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PAPER

Nonresonant high frequency excitation of mechanical vibrations in a graphene based nanoresonator

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Abstract

We theoretically analyze the dynamics of a suspended graphene membrane which is in tunnel contact with grounded metallic electrodes and subjected to ac-electrostatic potential induced by a gate electrode. It is shown that for such a system the retardation effects in the electronic subsystem generate an effective pumping for the relatively slow mechanical vibrations if the driving frequency exceeds the inverse charge relaxation time. Under this condition there is a critical value of the driving voltage amplitude above which the pumping overcomes the intrinsic damping of the mechanical resonator, leading to a mechanical instability. This nonresonant instability is saturated by nonlinear damping and the system exhibits self-sustained oscillations of relatively large amplitude.

Rapid progress in carbon nanostructures manufacturing has stimulated new experimental and theoretical efforts in studying their unique optical, electrical, and mechanical properties[1, 2]. In particular, the very high stiffness and low density of graphene make it an ideal material for constructing nanoelectromechanical resonators. These features of graphene are of great interest both for fundamental studies of mechanics at the nanoscale level and a variety of applications, including force, position and mass sensing [3–7]. In particular, it was demonstrated [8] that a graphene-based nanomechanical resonator can be employed as an active element for frequency-modulated signal generation and efficient audio signal transmission. The operation of many nanomechanical devices is based on the excitation of mechanical vibrations by an external periodic force, of electrostatic or optic origin, with a frequency comparable to the vibrational frequency of the mechanical resonator [8–13]. At the same time it was shown [14, 15] that, in certain nanoelectromechanical systems, self-sustained mechanical oscillations with relatively large amplitude may also be actuated by 'shuttle instability'. In the shuttle structures described in [14, 15], the instability was found to occur at driving frequencies that are much smaller than the eigenfrequency of the mechanical subsystem. In the present work we seek to answer the question of whether it is possible to achieve a regime of self-sustained oscillations in a graphene-based nanoresonator by using an electromechanical instability effect caused by a nonresonant high frequency driving field. In the paper we demonstrate that such a feat is possible. However, in contrast to the shuttle instability, electromechanical instability in graphene-based resonators similar to those considered in the publication [3–6, 8] occurs when the driving frequency is much greater than the eigenfrequency of the mechanical subsystem. In this regard, nonresonant excitations of mechanical resonators may be achieved in optomechanical systems where the time delayed radiation pressure controls the mechanical damping [16, 17]. In particular, such a phenomenon was demonstrated for a graphene resonator where the mechanical motion was both activated and cooled by a laser beam [18]. However, the instability considered below is of electromechanical rather than optomechanical origin. More specifically, it is controlled by the relaxation time in the electronic subsystem rather than resonant properties of a microwave cavity [16, 18] or external circuit [19, 20].

The system under consideration in this paper is shown in figure 1. A doped graphene membrane is suspended over a trench and is in contact with grounded metallic electrodes through the oxide layer. An oscillating voltage $\tilde{V}_G(t) = V \cos(\Omega t)$ is applied on a side-gate electrode which is positioned below the suspended part of the sheet.

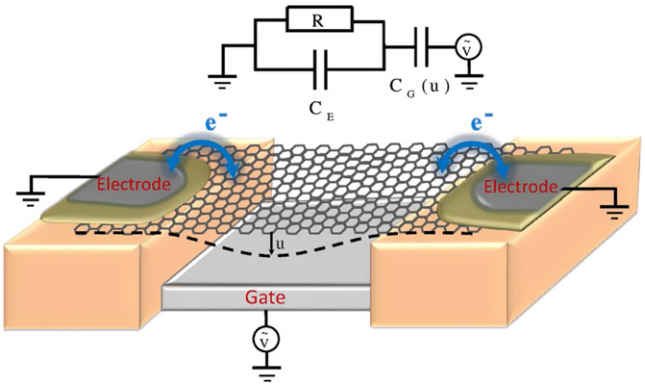

Figure 1. A sketch of the graphene membrane resonator suspended over the trench and separated from the grounded metallic electrodes by an oxide layer. A side-gate subjected to an AC voltage induces an electrostatic potential on the graphene sheet. It is assumed that the potential is dependent on the membrane deflection u. The schematic RC-circuit is shown above.

The potential φ on the membrane, and thus the electronic flow through the tunneling barrier, depends on the charge q accumulated on the membrane, gate voltage and mutual capacitance. The latter depends on the membrane deflection. At the same time, a high frequency electrical field produced by the gate electrode sets up a time-varying force acting on the charged membrane. This force provides a feedback between the mechanical and electronic subsystems. We show that such feedback may generate electromechanical instability, resulting in high amplitude mechanical self-oscillations even for the case when the external frequency Ω is much higher than the characteristic mechanical frequency ω_m.

For a quantitative description of the above-mentioned phenomena, we suggest the following model. We represent the graphene sheet as an elastic thin membrane whose motion is described within the continuum mechanics approach and completely characterized by the amplitude of its fundamental bending mode $u(t)$. Since the instability, as it is shown below, is a nonresonant phenomenon, we disregard the geometric nonlinearity of the graphene membrane. The time evolution of the membrane deflection u is described as a damped oscillator subjected to an external electrostatic force induced by the side-gate voltage $\tilde{V}_G(t)$:

$$\ddot{u} + \frac{\omega_m}{Q}\dot{u} + \omega_m^2 u = \frac{1}{2m}\frac{\partial C_G}{\partial u}\left(\varphi - \tilde{V}_G(t)\right)^2, \tag{1}$$

where Q is the quality factor of the oscillator and $C_G(u)$ is the mutual capacitance between the graphene and the side-gate. Since the membrane deflection $u(t)$ is much smaller than the distance d between the side-gate and the membrane, we set $C_G(u) \approx C_G(1 + u(t)/\tilde{d})$, where $\tilde{d} = C_G(0)/C_G'(0) \approx d$.

The electrostatic potential on the membrane $\varphi(q, \tilde{V}_G(t), u)$ is given by the expression $\varphi = (C_G(u)\tilde{V}_G(t) + q)/C(u)$, where $C(u) = C_G(u) + C_E$ is the total capacitance of the membrane and C_E is the mutual capacitance between the graphene and the grounded electrodes which is independent of the membrane deflection.

The time evolution of the charge $q(t)$ may be described by the following equation for the equivalent RC-circuit, shown in figure 1:

$$\dot{q} = -\nu_{RC}(u)\left(q + C_G(u)\tilde{V}_G(t)\right) \tag{2}$$

where $\nu_{RC}(u) = 1/RC(u)$ is the charge relaxation frequency and R is the total resistance dominated by the tunnel resistance between the graphene membrane and leads. The set of equations (1) and (2) describes a coupled dynamics of the electronic and mechanical subsystems.

To analyze these equations analytically, let us introduce dimensionless variables for the displacement $x = u/\tilde{d}$, time $\tau = t\nu_{RC}(0)$, mechanical frequency $\tilde{\omega}_m = \omega_m/\nu_{RC}(0)$, external frequency $\tilde{\Omega} = \Omega/\nu_{RC}(0)$ and charge $\mathrm{q} = q/C_G(0)V$. Then, by solving (equation 2) and keeping only linear terms with respect to x, we get the following system of equations:

$$x_{\tau\tau}(\tau) + \frac{\tilde{\omega}_m}{Q}x_\tau + \tilde{\omega}_m^2 x = \varepsilon\frac{C(0)^2}{C(x)^2}\left[\eta\mathrm{q} - (1-\eta)\cos\left(\tilde{\Omega}\tau\right)\right]^2 \tag{3}$$

$$q = -\frac{\cos\left(\tilde{\Omega}\tau - \vartheta\right)}{\sqrt{1+\tilde{\Omega}^2}} - \int_{-\infty}^{\tau} d\tau' e^{\tau-\tau'}\left[\frac{\cos\left(\tilde{\Omega}\tau' - \vartheta\right)}{\sqrt{1+\tilde{\Omega}^2}} + (1-\eta)\cos\left(\tilde{\Omega}\tau'\right)\right]x(\tau') \quad (4)$$

with capacitance ratio $\eta = C_G(0)/C(0)$, and the phase shift of the component independent of the displacement given by $\tan\vartheta = \tilde{\Omega}$. The parameter $\varepsilon = \Omega_V^2/\nu_{RC}^2(0) = C_G V^2/2m\tilde{d}^2\nu_{RC}^2(0)$ characterizes the electromechanical coupling strength. Therein, the coupling frequency Ω_V may be estimated as $\Omega_V \approx V\sqrt{\varepsilon_0/2\varrho\tilde{d}^3}$ where ε_0 is the vacuum permittivity and ϱ is the 2D-mass density of the graphene. For the effective distance $\tilde{d} = 10$ nm, one obtains $\Omega_V \approx 10^9\,[\mathrm{Hz/V}]\,V$. Therefore, for the voltage range $V < 10$ mV, one can find that the frequency Ω_V is less than 1 MHz, which is much smaller than the typical mechanical frequency $\omega_m \approx 100$ MHz. From equation (4) it follows that the instant charge on the membrane exhibits a time-delayed response, with exponential memory decay, to the mechanical displacement of the membrane. To study the stability of the system we consider the linear regime with respect to x for the small parameter $\varepsilon \ll 1$ case. Under such conditions the driving field introduces high frequency components of small order ε to the mechanical vibration and we can write $x(\tau) = \bar{x}(\tau)(1 + \varepsilon\chi(\tau))$, with $\chi(\tau) = \chi(\tau + \pi\tilde{\Omega}^{-1})$ and $\bar{x}$ changing slowly on the time scale $\tilde{\Omega}^{-1}$. Substituting equation (4) into equation (3), we get the equation for the slow component $\bar{x}$:

$$\bar{x}_{\tau\tau}(\tau) + \frac{\tilde{\omega}}{Q}\bar{x}_{\tau}(\tau) + \left(\tilde{\omega}_m^2 + \varepsilon\alpha\right)\bar{x}(\tau) - \varepsilon\alpha\int_{-\infty}^{0} d\tau' e^{\tau'}\cos(\tau')\bar{x}(\tau+\tau') = \frac{\varepsilon\alpha}{2\eta} \quad (5)$$

with coupling coefficient $\alpha = \eta(1 + (1-\eta)^2\tilde{\Omega}^2)/(1+\tilde{\Omega}^2)$. We seek the solution of this equation in the form $\bar{x} = \bar{x}_0 + \delta\bar{x}(\tau)$, where $\bar{x}_0 = \varepsilon\alpha/2\eta\tilde{\omega}^2$ and $\delta\bar{x}(\tau) = \sum_i A_i \exp\lambda_i\tau$ is the general solution of the corresponding homogeneous equation. The dimensionless complex frequencies λ_i are the solutions to the dispersion equation:

$$\lambda^2 + \frac{\tilde{\omega}}{Q}\lambda + \left(\tilde{\omega}_m^2 + \varepsilon\alpha\right) = \frac{\varepsilon\alpha(1+\lambda)}{(1+\lambda)^2 + \tilde{\Omega}^2}. \quad (6)$$

To this end, we restrict the analysis to the regime in which the external frequency is of the order of the relaxation frequency but essentially exceeds the mechanical frequency $\tilde{\Omega} \approx 1 \gg \tilde{\omega}_m$ and the electromechanical coupling is considered to be weak so $\tilde{\omega}_m \gg \varepsilon \sim \tilde{\omega}_m/Q$. Under these assumptions, the solution of equation (6) in the first order approximation on small parameters is:

$$\lambda = \tilde{\gamma} \pm i\sqrt{\tilde{\omega}_m^2 + \epsilon\alpha\frac{\tilde{\Omega}^2}{1+\tilde{\Omega}^2}}$$
$$\tilde{\gamma} = -\frac{1}{2}\left(\frac{\tilde{\omega}_m}{Q} + \epsilon\zeta\right),\quad \zeta = \alpha\frac{1-\tilde{\Omega}^2}{\left(1+\tilde{\Omega}^2\right)^2}. \quad (7)$$

From equation (7) one can see that the electromechanical coupling shifts the mechanical frequency and the effective mechanical damping of the membrane with respect to the intrinsic values. If the effective damping is negative, the static state $u(t) = \tilde{d}x_0$ is unstable and $\bar{x}$ performs oscillations with frequency $\sim\omega_m$ and exponentially growing amplitude. This situation is possible when the normalized damping shift $\zeta < 0$, i.e., when the external frequency exceeds the charge relaxation frequency $\Omega > \nu_{RC}(0)$. Under this condition, the external high frequency electromagnetic field generates an effective pumping of the mechanical oscillations despite the fact that the resonance condition is strongly violated $\Omega \gg \omega_m$. The behavior of the function $\zeta(\tilde{\Omega})$ for different values of η is shown in figure 2. Further, instability occurs when the amplitude of the side-gate voltage is larger than a critical value $V > V_c$, satisfying $\Omega_V^2(V_c) = -\tilde{\omega}_m\nu_{RC}(0)^2/\zeta Q$ so that the pumping overcomes the intrinsic damping.

To estimate the amplitude of the gate voltage oscillations needed to initiate the instability, we set $\eta = C_G/C = 0.5$. From figure 2 one can deduce that under this condition, the minimum value of $\zeta \sim -0.03$. Then, by using the following values for the distance $\tilde{d} \approx 100$ nm, vibrational frequency $\omega_m \approx 10^8$Hz, quality factor $Q \approx 10^4$, charge relaxation frequency $\nu_{RC}(0) \approx 10^9$ Hz, membrane tunneling resistance $R \approx 10^7\ \Omega$, and capacitance of the membrane $C \approx 10^{-16}$ F, we get an estimation for the threshold voltage $V_c = \sqrt{2\omega_m\nu_{RC}(0)\rho\tilde{d}^3(|\zeta|Q\varepsilon_0)^{-1}} \approx 10$ mV. Now, let us analyze the saturation mechanism of the instability. If the system is driven resonantly, the nonlinearity in the mechanical subsystem will saturate the system at a stationary oscillation amplitude. However, since in our case the excitation phenomena is not of resonance nature, the saturation mechanism should be of different origin. One of the most likely candidates is the intrinsic nonlinear damping [21, 22]. This may be taken into account by adding the term $\omega_m Q^{-1}\dot{u}(u-u_{st})^2/a^2$ into equation (1), where a is the characteristic length of damping variations. Such nonlinear damping saturates the amplitude of the mechanical oscillations at

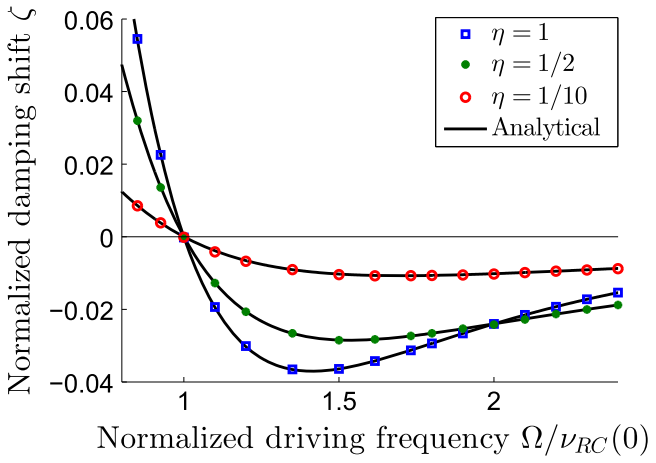

Figure 2. Markers indicate numerical simulations of the normalized damping shift from equations (1) and (2) for different values of η and parameter values $\omega_m/\nu_{RC}(0) = 10^{-2}$ and $u/d \sim 10^{-2}$. The solid black lines are the corresponding analytical solutions ζ from equation (7). When the driving frequency exceeds the inverse retardation time $\nu_{RC}(0)$ the damping shifts becomes negative.

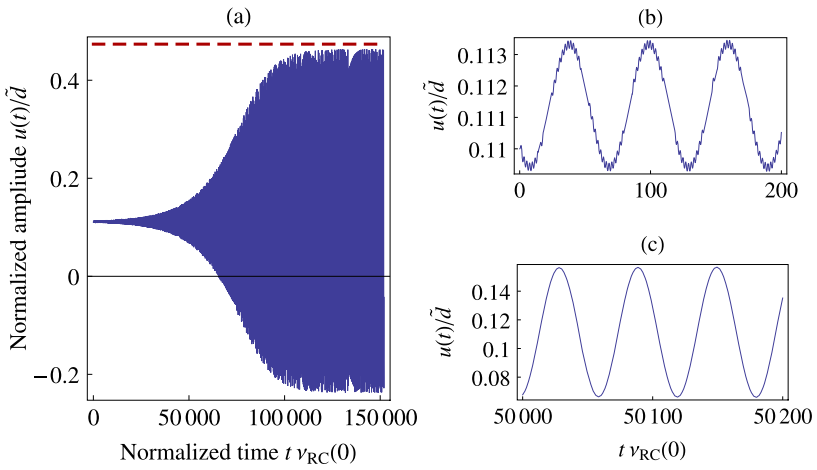

Figure 3. Time evolution of the membrane deflection obtained by direct numerical simulations of equations (1) and (2), including the nonlinear damping in the form discussed in the text. The parameters $\epsilon = 10^{-2}/2$, $\Omega/\nu_{RC}(0) = 3/2$, $\omega_m/\nu_{RC}(0) = 10^{-1}$, $Q = 10^4$, $\eta = 1/2$ and $a = d/20$ were chosen in order to clearly demonstrate (a) the exponential amplitude growth and saturation due to the nonlinear damping and (b) small amplitude modulation of fast oscillations with frequency 2Ω. This modulation is not visible in (c) since the ω_m component has been pumped to a relatively large amplitude. The dashed line corresponds to the saturated value of the amplitude of oscillations predicted by equation (8).

$$A_{st} = 2a\sqrt{\left(\frac{V}{V_c}\right)^2 - 1}\,. \tag{8}$$

To confirm the above mentioned phenomena, we perform direct numerical simulations of equations (1) and (2), shown in figure 3.

Finally, note that the membrane deflections induced by the ac-voltage give rise to a nonlinear correction to the impedance Z_Ω of the equivalent RC-curcuit, see figure 1. In particular, one can show that the instability discussed above manifests itself as a discontinuous jump at $V = V_c$ in the first derivative of the average active ac-power W(V)

$$\frac{\partial W(V)}{\partial V}\bigg|_{V_c^{-0}}^{V_c^{+0}} = I_0\left(V_c\right)\beta\left(\frac{a}{d}\right)^2 \tag{9}$$

where $I_0(V_c) = Re\ Z_c^{-1}V_c$, and $\beta(\eta, \tilde{\Omega})$ is a numerical factor. Modeling C_G as a parallel plate capacitor and considering the case $\tilde{\Omega} = 3/2$, which yields an almost optimal pumping regime, we obtain $\beta \approx 12(1 - 0.7\eta)^2$. Therefore, measurement of the ac-power may provide information about the electromechanical instability, in particular, to estimate the nonlinear damping of the mechanical vibrations.

To conclude, we have found that the mechanical vibrations of a graphene-based oscillator may be actuated by nonresonant, high frequency electromagnetic fields. This is due to the fact that the effective damping of the oscillator is reduced when the frequency of the electromagnetic field exceeds the inverse response time of the charge oscillations in the graphene membrane. If the field strength is strong enough to overcome the intrinsic damping, the mechanical vibrations become unstable and saturate due to the nonlinear damping. The phenomena should be detectable with the available experimental techniques not only for the graphene membranes but also for other electromechanical oscillators due to the robustness of the predicted mechanism.

Acknowledgments

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