# Crop Insurance Valuation Under Alternative Yield Distributions 

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#### Abstract

: Considerable disagreement exists about the most appropriate characterization of farm-level yield distributions. Yet, the economic importance of alternate yield distribution specifications on insurance valuation, product designs and farm-level risk management has not been investigated or documented. The results of this study demonstrate that large differences in expected payments from popular crop insurance products can arise solely from the parameterization chosen to represent yields. The results suggest that the frequently unexamined yield distribution specification may lead to incorrect conclusions in important areas of insurance and risk management research such as policy rating, and assessment of expected payments from policies.


Keywords: crop yield distributions, crop insurance, risk and uncertainty.

## Introduction

Successful risk management strategies depend on accurate characterization of the uncertainties being faced. Of primary importance for crop farmers is revenue variability arising from uncertain crop yields and prices. In response to the difficulty in managing this risk, the Federal government has encouraged the development of new crop yield and revenue insurance products, and has provided incentives to purchase crop insurance by subsidizing many insurance premiums. However, crop insurance participation rates have been cited as disappointingly low and uneven, and loss rates have not been easily or accurately estimated. Attempts to better understand participation, and loss rating, and to further improve product designs have been hampered by the considerable disagreement that remains about the most appropriate characterization of farm-level yield and revenue distributions.

Crop revenue risks arise from both price and yield components. While crop price distributions are reasonably well understood and can be fairly accurately recovered from futures and options data (e.g., Fackler and King; Sherrick, Garcia and Tirupattur; Zulauf and Irwin), alternative conceptual frameworks and methodologies have led to substantial divergence in cropyield models (e.g., Day; Gallagher; Goodwin and Ker; Heifner; Just and Weninger; Marra and Schurle; Moss and Shonkwiler; Nelson and Preckel; Pease; Ramirez; Schurle; Wang et al.; Ker and Goodwin). Some yield models have taken advantage of parametric distributions with known attributes, such as the beta distribution (e.g., Nelson and Preckel; Tirupattur, Hauser and Chaherli) or the lognormal distribution (e.g., Jung and Ramezani; Stokes; Tirupattur, Hauser and Chaherli). Other yield models have explored distributional aspects of crop yields using nonparametric approaches (e.g., Ker and Goodwin). A recent article by Just and Weninger even challenges the consensus view that yields are distributed non-normally, arguing instead that methodological and data limitations may have led to inappropriate rejection of normality in favor of other parametric representations.

While different yield-model specifications could reasonably be expected to significantly impact quantitative assessments of revenue risk, insurance values, and other components that enter farmers' risk management decisions, the economic importance of alternate yield distribution assumptions has not been formally examined. Previous empirical studies rarely use more than one yield model, and research that has considered at least two yield-model specifications simultaneously provides mixed evidence (e.g. Skees and Reed; Buccola; Nelson; Tirupattur, Hauser and Chaherli). In response, this study evaluates five alternative parametric yield specifications that have been suggested as candidates by previous work or empirical evidence, and assesses their implications for valuation of two popular crop-insurance products. One of the factors most responsible for the uncertainty surrounding yield model specification in the past has been the lack of reliable farm-level yield data (Taylor; Just and Weninger). This analysis utilizes highly unique farm-level data from the University of Illinois Endowment Farms containing same-site yield records from 1972 to 1999 , including 26 and 25 corn and soybeans fields, respectively. This relatively long sample period, and reasonable large number of sites, permits a broader assessment of alternative distributions than has been possible with single site, or more aggregated data that have been commonly used in earlier studies.

The remainder of the paper is organized as follows: the subsequent section provides a general description and summary statistics of the data. Thereafter, potential candidate parameterizations are discussed with the normal, logistic, Weibull, beta and lognormal distributions selected for analysis. The next section summarizes the maximum likelihood estimation results. Then, actuarial tables and Average Production History (APH) insurance are evaluated under the alternative yield distributions, quantifying the economic implications of alternative crop-yield representations. The price component of farmers' revenue distribution is then brought into the analysis through an evaluation of Crop Revenue Coverage (CRC) insurance payments. The final section of the paper summarizes the results and offers potential implications of the work, and guidance for further research.

## Data

The yield data used in this study are from the University of Illinois Endowment Farm Division. This Division manages over 11,000 acres distributed among numerous farms ranging in size from 40 acres up to 1,200 acres. These farms were conveyed to the University by donors with the intent of providing support to various academic, research, fellowship, and scholarship programs. The objective of Division managers is to maximize funds for these programs by managing farms in the most profitable manner possible. Division managers accomplish this objective by renting farms to over 40 commercial farm operators predominately using 50-50 share rental arrangements, a common leasing arrangement in northern and central Illinois. Hence, these farms are operated and managed in a manner similar to most commercial operations in Illinois and provide highly representative yield data under accurate and consistent recordkeeping practices.

The Endowment Farms studied are located in 12 counties throughout northern and central Illinois. The farms are distributed throughout the state in an area approximately 200 miles north to south, and 150 miles from east to west. The farmland in this region is generally very high
quality, with above average corn and soybean yields relative to the remainder of the nonirrigated farmland in the United States. As is typical of commercial grain operations in Illinois, approximately one-half of a farm's acreage is devoted to corn production while the other onehalf is devoted to soybean production.

Annual corn and soybean yields were obtained from each farm. As is the case on any commercial farm, these yields are not necessarily field specific. Under the typical yield rotations employed, a farm's corn yield in one year comes from a mixture of fields, most of which were planted to soybeans in the previous year and in corn two years prior, and vice versa for soybeans. This manner of constructing yield series is the same as is used in determining farm-specific Actual Production History (APH) proven yields, and for other crop insurance purposes.

To be included in this study the data were screened to have at least 20 crop yield observations over the period 1972 through 1999. By comparison to previous studies, the sample period tends to be much longer, thereby overcoming a data limitation often mentioned when conducting yield-fitting exercises (e.g., Taylor). Application of this data screen resulted in 26 farms with corn yield series and 25 farms with soybean yield series. For identification purposes, the farms are designated with single letters " $a$ " through " $z$ ", generally sorted from northern to southern Illinois.

The use of time series data to represent a point-in-time distribution requires that the time component, if any, be controlled in the data. Commonly employed approaches for detrending yield data include the use of deterministic trend models and stochastic trend models. A fairly flexible approach that is consistent with most previous studies is to consider a polynomial time trend and test for the order of the polynomial. As proposed by Just and Weninger, yields are fitted to time trend models and tested downward from a fifth order (i.e., $y_{t}=\alpha_{1}+\alpha_{2} t+\alpha_{3} t^{2}+$ $\alpha_{4} t^{3}+\alpha_{5} t^{4}+\alpha_{6} t^{5}$ where $y_{t}$ is yield in year $t$, $t$ is a year index, and $\alpha$ are parameters to be estimated), using standard F-tests to judge parameter significance. Unlike Just and Weninger the results from this much richer data set suggest that a linear model adequately captures yield trends. For corn, no third or higher order parameters were significant and the second order term was significant on only one farm. For soybeans, second order terms were not rejected for only six farms (farms $a, c, f, h, l$, and $w$ ), but with no obvious consistency in the parameters. Because a linear model fit the data reasonably well, linear models are used to detrend data for all farms in this study. Moreover, a quadratic trend model is fundamentally inconsistent in its implication that the trend is actually decreasing over some range and has either a maximum or a minimum depending on its parameter sign. Third and higher order trends that have been found in previous work are therefore likely to have been sample period specific or simply overfit. Thus, the yield data from each farm for both crops were detrended using a linear specification before fitting to alternate distributional specifications. The year 2000 is used as the as the base time period in detrending the data. ${ }^{1}$

Summary statistics of detrended yields for corn are shown in table 1. The means of detrended yields across all farms average 155.4 bushel, indicating highly productive farmland. The means vary considerably, ranging from a low of 117.6 bushels (farm $u$ ) to a high of 180.6 bushels (farm $h$ ). However, not all of this variability is strictly due to land productivity. Farms $o$ to $s$ are actually contiguous to one another in Piatt county. Even on these farms, which are generally viewed as being of roughly similar quality, the means of yields range from 144.9 (farm
$r$ ) bu./acre to 167.8 bu./acre (farm $q$ ). Obviously, factors other than productivity, such as quality of farm operator and random chance, influence yield estimates. The variability of yields also differs across the farms. Standard deviations range from 20.3 (farm $i$ ) to 37.1 (farm $x$ ). The Piatt county farms also have a meaningful range in standard deviations ( 24.0 for farm $q$ to 30.3 for farm $p$ ). All but one of the samples exhibit negative coefficients of skewness. The coefficients of skewness range from the low of -1.6999 (farm $a$ ) to .0074 (farm $n$ ), with an average of -.811 across all farms (see table 1). Negative skewness is consistent with results from other studies, but calls into question the use of symmetric distributions to model yields. Sample kurtosis coefficients range from $2.164($ farm $n$ ) to 6.568 (farm $a$ ), with an average value of 3.720 . Thus, the farm level corn yields are considerably "fatter tailed" on average than would be implied by normal distributions of yields. The summary statistics for soybeans (shown in table 2) indicate the same general features as for corn: distributions differ meaningfully across farms, and yields display predominantly negative skewness and fatter tails than would be implied by normal distributions.

## Crop-yield distributions

In selecting candidate parameterizations of the yield distributions, consideration was given to (i) the preponderance of empirical evidence generated through sample statistics and moment-ratio diagrams, (ii) stylized features of crop yields (i.e., bounded at zero, potentially asymmetric), and (iii) past modeling efforts reflected in the literature. In this study, only parametric approaches are considered, and the class of distributions restricted to finite variance, and uni-modal or l-shaped distributions. The result is a manageable scope while allowing reasonably broad and flexible distributions to be examined.

Considerable empirical evidence is provided through the use of moment-ratio diagrams in which sample moments are plotted and compared to plausible ranges permitted by different parameterizations. The use of moment ratio diagrams has been shown to provide a useful means of comparing alternative distributions and qualitative assessing their abilities to fit the data (D'Agostino and Stephens; Day and Nelson). The moment ratio diagram in figure 1 shows skewness $(\sqrt{b 1})$ and kurtosis ( $b 2$ ) points or regions permitted under several distributions. The normal distribution is represented by a point located at $(0,3)$ in the skewness-kurtosis plane. The logistic has skewness of 0 (symmetric) and kurtosis of 4.2 (fatter tailed than normal). The lognormal distribution has the feature that, while permitting varying magnitudes of positive skewness, the third and fourth moments can be written as functions of each other. The result of this fact is that the region occupied in the skewness-kurtosis plane by the lognormal distribution is a curved line segment. The Weibull permits positive or negative skewness with associated kurtosis values resulting in a curved segement in the skewness-kutosis plane. The beta distribution, represented by the shaded areas in figures 1 and 2, covers a fairly wide region in the skewness-kurtosis plane. The general use of the moment ratio diagrams is to plot the sample moments and use the location to identify candidate parameterizations that are more likely to have generated the data. For some distributions, it is also possible to construct formal tests of the "distance" between sample moments and implied measures. For example, the Jarque-Bera test can be represented by an ellipse around the point $(0,3)$ in the skewness-kurtosis plane with
rejections of normality when the sample data plot outside the boundary of the ellipse representing a given level of significance (note that these tests are not shown here).

Figure 1 contains points marking the sample skewness-kurtosis pairs for each of the farms' corn series. The locations of the points relative to those implied by distributions strongly suggest that the yield distributions are not generated by a normal distribution. If yield data were generated by normal distributions, the points would be distributed with a declining density around the point $(0,3)$. Instead, the corn samples display prominent negative skewness and wide variation in the kurtosis. The sample point are not clustered around the lognormal line segment or the logistic point with the implication that they may not serve well to characterize the sample data as the imposition of their functional restrictions results in "relocating" the mass in the distribution to satisfy their skewness-kurtosis relationships. Based on the moment ratio diagrams, the beta and Weibull distribution appear to be plausible parameterizations of the corn yield series. Similar qualitative results are obtained for soybeans, as shown in figure 2. As a result of the empirical features and to reflect important stylized considerations, the beta and Weibull distributions will be included in further tests. The logistic is also retained as a fattertailed alternative to the normal, and because it also fits well in some specific circumstances.

In addition to the empirical guidance, there are recent cases in the literature to which a comparison of alternate parameterizations could prove to be important. Most prominently, the normal has been frequently discussed and examined, with its most recent and strongest defense by Just and Weninger. The lognormal is also often used as it is convenient and well-understood and exhibits several desirable features for representing yields (non-negative, flexible, easily estimated). Further, the lognormal results from the application of popular diffusion processes (Gauss-Weiner diffusion, or Brownian motion) and has been frequently used in studies evaluating insurance products associated with crop yields.

Thus, in addition to the beta, Weibull, and logistic, the normal and lognormal distributions will be considered from here forward. Each distribution is presented below with its probability density function and brief restatements of reasons for inclusion to provide context for interpretation of the results.

## Distributions examined:

1. Normal: parameters $\sigma>0,-\infty<\mu<\infty$

Probability density function, $f(x)=\frac{1}{\sigma(2 \pi)^{1 / 2}} \exp \left[\frac{-(x-\mu)^{2}}{2 \sigma^{2}}\right]$
The normal distribution is defended by Just and Weninger in their study of yield distributions. Just and Weninger suggest that the normal distribution cannot be rejected for most yield distributions. The normal distribution is symmetric, has a fixed kurtosis below that observed from many yield samples, and is not bounded below by zero.
2. Lognormal: parameters $\sigma_{l}>0,-\infty<\mu_{l}<\infty$

Probability density function, $f(x)=\frac{1}{x \sigma_{l}(2 \pi)^{1 / 2}} \exp \left\{\frac{-\left(\ln (x)-\mu_{l}\right)^{2}}{2 \sigma_{l}{ }^{2}}\right\}$
The lognormal is used by Stokes, and Jung and Ramezani, and others in studies evaluating crop insurance. The lognormal is bounded below by zero and allows for varying degrees of positive skewness and kurtosis. It also results from the application of a popular and tractable diffusion specification that is commonly employed in option pricing studies.
3. Logistic: parameters $-\infty<a<\infty, b>0$

Probability density function, $\quad f(x)=\frac{\exp [(-x-a) / b]}{b\{1+\exp [-(x-a) / b]\}^{2}}$
The logistic distribution is used as an alternative to the normal, particularly in cases in which excess kurtosis exists. It is a symmetric distribution and is not bounded by zero.
4. Beta: parameters $\alpha, \gamma>0$

The beta distribution is used by Nelson and Preckel; and Tirupattur, Hauser, and Chaherli, and others to represent yields. The beta distribution has many desirable properties for modeling yields: it can be bounded by zero and allows for a wide range of skewed and kurtotic distributions. Conceptually, the upper bound can be endogenized and jointly estimated with its other parameters.

Probability density function, $\frac{f(x)=x^{\alpha-1}(1-x) \gamma-1}{B(\alpha, \gamma)}$
where $B(\alpha, \gamma)$ is the beta function.
5. Weibull: parameters $\alpha, \beta>0$

Probability density function, $f(x)=\frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} \exp \left[-(x / \beta)^{\alpha}\right]$
The Weibull distribution has many of the same desirable properties as does the beta distribution in its flexibility, that it is bounded by zero, it is not symmetric, and it allows for a wide range of skewness and kurtosis.

## Maximum Likelihood Estimates

Maximum likelihood methods were used to solve for the parameters of each of the five distributions for each farm sample. ${ }^{2}$ Specifically, all five distributions were separately estimated with the data from farm $a$, and then for farm $b$, and so on through each sample so that comparisons of the economic impacts across distributions can be made at the farm level (same data, different distributions) across farms (same distributional specifications, different data) and between crops. The log-likelihood functions are specified assuming that the detrended yield samples are independently and identically distributed (i.i.d.), a fact generally supported by the heteroskedasticity and autocorrelation diagnostic tests for the linear regression results. Each of these distributions has two estimated parameters. Hence, all distributions have the same degrees of freedom when estimating the maximum likelihood functions for individual fields. The resulting differences thus relate primarily to the underlying characteristics of each distribution rather than to differences in the estimators. ${ }^{3}$

The fitted Weibull and beta distributions are negatively skewed, while the lognormal distributions are positively skewed by definition for all fields and both commodities. Thus, while the beta and Weibull distribution capture the sample skewness reasonably well, the restricted (positive) skewness of the lognormal results in a fitted distribution with positive skewness even though the samples exhibit negative skewness. Although the evidence suggests that the statistical properties of alternative distributions may be distinguished, the more important question that remains is the economic significance of using alternative characterizations in riskdecision contexts. To address this issue, actuarial tables and insurance values implied under each distribution are calculated and compared.

## Actuarial tables

Actuarial tables relating various levels of yields to the implied probability under each distribution for each farm are provided to help compare implications of alternate parameterizations. Table 3 contains results for corn organized in panels from top to bottom corresponding to farms, and in panels left to right that differ by whether the probability or yield level is fixed. The corn yields implied by each fitted distribution are tabulated from 70 to 200 bu./acre in 30 bu./acre increments and at quantiles of $1 \%, 5 \%, 10 \%, 25 \% 50 \%, 75 \%$ and $90 \%$. Table 4 contains analogous results for soybeans. The results convey the degree to which the alternative distributional assumptions result in different measures of risk. To better understand the relationship between the left and right panels in the table, consider the beta distribution for farm $a$. In the left panel of the table, it can be seen that there is approximately $9.9 \%$ probability associated with a yield of 130 bu ./acre or below. In the right hand side of the table, the $10 \%$ quantile can be seen to be associated with a yield of 130.2 bu./acre indicating that the likelihood implied by the beta distribution for the yields to be below 130.2 bu./acre is $10 \%$. Thus, while both panels of the actuarial schedule tabulate points on the cumulative distribution function, the left permits comparisons at prespecified yields and the right at prespecified probability levels.

Of interest is the degree to which the parameterization can lead to differing estimates of the risk. For example in table 3, the results for farm $a$ indicate that the probability implied for
yield to be 130 bu./acre or less ranges from $5.1 \%$ under the logistic parameterization to $13.4 \%$ under the lognormal. As expected, the differences among distributions are not as pronounced in the lower tail of the distributions where all imply very low likelihoods of occurrence. Likewise, the means (not shown in table) are in close agreement across different parameterizations. Perhaps more striking, in the right hand side of the table, are the differences in yields, which at the 25 th percentile range from 140.9 to 152.2 bu./acre. The importance of that difference can be appreciated in the context of an insurance product that is expected to pay in one of four years. Under such insurance, a range of 11 bu ./acre would be valued at $\$ 22$ per acre, at a $\$ 2 / \mathrm{bu}$. price of corn. Other farm results show similar patterns throughout the remaining sections of table 3 .

Table 4 contains the actuarial schedule for soybean results from 30 to 80 bu./acre in 10 bu./acre increments based on the fitted normal, logistic, Weibull, beta and lognormal distributions. Again, large differences in risk estimates are found across the fields and fitted distributions. For example, probability of obtaining 50 bu./acre or less in field " $O$ " implied by each distribution is $47.7 \%, 42.1 \%, 42.5 \%, 44.7 \%$ and $50.9 \%$ under the normal, logistic, Weibull, beta and lognormal distributions, respectively. While the bushel differences are not as great in the estimated soybean distributions across parameterizations, the patterns are similar to those as were found in corn, and may have significant economic implications.

While the differences in the estimated distributions due to the choice of parameterization are meaningful results on their own, the more important issue is whether there are economically meaningful differences within actual decision contexts. To address that issue, the study next considers two popular crop insurance products and examines the differences in calculated insurance values that arise under alternate representations of yield distributions. In the first case, Actual Production History (APH) insurance is examined to see if meaningful differences would arise due to yield characterization alone. Then, Crop Revenue Coverage (CRC) insurance is also examined. In the case of CRC, the impacts of "mistakes" induced from the distributional representations are expected to be muted due to the relationship of the price component. Thus, this insurance product provides a more demanding case to examine the economic significance of potential differences, but one that closely matches decision makers' use of estimated measures of risk.

## Average Production History (APH) Insurance

APH insurance makes an indemnity payment if realized yields fall below the insured yield. Producers can choose coverage levels commonly defined as fractions of a "proven yield" taken from actual production histories. The available fractions of proven yield vary from $55 \%$ to $85 \%$. The highest coverage level, $85 \%$, is chosen for the purpose of illustrating the economic implications of alternative crop-yield representations. ${ }^{4}$ To formalize the presentation, the guaranteed yield, $Y_{g}$, is defined as a function of a proven yield, $\bar{y}$, and an election level as a fraction of proven yield, $h$ :

$$
\begin{equation*}
Y_{g}=h^{*} \bar{y} \tag{1}
\end{equation*}
$$

Indemnity payments are triggered when actual yields, y , fall below $Y_{g}$, with producers receiving the difference between $Y_{g}$ and the actual yields at a price guaranteed and defined at the time of planting by the FCIC. Thus, the guarantee of yields below $h$ percent of the expected production level $\bar{y}$ is equivalent to

$$
\begin{equation*}
G=\max \left\{0, \mathrm{P}_{g}\left(Y_{g}-y\right)\right\} \tag{2}
\end{equation*}
$$

where $\mathrm{P}_{\mathrm{g}}$ is the price defined at the beginning of the crop season (for the crop-year 2000, the FCIC prices are $\$ 1.90 / \mathrm{bu}$. for corn and $\$ 5.16 / \mathrm{bu}$. for soybeans). The stochastic variable at the time the insurance decision is made is the yield outcome. Thus, the actuarily fair payment (AFP) from insurance can be obtained by characterizing yields by a given probability density function, $f(y)$ :

$$
\begin{equation*}
E(G)=\int_{0}^{Y_{g}} \mathrm{P}_{g}\left(Y_{g}-y\right)^{*} f(y) d y \tag{3}
\end{equation*}
$$

The fitted normal, logistic, Weibull, beta and lognormal distributions from each farm sample are used alternatively to define $f(y)$ and calculate the corresponding expected APH payments for each corn and soybean field. The differences in expected payments across distributions are then direct measures of the economic significance of the choice of distribution.

Proven yields, $\bar{y}$, are defined based on the estimated trends in a manner that closely reflects a farmer's likely behavior. The provisions for defining $\bar{y}$ allow producers to use at least the last four years of actual historic yields. Skees and Reed point out that $\bar{y}$ will generally be below predicted trend values for the same year if actual historic yields have been increasing through time. As a result, for any fraction $h$, the $Y_{g}$ based on actual historic yields will be lower on average than the $Y_{g}$ based on the 2000 predicted yields. Thus, to define the coverage levels, the mid-point of the yield trend between the years 1995 to 1999 is used to define $\bar{y}$, as would be the typical case for a farmer based on their own historical yield records.

Table 5 shows actuarially fair payments (AFPs) estimated for each farm's corn yield series. The farms are ordered in this table from greatest to least dispersion in implied insurance values. In addition to the calculated insurance values across distributions, this table shows the range in payments, the actual 2000 insurance premium, and the ranking of AFPs across distributions. For example, it can be seen in the table that the AFP for the normal distribution for farm $x$ is $\$ 9.28$, the range in AFPs across distributions is $\$ 8.51$, the 2000 insurance premium is $\$ 13.20$, and the rank of payments from high to low is the lognormal, beta, normal, Weibull, and logistic distributions. From the range in AFPs, it is glaringly apparent that the choice of distribution matters in generating AFPs, and thus to the assessment of risk, and to making management decisions related to these risks. The highest range in AFPs across a farm is $\$ 8.51$, the lowest range is $\$ .77$, with an average range of $\$ 1.86$.

The somewhat more flexible distributions (i.e., the beta and Weibull distributions) tend to have higher AFPs than do the symmetric distributions (i.e., normal and logistic) suggesting that
they contain relatively higher mass and lower conditional expected values in the region for which insurance payments are triggered. For each farm, the beta distribution always generates a higher AFP than do the normal and logistic distributions. Similarly, the AFP for the Weibull distribution is usually higher than the normal distribution (16 out of 26 farms) and always higher than the logistic distribution. Across all farms, the average AFPs for the beta (\$5.18) and Weibull (\$4.65) distributions are higher than the normal (\$4.48) and logistic (\$3.87).

Between the two symmetric distributions, the normal distribution's average implied AFP of $\$ 4.48$ is higher than the logistic distribution's average AFP of $\$ 3.87$ (see table 5). Across all farms, the normal distribution generates higher AFPs on 16 out of 26 fields. The logistic distribution has higher kurtosis than does the normal distribution, more closely matching the sample kurtosis estimates from the data. Yet higher kurtosis on its own does not result in higher yield risk assessment in this context.

Between the distributions that permit negative skewness, the beta distribution's AFP is typically higher than the Weibull distribution's AFP. Average AFP across all farms is $\$ 5.18$ for the beta and $\$ 4.65$ for the Weibull (see table 5). Moreover, the beta's distribution's AFP is higher than the Weibull's AFP on 20 of the 26 fields. The beta distribution has an upper bound while the Weibull does not have an upper bound. Hence, under the beta distribution the probability is distributed more compactly, leading to higher probability at lower yields, and associated higher yield insurance values.

The lognormal distribution's AFPs are inconsistently ranked among the other distributions. Overall, the average AFP of the lognormal across all farms (\$4.88) is higher than the symmetric distributions (normal (\$4.48), logistic (\$3.87)) and between the distributions that permit negative skewness (Weibull (\$4.65) and beta (\$3.87)). However, its ranking is not consistent for any given field: the lognormal results in the highest AFP in 9 fields, second highest in 2 fields, third highest in 6 fields, fourth highest in 1 field, and lowest in 8 fields (see bottom left corner of table 5).

Table 5 also shows the 2000 premium for yield insurance. Interestingly, the actual premiums display nearly zero correlation to the AFPs generated by the alternative distributions. The correlations between the actual premiums and the AFP are as follows: -. 09 under the normal; -. 18 under the logistic; -. 10 under the Weibull; -. 08 under the beta; and -. 01 under the lognormal. Because the data series are quite long, estimates of yield distributions used in this study are likely more accurate than those used in rating crop insurance products - yet the results raise questions about the efficacy of crop insurance rating procedures. In particular, if actual premiums are not closely correlated with expected payments, that significant problems in controlling payout ratios would be expected.

Table 6 shows AFPs for soybean yield series. The qualitative results are similar for soybeans as for corn in that: 1) the symmetric distributions give lower estimates of AFPs than do negatively skewed distributions, 2) the normal distribution's AFPs are generally higher than the logistic distribution's AFPs, 3) the lognormal distribution's AFP ranks are sporadic compared to the other distributions, and 4) estimated AFPs have roughly zero correlation with actual premiums. Unlike corn, the Weibull distribution consistently has higher AFPs than does
the beta distribution. This result likely arises due to the more compact distributions associated with soybeans compared to corn.

Results are summarized in tables 7 and 8 based on the mean absolute-percentage differences between APH expected payments calculated based on alternative distributions. The absolute values are used simply because the relative expected payments have biases in different directions across the fields and would be somewhat offset if simple averages were taken. Each cell in tables 7 and 8 represents the absolute-percentage differences between two distributions. For example, the mean absolute-percentage differences in the cell on the column labeled "logistic" and on the row labeled "over normal" is $10 \%$, which is the average of the absolute differences between the APH expected under the logistic and normal distributions divided by the APH expected under the normal distributions.

Results in tables 7 and 8 demonstrate strikingly large percentage differences across distributions in the expected value of APH insurance due solely to distributional assumptions. The differences are perhaps more surprising given that relatively high yielding and reasonably similar University of Illinois Endowment Farms are examined. For example, the mean absolutepercentage differences between APH expected payments calculated based on the logistic, Weibull, beta and lognormal distributions versus the normal are $19 \%, 20 \%, 18 \%$ and $14 \%$ for corn and $10 \%, 25 \%, 16 \%$ and $10 \%$ for corn and soybeans, respectively. Pairwise comparison among the other distributions shows similarly large mean absolute-percentage differences in APH payments.

## Revenue simulation and Crop Revenue Coverage insurance

Crop Revenue Coverage (CRC) is examined next to provide a complementary case to the APH in which the potentially mitigating effects of a negatively correlated price distribution are included. CRC makes payments when crop revenue falls below an insured level due to low prices, low yields, or a combination of both. Analogously to APH, the farmer chooses an election level and is paid based on the fraction of the shortfall from the insured revenue (see Schnitkey for details of the features of CRC). Valuation of CRC requires that the product of two correlated random variables be evaluated. Analytical expressions for the product of two random variables are only available under special assumptions about the price and yield distributions (Buccola), and, as a result, researchers have commonly used simulation methods based on the empirical estimation of the price and yield distributions (Taylor; Tirupattur, Hauser and Chaherli; Ramirez; Miranda). In this study, the same yield representations as under APH were examined while the price distributions are recovered from futures and options data using standard Black-Scholes method (see Fackler and King; Sherrick, Garcia and Tirupattur; Zulauf and Irwin). Implied lognormal distributions are computed from market data from both puts and calls and averaging the standard deviation between the nearest-to-the-money options. The required discount rate on three-month Treasury Bills on March 1, 2000, is used as the discount rate. ${ }^{5}$

CRC insurance reflects the fact that prices and yields are typically negatively correlated, with the price-yield offsetting effects known to be an important factor in solving for optimal risk-
management decisions (Plato; Tirupattur, Hauser and Chaherli; Heifner and Coble). Debate still exists regarding the quantitative measure of price-yield correlation, especially when considering that producers do not receive a uniform price because they market their crop differently. Therefore, results are computed across a range of plausible price-yield correlation values and reported for a value thought to be typical in the region examined ( -0.4 ). Results for other correlation values were examined for sensitivity and were qualitatively identical.

The simulation procedures used were: (i) generate two correlated normal random samples with 10,000 observations each, (ii) transform the normal random samples to uniformly distributed samples through the inverse transformation method (see Law and Kelton), and (iii) transform the uniformly distributed back to the samples following the alternative distributions of interest (normal, logistic, Weibull, beta and lognormal distributions) by passing the correlated uniform variates through the associated inverse CDF function in conjunction with the lognormal price distribution.

The expected payments from CRC insurance for corn and soybeans are presented in tables 9 and 10 , respectively. As was the case under APH, the expected payments from buying CRC insurance are often less than the (highly subsidized) insurance premiums. Expected payments across all corn fields average a low of $\$ 14.60$ for the logistic distribution to a high of $\$ 17.18$ for the lognormal distribution, a difference of $\$ 2.58$, as shown in table 9. Expected payments across all soybean fields average a low of $\$ 7.57$ for the logistic distribution to a high of $\$ 8.46$ for the Weibull distribution, a difference of $\$ 0.89$, as shown in table 10 . Thus, the CRC results differ across distributions by as much as $18 \%$ for corn and $12 \%$ for soybeans. These percentages are lower than the APH average differences of $34 \%$ for corn and $37 \%$ for soybeans, showing that the price-yield correlation mitigates the effects of distributional choices.

AFPs for corn under CRC are shown in table 9. As was the case under APH, the expected payments from buying CRC insurance are often less than the (highly subsidized) insurance premiums. Expected payments across all corn fields average a low of $\$ 14.60$ for the logistic distribution to a high of $\$ 17.18$ for the lognormal distribution, a difference of $\$ 2.58$. Similar to the APH results, AFPs for CRC display the following general results: 1) the normal distribution generates higher AFPs than does the logistic distribution, and 2) the lognormal distribution's AFP ranks are sporadically ranked relative to the other distributions.

Distributions that permit negative skewness generally have higher AFPs than do symmetric distributions. However, the relationship is not as strong for CRC as for APH. Across farms, the beta distribution's implied AFPs are always greater than the normal distributions AFPs and the beta distributions AFPs are usually greater than the logistic distribution (24 out of 26 farms). The Weibull distribution's AFPs are usually greater than the logistic distributions AFPs. However, the Weibull's distribution's AFPs are greater than the normal distributions in 11 out of 26 cases. Across all farm's, the average Weibull distribution's AFP of $\$ 15.95$ is less than the average of the normal distribution's AFP of $\$ 16.20$ (see table 9). The results indicate that relative economic differences between symmetric distributions and distributions that allow for negative skewness are less when revenue risk is assessed.

As was the case with APH, there are actually negative correlations between the implied insurance values and the premiums charged with correlations between the actual premium and
normal distribution's AFPs of -.34 , the logistic with correlation of -.42 , the Weibull with correlations of -.35 , the beta with -.32 , and the lognormal of .09 . Again, these results call into question methodologies used to rate crop insurance.

AFPs for soybeans are shown in table 10. Unlike the corn case, distributions allowing negative skewness have higher AFPs than symmetric distributions. Across farms, the beta distribution's AFPs are always higher than the normal and logistic distribution's AFPs. The Weibull's AFPs are always higher than logistic distribution's AFPs and usually higher than the normal distribution's AFPs ( 20 out of 25 cases). Unlike the corn case, the AFPs from different distributions have approximately zero correlation with actual premiums.

The mean absolute-percentage differences across distributions in tables 11 and 12 are not as strong when evaluating the CRC insurance compared to the APH results. Still, the mean absolute-percentage differences between CRC expected payments calculated based on the logistic, Weibull, beta and lognormal distributions versus the normal distribution are $11 \%, 6 \%$, $3.8 \%$ and $6.2 \%$ for corn and $8.1 \%, 8.6 \%, 4 \%$ and $41 \%$ for soybeans, respectively. Pairwise, a comparison with the other distributions shows mean-percentage differences as large as $20.8 \%$ for the lognormal distribution versus the logistic distribution. Therefore, despite the mitigating influence of the price distribution and its interaction with yield, the range of expected payments across alternative yield distributions represents a significant fraction of the estimated insurance values.

## Summary and Conclusions

Considerable disagreement exists about the most appropriate characterization of farmlevel yield distributions. This study assesses the economic importance of alternate yield distribution specifications by comparing implied distributional characteristics under alternative specifications, and by demonstrating the impact of the distributional assumptions on implied yields and crop insurance values. The study takes advantage of a high quality data set that contains a relatively large number of farms with over a long time period in a major corn and soybean producing region of the United States. The candidate parameterizations were chosen to reflect empirical evidence, and to broadly encompass previous efforts reflected individually in the literature. Each of the candidate distributions was fitted to the data from each case farm to permit isolation of differences in implied yields and insurance values that arise due to parameterization choices.

The results of this study demonstrate that meaningful differences in implied yields result solely from parameterization choices. Actuarial tables that tabulate quantiles from cumulative distribution functions under each fitted distribution provide evidence about the magnitude of differences in implied yields. While means and the extreme tails eventually agree fairly well, the greatest differences occur at yield levels most important for risk management applications such as crop insurance evaluation.

To more directly assess the economic importance of the differences, payouts to APH and CRC insurance products are calculated under each parameterization and compared, both within farm sample, and across farms. Again, the results demonstrate that distributional choice can have a significant impact in the actuarial values of insurance. As expected, the APH results are more pronounced than under CRC due to the mitigating influence of the price distribution and its negative correlation with yield. Interestingly, all distributions exhibited the feature that implied payments were not highly correlated with actual premiums (and in fact, often negatively related). These results also call into question insurance rating procedures and their relationships on individual farm units. Because the estimated insurance payments at the farm level do not bear a positive relationship to actual insurance premiums, the actual rating procedures could lead to adverse selection problems, or other difficulties in managing payout rates. Further research in this area is therefore warranted.

This study addresses a limited set of distributions and only the two most popular crop insurance programs. Future work that would examine a wider set of distributional choices is therefore of value. In particular, an examination of non-parametric distributional analysis seems a fruitful avenue of future research. At the same time, the advisability of using non-parametric approaches in circumstances that are difficult to generalize due to data concerns remains questionable. What is evident is from this work is that distribution choice has a large impact on rating crop insurance products and on yield risk assessment and therefore should not simply be accepted as an unexamined premise or chosen according to convenience without testing the economic significance of the assumption.

Table 1. Corn farm locations and yield-sample moments, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| Farm | Counties | Number of observations | Sample <br> Mean | Standard Deviation | Skewness | Kurtosis |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| a. | De Kalb | 28 | 162.1 | 25.030 | -1.699 | 6.568 |
| b. | De Kalb | 20 | 135.2 | 22.379 | -1.681 | 6.224 |
| c. | De Kalb | 28 | 148.1 | 21.248 | -0.305 | 2.558 |
| d. | LaSalle | 28 | 159.1 | 26.435 | -1.127 | 5.290 |
| e. | Wabash | 23 | 171.3 | 26.473 | -1.098 | 4.708 |
| f. | De Witt | 23 | 154.0 | 31.548 | -0.122 | 2.170 |
| g . | De Witt | 27 | 177.2 | 24.347 | -1.411 | 4.626 |
| h. | Macon | 28 | 180.6 | 31.490 | -0.363 | 2.340 |
| i. | LaSalle | 28 | 164.1 | 20.256 | -0.594 | 4.494 |
| j. | Champaign | 26 | 167.3 | 28.377 | -0.595 | 2.501 |
| k. | Champaign | 23 | 137.6 | 27.253 | -0.730 | 3.616 |
| 1. | Champaign | 25 | 137.5 | 24.138 | -0.086 | 2.922 |
| m. | Champaign | 28 | 159.6 | 27.728 | -0.735 | 3.298 |
| n. | Douglas, Moultrie | 28 | 152.0 | 23.343 | 0.074 | 2.164 |
| o | Piatt | 28 | 156.6 | 29.165 | -0.493 | 2.402 |
| p. | Piatt | 28 | 159.5 | 30.290 | -0.618 | 2.555 |
| q. | Piatt | 27 | 167.8 | 24.000 | -0.921 | 2.950 |
| r. | Piatt | 24 | 144.9 | 26.313 | -1.026 | 3.436 |
| s. | Piatt | 26 | 155.8 | 27.787 | -0.548 | 3.452 |
| t. | Moultrie | 25 | 159.5 | 23.688 | -1.127 | 3.874 |
| u. | Vermilion | 27 | 117.6 | 28.923 | -0.710 | 2.821 |
| v. | Sangamon | 25 | 158.7 | 23.091 | -1.395 | 5.434 |
| w. | Sangamon | 25 | 173.6 | 24.470 | -1.006 | 5.289 |
| x . | Menard | 20 | 152.2 | 37.109 | -1.511 | 4.317 |
| $y$. | Sangamon, Mccoupin | 25 | 167.9 | 27.960 | -0.790 | 4.059 |
| z. | Vermilion | 26 | 120.2 | 32.626 | -0.472 | 2.639 |
|  | Average | 25.7 | 155.4 | 26.749 | -0.811 | 3.720 |
|  | Minimum | 20 | 117.6 | 20.256 | -1.699 | 2.164 |
|  | Maximum | 28 | 180.6 | 37.109 | 0.074 | 6.568 |

Table 2. Soybean field locations and yield-sample moments, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

|  |  | Number of |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| Farm | Counties | Sample <br> Mebservations | Standard <br> Deviation | Skewness | Kurtosis |  |
| a. | De Kalb | 28 | 53.7 | 8.362 | 0.236 | 3.531 |
| c. | De Kalb | 25 | 45.9 | 6.603 | -0.358 | 3.322 |
| d. | LaSalle | 28 | 57.3 | 7.271 | -0.719 | 5.498 |
| e. | Wabash | 23 | 46.2 | 6.263 | -0.421 | 2.368 |
| f. | De Witt | 23 | 46.8 | 6.621 | -0.550 | 3.923 |
| g. | De Witt | 27 | 49.2 | 5.808 | -0.473 | 3.691 |
| h. | Macon | 27 | 56.6 | 6.875 | -0.040 | 2.869 |
| i. | LaSalle | 28 | 45.2 | 6.007 | -0.617 | 3.016 |
| j. | Champaign | 26 | 51.1 | 7.486 | -1.859 | 7.547 |
| k. | Champaign | 24 | 49.5 | 7.086 | -0.455 | 2.761 |
| 1. | Champaign | 25 | 45.2 | 6.308 | -0.027 | 2.447 |
| m. | Champaign | 27 | 49.0 | 6.726 | -0.504 | 3.414 |
| n. | Douglas, | 28 | 45.7 | 7.402 | 0.041 | 2.054 |
|  | Moultrie |  |  |  |  |  |
| o | Piatt | 28 | 50.4 | 7.192 | -1.365 | 5.198 |
| p. | Piatt | 28 | 54.6 | 9.183 | -0.614 | 3.737 |
| q. | Piatt | 27 | 55.6 | 7.488 | -0.757 | 3.311 |
| r. | Piatt | 25 | 54.9 | 6.561 | -1.063 | 4.423 |
| s. | Piatt | 27 | 52.4 | 7.487 | -0.731 | 3.631 |
| t. | Moultrie | 25 | 50.2 | 8.676 | -1.419 | 6.044 |
| u. | Vermilion | 28 | 38.0 | 8.178 | -0.435 | 2.860 |
| v. | Sangamon | 25 | 48.6 | 6.570 | -2.344 | 9.571 |
| w. | Sangamon | 25 | 49.2 | 5.516 | -0.804 | 2.895 |
| x. | Menard | 20 | 42.8 | 5.915 | -1.309 | 3.902 |
| y. | Sangamon, | 25 | 48.9 | 5.918 | -0.180 | 2.554 |
| z. | Mccoupin | Vermilion | 26 | 33.9 | 6.680 | -0.172 |
|  | Average | 25.9 | 48.8 | 6.967 | -0.678 | 3.389 |
|  | Minimum | 20 | 33.9 | 5.516 | -2.344 | 2.054 |
|  | Maximum | 28 | 57.3 | 9.183 | 0.236 | 9.571 |

Figure 1. Representation of the distributions in the skewness-kurtosis plane and corn-yield linear-trend predicted values' skewness ( $\sqrt{b 1}$ ) and kurtosis ( $b 2$ ), University of Illinois Endowment Farms, 1972 to 1999. Each dot represents one farm.
b2


Figure 2. Representation of the distributions in the skewness-kurtosis plane and soybean-yield linear-trend predicted values' skewness ( $\sqrt{b 1}$ ) and kurtosis ( $b 2$ ), University of Illinois Endowment Farms, 1972 to 1999. Each dot represents one farm.

Table 3. Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| Field a. Normal | 70 bu./ac | 100 bu./ac | 130 bu./ac | 160 bu./ac | 190 bu./ac | 220 bu./ac | 1\% | 5\% |  |  | 50\% | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | probabilities |  |  |  |  |  | yields per acres |  |  |  |  |  |  |
|  | 0.000 | 0.006 | 0.096 | 0.466 | 0.872 | 0.974 | 104.9 | 121.7 | 130.6 | 145.5 | 162.1 | 178.7 | 193.6 |
| Logistic | 0.000 | 0.004 | 0.051 | 0.389 | 0.883 | 0.975 | 109.8 | 129.8 | 138.8 | 152.2 | 165.5 | 178.8 | 192.1 |
| Weibull | 0.000 | 0.007 | 0.074 | 0.412 | 0.928 | 0.999 | 104.5 | 124.5 | 134.5 | 149.8 | 164.7 | 177.4 | 187.3 |
| Beta | 0.000 | 0.009 | 0.099 | 0.440 | 0.901 | 0.999 | 100.9 | 120.0 | 130.2 | 146.7 | 163.6 | 178.6 | 189.9 |
| Lognormal | 0.000 | 0.006 | 0.134 | 0.504 | 0.825 | 0.930 | 103.6 | 117.6 | 125.9 | 140.9 | 159.7 | 181.0 | 202.7 |
| Field b. Normal | 0.001 | 0.053 | 0.405 | 0.872 | 0.994 | 1.000 | 84.5 | 99.4 | 107.3 | 120.5 | 135.2 | 150.0 | 163.2 |
| Logistic | 0.002 | 0.031 | 0.323 | 0.876 | 0.991 | 0.998 | 87.1 | 105.5 | 113.8 | 126.0 | 138.2 | 150.4 | 162.7 |
| Weibull | 0.002 | 0.039 | 0.342 | 0.931 | 1.000 | 1.000 | 85.6 | 102.8 | 111.4 | 124.6 | 137.5 | 148.6 | 157.3 |
| Beta | 0.003 | 0.059 | 0.376 | 0.911 | 1.000 | 1.000 | 80.7 | 97.9 | 107.0 | 121.8 | 136.8 | 149.7 | 159.1 |
| Lognormal | 0.001 | 0.077 | 0.455 | 0.823 | 0.963 | 0.989 | 83.5 | 95.7 | 102.9 | 116.2 | 132.9 | 152.1 | 171.8 |
| Field c. Normal | 0.000 | 0.011 | 0.192 | 0.715 | 0.978 | 0.998 | 99.6 | 113.8 | 121.4 | 134.1 | 148.1 | 162.2 | 174.9 |
| Logistic | 0.001 | 0.017 | 0.169 | 0.709 | 0.967 | 0.994 | 93.7 | 113.7 | 122.7 | 136.0 | 149.2 | 162.5 | 175.8 |
| Weibull | 0.001 | 0.024 | 0.190 | 0.688 | 0.992 | 1.000 | 89.8 | 109.5 | 119.5 | 135.0 | 150.2 | 163.4 | 173.8 |
| Beta | 0.000 | 0.018 | 0.193 | 0.689 | 0.993 | 1.000 | 94.4 | 111.1 | 120.0 | 134.5 | 149.8 | 163.5 | 174.2 |
| Lognormal | 0.000 | 0.005 | 0.207 | 0.724 | 0.961 | 0.993 | 104.1 | 115.1 | 121.4 | 132.8 | 146.6 | 161.9 | 177.0 |
| Field d. Normal | 0.000 | 0.011 | 0.131 | 0.514 | 0.883 | 0.975 | 98.7 | 116.4 | 125.8 | 141.6 | 159.1 | 176.6 | 192.4 |
| Logistic | 0.001 | 0.011 | 0.089 | 0.474 | 0.892 | 0.973 | 99.3 | 121.6 | 131.7 | 146.6 | 161.4 | 176.3 | 191.1 |
| Weibull | 0.001 | 0.018 | 0.124 | 0.478 | 0.911 | 0.995 | 92.8 | 114.8 | 126.1 | 143.8 | 161.3 | 176.6 | 188.7 |
| Beta | 0.001 | 0.018 | 0.139 | 0.491 | 0.894 | 0.993 | 93.8 | 113.5 | 124.2 | 141.9 | 160.6 | 177.5 | 190.7 |
| Lognormal | 0.000 | 0.010 | 0.168 | 0.545 | 0.842 | 0.936 | 99.9 | 113.9 | 122.2 | 137.4 | 156.5 | 178.3 | 200.4 |
| Field e. Normal | 0.000 | 0.003 | 0.056 | 0.332 | 0.766 | 0.933 | 111.0 | 128.7 | 138.1 | 153.8 | 171.3 | 188.7 | 204.4 |
| Logistic | 0.000 | 0.003 | 0.033 | 0.254 | 0.772 | 0.940 | 114.1 | 135.7 | 145.4 | 159.7 | 174.1 | 188.4 | 202.7 |
| Weibull | 0.000 | 0.007 | 0.062 | 0.298 | 0.768 | 0.965 | 104.0 | 126.7 | 138.2 | 156.1 | 173.6 | 188.8 | 200.7 |
| Beta | 0.000 | 0.005 | 0.065 | 0.319 | 0.756 | 0.952 | 106.3 | 126.1 | 136.8 | 154.3 | 172.8 | 189.5 | 202.8 |
| Lognormal | 0.000 | 0.001 | 0.064 | 0.376 | 0.752 | 0.897 | 113.1 | 127.2 | 135.5 | 150.4 | 168.9 | 189.8 | 210.7 |

Table 3 (continuation). Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

Table 3 (continuation). Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| Field k. Normal | 70 bu./ac | $100 \mathrm{bu} . / \mathrm{ac}$ | 130 bu./ac | 160 bu./ac | 190 bu./ac | 220 bu./ac | 1\% | 5\% | 10\% | 25\% | 50\% | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | probabilities |  |  |  |  |  | yields per acres |  |  |  |  |  |  |
|  | 0.006 | 0.079 | 0.387 | 0.799 | 0.975 | 0.997 | 75.6 | 93.8 | 103.5 | 119.7 | 137.6 | 155.6 | 171.8 |
| Logistic | 0.009 | 0.067 | 0.351 | 0.803 | 0.969 | 0.992 | 70.9 | 95.4 | 106.5 | 122.8 | 139.1 | 155.4 | 171.7 |
| Weibull | 0.009 | 0.080 | 0.355 | 0.804 | 0.992 | 1.000 | 71.5 | 92.6 | 103.7 | 121.6 | 139.8 | 156.0 | 169.0 |
| Beta | 0.010 | 0.092 | 0.364 | 0.789 | 0.999 | 1.000 | 69.8 | 90.1 | 101.4 | 120.2 | 139.9 | 157.1 | 169.8 |
| Lognormal | 0.002 | 0.093 | 0.439 | 0.780 | 0.938 | 0.976 | 79.9 | 93.1 | 101.0 | 115.7 | 134.6 | 156.5 | 179.4 |
| Field 1. Normal | 0.002 | 0.056 | 0.376 | 0.829 | 0.987 | 0.999 | 82.5 | 98.6 | 107.2 | 121.5 | 137.5 | 153.4 | 167.8 |
| Logistic | 0.005 | 0.051 | 0.346 | 0.839 | 0.981 | 0.996 | 78.1 | 99.7 | 109.5 | 124.0 | 138.4 | 152.8 | 167.2 |
| Weibull | 0.008 | 0.080 | 0.361 | 0.816 | 0.994 | 1.000 | 71.9 | 92.7 | 103.7 | 121.3 | 139.2 | 155.1 | 167.8 |
| Beta | 0.004 | 0.070 | 0.363 | 0.812 | 0.995 | 1.000 | 77.7 | 95.4 | 105.1 | 121.5 | 139.0 | 155.3 | 168.3 |
| Lognormal | 0.000 | 0.046 | 0.411 | 0.824 | 0.971 | 0.993 | 89.1 | 100.7 | 107.5 | 119.9 | 135.4 | 152.8 | 170.4 |
| Field m. Normal | 0.000 | 0.014 | 0.138 | 0.506 | 0.868 | 0.968 | 96.3 | 114.8 | 124.7 | 141.3 | 159.6 | 178.0 | 194.5 |
| Logistic | 0.002 | 0.015 | 0.104 | 0.466 | 0.868 | 0.962 | 93.8 | 118.3 | 129.4 | 145.7 | 162.1 | 178.4 | 194.7 |
| Weibull | 0.002 | 0.021 | 0.131 | 0.468 | 0.887 | 0.989 | 90.3 | 113.1 | 124.9 | 143.5 | 162.1 | 178.4 | 191.4 |
| Beta | 0.001 | 0.020 | 0.144 | 0.480 | 0.874 | 0.988 | 91.6 | 112.1 | 123.3 | 141.8 | 161.4 | 179.1 | 192.8 |
| Lognormal | 0.000 | 0.009 | 0.160 | 0.540 | 0.843 | 0.937 | 101.0 | 114.9 | 123.1 | 138.1 | 157.0 | 178.4 | 200.2 |
| Field n. Normal | 0.000 | 0.012 | 0.168 | 0.636 | 0.951 | 0.994 | 98.7 | 114.3 | 122.7 | 136.6 | 152.0 | 167.5 | 181.4 |
| Logistic | 0.002 | 0.021 | 0.166 | 0.645 | 0.943 | 0.986 | 89.6 | 111.9 | 122.1 | 137.0 | 151.9 | 166.8 | 181.7 |
| Weibull | 0.002 | 0.029 | 0.181 | 0.599 | 0.960 | 0.999 | 86.5 | 108.0 | 119.2 | 136.7 | 154.1 | 169.3 | 181.5 |
| Beta | 0.001 | 0.021 | 0.175 | 0.600 | 0.963 | 1.000 | 92.3 | 110.7 | 120.6 | 136.9 | 154.0 | 169.4 | 181.4 |
| Lognormal | 0.000 | 0.004 | 0.172 | 0.659 | 0.937 | 0.986 | 105.3 | 116.9 | 123.5 | 135.6 | 150.3 | 166.6 | 182.8 |
| Field o. Normal | 0.001 | 0.024 | 0.176 | 0.547 | 0.878 | 0.969 | 90.0 | 109.5 | 119.9 | 137.3 | 156.6 | 176.0 | 193.4 |
| Logistic | 0.005 | 0.030 | 0.155 | 0.524 | 0.868 | 0.956 | 81.4 | 109.1 | 121.6 | 140.0 | 158.4 | 176.8 | 195.2 |
| Weibull | 0.003 | 0.030 | 0.163 | 0.510 | 0.895 | 0.988 | 84.7 | 108.0 | 120.3 | 139.7 | 159.3 | 176.7 | 190.6 |
| Beta | 0.003 | 0.034 | 0.176 | 0.509 | 0.891 | 0.996 | 83.7 | 106.0 | 118.2 | 138.4 | 159.3 | 177.6 | 191.0 |
| Lognormal | 0.000 | 0.015 | 0.199 | 0.579 | 0.857 | 0.942 | 96.9 | 111.0 | 119.2 | 134.5 | 153.8 | 175.8 | 198.3 |

Table 3 (continuation). Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| Field p. Normal | 70 bu./ac | $100 \mathrm{bu} . / \mathrm{ac}$ | 130 bu./ac | 160 bu./ac | 190 bu./ac | 220 bu./ac | 1\% | 5\% | $10 \%$ | 25\% | 50\% | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | probabilities |  |  |  |  |  | yields per acres |  |  |  |  |  |  |
|  | 0.001 | 0.023 | 0.160 | 0.506 | 0.847 | 0.955 | 90.3 | 110.6 | 121.4 | 139.5 | 159.5 | 179.6 | 197.7 |
| Logistic | 0.004 | 0.024 | 0.128 | 0.465 | 0.837 | 0.944 | 84.8 | 112.7 | 125.3 | 143.8 | 162.4 | 180.9 | 199.4 |
| Weibull | 0.003 | 0.027 | 0.146 | 0.468 | 0.863 | 0.979 | 86.0 | 109.8 | 122.3 | 142.2 | 162.3 | 180.0 | 194.2 |
| Beta | 0.002 | 0.030 | 0.163 | 0.477 | 0.852 | 0.981 | 85.4 | 107.7 | 119.9 | 140.3 | 161.8 | 180.9 | 195.3 |
| Lognormal | 0.000 | 0.015 | 0.184 | 0.544 | 0.828 | 0.924 | 97.1 | 111.6 | 120.3 | 136.2 | 156.4 | 179.6 | 203.5 |
| Field q. Normal | 0.000 | 0.002 | 0.054 | 0.371 | 0.827 | 0.963 | 113.0 | 129.0 | 137.6 | 151.9 | 167.8 | 183.7 | 198.0 |
| Logistic | 0.001 | 0.005 | 0.045 | 0.311 | 0.813 | 0.952 | 109.7 | 131.6 | 141.5 | 156.0 | 170.5 | 185.1 | 199.6 |
| Weibull | 0.000 | 0.004 | 0.049 | 0.310 | 0.856 | 0.994 | 110.0 | 130.3 | 140.4 | 155.8 | 170.7 | 183.5 | 193.4 |
| Beta | 0.000 | 0.005 | 0.062 | 0.335 | 0.844 | 0.998 | 107.3 | 127.0 | 137.3 | 153.7 | 170.2 | 184.1 | 194.2 |
| Lognormal | 0.000 | 0.000 | 0.056 | 0.406 | 0.811 | 0.938 | 116.1 | 128.9 | 136.3 | 149.6 | 165.9 | 184.0 | 202.0 |
| Field r. Normal | 0.002 | 0.041 | 0.281 | 0.721 | 0.960 | 0.994 | 85.0 | 102.6 | 111.9 | 127.6 | 144.9 | 162.3 | 177.9 |
| Logistic | 0.004 | 0.031 | 0.213 | 0.699 | 0.952 | 0.988 | 84.1 | 107.1 | 117.6 | 132.9 | 148.2 | 163.5 | 178.9 |
| Weibull | 0.003 | 0.037 | 0.235 | 0.720 | 0.990 | 1.000 | 83.8 | 104.2 | 114.7 | 131.2 | 147.6 | 161.9 | 173.2 |
| Beta | 0.004 | 0.049 | 0.264 | 0.706 | 0.995 | 1.000 | 80.2 | 100.2 | 111.0 | 128.8 | 147.1 | 162.9 | 174.4 |
| Lognormal | 0.000 | 0.042 | 0.330 | 0.718 | 0.922 | 0.972 | 88.4 | 101.6 | 109.4 | 123.9 | 142.2 | 163.2 | 184.8 |
| Field s. Normal | 0.001 | 0.020 | 0.172 | 0.562 | 0.896 | 0.977 | 92.4 | 110.9 | 120.8 | 137.4 | 155.8 | 174.1 | 190.7 |
| Logistic | 0.003 | 0.022 | 0.141 | 0.545 | 0.897 | 0.970 | 88.0 | 112.9 | 124.1 | 140.7 | 157.3 | 173.9 | 190.5 |
| Weibull | 0.003 | 0.031 | 0.170 | 0.531 | 0.910 | 0.991 | 84.3 | 107.3 | 119.4 | 138.6 | 157.9 | 175.1 | 188.8 |
| Beta | 0.002 | 0.029 | 0.178 | 0.535 | 0.900 | 0.992 | 87.3 | 107.7 | 118.9 | 137.6 | 157.5 | 175.7 | 189.9 |
| Lognormal | 0.000 | 0.014 | 0.199 | 0.590 | 0.868 | 0.949 | 97.6 | 111.4 | 119.5 | 134.4 | 153.1 | 174.4 | 196.1 |
| Field t. Normal | 0.000 | 0.005 | 0.102 | 0.508 | 0.905 | 0.985 | 105.5 | 121.3 | 129.8 | 143.9 | 159.5 | 175.2 | 189.3 |
| Logistic | 0.001 | 0.007 | 0.070 | 0.452 | 0.900 | 0.978 | 104.9 | 125.5 | 134.9 | 148.7 | 162.4 | 176.2 | 189.9 |
| Weibull | 0.000 | 0.008 | 0.084 | 0.456 | 0.952 | 1.000 | 103.1 | 122.8 | 132.6 | 147.6 | 162.2 | 174.7 | 184.5 |
| Beta | 0.000 | 0.009 | 0.104 | 0.473 | 0.943 | 1.000 | 100.7 | 119.5 | 129.4 | 145.3 | 161.5 | 175.5 | 185.8 |
| Lognormal | 0.000 | 0.003 | 0.120 | 0.537 | 0.874 | 0.960 | 107.6 | 120.3 | 127.7 | 141.1 | 157.6 | 176.0 | 194.4 |

Table 3 (continuation). Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| Field u. Normal | 70 bu./ac | $100 \mathrm{bu} . / \mathrm{ac}$ | 130 bu./ac | 160 bu./ac | 190 bu./ac | 220 bu./ac | 1\% | 5\% |  | 25\% | 50\% | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | probabilities |  |  |  |  |  | yields per acres |  |  |  |  |  |  |
|  | 0.047 | 0.267 | 0.669 | 0.932 | 0.995 | 0.999 | 51.6 | 70.9 | 81.3 | 98.5 | 117.6 | 136.8 | 154.0 |
| Logistic | 0.040 | 0.218 | 0.648 | 0.924 | 0.988 | 0.996 | 47.3 | 73.5 | 85.4 | 102.8 | 120.3 | 137.8 | 155.2 |
| Weibull | 0.044 | 0.243 | 0.657 | 0.956 | 0.999 | 1.000 | 52.4 | 72.0 | 82.8 | 100.7 | 119.4 | 136.7 | 150.9 |
| Beta | 0.058 | 0.262 | 0.640 | 0.963 | 1.000 | 1.000 | 48.2 | 67.8 | 79.2 | 98.8 | 119.8 | 138.3 | 151.6 |
| Lognormal | 0.045 | 0.329 | 0.684 | 0.886 | 0.965 | 0.985 | 58.5 | 71.0 | 78.8 | 93.6 | 113.5 | 137.5 | 163.4 |
| Field v. Normal | 0.000 | 0.005 | 0.102 | 0.523 | 0.917 | 0.988 | 106.1 | 121.5 | 129.7 | 143.4 | 158.7 | 174.0 | 187.7 |
| Logistic | 0.000 | 0.006 | 0.066 | 0.471 | 0.919 | 0.984 | 107.1 | 126.6 | 135.4 | 148.4 | 161.4 | 174.3 | 187.3 |
| Weibull | 0.000 | 0.007 | 0.085 | 0.476 | 0.965 | 1.000 | 103.6 | 122.8 | 132.4 | 147.1 | 161.2 | 173.3 | 182.7 |
| Beta | 0.000 | 0.008 | 0.105 | 0.494 | 0.952 | 1.000 | 101.8 | 119.8 | 129.3 | 144,6 | 160.4 | 174.1 | 184.5 |
| Lognormal | 0.000 | 0.003 | 0.130 | 0.549 | 0.877 | 0.961 | 106.5 | 119.3 | 126.7 | 140.1 | 156.7 | 175.3 | 193.9 |
| Field w. Normal | 0.000 | 0.001 | 0.034 | 0.285 | 0.753 | 0.935 | 117.9 | 134.2 | 142.9 | 157.5 | 173.6 | 189.8 | 204.4 |
| Logistic | 0.000 | 0.003 | 0.029 | 0.240 | 0.770 | 0.942 | 116.2 | 137.2 | 146.7 | 160.7 | 174.6 | 188.6 | 202.6 |
| Weibull | 0.000 | 0.005 | 0.046 | 0.257 | 0.746 | 0.964 | 109.3 | 131.3 | 142.4 | 159.4 | 176.0 | 190.3 | 201.4 |
| Beta | 0.000 | 0.003 | 0.047 | 0.270 | 0.736 | 0.961 | 111.2 | 130.9 | 141.4 | 158.3 | 175.7 | 190.9 | 202.4 |
| Lognormal | 0.000 | 0.000 | 0.037 | 0.325 | 0.742 | 0.901 | 119.4 | 132.8 | 140.6 | 154.6 | 171.7 | 190.8 | 209.7 |
| Field x. Normal | 0.012 | 0.075 | 0.270 | 0.585 | 0.852 | 0.945 | 68.0 | 92.7 | 105.8 | 127.8 | 152.2 | 176.6 | 198.5 |
| Logistic | 0.007 | 0.036 | 0.166 | 0.514 | 0.849 | 0.945 | 76.3 | 106.0 | 119.4 | 139.2 | 159.0 | 178.8 | 198.5 |
| Weibull | 0.006 | 0.049 | 0.215 | 0.569 | 0.905 | 0.986 | 76.4 | 100.3 | 113.1 | 133.8 | 154.9 | 173.9 | 189.3 |
| Beta | 0.015 | 0.084 | 0.262 | 0.566 | 0.891 | 0.997 | 64.1 | 89.3 | 103.9 | 128.5 | 154.2 | 176.0 | 191.1 |
| Lognormal | 0.011 | 0.118 | 0.358 | 0.613 | 0.795 | 0.873 | 69.5 | 86.4 | 97.0 | 117.7 | 146.0 | 181.1 | 219.8 |
| Field y. Normal | 0.000 | 0.007 | 0.083 | 0.387 | 0.790 | 0.938 | 104.2 | 122.8 | 132.8 | 149.4 | 167.9 | 186.4 | 203.0 |
| Logistic | 0.001 | 0.010 | 0.067 | 0.349 | 0.799 | 0.938 | 100.6 | 125.3 | 136.5 | 152.9 | 169.3 | 185.8 | 202.2 |
| Weibull | 0.001 | 0.013 | 0.087 | 0.351 | 0.792 | 0.964 | 96.8 | 120.4 | 132.5 | 151.5 | 170.4 | 186.9 | 200.0 |
| Beta | 0.001 | 0.012 | 0.095 | 0.363 | 0.779 | 0.963 | 97.5 | 119.2 | 130.9 | 150.2 | 170.1 | 187.7 | 201.0 |
| Lognormal | 0.000 | 0.003 | 0.096 | 0.430 | 0.775 | 0.903 | 107.7 | 122.1 | 130.5 | 146.0 | 165.3 | 187.2 | 209.3 |

Table 3 (continuation). Corn, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

Table 4. Soybeans, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

|  | $30 \mathrm{bu} . \mathrm{ac}$ | 40 bu./ac | 50 bu./ac <br> prob | 0 bu./ac <br> ilities | 70 bu.ac | 80 bu./ac | 1\% | 5\% | $10 \%$ yie | $25 \%$ | $50 \%$ | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Field a. Normal | 0.002 | 0.048 | 0.327 | 0.780 | 0.977 | 0.999 | 34.6 | 40.2 | 43.1 | 48.1 | 53.7 | 59.2 | 64.2 |
| Logistic | 0.006 | 0.050 | 0.322 | 0.811 | 0.975 | 0.997 | 32.5 | 40.0 | 43.4 | 48.4 | 53.4 | 58.4 | 63.4 |
| Weibull | 0.013 | 0.085 | 0.331 | 0.748 | 0.980 | 1.000 | 29.0 | 36.9 | 41.0 | 47.6 | 54.2 | 60.0 | 64.7 |
| Beta | 0.005 | 0.064 | 0.321 | 0.751 | 0.982 | 1.000 | 32.5 | 38.8 | 42.3 | 48.0 | 54.2 | 60.0 | 64.6 |
| Lognormal | 0.000 | 0.035 | 0.353 | 0.786 | 0.963 | 0.996 | 36.9 | 41.0 | 43.4 | 47.7 | 53.0 | 58.9 | 64.8 |
| Field c. Normal | 0.007 | 0.180 | 0.735 | 0.985 | 1.000 | 1.000 | 30.9 | 35.3 | 37.6 | 41.6 | 45.9 | 50.3 | 54.2 |
| Logistic | 0.011 | 0.155 | 0.744 | 0.979 | 0.999 | 1.000 | 29.5 | 35.5 | 38.2 | 42.2 | 46.1 | 50.1 | 54.1 |
| Weibull | 0.020 | 0.185 | 0.711 | 0.996 | 1.000 | 1.000 | 27.5 | 33.7 | 36.8 | 41.7 | 46.5 | 50.7 | 54.0 |
| Beta | 0.013 | 0.184 | 0.712 | 0.996 | 1.000 | 1.000 | 29.3 | 34.4 | 37.2 | 41.7 | 46.4 | 50.7 | 54.1 |
| Lognormal | 0.003 | 0.196 | 0.739 | 0.969 | 0.998 | 1.000 | 32.1 | 35.6 | 37.5 | 41.1 | 45.4 | 50.2 | 55.0 |
| Field d. Normal | 0.000 | 0.008 | 0.154 | 0.648 | 0.963 | 0.999 | 40.7 | 45.5 | 48.1 | 52.5 | 57.3 | 62.1 | 66.4 |
| Logistic | 0.001 | 0.008 | 0.113 | 0.658 | 0.967 | 0.998 | 40.7 | 46.7 | 49.5 | 53.5 | 57.6 | 61.6 | 65.7 |
| Weibull | 0.002 | 0.025 | 0.171 | 0.617 | 0.978 | 1.000 | 36.1 | 43.3 | 46.9 | 52.4 | 57.9 | 62.5 | 66.2 |
| Beta | 0.000 | 0.014 | 0.162 | 0.624 | 0.974 | 1.000 | 38.9 | 44.6 | 47.6 | 52.5 | 57.7 | 62.5 | 66.4 |
| Lognormal | 0.000 | 0.005 | 0.175 | 0.657 | 0.938 | 0.994 | 41.4 | 45.4 | 47.7 | 51.8 | 56.8 | 62.2 | 67.6 |
| Field e. Normal | 0.004 | 0.155 | 0.732 | 0.988 | 1.000 | 1.000 | 32.0 | 36.1 | 38.4 | 42.1 | 46.2 | 50.3 | 54.1 |
| Logistic | 0.010 | 0.139 | 0.724 | 0.977 | 0.999 | 1.000 | 30.0 | 36.0 | 38.6 | 42.6 | 46.5 | 50.5 | 54.4 |
| Weibull | 0.012 | 0.150 | 0.711 | 0.999 | 1.000 | 1.000 | 29.5 | 35.3 | 38.2 | 42.6 | 46.9 | 50.6 | 53.5 |
| Beta | 0.009 | 0.157 | 0.709 | 1.000 | 1.000 | 1.000 | 30.2 | 35.3 | 38.0 | 42.3 | 46.8 | 50.7 | 53.6 |
| Lognormal | 0.001 | 0.165 | 0.737 | 0.974 | 0.999 | 1.000 | 33.1 | 36.4 | 38.3 | 41.7 | 45.8 | 50.3 | 54.7 |
| Field f. Normal | 0.005 | 0.147 | 0.690 | 0.979 | 1.000 | 1.000 | 31.7 | 36.1 | 38.5 | 42.4 | 46.8 | 51.2 | 55.1 |
| Logistic | 0.008 | 0.124 | 0.702 | 0.975 | 0.998 | 1.000 | 30.6 | 36.5 | 39.1 | 43.1 | 47.0 | 50.9 | 54.8 |
| Weibull | 0.015 | 0.154 | 0.663 | 0.993 | 1.000 | 1.000 | 28.6 | 34.7 | 37.9 | 42.7 | 47.4 | 51.5 | 54.7 |
| Beta | 0.011 | 0.155 | 0.660 | 0.995 | 1.000 | 1.000 | 29.8 | 35.1 | 38.0 | 42.6 | 47.3 | 51.6 | 54.9 |
| Lognormal | 0.002 | 0.164 | 0.697 | 0.959 | 0.997 | 1.000 | 32.7 | 36.2 | 38.2 | 41.9 | 46.3 | 51.2 | 56.1 |

Table 4 (continuation). Soybeans, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

| oి |  |  | $\underset{\sim}{n} \underset{\sim}{3} \underset{\sim}{n}$ |  |  | $\begin{array}{cccc} \underset{0}{0} & \infty & \infty & 0 \\ i & \cdots \\ i & \infty \\ n & i n & \infty \\ i n & \dot{n} \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { ì } \\ & \text { in } \end{aligned}$ |  |  | $\underset{\sigma}{9} \frac{9}{0} \frac{0}{6} \frac{0}{0}$ |  | $\begin{array}{cccc} \overrightarrow{0} & 0 & 0 & 0 \\ \dot{0} & \infty \\ i n & 0 \\ i n & i \end{array}$ |  |
| $\begin{aligned} & \text { oे } \\ & \text { in } \end{aligned}$ |  |  |  | $\underset{\sim}{\underset{\sim}{r}} \underset{\sim}{\underset{\sim}{f}} \underset{\sim}{\sim} \underset{\sim}{\infty}$ | $\underset{i n}{\vec{i}} \underset{\sim}{i} \frac{\pi}{i}$ |  |
| ì | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $$ |  |  | $\begin{array}{lllll} \infty & n & \cdots & 0 & m \\ \dot{f} & \underset{\sim}{f} & \underset{f}{f} & \underset{f}{f} & \underset{f}{f} \end{array}$ |
| oి | $\stackrel{\square}{\square}$ |  |  | $\underset{n}{\underset{n}{n}} \underset{n}{\infty} \underset{n}{n} \underset{\sim}{\underset{n}{n}} \underset{\sim}{n}$ |  |  |
| in |  | $\underset{\sim}{\dot{m}} \underset{\sim}{\dot{q}} \underset{\sim}{\infty} \underset{\sim}{c} \dot{q}$ |  |  |  | $\vec{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty} \underset{\sim}{n} \underset{\sim}{n} \underset{\sim}{\infty}$ |
| $\bigcirc$ |  |  |  |  |  <br> $\dot{m} \dot{m} \dot{m}$ | $\underset{m}{m} \underset{m}{\dot{m}} \underset{m}{n} \underset{m}{\dot{m}}$ |
| $\begin{aligned} & 0 \\ & \text { g } \\ & \text { İ } \\ & 0 \\ & 0 \end{aligned}$ |  | $\underset{i}{8} 8$ |  | $8888$ | 응 응 | $\begin{aligned} & 8888 \\ & 8 \\ & 0 \\ & 0 \end{aligned}$ |
|  |  |  |  |  |  |  |
| $\begin{aligned} & \text { o } \\ & \text { z. } \\ & \text { 0. } \\ & 8 \end{aligned}$ | $\begin{gathered} 0 \\ \frac{0}{n} \\ \frac{0}{0} \\ \hline \end{gathered}$ |  |  |  |  |  |
| $\begin{aligned} & 0 \\ & \text { g } \\ & \text { jin } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{llll} n & \pi & 0 \\ n & n & n \\ n & n & n & n \\ 0 & 0 & 0 \end{array}$ | $\frac{?}{\frac{n}{0}} \frac{0}{3} \frac{N}{0} \frac{N}{0} \frac{8}{0}$ |  |  | 0 $\hat{0}$ $\infty$ $\infty$ <br> $n$ $n$   <br> 0 0 0 $n$ <br> 0 0 0  |
| $\begin{aligned} & \text { g } \\ & \text { g } \\ & \text { B } \\ & \text { o } \end{aligned}$ |  |  | $$ | $\begin{array}{lll} \frac{\infty}{\infty} & \frac{\pi}{5} & \frac{\infty}{0} \\ 0 & \stackrel{N}{0} \\ 0 \end{array}$ | $\begin{array}{lllll} n & m & \ddots & \ddots & n \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$ | $\begin{array}{llll} \circ & 0 & 0 & 0 \\ 0 & 0 & \circ & 0 \\ 0 & 0 \\ 0 & 0 & 0 \end{array}$ |
| $\begin{aligned} & \text { O } \\ & \text { ت̃ } \\ & \text { In } \end{aligned}$ |  |  |  | $\begin{array}{llll} n & 0 & 0 & \hat{0} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$ |  | $\begin{array}{llll} \text { N } & \hat{8} & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & \end{array}$ |
|  |  |  |  |  |  |  |

Table 4 （continuation）．Soybeans，actuarial yield table，University of Illinois Endowment Farms， 1972 to 1999，base year 2000.

| oి |  | $\vec{n} \vec{n} \vec{n} \times \vec{n}$ | $\begin{array}{lc} \underset{n}{n} & 0 \\ \text { in in } & \text { n } \\ \text { in } \end{array}$ | $\begin{array}{llll} 0 & 0 & 0 \\ n & n \\ n & n & n & n \end{array}$ | $\begin{array}{llll} n & 0 & n & n \\ i & 0 \\ i n & i n & 0 \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\circ}{i}$ |  |  | $\begin{array}{ccc} \stackrel{\infty}{n} \\ \underset{n}{n} & \stackrel{\infty}{n} & \stackrel{n}{n} \end{array}$ |  | $$ | $\hat{i}$ |
| o̊ |  |  |  |  | $\begin{array}{cccc}  \pm & \text { y } & 0 \\ \text { in in } \\ \text { in } & \text { in } & 0 \end{array}$ | ○ O ナ N $\infty$ <br> $\dot{\sim} \dot{\sim} \dot{n} \dot{n}$ |
| $\stackrel{\stackrel{i}{n}}{~}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{5}{0} \end{aligned}$ | $\stackrel{0}{\dot{F}} \underset{\vec{F}}{\vec{F}} \underset{子}{\vec{F}} \underset{子}{\infty}$ |  |  |  |  |
| o̊ | $\cdots$ |  | $\mathfrak{n} \ddagger$ | $\begin{array}{ccc} \underset{j}{0} & \vec{b} & n \\ \cdots & \hat{m} & \hat{j} \end{array}$ |  |  |
| in |  |  |  | $\wedge a \circ$ no <br> m் ぶ |  |  |
| $\bigcirc$ |  | $\begin{array}{cccc} \infty \\ \stackrel{\infty}{\infty} & \stackrel{\infty}{\infty} & \stackrel{\infty}{\sim} & \stackrel{\infty}{\sim} \\ \underset{\sim}{N} \end{array}$ |  | $\infty \infty$ ナ $\quad$ ． <br>  |  |  |
| $\begin{aligned} & \text { U } \\ & \text { İ } \\ & \text { B } \\ & 0 \end{aligned}$ |  | 88888 | $88888$ | 88888 |  |  |
| $\begin{aligned} & 0 \\ & \text { O } \\ & \text { j} \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{array}{lll} 8 & \circ \\ 8 & 8 & \circ \\ 0 & 2 \\ \hline \end{array}$ |  |  |  |  |
| $\begin{aligned} & 0 \\ & \text { g } \\ & \text { 3 } \\ & 0 \\ & 8 \end{aligned}$ | $\begin{gathered} \stackrel{0}{3} \\ \stackrel{0}{0} \\ \end{gathered}$ |  |  |  |  |  |
| $\begin{aligned} & \text { y } \\ & \text { ji } \\ & \text { 0 } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $$ |  |  |  |
| $\begin{aligned} & \text { o } \\ & \text { ju } \\ & \text { b } \\ & \text { of } \end{aligned}$ |  |  | $\begin{array}{llll} \infty & \cdots & \cdots & 2 \\ 0 & \hat{0} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$ | $$ | $$ | $\begin{array}{llll} n & \infty & t & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 \end{array}$ |
|  |  |  | $\begin{array}{llll}\text { N } & 6 & 0 & \ddots \\ 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0\end{array}$ | $\begin{array}{llll} n & 0 & N & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline \end{array}$ | $\begin{array}{lllll} 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$ | $\begin{array}{llll} \substack{0} & \infty & 0 & 0 \\ 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 \\ \hline \end{array}$ |
|  |  |  |  |  |  |  |


Table 4 (continuation). Soybeans, actuarial yield table, University of Illinois Endowment Farms, 1972 to 1999, base year 2000.

|  | $30 \mathrm{bu} . \mathrm{ac}$ | $40 \mathrm{bu} . / \mathrm{ac}$ | $\begin{array}{r} 50 \mathrm{bu} . / \mathrm{ac} \\ \text { proba } \end{array}$ | 60 bu./ac <br> abilities | $70 \mathrm{bu} . / \mathrm{ac}$ | $80 \mathrm{bu} . / \mathrm{ac}$ | 1\% | 5\% | $10 \%$ yiel | 25\% | 50\% | 75\% | 90\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Field v. Normal | 0.002 | 0.091 | 0.587 | 0.962 | 1.000 | 1.000 | 33.6 | 38.0 | 40.3 | 44.2 | 48.6 | 52.9 | 56.8 |
| Logistic | 0.001 | 0.037 | 0.543 | 0.974 | 0.999 | 1.000 | 36.2 | 40.9 | 43.1 | 46.3 | 49.5 | 52.7 | 55.9 |
| Weibull | 0.002 | 0.057 | 0.560 | 0.999 | 1.000 | 1.000 | 34.4 | 39.5 | 42.0 | 45.8 | 49.3 | 52.3 | 54.6 |
| Beta | 0.004 | 0.088 | 0.567 | 0.999 | 1.000 | 1.000 | 32.7 | 37.9 | 40.5 | 44.8 | 49.0 | 52.7 | 55.3 |
| Lognormal | 0.002 | 0.137 | 0.596 | 0.909 | 0.988 | 0.999 | 32.6 | 36.5 | 38.8 | 42.9 | 48.0 | 53.7 | 59.5 |
| Field w. Normal | 0.000 | 0.045 | 0.560 | 0.977 | 1.000 | 1.000 | 36.6 | 40.3 | 42.3 | 45.5 | 49.2 | 52.8 | 56.1 |
| Logistic | 0.002 | 0.041 | 0.524 | 0.966 | 0.999 | 1.000 | 35.6 | 40.6 | 42.9 | 46.3 | 49.7 | 53.1 | 56.5 |
| Weibull | 0.002 | 0.048 | 0.508 | 0.998 | 1.000 | 1.000 | 35.0 | 40.1 | 42.6 | 46.4 | 49.9 | 52.9 | 55.2 |
| Beta | 0.001 | 0.054 | 0.522 | 0.999 | 1.000 | 1.000 | 35.1 | 39.8 | 42.1 | 45.9 | 49.7 | 53.0 | 55.4 |
| Lognormal | 0.000 | 0.043 | 0.579 | 0.961 | 0.999 | 1.000 | 37.3 | 40.3 | 42.1 | 45.2 | 48.9 | 52.9 | 56.7 |
| Field x. Normal | 0.013 | 0.314 | 0.895 | 0.999 | 1.000 | 1.000 | 29.4 | 33.3 | 35.4 | 38.9 | 42.8 | 46.7 | 50.2 |
| Logistic | 0.011 | 0.228 | 0.886 | 0.995 | 1.000 | 1.000 | 29.7 | 34.7 | 37.0 | 40.4 | 43.7 | 47.1 | 50.5 |
| Weibull | 0.012 | 0.239 | 0.956 | 1.000 | 1.000 | 1.000 | 29.6 | 34.3 | 36.7 | 40.2 | 43.6 | 46.4 | 48.6 |
| Beta | 0.019 | 0.281 | 0.945 | 1.000 | 1.000 | 1.000 | 28.3 | 33.1 | 35.5 | 39.4 | 43.3 | 46.6 | 49.0 |
| Lognormal | 0.011 | 0.354 | 0.864 | 0.989 | 1.000 | 1.000 | 29.8 | 33.0 | 34.9 | 38.2 | 42.3 | 46.9 | 51.4 |
| Field y. Normal | 0.001 | 0.062 | 0.573 | 0.972 | 1.000 | 1.000 | 35.4 | 39.4 | 41.5 | 45.0 | 48.9 | 52.8 | 56.4 |
| Logistic | 0.004 | 0.066 | 0.572 | 0.962 | 0.998 | 1.000 | 33.4 | 39.0 | 41.5 | 45.3 | 49.0 | 52.7 | 56.5 |
| Weibull | 0.006 | 0.086 | 0.530 | 0.987 | 1.000 | 1.000 | 31.8 | 37.7 | 40.7 | 45.2 | 49.5 | 53.3 | 56.2 |
| Beta | 0.002 | 0.075 | 0.541 | 0.987 | 1.000 | 1.000 | 33.8 | 38.6 | 41.1 | 45.2 | 49.4 | 53.2 | 56.2 |
| Lognormal | 0.000 | 0.055 | 0.594 | 0.959 | 0.999 | 1.000 | 36.6 | 39.8 | 41.6 | 44.8 | 48.6 | 52.7 | 56.8 |
| Field z. Normal | 0.276 | 0.824 | 0.993 | 1.000 | 1.000 | 1.000 | 18.7 | 23.1 | 25.5 | 29.5 | 33.9 | 38.3 | 42.3 |
| Logistic | 0.254 | 0.826 | 0.985 | 0.999 | 1.000 | 1.000 | 16.6 | 22.9 | 25.7 | 29.9 | 34.1 | 38.3 | 42.4 |
| Weibull | 0.265 | 0.818 | 0.998 | 1.000 | 1.000 | 1.000 | 16.9 | 22.2 | 25.0 | 29.7 | 34.4 | 38.6 | 42.1 |
| Beta | 0.271 | 0.810 | 1.000 | 1.000 | 1.000 | 1.000 | 17.5 | 22.3 | 25.0 | 29.5 | 34.4 | 38.8 | 42.2 |
| Lognormal | 0.308 | 0.818 | 0.977 | 0.998 | 1.000 | 1.000 | 20.7 | 23.7 | 25.6 | 29.0 | 33.2 | 38.1 | 43.2 |

Table 5. Corn, expected payments from buying APH insurance at $85 \%$ election, University of Illinois Endowment Farms, 1972 to 1999, based on 2000 crop year, dollars per acre.

| Farm | Normal | Logistic Weibull |  | Beta | Lognormal | Range | Actual <br> Premiums | Payment Rank |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1 |  |  |  | 2 | 3 | 4 | 5 |
| x . | 9.28 | 4.95 | 6.43 |  | 10.04 | 13.46 | 8.51 | 13.20 | Ln | B | N | W | L |
| b. | 3.47 | 2.16 | 2.62 | 3.80 | 4.80 | 2.64 | 11.70 | Ln | B | N | W | L |
| a. | 3.21 | 1.81 | 2.66 | 3.61 | 4.40 | 2.60 | 13.40 | Ln | B | N | W | L |
| u. | 8.09 | 6.75 | 7.37 | 8.88 | 9.24 | 2.49 | 14.10 | Ln | B | N | W | L |
| n. | 2.92 | - 3.69 | 4.52 | 3.79 | 2.25 | 2.27 | 12.20 | W | B | L | N | Ln |
| 1. | 4.96 | 4.60 | 6.63 | 5.84 | 4.50 | 2.12 | 11.30 | W | B | N | L | Ln |
| z. | 9.76 | 9.04 | 9.35 | 10.65 | 10.91 | 1.87 | 14.60 | Ln | B | N | W | L |
| d. | 3.80 | - 2.77 | 4.18 | 4.51 | 4.63 | 1.86 | 14.70 | Ln | B | W | N | L |
| r. | 4.97 | 3.74 | 4.33 | 5.50 | 5.60 | 1.86 | 11.20 | Ln | B | N | W | L |
| f. | 7.02 | 7.84 | 8.00 | 7.94 | 6.37 | 1.64 | 10.20 | W | B | L | N | Ln |
| h. | 4.19 | - 4.80 | 5.00 | 5.29 | 3.67 | 1.62 | 12.30 | B | W | L | N | Ln |
| k. | 6.58 | - 5.89 | 6.64 | 7.49 | 7.50 | 1.61 | 11.60 | Ln | B | W | N | L |
| m. | 4.74 | - 3.92 | 5.10 | 5.41 | 4.92 | 1.50 | 11.50 | B | W | Ln | N | L |
| c. | 2.08 | - 2.37 | 3.14 | 2.71 | 1.72 | 1.42 | 12.70 | W | B | L | N | Ln |
| s. | 5.18 | 4.67 | 6.07 | 6.03 | 5.32 | 1.40 | 11.40 | W | B | Ln | N | L |
| e. | 2.38 | 1.55 | 2.93 | 2.94 | 2.57 | 1.39 | 22.30 | B | W | Ln | N | L |
| y. | 4.14 | - 3.70 | 4.73 | 5.03 | 4.46 | 1.33 | 12.40 | B | W | Ln | N | L |
| v. | 2.65 | 1.88 | 2.47 | 3.06 | 3.20 | 1.32 | 11.10 | Ln | B | N | W | L |
| i. | 1.20 | 1.15 | 2.36 | 1.79 | 1.05 | 1.30 | 14.60 | W | B | N | L | Ln |
| p. | 5.14 | 4.67 | 5.21 | 5.84 | 5.05 | 1.17 | 11.00 | B | W | N | Ln | L |
| t. | 3.13 | - 2.41 | 2.86 | 3.58 | 3.42 | 1.17 | 11.10 | B | Ln | N | W | L |
| g. | 2.17 | 1.35 | 1.83 | 2.50 | 2.44 | 1.15 | 11.20 | B | Ln | N | W | L |
| w. | 2.66 | 2.30 | 3.42 | 3.44 | 2.94 | 1.14 | 12.30 | B | W | Ln | N | L |
| j. | 4.50 | 4.48 | 4.72 | 5.41 | 4.34 | 1.07 | 11.40 | B | W | N | L | Ln |
| o. | 5.50 | 5.64 | 5.74 | 6.35 | 5.33 | 1.01 | 11.70 | B | W | L | N | Ln |
| q. | 2.72 | - 2.47 | 2.59 | 3.23 | 2.70 | 0.77 | 11.50 | B | N | Ln | W | L |


| Mean | 4.48 | 3.87 | 4.65 | 5.18 | 4.88 | 1.86 | Normal | Count |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | 0 | 1 | 13 | 12 | 0 |
| St. Dev. | 2.20 | 2.02 | 1.96 | 2.31 | 2.81 | 1.45 | Logistic | 0 | 0 | 5 | 3 | 18 |
| Min. | 1.20 | 1.15 | 1.83 | 1.79 | 1.05 | 0.77 | Weibull | 6 | 8 | 2 | 10 | 0 |
| Max. | 9.76 | 9.04 | 9.35 | 10.65 | 13.46 | 8.51 | Beta | 11 | 15 | 0 | 0 | 0 |
|  |  |  |  |  |  |  | Lognormal | 9 | 2 | 6 | 1 | 8 |

Note: N, L, W, B and Ln denotes Normal, Logistic, Weibull, Beta and Lognormal distributions, respectively. Premium Calculation Program version PY2000.02 by the FCIC, http://www.rma.usda.gov/tools/.

Table 6. Soybeans, expected payments from buying APH insurance at $85 \%$ election, University of Illinois Endowment Farms, 1972 to 1999, based on 2000 crop year, dollars per acre.

| Farm | Normal | Logistic Weibull |  | Beta | Lognormal | Range | Actual <br> Premiums | Payment Rank |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1 |  |  |  | 2 | 3 | 4 | 5 |
| a. | 2.84 | 3.12 | 5.17 |  | 3.74 | 2.21 | 2.96 | 9.00 | W | B | L | N | Ln |
| t. | 3.74 | 2.72 | 3.48 | 4.34 | 5.12 | 2.41 | 8.30 | Ln | B | N | W | L |
| v. | 1.87 | 0.75 | 1.23 | 1.99 | 2.90 | 2.16 | 8.40 | Ln | B | N | W | L |
| j. | 2.40 | 1.32 | 1.98 | 2.73 | 3.42 | 2.10 | 8.20 | Ln | B | N | W | L |
| n. | 2.90 | 3.62 | 4.12 | 3.71 | 2.31 | 1.81 | 8.50 | W | B | L | N | Ln |
| h. | 0.99 | 1.31 | 2.39 | 1.49 | 0.68 | 1.71 | 7.80 | W | B | L | N | Ln |
| 1. | 2.03 | 2.40 | 3.30 | 2.74 | 1.64 | 1.66 | 7.10 | W | B | L | N | Ln |
| d. | 1.34 | 1.11 | 2.69 | 1.84 | 1.28 | 1.57 | 10.90 | W | B | N | Ln | L |
| c. | 2.14 | 2.18 | 3.24 | 2.72 | 1.93 | 1.32 | 9.20 | W | B | L | N | Ln |
| y. | 1.25 | 1.65 | 2.29 | 1.73 | 0.98 | 1.31 | 8.40 | W | B | L | N | Ln |
| g. | 1.10 | 1.06 | 2.17 | 1.50 | 0.94 | 1.23 | 9.50 | W | B | N | L | Ln |
| u. | 5.31 | 4.79 | 5.62 | 5.98 | 5.44 | 1.20 | 9.70 | B | W | Ln | N | L |
| m. | 1.89 | 1.91 | 2.78 | 2.48 | 1.72 | 1.06 | 7.10 | W | B | L | N | Ln |
| f. | 2.12 | 2.08 | 3.08 | 2.78 | 2.04 | 1.04 | 8.90 | W | B | N | L | Ln |
| o. | 2.10 | 1.43 | 2.05 | 2.45 | 2.45 | 1.03 | 8.30 | Ln | B | N | W | L |
| k. | 2.25 | 2.34 | 3.05 | 2.83 | 2.02 | 1.03 | 7.10 | W | B | L | N | Ln |
| z. | 4.11 | 4.23 | 4.76 | 4.67 | 3.81 | 0.95 | 10.40 | W | B | L | N | Ln |
| p. | 3.03 | 2.98 | 3.85 | 3.72 | 3.01 | 0.87 | 8.20 | W | B | N | Ln | L |
| e. | 1.23 | 1.56 | 1.82 | 1.70 | 0.98 | 0.84 | 15.10 | W | B | L | N | Ln |
| s. | 2.22 | 2.09 | 2.81 | 2.83 | 2.17 | 0.74 | 8.20 | B | W | N | Ln | L |
| i. | 1.52 | 1.58 | 2.08 | 1.90 | 1.37 | 0.70 | 9.50 | W | B | L | N | Ln |
| x. | 1.57 | 1.11 | 1.20 | 1.77 | 1.71 | 0.66 | 9.10 | B | Ln | N | W | L |
| q. | 2.03 | 1.92 | 2.55 | 2.54 | 1.93 | 0.63 | 8.20 | W | B | N | Ln | L |
| w. | 0.92 | 1.02 | 1.19 | 1.26 | 0.81 | 0.45 | 8.30 | B | W | L | N | Ln |
| r. | 0.98 | 0.94 | 1.31 | 1.32 | 0.94 | 0.38 | 8.40 | B | W | N | Ln | L |


|  |  |  |  |  |  |  |  | Count |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean | 2.16 | 2.05 | 2.81 | 2.67 | 2.15 | 1.27 | Normal | 0 | 0 | 12 | 13 | 0 |
| St. Dev. | 1.06 | 1.05 | 1.20 | 1.16 | 1.25 | 0.64 | Logistic | 0 | 0 | 12 | 2 | 11 |
| Min | 0.92 | 0.75 | 1.19 | 1.26 | 0.68 | 0.38 | Weibull | 16 | 4 | 0 | 5 | 0 |
| Max | 5.31 | 4.79 | 5.62 | 5.98 | 5.44 | 2.96 | Beta | 5 | 20 | 0 | 0 | 0 |
|  |  |  |  |  |  |  | Lognormal | 4 | 1 | 1 | 5 | 14 |

Note: N, L, W, B and Ln denotes Normal, Logistic, Weibull, Beta and Lognormal distributions, respectively. Premium Calculation Program version PY2000.02 by the FCIC, http://www.rma.usda.gov/tools/.

Table 7. Corn, mean across fields of the percentage-differences in expected payments from buying APH insurance at $85 \%$ election, 1972 to 1999 , based on 2000 crop year.

|  | Normal | Logistic | Weibull | Beta | LogNormal |
| ---: | :---: | :---: | :---: | :---: | :---: |
| over Normal | ---- | $19 \%$ | $20 \%$ | $18 \%$ | $14 \%$ |
| over Logistic | $26 \%$ | ---- | $28 \%$ | $42 \%$ | $43 \%$ |
| over Weibull | $17 \%$ | $20 \%$ | ---- | $17 \%$ | $28 \%$ |
| over Beta | $15 \%$ | $27 \%$ | $15 \%$ | ---- | $16 \%$ |
| over Lognormal | $13 \%$ | $28 \%$ | $30 \%$ | $20 \%$ | ----- |

Table 8. Soybeans, mean across fields of the percentage-differences in expected payments from buying APH insurance at $85 \%$ election, 1972 to 1999 , based on 2000 crop year.

|  | Normal | Logistic | Weibull | Beta | LogNormal |
| ---: | :---: | :---: | :---: | :---: | :---: |
| over Normal | ----- | $10 \%$ | $25 \%$ | $16 \%$ | $10 \%$ |
| over Logistic | $17 \%$ | ---- | $27 \%$ | $28 \%$ | $32 \%$ |
| over Weibull | $19 \%$ | $19 \%$ | ---- | $12 \%$ | $28 \%$ |
| over Beta | $13 \%$ | $16 \%$ | $12 \%$ | ---- | $18 \%$ |
| over Lognormal | $9 \%$ | $18 \%$ | $36 \%$ | $22 \%$ | ---- |

Table 9. Corn, expected payments from buying Crop Revenue Coverage at $85 \%$ election assuming price-yield correlation of -0.4, University of Illinois Endowment Farms, 1972 to 1999 , based on 2000 crop year.

| Fields | Normal | Logistic | Weibull | Beta | Lognormal | Range | Premiums | Payment Rank |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | 1 | 2 | 3 | 4 | 5 |
| x. | 21.77 | 14.16 | 17.44 | 22.68 | 27.96 | 13.79 | 24.08 | Ln | B | N | W | L |
| a. | 15.03 | 11.63 | 13.75 | 15.41 | 16.99 | 5.36 | 24.54 | Ln | B | N | W | L |
| b. | 13.95 | 11.01 | 12.37 | 14.28 | 15.99 | 4.99 | 21.74 | Ln | B | N | W | L |
| u. | 18.61 | 16.39 | 17.34 | 19.35 | 20.56 | 4.17 | 25.18 | Ln | B | N | W | L |
| r. | 16.67 | 13.90 | 15.33 | 17.09 | 17.91 | 4.01 | 20.43 | Ln | B | N | W | L |
| d. | 15.30 | 12.98 | 15.41 | 16.03 | 16.86 | 3.89 | 25.83 | Ln | B | W | N | L |
| g. | 14.11 | 11.21 | 12.90 | 14.31 | 14.93 | 3.72 | 21.61 | Ln | B | N | W | L |
| v. | 14.28 | 11.99 | 13.48 | 14.60 | 15.38 | 3.39 | 21.96 | Ln | B | N | W | L |
| e. | 12.24 | 9.80 | 12.55 | 12.80 | 13.08 | 3.28 | 38.50 | Ln | B | W | N | L |
| t. | 15.59 | 13.32 | 14.62 | 15.89 | 16.34 | 3.02 | 21.18 | Ln | B | N | W | L |
| z. | 20.78 | 20.00 | 19.96 | 21.63 | 22.78 | 2.82 | 25.58 | Ln | B | N | L | W |
| k. | 19.04 | 17.80 | 18.78 | 19.88 | 20.58 | 2.77 | 20.88 | Ln | B | N | W | L |
| m. | 17.20 | 15.21 | 17.20 | 17.78 | 17.91 | 2.70 | 22.43 | Ln | B | N | W | L |
| 1. | 17.11 | 16.40 | 18.96 | 17.86 | 16.94 | 2.56 | 20.88 | W | B | N | Ln | L |
| n. | 13.87 | 14.90 | 15.61 | 14.62 | 13.44 | 2.16 | 21.47 | W | L | B | N | Ln |
| y. | 16.55 | 15.38 | 16.85 | 17.35 | 17.47 | 2.10 | 23.15 | Ln | B | W | N | L |
| p. | 16.47 | 14.98 | 16.03 | 17.05 | 16.95 | 2.07 | 21.99 | B | Ln | N | W | L |
| q. | 15.15 | 13.53 | 14.25 | 15.47 | 15.54 | 2.01 | 22.43 | Ln | B | N | W | L |
| s. | 17.82 | 16.64 | 18.60 | 18.59 | 18.46 | 1.96 | 21.99 | W | B | Ln | N | L |
| w. | 15.47 | 14.42 | 16.00 | 16.14 | 16.24 | 1.83 | 24.06 | Ln | B | W | N | L |
| i. | 11.44 | 11.00 | 12.71 | 11.99 | 11.58 | 1.71 | 26.39 | W | B | Ln | N | L |
| j. | 17.22 | 16.39 | 16.90 | 17.93 | 17.54 | 1.53 | 22.33 | B | Ln | N | W | L |
| f. | 19.79 | 20.92 | 20.68 | 20.54 | 19.50 | 1.42 | 19.57 | L | W | B | N | Ln |
| c. | 12.06 | 11.90 | 13.06 | 12.59 | 11.97 | 1.16 | 24.60 | W | B | N | Ln | L |
| h. | 15.86 | 16.22 | 16.38 | 16.82 | 15.80 | 1.02 | 24.08 | B | W | L | N | Ln |
| o. | 17.79 | 17.47 | 17.58 | 18.46 | 18.10 | 0.99 | 21.96 | B | Ln | N | W | L |
|  |  |  |  |  |  |  |  | Count |  |  |  |  |
| Mean | 16.20 | 14.60 | 15.95 | 16.81 | 17.18 | 3.09 | Normal | 0 | 0 | 17 | 9 | 0 |
| Std. | 2.57 | 2.79 | 2.38 | 2.70 | 3.38 | 2.48 | Logistic | 1 | 1 | 1 | 1 | 22 |
| Min. | 11.44 | 9.80 | 12.37 | 11.99 | 11.58 | 0.99 | Weibull | 5 | 2 | 4 | 14 | 1 |
| Max. | 21.77 | 20.92 | 20.68 | 22.68 | 27.96 | 13.79 | Beta | 4 | 20 | 2 | 0 | 0 |
|  |  |  |  |  |  |  | Lognormal | 16 | 3 | 2 | 2 | 3 |

Note: N, L, W, B and Ln denotes Normal, Logistic, Weibull, Beta and Lognormal distributions, respectively. Premium Calculation Program version PY2000.02 by the FCIC, http://www.rma.usda.gov/tools/.

Table 10. Soybeans, expected payments from buying Crop Revenue Coverage at $85 \%$ election assuming price-yield correlation of -0.4, University of Illinois Endowment Farms, 1972 to 1999 , based on 2000 crop year.

| Farm | Normal | Logistic | Weibull | Beta | Lognormal | Range | Premiums | Payment Rank |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | 1 | 2 | 3 | 4 | 5 |
| t. | 9.81 | 8.13 | 9.20 | 10.28 | 11.63 | 3.50 | 11.77 | Ln | B | N | W | L |
| v. | 8.14 | 6.02 | 7.12 | 8.18 | 9.44 | 3.42 | 11.50 | Ln | B | N | W | L |
| j. | 8.73 | 6.72 | 7.92 | 8.93 | 10.09 | 3.37 | 11.67 | Ln | B | N | W | L |
| a. | 9.12 | 9.56 | 11.41 | 9.80 | 8.83 | 2.58 | 11.80 | W | B | L | N | Ln |
| o. | 8.19 | 6.77 | 7.76 | 8.37 | 8.86 | 2.09 | 11.81 | Ln | B | N | W | L |
| d. | 7.66 | 6.97 | 8.88 | 8.03 | 7.89 | 1.92 | 23.30 | W | B | Ln | N | L |
| x. | 6.78 | 5.43 | 5.95 | 6.80 | 7.16 | 1.73 | 10.68 | Ln | B | N | W | L |
| h. | 6.85 | 6.97 | 8.16 | 7.22 | 6.73 | 1.44 | 14.20 | W | B | L | N | Ln |
| g . | 7.33 | 6.90 | 8.21 | 7.58 | 7.37 | 1.31 | 14.11 | W | B | Ln | N | L |
| n. | 8.49 | 9.26 | 9.49 | 9.04 | 8.20 | 1.29 | 12.21 | W | L | B | N | Ln |
| u. | 10.21 | 9.49 | 10.34 | 10.71 | 10.66 | 1.23 | 10.49 | B | Ln | W | N | L |
| 1. | 8.04 | 8.32 | 9.07 | 8.49 | 7.90 | 1.17 | 10.91 | W | B | L | N | Ln |
| c. | 8.02 | 7.83 | 8.91 | 8.41 | 8.08 | 1.08 | 13.83 | W | B |  | N | L |
| q. | 9.04 | 8.31 | 9.16 | 9.32 | 9.24 | 1.01 | 12.32 | B | Ln | W | N | L |
| f. | 8.23 | 7.97 | 8.94 | 8.65 | 8.41 | 0.97 | 13.48 | W | B | Ln | N | L |
| p. | 8.66 | 8.27 | 9.24 | 9.19 | 9.06 | 0.97 | 14.40 | W | B | Ln | N | L |
| s. | 8.61 | 8.02 | 8.86 | 8.98 | 8.87 | 0.96 | 12.32 | B | Ln | W | N | L |
| r. | 6.97 | 6.29 | 6.94 | 7.16 | 7.19 | 0.90 | 11.96 | Ln | B | N | W | L |
| m. | 8.00 | 7.73 | 8.62 | 8.36 | 8.10 | 0.88 | 11.17 | W | B | Ln | N | L |
| y. | 7.58 | 7.81 | 8.35 | 7.86 | 7.51 | 0.84 | 13.71 | W | B | L | N | Ln |
| k. | 8.56 | 8.24 | 9.04 | 8.90 | 8.61 | 0.80 | 11.34 | W | B | Ln | N | L |
| w. | 7.25 | 6.73 | 7.13 | 7.41 | 7.32 | 0.68 | 11.62 | B | Ln | N | W | L |
| i. | 7.10 | 6.69 | 7.36 | 7.31 | 7.19 | 0.67 | 14.04 | W | B | Ln | N | L |
| z. | 8.70 | 8.76 | 9.19 | 9.08 | 8.65 | 0.53 | 14.41 | W | B | L | N | Ln |
| e. | 6.01 | 6.02 | 6.32 | 6.30 | 5.99 | 0.33 | 20.92 | W | B | L | N | Ln |
|  |  |  |  |  |  |  |  |  |  | Count |  |  |
| Mean | 8.08 | 7.57 | 8.46 | 8.41 | 8.36 | 1.43 | Normal | 0 | 0 | 7 | 18 | 0 |
| Std. | 0.97 | 1.12 | 1.22 | 1.07 | 1.26 | 0.91 | Logistic | 0 | 1 | 6 | 0 | 18 |
| Min. | 6.01 | 5.43 | 5.95 | 6.30 | 5.99 | 0.33 | Weibull | 15 | 0 | 3 | 7 | 0 |
| Max. | 10.21 | 9.56 | 11.41 | 10.71 | 11.63 | 3.50 | Beta | 4 | 20 | 1 | 0 | 0 |
|  |  |  |  |  |  |  | Lognormal | 6 | 4 | 8 | 0 | 7 |

Note: N, L, W, B and Ln denotes Normal, Logistic, Weibull, Beta and Lognormal distributions, respectively. Premium Calculation Program version PY2000.02 by the FCIC, http://www.rma.usda.gov/tools/.

Table 11. Corn, mean across fields of the percentage-differences in expected payments from buying CRC insurance at $85 \%$ election, 1972 to 1999, based on 2000 crop year.

|  |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
|  | Normal | Logistic | Weibull | Beta | LogNormal |
| over Normal | ---- | $11.0 \%$ | $6.0 \%$ | $3.8 \%$ | $6.2 \%$ |
| over Logistic | $13.4 \%$ | ---- | $10.3 \%$ | $16.7 \%$ | $20.8 \%$ |
| over Weibull | $6.2 \%$ | $9.0 \%$ | ---- | $7.2 \%$ | $11.9 \%$ |
| over Beta | $3.6 \%$ | $13.5 \%$ | $6.5 \%$ | ---- | $4.8 \%$ |
| over Lognormal | $5.6 \%$ | $15.6 \%$ | $10.2 \%$ | $4.6 \%$ | ----- |

Table 12. Soybeans, mean across fields of the percentage-differences in expected payments from buying CRC insurance at $85 \%$ election, 1972 to 1999 , based on 2000 crop year.

|  |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
|  | Normal | Logistic | Weibull | Beta | LogNormal |
| over Normal | ---- | $8.1 \%$ | $8.6 \%$ | $4.0 \%$ | $4.1 \%$ |
| over Logistic | $9.4 \%$ | ---- | $12.0 \%$ | $12.1 \%$ | $14.3 \%$ |
| over Weibull | $8.0 \%$ | $10.5 \%$ | ---- | $6.1 \%$ | $10.8 \%$ |
| over Beta | $3.9 \%$ | $10.2 \%$ | $6.0 \%$ | ---- | $5.0 \%$ |
| over Lognormal | $3.8 \%$ | $11.4 \%$ | $10.7 \%$ | $4.9 \%$ | ---- |

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[^1]:    ${ }^{1}$ Complete results of farm-level detrending regressions, tests for heteroskedasticity, and autocorrelation are available from the authors upon request. Computed Durbin-Watson indicated little concern with autocorrelation, with inconclusive or significant for only three farms. Similarly, F tests of squared residual were not significant for any farm or crop, suggesting that heteroscedasticity is not a concern.
    ${ }^{2}$ Mathematica 4.0 was used in the estimation. Tables containing results of the parameters by distribution, farm, and crop are available from the authors upon request.
    ${ }^{3}$ Maximum likelihood estimates of the beta distribution over the two parameters of the distribution with the upper limit endogenized as the third parameter failed to converge for many of the samples. Thus, the maximum parameter of the beta distribution was set to $10 \%$ above the maximum yield recorded. Ker and Coble suggested that further sensitivity analysis should be considered when upper limits on the beta distribution are defined arbitrarily. Alternatively, the importance of imposing an upper yield on the beta distribution can be assessed by comparing the probability mass above the maximum yields for the five fitted distribution and across the corn and soybean fields. The average implied probability of obtaining yields above the maximum yields across fields are $3.2 \%$ for corn and $2.5 \%$ for soybean fields using the beta distribution, probabilities using the Weibull distribution ( $3.1 \%$ for corn and $2.2 \%$ for soybeans). More importantly for purposes of examining insurance, there is almost no impact on the mass location in the lower tail of the distribution.
    ${ }^{4}$ While the percentage differences among the distributions is likely to be greater at lower election levels, $85 \%$ is the most commonly chosen election level in Illinois under APH insurance and thus most representative.
    ${ }^{5}$ Interest rates are available at $h t t p: / / w w w . b o g . f r b . f e d . u s / R e l e a s e s / H 15 / d a t a / b / t c m 3 m . t x t . ~$

