, (I-7) and (I-8) state that only those activities which do not incur any losses at the optimum must actually be carried out at positive levels.

Thus, the linear programming approach provides interesting insights for the general equilibrium relationships of the modelled economies. However, the fact that the dual multipliers share the marginality conditions of market prices at the optimum, should not be taken to imply that they share other properties of market prices as well. First of all, such models are based on the heuristic assumption that a fictitious planner, in command of all physical productive activities of the economy, yet subject to certain technological and natural constraints, seeks to maximize a well-defined welfare function for the whole society. They are, thus, not well-suited, nor designed for the state-capitalist, mixed economies where individual agents independently try to maximize their own well-being subject to a budget constraint in an environment regulated by the government bureaucracy at varying degrees. In this environment, individual agents taken together determine certain outcomes that can be affected only indirectly by the planner. The planner does not have real command on all the productive activities of the economy, but relies on the decisions of many other independent "optimizers", each exerting an influence on the specific development path of the economy.

The remedy to this observation is, of course, to construct a model where endogenous prices and quantities are allowed to transmit market information through different sectors of the economy, thereby simulating the workings of a perhaps regulated and intervened, yet absolutely decentralized (i.e. not commanded by a social planner) markets. Yet, this level of price endogeneity cannot be designed within the realm of the linear programming models. To cite the main problem, "the crucial difficulty lies in the fact that economic behavior and relations such as budget constraints, consumption functions, and saving functions must be expressed in current endogenous factor and commodity prices. But the standard primal constraint equations of a linear program cannot include the "shadow" prices that result as a by-product of the maximization. Or, to put it differently, one cannot in general expect that the resource allocation and production structure determined by the solution of a linear program is consistent with the incomes and budgets that result from its dual solution. Indeed, if factor prices have any impact on the structure of demand, the quantities supplied that are the outcome of the primal solution will in general not equal the quantities demanded that are implied by the dual solution".[4]

In general, those models which enable this price endogeneity on the one hand, and incorporate the fundamental general equilibrium linkages between the incomes of various worker-consumer groups and the resulting patterns of demand on the other, are termed Computable General Equilibrium (CGE) models. This paper is about one such model that incorporates the international economy as well as the domestic markets into the analysis. Given an arbitrary set of prices, the model solves for the output levels across sectors and finds the market clearing wage/rental rates. These in turn become the sources of income generation for various household groups and determine the pattern of demand. Quantities imported and exported are solved as a function of domestic production costs, international prices and relevant elasticities. The investment behavior is also endogenized through the saving patterns and sectoral investment share parameters, which, in turn are determined as a function of differential profit rates across sectors. After calculating excess demands in this manner, the model updates the initial guess of domestic prices through a Walrasian tâtonnement algorithm and iterates the whole process until convergence is achieved.

It should be noted, however, that the model designed in this paper can only solve for the <u>relative</u> prices and the <u>real</u> variables of the economy. Yet to achieve this, the planner has to feed a normalization rule into the model, a completely exogenous practice. The rule most commonly resorted to and which will also be used here is to employ a no-inflation benchmark by defining a constant level of the price index, which is set exogenously by the modeller. This choice is quite consistent with the early treatment of Walrasian General Equilibrium Models, in which only relative prices and real variables would matter, without much concern devoted to monetary problems. Thus, using such a normalization rule precludes the treatment of monetary phenomena as well as the possibility of using such models for very short-run, stabilization analyses.

On the one hand, incorporation of the interactions between the real and monetary spheres of the economy in a general equilibrium framework is still a very difficult branch of economic theory; and building an ad hoc macro-monetary superstructure interwoven with the microeconomic general equilibrium system through simple behavioral equations will have its own drawbacks as an analytical tool. Further, such an exercise may turn out to be too general and cumbersome to be a useful model for focusing on a vast array of development issues.

These arguments should not be taken, however, to imply that planning monetary phenomena is not possible or ill-advised all together. Depending on the question in hand and the time horizon to be analyzed, the monetary sphere of the economy can be incorporated into the CGE framework in various ways. For example, a very elegant model that tackled this task quite effectively is provided by Adelman and Robinson in their 1978 work, which focuses on the income distribution consequences of different development strategies in South Korea.

Thus, to recapilulate, the reader has to appreciate the fact that in applied policy analyses much depends on the specific purpose of the model-building effort and the access to realistic data supplies, which is still a major restraint on students of Development Economics.

The Model constructed in this paper is adapted and updated from the works of Dervis et. al. (1982); Dervis and Robinson (1978); Lewis and Urata (1983); Adelman and Robinson (1978) and Lundborg (1984). Its distinguishing features are: (1) explicit specification of the public enterprises as distinct from private enterprises; (2) recognition of monopoly power in certain product markets; (3) recognition of inter-sectoral wage differences for the same type of labor; and (4) endogenous calculation of the sectoral export subsidies arising from export-incentive-packages granted by the government.

The Model is constructed and designed to be run in two stages. The first stage is a within-period general equilibrium construction which is static in its equations and variables. Given certain exogenous government policy variables and other parameters, the Stage 1 Model, as will be called hereafter, finds the relative prices and solves for all real/structural variables of the economy. In other words, it comprises the core system of the overall Model.

The second stage, on the other hand, is designed to up-date the exogenous variables of the first stage. It is a dynamic system and basically used for the purpose of "aging" the Model.

Armed with this background we can begin constructing our model. I first introduce the system of notation that will be used throughout the entire Model. Unless otherwise specified, I adhered to the following legend of principles:

(1) Endogenous variables are denoted by capital letters without any bar (¯) on them. All capital letters with a bar, and lower case letters (with the exception of d_i and m_i) are exogenous variables or fixed parameters in the Stage 1 Model which needs to be updated in the second stage.

(2) All Greek letters are parameters not variables.

(3) Letters with a circumflex (^) are policy variables to be set exogenously by the government.

(4) Time subscripts are omitted for all variables unless there are time lags involved. Thus, unless otherwise specified explicitly, all variables refer to the current period.

(5) The subscripts i and j are used for sectors. They always range from 1 to n. When these two are used together (e.g. a_{ij} or b_{ji}), the first subscript always refers to the sector of origin and the second to the sector of destination.

(6) The subscript s refers to different skill types of labor and ranges from 1 to m.

(7) Subscript f is used to distinguish between the private and the public firm (p: private; g: public).

II – THE OPEN CGE MODEL: STAGE 1

The core equations of the Stage 1 Model in their explicit functional forms are constructed in this section. We first begin with the presentation of the price system.

PRICES

The specification of the price system in an open economy model presents some interesting problems to the applied model builder. To begin with, in the absence of any trade restrictions, invoking the neo-classical assumptions, that the tradables are perfect substitutes and that the country being modelled is too small to affect the world prices, implies that the domestic relative prices are set by the given world price ratios. Thus, there remains no independent, or endogenous, price system for the model to solve at all. The prices for the open economy model are determined in the international markets and these should be fed into the model as given, fixed variables. However, in practice, especially when we are trying to build models with limited degrees of disaggregation of the productive sectors, the perfect substitutability assumption greatly exaggerates the role of the international price system and the domestic trade policy over determination of the domestic price system. The applied macro models, due to the understable reasons of computation, data limitations, etc., involve a fair amount of aggregation of sectoral activities, and at such levels of aggregation the perfect substitutability assumption may lead to quite misleading results.

Another difficulty, as illustrated by Dervis et. al. (1982, Chapter 3) is that assuming the above mentioned neo-classical hypotheses along with the specification of the productive technology as one of constant returns to scale, result in extreme specialization in the sectors that the domestic economy has comparative advantage, with no home production ever on the sectors that it doesn't have. Obviously, this is a very crude portrayal of the way economies engage into international trade and is not supported by empirical evidence. Two-way sectoral trade is abound, especially at high levels of aggregation.

A formulation to handle these problems has been proposed in a 1969 paper by Armington which distinguishes commodities not only by their kind - e.g. machinery,chemical - but also by their place of production. In Armington's commodity system, not only is each good different from any other good, but also each good is assumed to be differentiated by the country of origin of supply.

Following Armington's hypothesis, domestically produced goods and imports are assumed to be imperfect substitutes. To reflect this, we define a tradable composite commodity CC_i, which is a CES aggregation of the domestic commodity DC_i, and the imported foreign good, M_i. The elasticity of substitution of the CES function, σ_i, reflects the differences between the domestic and imported good from the buyer's viewpoint (the smaller the σ_i, the greater the difference between DC_i and M_i and the harder to substitute them with each other). Plausibly in sectors such as agriculture, food processing or textiles, σ_i is fairly large, whereas for the capital goods sectors it is quite low.

The explicit formulation of the composite commodity in the i-th sector is:

$$CC_i = \bar{B}_i [\delta_i M_i^{-\rho_i} + (1-\delta_i) DC_i^{-\rho_i}]^{-1/\rho_i} \qquad \text{(II-1)}$$

where $\bar{B}_i$, δ_i and ρ_i are parameters; with δ_i giving the share of the imported good in CC_i and ρ_i is related to the elasticity of substitution, σ_i by the expression $\sigma_i = 1/1+\rho_i$.

The consumers are then hypothesized as minimizing a cost function subject to the CES composite commodity "technology", just like a firm trying to produce a specified level of output at minimum cost. Accordingly M_i and DC_i are like "inputs" producing the aggregate output CC_i. Therefore, the composite good price, PC_i, can be expressed using the cost function of the CES technolgy(5),

$$PC_i = 1/\bar{B}_i [\delta_i^{\sigma_i} PM_i^{(1-\sigma_i)} + (1-\delta_i)^{\sigma_i} PD_i^{(1-\sigma_i)}]^{1/1-\sigma_i} \qquad \text{(II-2)}$$

where PM_i is the domestic currency price of the imported good, which will be determined by the world price $\overline{PWM}_i$, the ad valorem tariff rate $\hat{tm}_i$, and the exchange rate $\hat{ER}$ (defined as units of domestic currency per unit of foreign currency, usually dollar).

$$PM_i = \overline{PWM}_i (1+\hat{tm}_i) \hat{ER} \qquad \text{(II-3)}$$

To complete the specification of the price system I will also introduce the net price, PN_{fi}, upon which the producers make their production plans. Thus;

$$PN_{fi} = PD_i - \sum_j PC_j a_{ji} - \hat{tn}_i PD_i + \hat{sn}_{fi} PD_i \qquad \text{(II-4)}$$

where $\hat{tn}_i$ is the indirect tax rate and $\hat{sn}_{fi}$ is the net production subsidy. Depending on the government's attitude towards private versus public enterprises, the granted subsidy rate is differentiated among firms as well as across sectors. Further, a_{ji} stands for the amount of intermediate input j used for the production of one unit of i. Hence $\sum_j PC_j a_{ji}$ gives the value of intermediate inputs used in the production of one unit of the i-th good.

The two other prices used in the Model, the price of capital, PK_i and the export price, PWE_i will be introduced below, at a later point of the Stage 1 Model. At this juncture, however, I will turn to the specification of the production technology of the economy.

PRODUCTION TECHNOLOGY AND FACTOR MARKETS

The crucial assumption in constructing the productive sphere of the Open CGE Model is that each sector is envisaged to produce a single commodity (may be thought as an aggregate-commodity in the Hicksian sense). Conversely, each such commodity is associated with a single production sector of the economy. This specification, very much in the tradition of early economy-wide models, enables us to continue to define the productive sectors as entries of an input-output table.

As hinted in the net-price equation (II-4), the intermediate input demands has been assumed to constitute a linear system with fixed-coefficient production technology for such input-usage. The retention of this specification is not necessary for the non-linear CGE Model and extensions of this technology have been tried in Lewis and Urata (1983) and also in Ahluwalia and Lysy (1979). For purposes of realism we may need to separate the technology of intermediate inputs from the production technology for primary inputs - capital and labor. Our specification is perhaps the simplest way to achieve this.

In particular, the production technology available to a firm can be thought to be given by either a one- or a two-level Cobb-Douglas function of capital and labor in each sector i:

$$X_{fi} = \bar{A}_{fi} \; \bar{K}_{fi}^{\alpha_{fi}} \prod_{m=1}^{m} L_{fim}^{\alpha_{fim}} \qquad \text{(II-5)}$$

or,

$$X_{fi} = \bar{A}_{fi} \; \bar{K}_{fi}^{\alpha_{fi}} \; L_{fi}^{(1-\alpha_{fi})} \qquad \text{(II-6)}$$

where L_{fi} is further formulated as a CES aggregation of different skill levels (II-7 below).

The modeller can choose either of the specification for the production technology for a particular firm or sector. However, the two-level Cobb-Douglas technology seems to be more realistic because it is very unlikely that the elasticity of substitution between all types of labor is the same and equal to that between labor types on the one hand, and to capital on the other, as was assumed in formulation (II-5).

Then, I will retain the two-level Cobb-Douglas technology for the Model. For such technology, capital is thought as a fixed-coefficients, composite good with elements b_{ij}, where b_{ij} is the amount of capital good originating from sector i that will be used to make up one unit of real capital in sector j.

Further, capital stock is assumed to be fixed in the within-period modelling of the first stage. This assumption tries to capture the fact that capital is not "malleable", i.e. that combine machines once installed cannot be converted into trucks easily.

The labor parameter, L_{fi}, of the Cobb-Douglas production technology is given by a further CES aggregation of m different skill types. Thus;

$$L_{fi} = L_{fi} \; (L_{fi1}, \ldots, L_{fim}) \qquad \text{(II-7)}$$

where $L_{fi}:R_+^m \rightarrow R_+$ is a CES function of skill categories. So we distinguish between m skill levels in the Model.

More detailed specifications of the production technology are of course possible. However, more detailed specifications of the production functions mean more parameters that need to be estimated, and the applied modeller always faces the trade off between vigorous functional specification and parameter estimation. The CES and two-level Cobb-Douglas functional forms used here require a "moderate" degree of parameter estimation and have, reportedly yielded quite realistic results in real world applications (see References).

Now, to turn to the mathematical properties of the production technology of our model, we should first distinguish between the gross production possibility set $X_f = \{X_{f1}, \ldots, X_{fn}\}$ from the <u>net</u> production possibility set, which is:

$$\overset{N}{X}_f = \{\overset{N}{X}_{fi} \mid \overset{N}{X}_{fi} = X_{fi} - \sum_j a_{ji} X_{fj}\}$$

The desirable property, of course, is that the set X_f^N be strictly convex. And this is achieved if the set X_f is strictly convex and the Hawkins-Simon conditions are satisfied.(6) We basically achieve convexity by assuming capital stocks to be fixed. Further, the degree of convexity is to be increased with the number of fixed factors of production in each firm, since this implies the well-celebrated hypothesis of diminishing returns to scale to the variable factors.

In contrast to retaining the neo-classical properties in its production technology, the Model recognizes two kinds of imperfections for the portrayal of market behavior. The first one is the explicit allowance of monopoly power in certain productive sectors; the other is the recognition of intersectoral wage differences for the same category of labor.

Incorporation of monopoly power into CGE type policy models has not been a common practice (with the exception of Adelman & Robinson 1978 study). Yet, in a recent paper Per Lundborg provides evidence from the Malaysian tin market showing that the frequent procedure of assuming competitive markets may lead to misleading results (see Lundborg, 1984). Especially, when analyzing the distributional effects of different policy packages, existance of monopoly power may have important consequences which the competitive markets cannot generate. The income flows arising from the monopoly profits may be substantial, and may further result in biased innovations (of the Binswanger - Hayami - Ruttan type) affecting the intertemporal growth path of the economy.(7)

Formally, monopoly power is introduced into the Model by the following condition:

$$MR_{fi} = PN_{fi} \left(1 + 1/\varepsilon_i^{XD}\right)^{up_{fi}} \qquad \text{(II-8)}$$

where ε_i^{XD} is the elasticity of total demand for commodity i, and up_{fi} is a parameter showing the rate at which the potential monopoly power is actually utilized. For $up_{fi} = 1$ we have the pure monopoly case. If $up_{fi} = 0$ we return to the competitive configuration. The utilization parameter, up_{fi}, may be interpreted as narrating the situation of a monopolist without full information about the demand curve and/or it may be interpreted as capturing the institutional and legal constraints faced by the enterprises.

The issue of inter-sectoral wage differences for labor of the same skill type, on the other hand, has been tackled in most of the CGE Models that focus on income distribution analysis (see, for example, Adelman & Robinson (1978); de Melo & Robinson (1980); Lewis & Urata (1983)).

In fact, evidence on inter-sectoral wage-spread is abound, yet there is no coherent theoretical explanation for this fact. Interestingly, this phenomenon is not solely an attribute of developing countries, but is also observed in the developed market economies, as well. For example Schuh (1976) argues that due to the loss of positive externalities in the migration process, the wage differentials and the associated labor migration from rural U.S.- South to urban U.S.-West has been continuing for over 100 years now, yet without any prospects of equilibrium being achieved.

The method for incorporating wage differentials has conventionally been to assume constants of proportionality between the location of labor and the economy-wide average wage for that category of labor, which is endogenously determined by the model to clear the labor markets. Our Model, also, will retain this method. Thus, denoting the wage rate for labor type s, employed in sector i, by the firm f, with W_{fis}; the economy-wide average wage rate for labor type s by W_s; and letting the proportionality coefficient be ψ_{fis} we have:

$$W_{fis} = \psi_{fis} \cdot W_s \qquad \text{(II-9)}$$

Given the specified production technology and the net prices from equation (II-4), using (II-8) and (II-9) one can derive the enterprise demands for labor of each skill category s. The private enterprise's demand for labor of the skill type s is given by the first order conditions of profit maximization:

$$PN_{pi} \ \partial X_{pi} / \partial L_{pis} = W_{pis} \qquad \text{(II-10)}$$

If one assumes a Cobb-Douglas formulation for the labor aggregation function in (II-7), using the two level Cobb-Douglas technology, equation (II-10) takes the following form:

$$L_{pis} = (1/W_{pis}) \ [(1-\alpha_{pi}) \cdot \lambda_{pis}] \ MR_{pi} \cdot XS_{pi} \qquad \text{(II-11)}$$

where λ_{pis} is the labor aggregation elasticity with respect to skill type s and XS_{pi} (profit-maximizing output supply of the private firm) is given by: (II-12)

$$XS_{pi} = [\bar{A}_{pi}\ \bar{K}_{pi}^{\alpha_{pi}}\ MR_{pi}^{(1-\alpha_{pi})}\ \bar{N}_{pi}^{(1-\alpha_{pi})}\ \prod_{m} [(1-\alpha_{pi})\lambda_{pim}/W_{pim}]^{(1-\alpha_{pi})\lambda_{pim}}]^{1/\alpha_{pi}}$$

The public enterprise, on the other hand, suffers from government intervention in its labor-hire decisions, and from inefficient management associated with the "politicization" of incentives.

In particular, public enterprise's labor-demand function is distorted by an interference factor, $\widehat{INT}_i$ $(0 < \widehat{INT}_i \leq 1)$; and is given by:

$$L_{gim} = (1/\widehat{INT}_i \cdot W_{gim})\ [\ (1-\alpha_{gi})\ \lambda_{gim}\ ]\ MR_{gi} \cdot XS_{gi} \qquad \text{(II-13)}$$

Note that, as the intensity of government interference increases, the value of $\widehat{INT}_i$ has to be reduced. Note also that when $\widehat{INT}_i = 1$, there is no government interference and the public firm is able to maximize its profits just like the private firm.

Due to the alleged interference and inefficient management, the public firm is run at sub-optimal capacity. The rate of capacity-utilization in the public firm is explicitly modelled as follows:

$$Q_i = \widehat{INT}_i^{\,bmc_i(LX_{gi})} \qquad \text{(II-14)}$$

where Q_i is the public enterprise capacity-utilization rate in sector i; $bmc_i \geq 0$ is the coefficient of "bad management"; and LX_{gi} is the sectoral labor-output ratio for the public firm. Note that since $0 < \widehat{INT}_i \leq 1$; $bmc_i \geq 0$ and $LX_{gi} > 0$, we have $0 < Q_i \leq 1$.

Again, using a Cobb-Douglas formulation for the labor aggregation function (II-7) along with the two-level Cobb-Douglas production technology, an explicit expression can be derived for the public firm's labor-output ratio:

$$LX_{gi} = L_{gi}/XS_{gi} \tag{II-15}$$

$$= (1/XS_{gi})\ \bar{N}_{gi} \prod_{s=1}^{m} [(1/INT_i \cdot \hat{W}_{gis})\ (1-\alpha_{gi})\ \lambda_{gis}\ MR_{gi}\ XS_{gi}]^{\lambda_{gis}}$$

$$= \bar{N}_{gi} \prod_{s=1}^{m} [(1/INT_i \cdot \hat{W}_{gis})\ (1-\alpha_{gi})\ \lambda_{gis}\ MR_{gi}\ ]^{\lambda_{gis}}$$

since $\sum_s \lambda_{gis} = 1$.

Sectoral output supply of the public enterprise is given by:

$$XS_{gi} = Q_i\ \{\bar{A}_{gi}\ \bar{K}_{gi}^{\alpha_{gi}}\ MR_{gi}^{(1-\alpha_{gi})}\ \bar{N}_{gi}^{(1-\alpha_{gi})} \cdot \prod_s [(1-\alpha_{gi})\lambda_{gis}/(INT_i \cdot \hat{W}_{gis})]^{(1-\alpha_{gi})\lambda_{gis}}\}^{1/\alpha_{gi}} \tag{II-16}$$

Total output in sector i then becomes:

$$XS_i = XS_{pi} + XS_{gi} \tag{II-17}$$

Given labor demands in each sector, total labor demand for each skill category s can be calculated. Thus,

$$DL_s = \sum_i (L_{pis} + L_{gis}) \tag{II-18}$$

In Stage 1, labor supplies by skill type are assumed to be fixed at $\overline{SL}_s$. These, however, will be endogenized by assuming a natural growth rate for labor and recognizing the possibility of migration from agricultural to urban sectors in the second, dynamic stage of the CGE Model.

Given fixed labor supplies for each skill category, the market clearing nominal wage rate, W_m, can be found via iteration on

$$DL_m - \overline{SL}_m = 0 \qquad \text{(II-19)}$$

Note that, for certain skill categories the nominal wage rate can be taken as given or having a lower bound, reflecting, for instance, government's policies on minimum wages. Then W_{fim} becomes a fixed variable and the level of employment is determined by the level of demand.

The model can further be enriched by specifying monopolistic factor markets reflecting labor unions' power and so on. The flow of the core model, however, will remain the same.

Having derived the wage bill, the profits of the enterprises can easily be calculated as residuals in the sectoral value added. Thus, the private enterprise profits become:

$$RP_i = PN_{pi} \cdot XS_{pi} - \sum_m W_{pim} \cdot L_{pim} \qquad \text{(II-20)}$$

and the public enterprise profits (losses if negative) are:

$$RG_i = PN_{gi} \cdot XS_{gi} - \sum_m W_{gim} \cdot L_{gim} \qquad \text{(II-21)}$$

FOREIGN TRADE

Now we can construct the trade equations of our model. On the import side, recall that we have specified the buyer's problem as one of cost minimization, where the relevant "cost function" was one of a CES formulation used to "produce" a composite commodity, CC_i, with imported good, M_i, and the domestic good, DC_i, taken as "inputs".

In Economics jargon, the buyer's problem is simply to find the import-domestic demand ratio which satisfies the condition that the marginal rate of substitution between M_i and DC_i be equal to their respective price ratios.

For convenience, I repeat here the composite commodity function (II-1):

$$CC_i = \bar{B}_i [\delta_i M_i^{-\rho_i} + (1-\delta_i) DC_i^{-\rho_i}]^{-1/\rho_i} \qquad \text{(II-1)}$$

The Lagrangean of the buyer's problem is:

$$S = PM_i \cdot M_i + PD_i \cdot DC_i + \lambda[\overline{\overline{CC}}_i - \bar{B}_i (\delta_i M_i^{-\rho_i} + (1-\delta_i) DC_i^{-\rho_i})^{-1/\rho_i}] \qquad \text{(II-22)}$$

where $\overline{\overline{CC}}_i$ is a pre-specified level of "output" of CC_i. The first order conditions of this problem yield:

$$m_i = M_i / DC_i = (PD_i / PM_i)^{\sigma_i} (\delta_i / 1-\delta_i)^{\sigma_i} \qquad \text{(II-23)}$$

Recall that $\sigma_i = 1/1+\rho_i$, is the elasticity of substitution. Import demand for commodity i can be found easily from (II-23):

$$M_i = (PD_i/PM_i)^{\sigma_i} (\delta_i/1-\delta_i)^{\sigma_i} DC_i \qquad \text{(II-24)}$$

However, at this point DC_i is not yet known. It needs to be calculated and be fed into (II-24). Yet, without knowing the import-quantities domestic production/consumption decisions cannot be realized and there is no way of solving for both DC_i and M_i simultaneously. A simple trick solves the problem, however, by using the identity that domestic supply for domestic market, DS_i, is given by total domestic supply minus exports. We get,

$$DS_i = XS_i - E_i \qquad \text{(II-25)}$$

Further, in product market equilibrium we must have

$$DC_i = DS_i \qquad \text{(II-26)}$$

Hence, using (II-26) we can derive the import demand for commodity i as:

$$M_i = (PD_i/PM_i)^{\sigma_i} (\delta_i/1-\delta_i)^{\sigma_i} DS_i \qquad \text{(II-27)}$$

which is a workable relation for the Model.

On the export side, we first need to formulate the export price for each commodity. Similar to the treatment of import prices as in equation (II-3), the export price relations can be formulated as follows:

$$PE_i = \overline{PWE}_i \cdot (1 + SE_i)\widehat{ER} \qquad \text{(II-28)}$$

where PE_i is the domestic currency receipts per unit exported from sector i; SE_i is the rate of export subsidy for the product of sector i; and $\overline{PWE}_i$ is the fixed world price in foreign currency.

Further, there are certain behavioral constraints that we have to impose on (II-28) to guarantee meaningful results in the rest of our model. Note that if PE_i happens to be greater than PD_i, the domestic price of commodity i, then the Model will instruct the productive enterprises to export all of the domestic output leaving nothing for domestic consumption. Such a situation should, of course exert upward pressure on PD_i until both prices are equalized. Thus, although there is the logical possibility that $PE_i > PD_i$ and that all domestic demand for commodity i might be satisfied through imports, while all that is domestically produced being sold abroad, we will rule out this extreme behavior by recognizing the following constraints on the export side.

$PD_i \geq PE_i$ such that

if $PD_i = PE_i$, $E_i \geq 0$

if $PD_i > PE_i$, $E_i = 0$

Yet, another problem is the very hypothesis we have invoked about the treatment of tradeables in general. Accordingly, we distinguish products by country of origin and hence, there is the possibility that the export demand functions for the home country's products may be less than infinitely elastic.

The export demand functions for our country's products must then be in the form:

$$E_i = E_i(\overline{AWP}_i, PWE_i)$$

where $\overline{AWP}_i$ is an "aggregated" world price for products in the sector i's output category which, as well reflects a <u>weighted average</u> of all production costs and trade policies of all countries.

In designing the specific form of the export demand function $E_i(\quad)$, we will retain the small country assumption in the special sense that $\overline{AWP}_i$ will be treated as fixed and given. However, PWE_i now becomes an endogenous price, determined by the domestic production costs, export policy as reflected in sectoral subsidy rates and the exchange rate:

$$PWE_i = PD_i/[(1+SE_i)\widehat{ER}] \qquad \text{(II-29)}$$

From (II-29) we can easily deduce that an increase in our production costs will increase PD_i and raise the price of our exportables as we present them into the world market. Also an increase in the export subsidy rate or a devaluation (an increase of $\widehat{ER}$) leads to a fall in PWE_i. In the latter case, if $\overline{AWP}_i$ were to remain constant there will be an increase for our country's export demand for product i and hence, an increase in our world market share.

Following Dervis and Robinson (1978) and also Dervis et. al. (1982) one can make the assumption that the world consumers as a whole behave according to the rules of cost minimization with a generalized CES function specifying the world commodities as a composite good. We can then specify the export demand functions in the single elasticity form:

$$E_i = \bar{E}_{oi} \; (\overline{AWP}_i / PWE_i)^{\eta_i} \qquad \text{(II-30)}$$

where η_i is the elasticity of export demand and $\bar{E}_{oi}$ is the normal trend level of the home country exports when $\overline{AWP}_i = PWE_i$.

Having thus constructed the export demand equations, what is left for us is to find an endogenous expression for the export subsidy rate, SE_i. Many governments, instead of granting a single ad valorem subsidy rate to exporters, provide a complex set of incentives for producers to encourage the exportation of their products. These incentives may range from beggar-thy-neighbor mercantilist policies, to a laissez-faire treatment on tradables. In this paper, a policy package consisting of four different export incentive schemes is recognized and explicitly modelled. These are: (1) rebates on production taxes, $\hat{tn}_i$, on the products destined for exports; (2) allowance on the corporate income tax at a certain percentage rate of export earnings; (3) permission of duty free intermediate imports used for the production of exports; and (4) a sectorally differentiated ad valorem export subsidy (tax if negative) rate which is directly paid out of the government budget.

It will further be assumed in the Model that the government sets sectorally differentiated "eligibility criteria" on exports that may benefit from the above schemes, such as exports destined for designated world markets, or export earnings exceeding some minimum value in foreign currency, etc. Depending on the strictness of these conditions it will be assumed that the eligibility rate for exports in the production tax rebate and allowance on corporate income tax schemes is historically around

$\hat{ee}_i$ percent of total exports. For the remaining two schemes $\hat{ee}_i$ is taken to be 100% .

Therefore, under the first scheme total subsidy granted to sector i will be:

$$TSR_i = \hat{tn}_i \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i \qquad (II-31)$$

which corresponds to subsidy equivalent of $\hat{tn}_i \cdot \hat{ee}_i$ percent.

Under the corporate income tax allowance scheme, total income tax allowance granted is:

$$TKA = \hat{kta} \cdot \sum_i \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i \qquad (II-32)$$

where $\hat{kta}$ is the granted corporate income tax allowance rate. Letting $\hat{tk}$ denote the capitalist (corporate) income tax rate, total subsidy on exports due to this scheme is:

$$TSA_i = \hat{tk} \cdot \hat{kta} \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i \qquad (II-33)$$

which corresponds to an ad valorem subsidy of $\hat{tk} \cdot \hat{kta} \cdot \hat{ee}_i$ percent.

As for the third scheme, observe that the domestic currency value of imported intermediate inputs used for export production is:

$$EMI_i = \sum_j \overline{PWM}_j \cdot \hat{ER} \cdot d_j \cdot m_j \cdot a_{ji} \cdot E_i \qquad (II-34)$$

where d_j is the domestic use ratio of the j-th composite good (see equation II-57 below); and m_j is the import-domestic good ratio introduced in equation II-23 above. Thus $d_j \cdot m_j \cdot a_{ji}$ gives the amount of imported intermediate good j, per unit of good i produced and exported.

Total import tax to be paid on such imports then becomes:

$$TEMI_i = \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot ER \cdot d_j \cdot m_j \cdot a_{ji} \cdot E_i \qquad \text{(II-35)}$$

which gives rise to an ad valorem subsidy rate of $\sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot d_j \cdot m_j \cdot a_{ji} \cdot (1/PWE_i)$ per unit of exports.

Combining with an explicit sectoral subsidy/tax rate of $\hat{te}_i$ on exports, the realized overall export subsidy rate becomes:

$$SE_i = (\hat{tn}_i + \hat{tk} \cdot \hat{kta})\, \hat{ee}_i \qquad \text{(II-36)}$$
$$+ \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot d_j \cdot m_j \cdot a_{ji}\ (1/PWE_i) + \hat{te}_i$$

which appears in PWE_i in equation (II-29). Note that since the above expression further entails PWE_i, it needs to be solved by numerical methods.

Balance of Payments Equilibrium is then achieved when,

$$\sum_i \overline{PWM}_i \cdot M_i - \sum_i PWE_i \cdot E_i - \bar{F} - \overline{WR} = 0 \qquad \text{(II-37)}$$

where $\bar{F}$ and $\overline{WR}$ stand for the exogenous válue of the net foreign resource inflow and workers' remittances, respectively. If the exchange rate is allowed to adjust freely (which is as well a policy decision, hence we continue to use a circumflex on ER), $\hat{ER}$ will need to be iterated until (II-37) is satisfied. Per contra, if the government chooses to fix $\hat{ER}$ at some value, there is no guarantee that Balance of Payments Equilibrium would be satisfied (a surprise to no one!); hence, the government would need either to ration imports or try to increase exports, or get more foreign resources. Such commercial policy analyses using CGE Models are abound and the interested reader may wish to consult with the works cited in the References to this paper.

INCOMES GENERATION, CONSUMER DEMANDS AND SAVINGS

The functional incomes of different consumer groups of the Model are generated using the results derived in the factor markets and the derivations are quite straight forward.

For labor skill-type s, assuming that the tax rate is $\hat{ts}$ total disposable income can be written as:

$$YL_s = (1-\hat{ts}) \sum_i (W_{pis} \cdot L_{pis} + W_{gis} \cdot L_{gis}) + \omega_s \cdot \overline{WR} \cdot \hat{ER} \qquad \text{(II-38)}$$

where ω_s is the share of workers' remittances captured by labor group s.

Capitalists' disposable income becomes:

$$YK = (1-\hat{tk}) \sum_i RP_i \qquad \text{(II-39)}$$

Public enterprises' aggregate after-tax income is:

$$YKG = (1-\hat{tk}) \sum_i RG_i \qquad \text{(II-40)}$$

Government's total income is:

(II-41)

$$\begin{aligned} YG = & \sum_s \hat{ts} \sum_i (W_{pis} \cdot L_{pis} + W_{gis} \cdot L_{gis}) \\ & + \hat{tk} \sum_i (RP_i + RG_i - \hat{kta} \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i) \\ & + \sum_i [\hat{tm}_i \cdot \overline{PWM}_i \cdot \hat{ER} \cdot M_i - \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot \hat{ER} \cdot d_j \cdot m_j \cdot a_{ji} E_i] \\ & + \sum_i \hat{tn}_i [PD_i XS_i - \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i] - \sum_i \hat{te}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i \\ & - \sum_i [\hat{sn}_{pi} \cdot PD_i \cdot XS_{pi} + \hat{sn}_{gi} \cdot PD_i \cdot XS_{gi}] + YKG + \bar{F} \cdot \hat{ER} \end{aligned}$$

Given total incomes for different socio-economic groups, our next task then, is to calculate the components of domestic demand for each sector.

In the absence of money markets and any specification of lending behavior, the investment demand is totally savings-determined. Hence, the savings-pool of the economy sets the limits of the investment demand, and capital formation in general. The model distinguishes between private and public savings/consumption behavior. Private agents are assumed to save a fraction of their disposable incomes, given corresponding savings parameters. The government on the other hand, is assumed to set an exogenous policy on the required public investments as a proportion of total GDP; and given this exogenous policy ratio, it withdraws the necessary fraction of its total income as savings.

In particular, total private savings, TPS is the sum of the savings of all workers of skill types and capitalists:

$$TPS = \sum_{s} \bar{S}_{s}\ YL_{s} + sk\ YK \qquad (II\text{-}42)$$

where $\bar{S}_s$ and sk are saving rates out of labor income of skill-type s and capitalist income, respectively.

The government is assumed first to establish a policy on the ratio of total public investment to gross domestic product. Denoting this policy ratio by $\hat{\theta}$, the necessary investment fund of the government, GIF, can be found:

$$GIF = \hat{\theta}\ (\sum_{i} PD_i \cdot XS_i - \sum_{i}\sum_{j} PC_j\ a_{ji}\ XS_i) \qquad (II\text{-}43)$$

where the expression in parantheses gives the nominal gross domestic product of the economy. Having stated GIF, the required savings rate for the government is given by:

$$Sg = GIF/YG \quad \text{(II-44)}$$

Once the saving decisions are made, what is left for the transactors is to determine the consumption demands for each product. The private and public consumption functions will again be distinguished, reflecting the state of affairs that private consumers' demand functions are derived by way of preference maximization, but the government's decision are exogenous in nature. One can, of course, specify a preference map for the government bureaucrats as well and derive government's demand functions from that map. This, however, would be a very complicated task involving heuristic assumptions, and requiring very specialized data sets which would, most probably, be beyond reach for most of the developing countries.

The private consumption demand function will be given by a linear expenditure system (LES) of the following form:
For labor-skill s:

$$CL_{si} = \tau_{si} + \beta_{si}/PC_i [(1-\bar{S}_s)YL_s - \sum_j PC_j \tau_{sj}] \quad \text{(II-45)}$$

where τ_{si} is some absolute minimum (subsistence) level of consumption of commodity i for group s. The expression in the parantheses gives the total income in excess of the expenditures over the subsistence-basket. The parameter β_{si} is the marginal budget share of product i, and it tells how consumer group s allocates its marginal income above the subsistence level across sectors. The derivation of the LES and its properties are further examined in Appendix A.

The capitalist consumption demands also follow the same format,

$$CK_i = \tau_{ki} + \beta_{ki}/PC_i[(1-sk)YK - \sum_j PC_j\tau_{kj}] \qquad (II-46)$$

The government consumption demand will be assumed to be of the following simple form:

$$CG_i = \hat{q}_i \ (1-Sg) \ YG/PC_i \qquad (II-47)$$

where $\hat{q}_i$ is the policy-induced public expenditure share for the product of sector i.

The total consumption demand for product i then, can be found as the sum of private and public consumption demands,

$$C_i = \sum_m CL_{mi} + CK_i + CG_i \qquad (II-48)$$

To construct investment demands, recall that we assume total investment demand to be constrained by total savings generated in the economy. The Model determines sectoral private investments through exogenous investment allocation coefficients, $\bar{H}_i$. These coefficients are then endogenized in the Stage II Model by using previous period's prices, production costs and profit rates.

Real private investment in sector i is:

$$NP_i = \bar{H}_i \ (TPS/PK_i) \qquad (II-49)$$

where PK_i is the price of capital. Since capital is a fixed-coefficients composite commodity its price is given by a weighted average of its components:

$$PK_i = \sum_j b_{ji} PC_j \quad \text{(II-50)}$$

Real public investment in sector i is found in analogous manner, yet here sectoral allocation coefficients, $\overline{\overline{HG}}_i$, are truly exogenous (i.e. not endogenized in the Stage 2 Model, but only up-dated). This treatment reflects government's sectoral priorities for investment. Thus,

$$NG_i = \hat{HG}_i (GIF/PK_i) \quad \text{(II-51)}$$

total real investment to sector i is then:

$$NT_i = NP_i + NG_i \quad \text{(II-52)}$$

Now, note that NT_i gives the amount of real investment to the i-th sector. Yet, for national income accounting purposes we need to know the amount of investment demand from sector i. In Planning Literature, the former is referred as "investment by sector of destination", and the latter as "investment by sector of origin".

Therefore, to get real investment demands by sector of origin, Z_i, we use the capital composition coefficients once again and arrive:

$$Z_i = \sum_j b_{ij}(NT_j) \quad \text{(II-53)}$$

The last component of total demand is the demand for intermediate inputs and can be calculated easily using the input-output coefficients. Letting V_i denote the amount of intermediate demand from sector i,

$$V_i = \sum_j a_{ij} XS_j \qquad \text{(II-54)}$$

The total demand for commodity i, TD_i, is then:

$$TD_i = C_i + Z_i + V_i \qquad \text{(II-55)}$$

Now, since we have found the magnitude of total demand for each composite product our final task is to decompose this magnitude into its two components: domestic and foreign. TD_i of equation (II-55) gives total demand for the overall composite good, as was defined in (II-1). In the meantime we have derived domestic output supplies and in order to find the market clearing domestic prices (PD_i's) we need domestic demands as well. Important as it is, this is not a complicated task, especially when we use certain mathematical properties of the CES function which defines the composite good in (II-1). It can easily be shown that the CES function in (II-1) is linearly homogenous in its variables, M_i and DC_i, and can be re-written as:

$$CC_i = f_i(m_i, 1) \cdot DC_i \qquad \text{(II-56)}$$

where $f_i(\ )$ specifies the CES function given in (II-1). We then have:

$$d_i = DC_i / CC_i = f_i^{-1}(m_i, 1) \qquad \text{(II-57)}$$

Since m_i is a function of the relative price ratio PD_i/PM_i (see equation II-23), d_i is uniquely determined by PD_i/PM_i as well. It must be noted, however, that the demand for the composite commodity, CC_i, depends on all of the relative prices.

We can use d_i and go from composite commodity demand to the domestic demand for the domestically produced commodity,

$$DC_i = d_i \cdot TD_i \qquad \text{(II-58)}$$

Adding export demand of foreign consumers to domestic demand, we get the total demand for domestic product i:

$$XD_i = DC_i + E_i \qquad \text{(II-59)}$$

Market-clearing conditions imply that all excess demands be zero,

$$XD_i - XS_i = 0 \qquad \text{(II-60)}$$

The solution strategy of our static CGE Model then starts with an initial guess of domestic prices, PD_i, the wage rates, W_m, and the exchange rate $\hat{ER}$. We first calculate the excess labor demand in factor markets and revise wages until these markets are cleared. Then, using the labor aggregation and production functions we calculate sectoral output supplies. In the meantime, from wage and profit incomes, labor and capitalist incomes are generated; while government's total income is given by the tax revenues, public enterprises' sectoral profits and net foreign resource inflows. From incomes generated, savings and consumption decisions are carried out and the savings-pool is turned into sectoral investment demands by the sectoral investment-allocation parameters and capital-composition coefficients. Import and export demands are derived as a function of domestic and world prices, tariff/subsidy rates and the exchange rate.

Once product excess demands and the Balance of Payments equation are determined, the initial guess of domestic prices and the exchange rate are revised so as to satisfy the condition that they will be sufficiently close to zero.[8] With the new set of domestic prices and the exchange rate the Model is solved once again and this process continues until convergence is achieved.

As we have discussed previously, however, we know that the Model obeys the Walras' Law that the value of all excess demands add up to zero. This means that if $PD^* = \{PD_1^*, \ldots, PD_n^*\}$ is a set of solution prices so is $t \cdot PD^*$ for any scalar $t>0$; and it is the Planner's job to specify a normalization rule to set the relative price system for the Model. I have adopted a "no-inflation" procedure and chose to normalize the price system around a given index $\bar{P}$:

$$\sum_i PC_i \cdot \Omega_i = \bar{P} \qquad \text{(II-61)}$$

where Ω_i are the weights defining the index $\bar{P}$.

Equation (II-61) closes the system of equations of Stage 1. A sketch of an existance proof for the core model is presented in Dervis et. al. (1982) and the interested reader can get an intuitive notion of the general equilibrium properties of the Model from the analysis presented there. The stability properties of the Model are actually implied in the solution algorithm used for the iteration of prices to clear the excess demand equations in (II-60). Thus, practically what is left for us is to go on to the Stage 2 Model and update and/or endogenize the exogenous variables of the static Stage 1 Model equations.

III - THE OPEN CGE MODEL: STAGE - 2

As stated earlier, the task of the Stage 2 Model is to up-date and endogenize the fixed variables of the first stage. Here, some of the variables may need to be up-dated using certain regression/trend-line modelling; while certain others may warrant some behavioral model specification.(9) For those variables of the former type, the "aging technique" is left to the planner. This may be accomplished using simple trend-line equations, forecasting methods, etc. Adelman and Robinson (1978), for example, provides an inspiring tabulation of such exercises, and the interested reader may wish to consult with their methods on revising the exogenous variables of the Stage 1 Model.

In this section, two variables are selected and modelled using behavioral equations, just to give an example on such modelling. The variables are: labor supplies; $\overline{SL}_s$ (equation reference, II-19) and private investment allocation parameters, $\overline{H}_i$ (equation reference, II-49) though, of course, this selection is by no means decisive for all types of constructs. The behavioral models are taken from Dervis and Robinson (1978) and Dervis et. al. (1982). Lewis and Urata (1983), also used them with some minor changes in their planning exercises on Turkey, for the period 1978-1990.

LABOR SUPPLIES AND RURAL-URBAN MIGRATION

Labor supplies of different skill categories are endogenized through exogenously specified natural rates of population growth and endogenous migration from rural to urban sectors. Assuming that agricultural labor is distinguished by skill type-1 and that all other skill types, s=2,..., m, correspond to different categories of urban labor, we have the following system of labor supply equations:

$$SL_1(t+1) = (1+\Gamma_1)\ SL_1(t) - MIG(t) \qquad \text{(III-1)}$$

$$SL_s(t+1) = (1+\Gamma_s)\ SL_s(t) + (\bar{\bar{SM}}_s)\ MIG(t) \qquad \text{(III-2)}$$

$$s = 2,\ldots,m$$

where Γ_s (s=1,...,m) is the exogenously specified natural growth rate of the labor force - type s; $\bar{\bar{SM}}_s$ is the share of agricultural labor that joins the ranks of the urban labor force type-s (plausibly $\bar{\bar{SM}}_s$, will get a smaller numerical value as one goes higher in the skill levels distinguished for the urban sector). MIG(t) is the number of agricultural workers leaving their occupations and joining to the ranks of urban labor force.

Following Harris and Todaro (1970), migration is seen as a function of the differences between the rural and expected urban wages. In particular,

$$MIG(t) = \mu\ [(EW_u - W_1)/W_1]\ SL_1(t) \qquad \text{(III-3)}$$

where μ is a parameter measuring the responsiveness of migration to the differential between agricultural and anticipated urban wages; and EW_u is the expected urban wage, which can be formulated in the following simple fashion:

$$EW_u = \sum_{i=1}^{n} \sum_{s=2}^{m} [W_{pis}(t) \cdot L_{pis}(t) + W_{gis}(t) \cdot L_{gis}(t)]\ 1/L_u(t) \qquad \text{(III-4)}$$

where $L_u(t)$ is the total urban labor force (note s=2,...,m).

SECTORAL ALLOCATION OF PRIVATE INVESTMENT

Private investment behavior is one of the hardest aspects of applied planning exercises. Theoretically, private investment demand is affected by a whole set of variables, such as expected sales; past, present and expected future profits; profit rate differentials across sectors, as well as by the availability of the investment funds. In the previous applications of CGE Models, one of the most elaborate formulations of private investment behavior has been used by Adelman and Robinson (1978). In their model, investment demand and supply of loanable-funds decisions are carried out by different sets of agents. Through a simple model of expectations, enterprises form their investment demand decisions and demand funds from the loanable-funds market. Further, supply of funds come from various sources such as organized banks and the unorganized, curb market.

General and realistic as it is, the data base and effort required for such a construction is really tremendous and makes it very difficult to be applicable in development policy analysis. For this reason, I will resort to a less ambitious construction, one that has been used and tested successfully in the applied experiments of Dervis and Robinson (1978) and Lewis and Urata (1983).

In the Model used here, investment shares are seen as a function of the relative profit rate of each sector compared to the average profit rate as a whole. Sectors that have higher than average profit rates, then capture a larger portion of the private savings-pool. According to this formulation, investment shares are given by:

$$H_i(t+1) = SR_i(t) + \epsilon \cdot SR_i(t)\ [\pi_i(t) - \tilde{\pi}(t)]/\tilde{\pi}(t) \qquad \text{(III-5)}$$

where: $SR_i(t) = RP_i(t)/\sum_i RP_i(t)$, is the sectoral share in aggregate private profits; ϵ is a parameter measuring the mobility of investment funds; $\pi_i(t)$ is the sectoral profit <u>rate</u> and $\tilde{\pi}(t)$ is the average profit rate.

Private enterprise profit rates are formulated as follows:

$$\pi_i(t) = [PN_{pi}(t) \cdot XS_{pi}(t) - \sum_m W_{pim}(t) \cdot L_{pim}(t)]/PK_i(t-1) \cdot K_{pi}(t-1)$$

$$+[PK_i(t) - PK_i(t-1) \cdot (1-dp_i)]/PK_i(t-1) \qquad \text{(III-6)}$$

where dp_i is the fixed sectoral depreciation rate of the private physical capital stock, and $K_{pi}(t-1)$ is the amount of capital stock bought at the end of the last period and used in the production in the current period.

The crucial point in the formulation of investment allocation coefficients is to guarantee that they add up to 1. This is achieved by expressing the private enterprise average profit rate as the sum of the sectoral private profit rates weighted by their shares in total private profits. Thus,

$$\tilde{\pi}(t) = \sum_i \pi_i(t) \cdot SR_i(t) \qquad \text{(III-7)}$$

Since $\sum_i SR_i(t) = 1$, then it is true that for any value of ϵ, $\sum_i H_i(t) = 1$, because: (dropping time indeces)

$$\sum_i SR_i [\pi_i - \tilde{\pi}]/\tilde{\pi}$$

$$= \sum[RP_i/\sum RP_i((\pi_i \sum RP_i - \sum \pi_i RP_i)/\sum \pi_i RP_i)]$$

$$= (\sum \pi_i RP_i \sum RP_i - \sum RP_i \sum \pi_i RP_i)/\sum RP_i \sum \pi_i RP_i = 0$$

What remains is to up-date the sectoral capital stocks of the enterprises. Here, for the purposes of realism, one can specify gestation lags for installation of the capital stocks. Thus, the capital stock that will be used in the next production period will be expanded by the investments of the previous years finished after a certain lagged period of time. Accordingly, private enterprises' stocks of physical capital which will be used in the next period's production process will be given by:

$$Kp_i(t) = Kp_i(t-1)\cdot(1-dp_i) + \sum_{r=0}^{T} \beta_{pir}\ NP_i(t-r) \qquad \text{(III-8)}$$

where β_{pir} is the proportion of capital goods bought in time t-r that will be installed to the private firm operating in sector i, by the end of the current period. It must be true that $\sum_{r=0} \beta_{pir} = 1$. The variable T is the longest gestation lag, whereas the minimum lag can be set at one year by letting $\beta_{pio} = 1$.

Public enterprises' physical capital stocks which will be employed in the next period will likewise be:

$$Kg_i(t) = Kg_i(t-1)\cdot(1-dg_i) + \sum_{r=0}^{T} \beta_{gir}\ NG_i(t-r) \qquad \text{(III-9)}$$

This completes the construction of our Open CGE Model. In the next section I present the equations of the Stage 1 Model in a compact form. The presentation is designed so as to encompass a variety of applied problems; yet the specific modelling effort should, of course, always be suited to the special characteristics of the problem at hand.

IV – EQUATIONS AND VARIABLES OF THE OPEN CGE MODEL – STAGE 1

In this section I lay down the core equations of the Stage 1 Model. Table 1 gives the equation summary on prices, Table 2 gives the equations of factor markets and product supplies, and Table 3 presents those of the product markets.

TABLE 1: Prices

	No. of Eqns.	Reference No.
Composite commodity	n	(II-1)
$CC_i = \bar{B}_i[\delta_i M_i^{-\rho_i} + (1-\delta_i) DC_i^{-\rho_i}]^{-1/\rho_i}$		
Composite good price	n	(II-2)
$PC_i = 1/\bar{B}_i[\delta_i^{\sigma_i} PM_i^{(1-\sigma_i)} + (1-\delta_i)^{\sigma_i} PD_i^{(1-\sigma_i)}]^{1/1-\sigma_i}$		
Price of the imported good in domestic currency	n	(II-3)
$PM_i = \overline{PWM}_i (1+\hat{tm}_i) \hat{ER}$		
Net price or value added (f=p,g)	2n	(II-4)
$PN_{fi} = PD_i - \sum_j PC_j a_{ji} - \hat{tn}_i PD_i + \hat{sn}_{fi} PD_i$		
Price of capital	n	(II-50)
$PK_i = \sum_j b_{ji} PC_j$		
Normalization	1	(II-61)
$\sum_i PC_i \cdot \Omega_i = \bar{P}$		

Total: 6n + 1

Endogenous Variables		Number
CC_i	Composite good	n
PC_i	Composite good price	n
PM_i	Domestic price of imports	n
PN_{fi}	Net price (value added) of the output of firm type f.	2n
PK_i	Price of capital	n

Total: 6n

Exogenous Variables and Parameters

ρ_i	Composite good - CES function parameters
$\bar{B}_i$	Composite good - CES function scale parameter
δ_i	Composite good - CES function share parameter
σ_i	Elasticity of substitution between import and domestic demand in composite good
$\overline{PWM}_i$	World price of imports
$\hat{tm}_i$	Tariff rate on imports
$\hat{tn}_i$	Indirect tax rate
$\hat{sn}_{fi}$	Production subsidy rate
a_{ij}	Input-output coefficients
Ω_i	Weights of the price index
$\bar{P}$	Level of price index

TABLE 2: Factor Markets and Product Supplies

	No. of eqns.	Reference no.
Production functions (f=p,g)	2n	(II-6)

$$X_{fi} = \bar{A}_{fi}\, \bar{K}_{fi}^{\alpha_{fi}}\, L_{fi}^{(1-\alpha_{fi})}$$

Labor Aggregation (f=p,g)	2n	(II-7)

$$L_{fi} = L_{fi}(L_{fi1},\ldots,L_{fim}) = \bar{N}_{fi} \prod_{m=1}^{m} L_{fim}^{\lambda_{fim}}$$

Demand for Intermediate Inputs	n	(II-54)

$$V_i = \sum_j a_{ij}\, XS_j$$

Marginal Revenue (f=p,g)	2n	(II-8)

$$MR_{fi} = PN_{fi}(1+1/\xi_i^{xp\ up_{fi}})$$

Sectoral wage rate of the labor types (f=p,g)	2m·n	(II-9)

$$W_{fim} = \psi_{fim}\, W_m$$

Private firm sectoral output supplies	n	(II-12)

$$XS_{pi} = [\bar{A}_{pi}\, \bar{K}_{pi}^{\alpha_{pi}}\, MR_{pi}^{(1-\alpha_{pi})}\, \bar{N}_{pi}^{(1-\alpha_{pi})} \prod_m [(1-\alpha_{pi})\lambda_{pim}/W_{pim}]^{(1-\alpha_{pi})\lambda_{pim}}]^{1/\alpha_{pi}}$$

Private firm labor demands	n·m	(II-11)

$$L_{pim} = (1/W_{pim})\ [\ (1-\alpha_{pi})\ \lambda_{pim}\ ]\cdot MR_{pi}\cdot XS_{pi}$$

Public firm labor-output ratio	n	(II-15)

$$LX_{gi} = \bar{N}_{gi} \prod_{m=1}^{m} [(1/\widehat{INT}_i\cdot W_{gim})\ (1-\alpha_{gi})\ \lambda_{gim}\ MR_{gi}]^{\lambda_{gim}}$$

Public firm capacity utilization rate	n	(II-14)

$$Q_i = \widehat{INT}_i^{\,bmc_i(LX_{gi})}$$

Public firm sectoral output supplies n (II-16)

$$XS_{gi} = Q_i \{\bar{A}_{gi} \bar{K}_{gi}^{\alpha_{gi}} MR_{gi}^{(1-\alpha_{gi})} \bar{N}_{gi}^{(1-\alpha_{gi})} \cdot \prod_m [(1-\alpha_{gi})\hat{\lambda}_{gim}/(INT_i \cdot W_{gim})]^{(1-\alpha_{gi})\lambda_{gim}}\}^{1/\alpha_{gi}}$$

Public firm labor demands n·m (II-13)

$$L_{gim} = (1/INT_i \cdot W_{gim}) [(1-\alpha_{gi}) \hat{\lambda}_{gim}] MR_{gi} \cdot XS_{gi}$$

Aggregate Labor Demands m (II-18)

$$DL_m = \sum_i (L_{pim} + L_{gim})$$

Excess Demands for labor m (II-19)

$$DL_m - \bar{\bar{SL}}_m = 0$$

Sectoral Profits of the private enterprise n (II-20)

$$RP_i = PN_{pi} \cdot XS_{pi} - \sum_m W_{pim} \cdot L_{pim}$$

Sectoral Profits of the public enterprise n (II-21)

$$RG_i = PN_{gi} \cdot XS_{gi} - \sum_m W_{gim} \cdot L_{gim}$$

Total sectoral output supply n (II-17)

$$XS_i = XS_{pi} + XS_{gi}$$

Total: $14n + 2m + 4n \cdot m$

Endogenous Variables		Number
X_{fi}	Sectoral production technology of the firm type f	2n
L_{fi}	Labor used in sectoral production, by the firm type f	2n

V_i	Demand for Intermediate inputs	n
MR_{fi}	Marginal Revenue of the firm type f	2n
W_{fis}	Sectoral wage rate of the labor type s employed by the firm type f	2n·m
LX_{gi}	Public firm labor-output ratio	n
Q_i	Public firm capacity utilization rate	n
XS_{fi}	Sectoral output of the firm type f	2n
L_{fis}	Labor demand by sector and type	2n·m
DL_s	Aggregate labor demand by skill-type	m
W_s	Average nominal wage rate by skill type	m
R_{fi}	Sectoral profits of enterprises	2n
XS_i	Total sectoral output supply	n
	Total: 14n + 2m + 4n·m	

Exogenous Variables and Parameters

$\bar{A}_{fi}$	Production function scale parameter
$\bar{K}_{fi}$	Sectoral capital stock of the firm type f
α_{fi}	Production function share parameter
$\bar{N}_{fi}$	Labor aggregation function scale parameter
λ_{fis}	Elasticity of labor aggregate in sector i with respect to labor skill type s, employed in firm type f
$\overline{SL}_s$	Aggregate labor supply by skill type
ξ_i^{xD}	Elasticity of total demand for the domestic product
up_{fi}	Firm type f, monopoly power utilization parameter
ψ_{fis}	Coefficient of proportionality of the sectoral wage rate to average wage rate of labor type s employed in firm type f
$\hat{INT}_i$	Coefficient of government intrference to the public firm
bmc_i	Coefficient of "bad management" in the public firm

TABLE 3: Product Markets and Foreign Trade

	No. of eqns.	Reference no.
Total export subsidy granted under the tax rebate scheme	n	(II-31)
$TSR_i = \hat{tn}_i \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i$		
Total tax allowance granted under the corporate income tax allowance scheme	1	(II-32)
$TKA = \hat{kta} \cdot \sum_i \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i$		
Total export subsidy granted under the corporate income tax allowance scheme	n	(II-33)
$TSA_i = \hat{tk} \cdot \hat{kta} \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i$		
Domestic currency value of imported intermediate inputs used for export production	n	(II-34)
$EMI_i = \sum_j \overline{PWM}_j \cdot \hat{ER} \cdot d_j \cdot m_j \cdot a_{ji} \cdot E_i$		
Total tariff cost on imported intermediate inputs used for export production	n	(II-35)
$TEMI_i = \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot \hat{ER} \cdot d_j \cdot m_j \cdot a_{ji} \cdot E_i$		
Realized combined export subsidy rate	n	(II-36)
$SE_i = (\hat{tn}_i + \hat{tk} \cdot \hat{kta})\hat{ee}_i + \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot d_j \cdot m_j \cdot a_{ji} \ (1/PWE_i) + \hat{te}_i$		
Export supply price of domestic commodity in foreign currency	n	(II-29)
$PWE_i = PD_i/(1+SE_i) \cdot \hat{ER}$		
Foreign demand for exports	n	(II-30)
$E_i = \bar{E}_{oi} \ (\overline{AWP}_i/PWE_i)^{\eta_i}$		

Supply of Domestic goods used in the domestic market n (II-25)

$$DS_i = XS_i - E_i$$

Domestic demand for imports n (II-27)

$$M_i = (PD_i/PM_i)^{\sigma_i} \, (\delta_i/1-\delta_i)^{\sigma_i} \, DS_i$$

Balance of Payments Equilibrium 1 (II-37)

$$\sum_i \overline{PWM}_i \cdot M_i - \sum_i PWE_i \cdot E_i - \overline{F} - \overline{WR} = 0$$

Labor income by skill type m (II-38)

$$YL_m = (1-\hat{ts}) \sum_i (W_{pim} \cdot L_{pim} + W_{gim} \cdot L_{gim}) + \omega_m \cdot \overline{WR} \cdot \hat{ER}$$

Capitalist Income 1 (II-39)

$$YK = (1-\hat{tk}) \sum_i RP_i$$

Public Enterprise income 1 (II-40)

$$YKG = (1-\hat{tk}) \sum_i RG_i$$

Government Income 1 (II-41)

$$YG = \sum_m \hat{ts} \sum_i (W_{pim} \cdot L_{pim} + W_{gim} \cdot L_{gim})$$

$$+ \hat{tk} \sum_i (RP_i + RG_i - \hat{kta} \cdot \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i)$$

$$+ \sum_i [\hat{tm}_i \cdot \overline{PWM}_i \cdot \hat{ER} \cdot M_i - \sum_j \hat{tm}_j \cdot \overline{PWM}_j \cdot \hat{ER} \cdot d_j \cdot m_j \cdot a_{ji} E_i]$$

$$+ \sum_i \hat{tn}_i [PD_i \, XS_i - \hat{ee}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i] - \sum_i \hat{te}_i \cdot PWE_i \cdot \hat{ER} \cdot E_i$$

$$- \sum_i [\hat{sn}_{pi} \cdot PD_i \cdot XS_{pi} + \hat{sn}_{gi} \cdot PD_i \cdot XS_{gi}] + YKG + \overline{F} \cdot \hat{ER}$$

Total private savings 1 (II-42)

$$TPS = \sum_m \overline{S}_m \, YL_m + sk \, YK$$

Government investment fund 1 (II-43)

$$GIF = \hat{\theta}[\sum_i PD_i \cdot XS_i - \sum_i\sum_j PC_j a_{ji} XS_i]$$

Government savings rate 1 (II-44)

$$Sg = G\dot{I}F/YG$$

Labor consumption m·n (II-45)

$$CL_{mi} = \tau_{mi} + \beta_{mi}/PC_i((1-\bar{S}_m)YL_m - \sum_j PC_j \tau_{mj})$$

Capitalist consumption n (II-46)

$$CK_i = \tau_{ki} + \beta_{ki}/PC_i \ ((1-sk)YK - \sum_j PC_j \cdot \tau_{kj})$$

Government consumption n (II-47)

$$CG_i = \hat{q}_i \ (1-Sg) \ YG/PC_i$$

Total consumption demand n (II-48)

$$C_i = \sum_m CL_{mi} + CK_i + CG_i$$

Real Private Investment n (II-49)

$$NP_i = \bar{H}_i \ (TPS/PK_i)$$

Real government investment n (II-51)

$$NG_i = \hat{HG}_i \ (GIF/PK_i)$$

Total real investment to sector i (investment by sector of destination) n (II-52)

$$NT_i = NP_i + NG_i$$

Real investment demand (investment by sector of origin) n (II-53)

$$Z_i = \sum_j b_{ij} \cdot NT_j$$

Total demand for commodity i n (II-55)

$$TD_i = C_i + Z_i + V_i$$

Domestic use ratio n (II-57)

$d_i = f_i^{-1}(m_i, 1)$

Total domestic demand for domestic production n (II-58)

$DC_i = d_i \cdot TD_i$

Total demand for domestic production n (II-59)

$XD_i = DC_i + E_i$

Market clearing n (II-60)

$XD_i - XS_i = 0$

Total: $21n + m + n \cdot m + 8$

Endogenous Variables

		Number
TSR_i, TSA_i	Total export subsidy granted under the tax rebate and corporate income tax allowance schemes, respectively.	2n
TKA	Total corporate income tax allowance granted	1
EMI_i	Value of imported intermediate inputs used for export production	n
$TEMI_i$	Total tariff cost paid on EMI_i	n
SE_i	Realized combined export subsidy rate	n
PWE_i	Price of the exported domestic good	n
E_i	Foreign demand for exports	n
DS_i	Domestic supply, consumed domestically	n
M_i	Import demand	n
YL_s	Labor income by type	m

YK	Capitalist income	1
YKG	Public enterprise income	1
YG	Government income	1
TPS	Total private savings	1
GIF	Government investment fund	1
Sg	Government savings rate	1
CL_{mi}	Consumer demand by labor type	m·n
CK_i	Consumer demand by capitalists	n
CG_i	Government's consumption demand	n
C_i	Total consumption demand	n
NP_i	Real private investment to sector i	n
NG_i	Real government investment to sector i	n
NT_i	Total real investment by sector of destination	n
Z_i	Real investment demand by sector of origin	n
TD_i	Total demand for product i	n
d_i	Domestic use ratio	n
DC_i	Total domestic demand for domestic production	n
XD_i	Total demand for domestic production	n
$\hat{ER}$	Exchange rate	1
PD_i	Price of the domestically produced good	n

Total: 21n + m + n·m + 8

Exogenous Variables and Parameters

$\hat{ee}_i$	Ratio of eligible exports in total exports benefitting from export incentives
$\hat{kta}$	Tax allowance rate

$\hat{te}_i$	Export subsidy/tax rate
$\bar{E}_{oi}$	Normal level of exports
$\overline{AWP}_i$	Average world price of aggregated commodity i
η_i	Elasticity of export demand
$\bar{F}$	Net foreign resource inflow
$\overline{WR}$	Workers' remittances
$\hat{t}_s$	Tax rate on labor income, type s
ω_s	Share of workers' remittances accruing to labor type s
$\hat{tk}$	Tax rate on capitalist income
$\bar{S}_s$	Saving rate of labor, type s
sk	Capitalist saving rate
$\hat{\theta}$	Ratio of total public investment to Gross Domestic Product
τ_{si}	Absolute minimum (subsistence) level of consumption of product i, by labor type s.
τ_{ki}	Absolute minimum (subsistence) level of capitalist consumption of product i.
β_{si}	Labor type s, marginal budget share of product i
β_{ki}	Capitalists' marginal budget share of product i
$\hat{q}_i$	Government's consumption expenditures - share of product i
$\bar{H}_i$	Private investment-allocation share
$\hat{HG}_i$	Public investment-allocation share

There are a total of $41n + 3m + 5n \cdot m + 9$ equations and $41n + 3m + 5n \cdot m + 8$ endogenous variables. However, not all of the equations are independent. The n excess demand equations can determine only n-1 relative prices, and to do so one has to specify a normalization equation to set the absolute price level.

Footnotes:

(1) For a recent survey on CGE-type Modelling see: Shoven & Whalley (1984).

(2) My sole purpose in this introduction is to provide a bird's-eye-view comparison of various multisector planning models. The interested reader can find a comprehensive survey of Linear Programming Models in Taylor (1979); and of planning models in general, in Blitzer, Clark & Taylor (1975).

(3) For a more formal discussion on this proposition, see for example, Intriligator (1971).

(4) Dervis, de Melo and Robinson, 1982, p. 132. This observation clearly applies to the general structure of most of the LP models. Yet, for purposes of completeness we need to stress the existance of a body of literature which attempts to compute the competitive market equilibria by means of extensions of a mathematical programming model (e.g., see Goreaux (1977); Manne et.al. (1978); Norton & Scandizzo (1981)). The Goreaux and Manne et. al. studies utilize successive recursive sequences of linear programming solutions, and can be regarded as lying halfway between the LP and CGE type models.

The Norton & Scandizzo study, on the other hand, tries to mimic competitive equilibria by directly constructing the essential conditions of such equilibria as inequality constraints in the primal problem; and thus, their procedure can be utilized to yield a non-recursive linear programming solution. To fit the model into the LP framework, they first employ grid linearization techniques on non-linear constraints. Their maximand is defined as the excess of expenditures over factor incomes and attains a value of zero at the optimum. Also at the optimum, the dual shadow prices of resource constraints turn out to be equal to the primal variables which represent the rate of return on factors.

Innovative as it is, their model suffers from the implicit condition that no resource is allowed to be underemployed; thus to have a shadow value of zero at the optimum leaving its owner with a nill level of income. Yet, it is a reality of life that in many developing countries, certain resources (particularly labor) remain to be underutilized. Thus, the core of the problem, that is, to incorporate the sets of price-incentives as essential tools of policy-makers in decentralized, mixed economies with a general equilibrium framework, still remains to be addressed in this model as well.

(5) For derivation, see e.g. Varian (1978). The above form is taken from Dervis et. al. (1982).

(6) See, Appendix to Chapter 2 in Dervis et. al. (1982) for a discussion of these conditions. For the original statement, see: Hawkins, D & H. A. Simon "Note: Some conditions of Macroeconomic Stability" (1949) Econometrica, 17, July-Oct., pp. 245-48.

(7) The Model captures such intertemporal innovations in the Dynamic Stage with the explicit specification of the endogenous private investment share parameters (see equations III-5).

(8) For a discussion of various solution strategies and solution algorithms on the iteration of prices, see Dervis et. al., (1982, Appendix B); and Adelman and Robinson (1978, Appendix B) and the references therein cited.

(9) For a discussion on the strategy for construction dynamic, economy-wide models of developing countries, see Robinson (1976).

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Appendix A: Derivation of the Linear Expenditure System

In this Appendix I will present the derivation of the private consumption equations (II-45,-46) in more detail. For applied work linearity of such equations provides a distinct advantage and this is one of the reasons of the popularity of linear expenditure systems (LES). The other advantage of the LES is that it is relatively easy to estimate its parameters with modest data requirements.

This presentation follows that of Taylor (1979); for another method on deriving linear consumption functions the reader may refer to the discussion in Adelman & Robinson (1978, Appendix A). The notation used here pertains only to this Appendix.

First, we begin with the conventional hypothesis that each consumer group (m - labor categories and capitalists, a total of m + 1) shares a common utility function and thus, can be analyzed by the actions of a representative consumer. Dropping the subscripts s and k that distinguish different consumer categories, the preference map is assumed to be represented by the following function:

$$U = \Sigma \beta_i \log(C_i - \tau_i) \qquad \text{(A-1)}$$

where C_i is the consumption of i'th commodity and β_i and τ_i are the parameters of the utility function. If you recall, I have let β_i to stand for the marginal budget share of product i and τ_i to denote the "subsistence minima" of product i in physical terms.

Assuming non-satiation we can set up the budget constraint of the representative consumer as an equality,

$$\sum_i P_i \cdot C_i = Y \qquad \text{(A-2)}$$

where Y is the total money income of the consumer and P_i is the market price of the i-th commodity. The first order conditions of the consumer's problem yield.

$$\beta_i = \lambda P_i \, (C_i - \tau_i) \qquad \text{(A-3)}$$

where λ is the familiar Lagrange multiplier. In our context we will interpret it as the "marginal utility of income."

To solve the system (A-3), we will resort to a normalization rule which states that summation of β_i over i adds up to unity. Thus,

$$1 = \sum_i \beta_i = \lambda \, (\sum_i P_i C_i - \sum_i P_i \tau_i) \qquad \text{(A-4)}$$

Let us denote $\sum_i P_i \tau_i$ by S, the total cost of subsistence consumption basket, then from (A-4) we can solve for λ,

$$\lambda = 1/Y-S \qquad \text{(A-5)}$$

Substituting (A-5) into (A-3) we get a specific formula for demand of commodity i,

$$C_i = \tau_i + (\beta_i / P_i) \, (Y-S) \qquad \text{(A-6)}$$

which is the consumption equation I have used in the Model.

Given data on <u>average</u> budget shares, α_i ($\alpha_i = P_i C_i / Y$), and income elasticities, η_i, we can get an estimate of the marginal budget shares, β_i. Since by definition,

$$\eta_i = \partial C_i / \partial Y \cdot Y/C_i \qquad \text{(A-7)}$$

and

$$\beta_i = \partial C_i / \partial Y \cdot P_i \qquad \text{(A-8)}$$

we have

$$\beta_i = \alpha_i \cdot \eta_i \qquad \text{(A-9)}$$

which provides a straightforward estimate of β_i on the basis of the estimates of η_i.

The LES specification also provides an expression of the minimum subsistence parameter, τ_i, as a function of the other parameters of the model. To do this, however, I first introduce another parameter, θ, which stands for elasticity of marginal utility of income with respect to income, first used by Ragner Frisch (1959). We thus have,

$$\theta = d\lambda/dY \; Y/\lambda = -Y/Y-S \qquad \text{(A-10)}$$

Substituting the consumer-demand function (A-6) into the income-elasticity equation (A-7), we get:

$$\eta_i = \beta_i \; [Y/(P_i \cdot \tau_i + \beta_i \; (Y-S))] \qquad \text{(A-11)}$$

solving for τ_i,

$$\tau_i = (Y/P_i) \; (\alpha_i + \beta_i/\theta) \qquad \text{(A-12)}$$

which can be derived easily. Expression (A-12) can be written in even a more compact form by letting $\sigma = -1/\theta = (Y-S)/Y$, which gives the ratio of the consumer's excess income over the subsistence level to the income level. Following this definition, σ has been usually referred as the supernumerary income ratio. Lance Taylor reports that conventionally σ takes a value of about 0.5 for most consumer groups. Thus, having information on σ, the subsistence parameter τ_i can be easily estimated using the estimate of β_i.

Using σ, we can also derive neat expressions for own- and cross- price elasticities of demand. Differentiating consumer demand equation (A-6) and using the definition of price elasticities we get,

$$\epsilon_{ii} = -\eta_i \; (P_i \cdot \tau_i/Y + \sigma) \qquad \text{(A-13)}$$

$$\epsilon_{ij} = -\eta_i \; (P_j \tau_j/Y) \qquad \text{(A-14)}$$

where ϵ_{ii} and ϵ_{ij} are own- and cross- price elasticities of demand, respectively. In equation (A-13) the first term, $\eta_i(P_i\tau_i/Y)$, gives the income effect of a change price, P_i. The second term, $\eta_i\sigma$ measures the substitution effect. As can be seen as σ gets smaller (i.e. the smaller the excess of income over subsistence level), so does the consumer's substitution response to a percentage change in P_i.

Appendix B: Endogenous Derivation of the Own-Price Elasticity of Domestic Demand

In this Appendix we seek for an expression for ε_i^{XD} -own domestic price elasticity of aggregate demand for the domestic commodity- which can be generated within the Model endogenously. The calculation of this parameter bears importance, because for sectors which entail monopoly power in their product markets, labor hire, and thus, output supply decisions will depend upon the numerical value of the elasticity of the total demand curve facing the monopolist. It needs to be noted that, for the most part, the Appendix is written in the spirit of a suggestive essay. It needs to be amended in the realm of the model construction and available data. Of course, ε_i^{XD} can as well be estimated econometrically in a separate study and can be fed into the Model directly. Yet, this method may be harder and less reliable than it seems, due to the high level of aggregation of products in each sector. At such high levels of aggregation, the econometric fitness and the validity of the numerical value estimated for ε_i^{XD} may be questionable. Further, since CGE type Models are actually "consistency" models, it will be a more sound strategy to let the Model derive its parameters endogenously as much as possible and to rely less on parameters derived from outside models which use a different methodology.

There is yet a third possible "method" for finding a value for ε_i^{XD}, and that is to make an "educated guess" on likely values of ξ_i^{XD}. For example, the Adelman & Robinson (1978) Model which has as well incorporated monopoly power in certain product markets, tried to handle the problem this way. Retaining the "consistency spirit" of the Model and asking it to originate the relevant parameter within its structure is certainly a superior method over this option as well.

Then, to recapitulate, aggregate demand of the i-th Domestic Commodity is:

$$XD_i = d_i[\sum_s CL_{si} + CK_i + CG_i + Z_i + V_i] + E_i \qquad \text{(B-1)}$$

Therefore the elasticity of aggregate demand of the domestic commodity i with respect to own domestic price is a weighted average of the elasticities of its components:

$$\xi_i^{XD} = d_i[\sum_s \xi_{i,s}^{CL}\, CL_{si}/XD_i + \xi_i^{CK}\, CK_i/XD_i + \xi_i^{CG}\, CG_i/XD_i \qquad \text{(B-2)}$$

$$+\xi_i^{Z}\, Z_i/XD_i + \xi_i^{V}\, V_i/XD_i] + \eta_i \cdot E_i/XD_i$$

Since the consumption functions are of the LES type, ξ_i^{CL} and ξ_i^{CK} can be calculated using the relevant results of the previous Appendix. Thus,

$$\xi_{is}^{CL} = -\xi_{Ys}^{CL}\, [PC_i \cdot \tau_{si}/YL_s + (YL_s - \sum_j PC_j \cdot \tau_{sj})/YL_s] \cdot \xi_{PD,i}^{PC} \qquad \text{(B-3)}$$

$$\xi_i^{CK} = -\xi_Y^{CK}\, [PC_i \cdot \tau_{Ki}/YK + (YK - \sum_j PC_j \cdot \tau_{Kj})/YK] \cdot \xi_{PD,i}^{PC} \qquad \text{(B-4)}$$

where ξ_{Ys}^{CL} and ξ_Y^{CK} are income elasticities of consumption demand of labor type-s, and of the capitalists, respectively. Note that the system of consumption demands is originally specified as a function of PC_i's (the composite good prices); and yet, we need to have an expression for the elasticity of consumption demand with respect to the <u>domestic price</u>. Thus, we use

$$\xi^{CL}_{im} = (\partial CL_{mi}/\partial PC_i)\cdot(PC_i/CL_{mi})\cdot(\partial PC_i/PD_i)\cdot(PD_i/PC_i) \qquad \text{(B-5)}$$

which explains the presence of $\xi^{PC}_{PD,i}$ (the elasticity of composite price with respect to the domestic price) in (B-3) and (B-4).

The sectoral government consumption demands are given by the fixed share-coefficients. Thus:

$$\xi^{CG}_{i} = (-1)\cdot\xi^{PC}_{PD,i} \qquad \text{(B-6)}$$

In order to find the elasticity of investment demand we first begin by writing the components of Z_i more clearly:

$$Z_i = \sum_j b_{ij}\,[(\bar{H}_j\cdot TPS + \hat{HG}_j\cdot GIF)/PK_j]$$

$$= \sum_j b_{ij}\,[(\bar{H}_j\cdot TPS + \hat{HG}_j\cdot GIF)/(\sum_k b_{kj}\cdot PC_k] \qquad \text{(B-7)}$$

Thus:

$$\partial Z_i/\partial PC_i = -\sum_j b^2_{ij}\,[\bar{H}_j\cdot TPS + \hat{HG}_j\cdot GIF]/(\sum_k b_{kj}\cdot PC_k)^2$$

which yields

$$\xi^{Z}_{i,PC} = (\partial Z_i/\partial PC_i)\cdot(PC_i/Z_i)$$

$$= -\sum_j [b^2_{ij}\,(\bar{H}_j\cdot TPS+\hat{HG}_j\cdot GIF)](PC_i/Z_i)/(\sum_k b_{kj}\cdot PC_k)^2$$

$$= -\sum_j [(b_{ij}/PK_j)\cdot Z_j]\cdot(PC_i/Z_i) \qquad \text{(B-8)}$$

Then, all we need is to solve for:

$$\xi_i = \xi^{z}_{i,PC} \cdot \xi^{PC}_{PD,i} \qquad \text{(B-9)}$$

The demand for intermediate goods is given by fixed input-output coefficients; and will be assumed to be non-responsive to price changes.

Actually, any change in the domestic relative prices will induce an output effect and the changing output supply decisions will result in different quantities of intermediate goods demanded. Thus, we must have:

$$\partial V_i/\partial PD_i = \sum_j a_{ij} \; (\partial XS_j/\partial PD_i) \qquad \text{(B-10)}$$

where the term in the parantheses (the indirect effect of the cross-prices on output supplies) can be deduced from the factor markets through:

$$\partial XS_{fj}/\partial PD_i = (\partial XS_{fj}/\partial L_{fj}) \prod_{m} [(\partial L_{fj}/\partial L_{fjm}) \; (dL_{fjm}/dW_{fjm} \cdot dW_{fjm}/dPD_i)] \qquad \text{(B-11)}$$

The latter component $(dL_{jm}/dW_{jm} \cdot dW_{jm}/dPD_i)$ is embedded in the solution algorithm for clearing the labor markets and yet is not observable in a functional form.[1] In what follows, we are not able to make use of the functional relationship (B-11).

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(1) I am indebted to Prof. T. Roe for his comments on this point.

The assumption that the elasticity of intermediate demand is zero, clearly puts a downward bias on the aggregate value of ξ_i^{XD}. Yet, as we argued, due to the fixed-coefficients technology, the overall price sensitivity of intermediate demands have to be very low and the incurred bias shouldnot be substantial.

Our final task is to derive an expression for $\xi_{PD,i}^{PC}$.

Since,

$$PC_i = 1/\bar{B}_i [\delta_i^{\sigma_i} PM_i^{(1-\sigma_i)} + (1-\delta_i)^{\sigma_i} PD_i^{(1-\sigma_i)}]^{1/1-\sigma_i} \quad (B-12)$$

we have

$$\partial PC_i/\partial PD_i = (PC_i/PD_i)^{\sigma_i} (1/\bar{B}_i)^{1-\sigma_i} (1-\delta_i)^{\sigma_i} \quad (B-13)$$

Thus,

$$\xi_{PD,i}^{PC} = (\partial PC_i/\partial PD_i)\ (PD_i/PC_i)$$

$$= (PC_i/PD_i)^{\sigma_i-1} (1/\bar{B}_i)^{1-\sigma_i} (1-\delta_i)^{\sigma_i} \quad (B-14)$$

which completes our derivation. It should be noted, however, that in spite of its general stance, this exercise still entails partial equilibrium characteristics. We have not sought for an expression on the likely general equilibrium effects of PD_i on overall budgets of the economic agents which work through the factor markets and derived factor incomes. These effects have to be found in the solution algorithm and the overall interaction of all markets with a complex set of interlinkages, and technically cannot be identified in a single -or set of- functional form(s). The incurred bias in this manner may or may not be substantial depending on the specific case at hand and the results should be interpreted with caution.

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