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The Use of Mean-Variance for Commodity Futures and Options Hedging Decisions

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This study provides additional evidence of the usefulness of mean-variance procedures in the presence of options which can truncate and skew the returns distribution. Using a simulation analysis, price hedging decisions are examined for hog producers when options are available. Mean-variance results are contrasted with optimal decisions based on negative exponential and Cox–Rubinstein utility functions over 56 ending price scenarios and two levels of risk aversion. The findings from our simulation, which considers discrete contracts, basis risk, lognormality in prices, transactions costs, and alternative utility specifications, affirm the usefulness of the mean-variance framework.

Key words: discrete contracts, hedging, mean-variance, options, utility specifications.

Introduction

Optimal hedging in the presence of commodity options was first considered by Wolf in a linear mean-variance framework. For traditional portfolio choices, mean-variance may approximate utility maximizing choices very well (Kroll, Levy, and Markowitz). However, the availability of options raises theoretical concern about the usefulness of the mean-variance approach for hedging decisions (Lapan, Moschini, and Hanson). The inclusion of commodity options can lead to a truncated or skewed distribution of returns. In addition, the use of options means that the random variables in the choice set are nonlinearly related to the strike price which violates the location-scale condition for consistency between mean-variance and expected utility (Lapan, Moschini, and Hanson).[1] Only limited information exists on the usefulness of the mean-variance framework in the presence of options. Hanson and Ladd show that when these conditions are violated, the mean-variance approach may still provide a good approximation within a framework of constant absolute risk aversion, normally distributed output price, no transaction costs, and no basis uncertainty.

The purpose of this article is to further explore the implications of the presence of options on the selection of marketing strategies. Using a simulation analysis, price hedging decisions are examined for hog producers when options are available. It is assumed that the producer maximizes expected utility in a two-period model based on expectations about the ending distribution of cash and futures prices. Mean-variance (MV) results are contrasted with optimal decisions based on negative exponential and Cox–Rubinstein utility functions over 56 ending price scenarios and two levels of risk aversion. Because options can result in skewed outcome distributions (Cox and Rubinstein, p. 318), we also examine a third-order approximation to the negative exponential utility function (MV3), which is nearly as straightforward to apply as the MV, and empirically allows skewness to influence the selection of the optimal strategy.

The analysis differs from previous research in other important dimensions. The discrete nature of futures and options contracts is recognized by permitting the producer to choose

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only their integer multiples. A much larger set of possible (simultaneous) futures and options positions than in previous studies is considered, and basis risk and transaction costs are incorporated into the hedging model. In addition, daily price relatives are specified to follow a lognormal distribution which is consistent with traditional option pricing models but which may lead to violation of the location-scale condition for consistency between MV and expected utility (Meyer).

The results affirm the usefulness of the mean-variance framework in the presence of options, particularly at or around market expectations. Overall, the MV framework identifies optimal strategies in a high percentage of the cases examined. When errors occur, often the magnitudes of the losses from using the strategies identifed by the MV framework measured in certainty equivalents are small.

Conceptual Framework

Theoretical Model

A two-period model is used to simulate a hog producer's choice of pricing strategies. In period 1, given a quantity of the cash commodity which in period 2 will equal the size of a futures contract, the producer formulates an expectation of the bivariate distribution of cash and futures prices. Expected utility is maximized by buying or selling puts, calls, and futures contracts. These contracts are offset at the time the cash commodity is sold.

Income (R) is represented as the sum of cash sales and the profits made in the futures and options markets. Formally, R is

$$\begin{aligned} R = Qy &+ \sum_j [p_j^2 - rp_j^1]NP_j + \sum_i [c_i^2 - rc_i^1]NC_i + [f^2 - f^1]NF \\ &- (to_j^2 + to_j^1)\text{abs}(NP_j) - (to_i^2 + to_i^1)\text{abs}(NC_i) - (tf)\text{abs}(NF), \end{aligned} \tag{1}$$

where R = income; Q = quantity of cash commodity to be sold in period 2; y = price per unit of cash commodity in period 2; r = risk-free rate of return + unity (r adjusts period 1 premium values to period 2 terms); p_j^t = price of put option at jth strike price in period t, $t = 1, 2$; c_i^t = price of call option at ith strike price in period t, $t = 1, 2$; f^t = price of futures contract in period t, $t = 1, 2$; NF, NP_j, and NC_i are integers representing contracts in futures, puts at the jth strike price, and calls at the ith strike price (positive values indicate long positions in period 1 and negative values indicate short positions); "abs" indicates the absolute value of the integer contracts; to_j^t is the transaction cost for put options at the jth strike price in period t; to_i^t is the transaction cost for call options at the ith strike price in period t; and tf is the transaction cost for the futures contracts.

In this framework, the producer's problem is:

$$\begin{aligned} &\max_{NF,NP_j,NC_i} \; EU(R) \\ &\text{s.t. } NF,\ NP_j, \text{ and } NC_i \text{ are integers}^2 \end{aligned} \tag{2}$$

or

$$\begin{aligned} &\max_{NF,NP_j,NC_i} \; \int U(R)G'(R)\, dR \\ &\text{s.t. } NF,\ NP_j, \text{ and } NC_i \text{ are integers,} \end{aligned} \tag{3}$$

where $U(R)$ is the producer's utility function and $G'(R)$ represents the producer's expectation of the probability density function of R.

Utility Considerations

Two utility functions are used to characterize the producer's preferences—the negative exponential and the Cox–Rubinstein. The negative exponential utility function can be expressed as $EU(R) = -\exp(-qR)$, where q is the Arrow–Pratt (AP) coefficient of absolute risk aversion, $-U''/U'$. As is well known, the negative exponential specification imposes constant absolute risk aversion (CARA) ($AP' = dAP/dR = 0$), which implies that changes in the level of wealth do not influence investment decisions. In addition, we examine the Cox–Rubinstein utility function which has been used in the analysis of options (Cox and Rubinstein, p. 318). The Cox–Rubinstein utility function is expressed as $EU(R) = (1/(1-d))R^{1-d}$, where d is the level of constant relative risk aversion. In this formulation, $AP = d/R$ and $AP' = -d/R^2 < 0$, which implies a decreasing absolute risk aversion utility function (DARA) for $d > 0$.

Decision Analytics

Two decision approaches, MV and the MV3, are used to identify their usefulness for risky pricing situations in the presence of options. The MV approach has been widely used in economic and financial analysis. The form of the mean-variance specification is

$$EU(R) = m - (q/2)v, \tag{4}$$

where m is the mean of the outcome distribution, v is the variance of the outcome distribution, and q is the level of constant absolute risk aversion (Robison and Barry). The MV model is consistent with expected utility when utility is quadratic, outcomes are normally distributed, and/or choices involve a single random variable or linear combinations of the random variable (Meyer; Robison and Barry). In the presence of options, it is likely that these conditions are violated. Options can skew and truncate the returns distribution. Also, unlike futures contracts, options contracts are exercised depending on whether the futures price is greater than or less than the strike price of the option. Thus, the random variables in the choice set depend not only on the other random variables, but also are nonlinearly dependent on the strike prices of the options (Lapan, Moschini, and Hanson). These shortcomings make the use of the MV dependent on the ability to approximate results obtained from more general utility specifications.

Option positions may cause highly skewed return distributions. Cox and Rubinstein suggest that an evaluation of option positions would be seriously incomplete if it focused only on mean and variance and neglected an assessment of skewness. Therefore, a third-order Taylor series expansion to the negative exponential utility function is specified. The MV3 specification explicitly considers the skewness, mean, and variance of the producer's choice set, or the distribution of returns. The specification is written as

$$EU(R) = -\exp(-qm) - (q^2/2)\exp(-qm)v + (q^3/6)\exp(-qm)v^{1.5}s, \tag{5}$$

where R = income, m = mean, v = variance, s = the third moment of R about its mean, and q is defined above. In equation (5), positive skewness is associated with higher expected utility (Cox and Rubinstein, pp. 318–19). While not necessarily consistent with expected utility, the use of the MV3 should permit a closer assessment of the importance of skewness in the decision framework, particularly within the context of the negative exponential specification.

Empirical Considerations

Producer Model

Following Wolf, and Hanson and Ladd, only price risk on a fixed quantity is considered. Given the confinement technology used in hog production, quantity risk is assumed to be minimal. The hog producer is assumed to farrow an amount of pigs in period 1 whose sale weight six months later (in period 2) will equal the size of a futures contract. Selecting the size of operation equal to the size of a futures contract highlights the use of options (through mitigating the fixed futures contract size) and their effects on the returns distribution. This cash position creates a situation with a relatively large number of opportunities for the substitution of options for futures and increases the likelihood that the returns distribution will not be consistent with a mean-variance framework. For example, hedging a portion of the contract size requires the use of options. With larger production (e.g., three contract sizes), hedging a portion of the output can be accomplished with discrete futures contracts (i.e., one contract would permit hedging of one-third of the output), reducing the importance of options and their likely effects on the returns distribution. The relationship between size and the usefulness of options was identified by Hauser and Andersen.[3]

Six months is the approximate lag between farrowing pigs and selling them for consumption. It is assumed that no trades take place between period 1 and period 2. Options and futures contracts are offset at the time the cash commodity is sold, and no time value remains in the option premium.

The commission costs of using futures and options contracts are considered in evaluating marketing alternatives. Here, the commission cost for futures is \$80/contract per round turn, or \$.27/cwt. For options, it is 5% of the premium on each purchase or sale (e.g., an option with a premium of \$2.76/cwt would cost \$.14/cwt if the option were allowed to expire, and \$.28/cwt if it were offset with another purchase or sale in the options market). The commission costs assumed are those that are commonly charged by a full-service broker to a producer who trades only one or a few contracts at a time. Since average commission costs typically decrease as the number of contracts traded increases, these costs may be higher than many producers would be required to pay. Also, because full-service quotes were used, discounts may be available. Thus, the commission costs assumed here may influence the results slightly in the direction of a cash-only marketing strategy.[4]

Given these assumptions, producer income, R, can be rewritten as

$$\begin{aligned} R = {} & Qy + \sum_j [\mathrm{Max}(xp_j - f^2, 0) - rp_j^1]NP_j + \sum_i [\mathrm{Max}(f^2 - xc_i, 0) - rc_i^1]NC_i \\ & + (f^2 - f^1)NF - (to_j^2 + to_j^1)\mathrm{abs}(NP_j) - (to_i^2 + to_i^1)\mathrm{abs}(NC_i) - (tf)\mathrm{abs}(NF), \end{aligned} \tag{6}$$

where xp_j = jth strike price for put options, xc_i = ith strike price for call options, and to_j^2 and to_i^2 are zero if the respective option is not exercised.

With an initial cash position, Q, the producer generates income by simultaneously choosing positions in futures and options. To make the simulation manageable, several assumptions are made about the producer's choice set. Three strike prices for puts and three for calls are considered: one at the money, one \$2 in the money, and one \$2 out of the money. Also, the producer is permitted to buy or sell only one futures contract, as well as one put and one call at each strike price.[5] The number of strategies involving integer multiples of contracts is given by 3^{i+j+1}, where 3 is the number of instruments traded (i.e., futures, put, and call options), i is the number of call strikes, and j is the number of put strikes, and with a futures contract adding an additional combination. This means that 2,187 marketing strategies (3^7) are permitted under the expectations of each ending price distribution.

Under these assumptions, expected utility can be written

$$EU(R) = \int_{LF^2}^{UF^2} \int_{LY}^{UY} U(R)L'(y, f^2)\, dy\, df^2, \tag{7}$$

where $L'(y, f^2)$ is the producer's expectation of the joint distribution of cash price and futures price, LF^2 and UF^2 are the lower and upper bounds of integration for the futures price, and LY and UY are the lower and upper bounds for the cash price.

Structure of the Simulations

We simulate a producer's choice 448 times. Specifically, two levels of risk aversion, 56 sets of producer price expectations of mean and volatility, and the two utility specifications (negative exponential and the Cox–Rubinstein) and their approximations (MV and MV3) are examined ($2 \cdot 56 \cdot 4 = 448$).[6]

Two levels of risk aversion are specified—risk averse and slightly risk averse. To compare the results from the different utility specifications, the risk parameters (q and d) should reflect similar levels of risk aversion. By equating the Arrow–Pratt measures of absolute risk aversion from each utility specification, the relationship $d = R \cdot q$ is derived. Using values of q specified from the ranges suggested by Holt and Brandt for hog producers, and setting R at the period 1 futures price of \$44/cwt, values of d are calculated. The risk parameters (q, d) were specified for the risk averse producer ($q = .030$, $d = 1.32$), and for the slightly risk averse producer ($q = .010$, $d = .44$).

The 56 sets of price expectations are built around a base scenario of mean and volatility which reflect prices and their variation for the 1980–88 period.[7] In the base scenario, the current (period 1) futures price for the contract expiring six months later (period 2) is \$44/cwt and is used as the producer's expectation of the mean of the price distribution. The producer's expectation of the annualized percentage standard deviation of log-price return in period 2 is 23, which reflects the annualized average six-month volatility of the futures contract. In other scenarios, the producer's expectation for the mean varies incrementally from \$40/cwt to \$48/cwt, a range of 9% in either direction from the market's expectation. Consistent with the variability of annualized volatilities found over this time period, the producer's expectation of volatility primarily varies from 16 to 30, a range of 30% in either direction from the market's expectation. This range was further expanded to examine the case where a producer's expectation of volatility is considerably lower than the market's, which is consistent with findings indicating that a farmer's elicited annualized volatilities may be markedly below the market's (Eales et al.).[8]

Cash and futures prices are specified to follow a bivariate lognormal distribution. This formulation is based on previous research (Hauser, Andersen, and Offutt) and the results of statistical testing performed here which could not reject lognormality of daily price relatives. The expected mean of the period 2 cash price is assumed to equal the expected mean of the period 2 futures price, with basis risk entering the model through the correlation coefficient of the bivariate distribution. The correlation coefficient between cash and futures is set at .95, which reflects the correlation of futures and cash prices in various cash markets (e.g., Omaha) on the last option trading day for each September and March futures contract over the 1980–88 period. The option premiums in period 1 are calculated from Black's model using a volatility of 23 and an underlying futures price of \$44/cwt, which should provide representative premiums for the analysis considered here (Hauser and Neff).

Solution Procedures and Calculation of Certainty Equivalence

A mean-variance specification of expected utility at times may provide analytical solutions for the hedging model (Wolf). However, in the presence of options, other utility specifications, in general, do not; numerical search procedures must be used to solve for optimal values (e.g., Hanson and Ladd). Solution procedures often specify the contracts purchased or sold in fractions, and do not reflect the restrictions implied by futures and options contracts of fixed sizes. This is especially important in analyzing models containing both futures and options, because options, through the selection of strike prices, can mitigate the effects of fixed futures contract sizes (Hauser and Andersen).

Here, numerical procedures are used to search for solutions of integer positions in the futures and option markets. For the negative exponential and Cox–Rubinstein utility functions, under each level of risk aversion and price scenario, the maximization procedure evaluates each possible combination of puts, calls, and futures contracts by numerically integrating the utility of return R (6) achieved over a joint probability distribution of cash and futures prices as indicated in (7). In the base scenario, the producer agrees with the market's expectations of mean and volatility of the ending price distribution. The parameters of the density function $L'(y, f^2)$ are changed in other scenarios in order to analyze the choice of marketing strategies as the producer's expectations differ from those of the market.

For MV and MV3 approximations, the expected utility for each combination of contracts is solved by first numerically integrating expressions (8) through (10), below, over the price distribution in period 2 for each level of risk aversion, so that the mean (m), variance (v), and skewness (s) for each combination of contracts can be determined:

$$m = E(R) = \int\int Rf(y, f^2)\, dy\, df^2, \tag{8}$$

$$v = E(R - m)^2 = \int\int (R - m)^2 f(y, f^2)\, dy\, df^2, \tag{9}$$

$$s = E(R - m)^3 = \int\int (R - m)^3 f(y, f^2)\, dy\, df^2. \tag{10}$$

Then, using the MV and MV3 specifications (4) and (5), respectively, the expected utility of each combination of contracts is calculated.[9] For a given set of risk preferences and set of price expectations, the marketing alternative with the highest expected utility is identified as the "best" strategy.

Certainty equivalence (CE) can be used to measure in monetary terms the differences in expected utility from alternative marketing strategies. CE is the difference between the expected value and the risk premium (Robison and Barry), and provides monetary values of alternative strategies discounted for risk. For a particular risky strategy, the certainty equivalent is the risk-free return necessary to achieve the same level of expected utility as that obtained from using the strategy. In the context of a negative exponential utility function, for a given strategy, the expected utility = $\boldsymbol{U} = E[-\exp(-q\boldsymbol{R})]$, where $\boldsymbol{R}$ is a random variable depending on prices and the market positions associated with the strategy. The certainty equivalent must provide the same level of expected utility, $\boldsymbol{U}$. As a result, $\boldsymbol{U} = E[-\exp(-qCE)]$, which is equal to $[-\exp(-qCE)]$ since CE is a particular value rather than a random variable. Solving for CE, $CE = -[\ln(-\boldsymbol{U})]/q$. Thus, CE gives a monetary value for the risk-free return that provides the same expected utility as the risky market strategy. For the Cox–Rubinstein utility function, CE can be found in a similar manner and is expressed as $CE = [(1 - d)\boldsymbol{U}]^{1/(1-d)}$.

Certainty equivalence and the difference in CE are used to calculate the loss from using the MV and MV3 procedures to approximate the underlying utility specifications. Consider a comparison between the negative exponential and the MV for a particular ending price scenario. First, the "best" strategy is selected using both the negative exponential specification and the MV. Then, under the negative exponential specification, the CE is calculated for the "best" strategy chosen with each procedure. The difference between the two CE calculations is defined as the loss in CE from choosing the strategy identified by the MV framework.

Simulation Results

The use of two utility functions was specified to identify the robustness of the MV approach to a DARA as well as a CARA specification. However, the selected marketing strategies resulting from the negative exponential and the Cox–Rubinstein functions were identical. Over the range of returns, levels of initial wealth considered, and the degree of risk aversion, the shape of the negative exponential and the Cox–Rubinstein functions were very similar, differing appreciably only at very low levels of wealth (fig. 1). In the appendix, we show that as the level of wealth increases, the functions more closely approximate each other. Simulations run in the most extreme case, assuming zero initial wealth, did not produce any difference in the findings. The similarity in the results also is likely attributable to the integer constraints, which do not permit fractions of futures, puts, and call contracts to be utilized. Hence, in this simulation, the MV and the MV3 approximate both the DARA and the CARA specifications to the same degree. Below, because of its use and

Table 1. Best Strategies and *CE* Loss in $/cwt when Approximations Are Used Rather than Negative Exponential: Risk Averse Producer

Price Expectations			Neg. Exponential		3rd-Order Approximation (MV3)		Mean-Variance (MV)	
Scenario	Volatility	Mean	Best	*CE*	Best	Loss	Best	Loss
11	9	40	460	53.53	460	.00	460	.00
12	9	42	28	51.30	28	.00	28	.00
13	9	43	1	51.76	1	.00	1	.00
14	9	44	730	52.52	730	.00	730	.00
15	9	45	1,459	53.44	1,459	.00	1,459	.00
16	9	46	1,468	54.81	1,468	.00	1,468	.00
17	9	48	1,483	59.76	1,483	.00	1,483	.00
21	16	40	460	51.21	460	.00	460	.00
22	16	42	109	47.35	109	.00	109	.00
23	16	43	28	46.39	28	.00	28	.00
24	16	44	757	46.24	757	.00	757	.00
25	16	45	739	47.07	739	.00	739	.00
26	16	46	1,468	48.85	1,468	.00	1,468	.00
27	16	48	1,472	54.21	1,472	.00	1,472	.00
31	19.5	40	460	49.48	460	.00	460	.00
32	19.5	42	352	45.29	352	.00	352	.00
33	19.5	43	1,081	44.29	1,081	.00	1,081	.00
34	19.5	44	1,090	44.17	1,090	.00	847	.03
35	19.5	45	850	44.82	850	.00	850	.00
36	19.5	46	770	46.12	770	.00	742	.21
37	19.5	48	1,472	50.83	1,472	.00	1,471	.21
41	21.25	40	676	48.48	676	.00	460	.03
42	21.25	42	361	44.60	361	.00	361	.00
43	21.25	43	364	43.91	364	.00	364	.00
44	21.25	44	1,093	43.75	1,093	.00	1,093	.00
45	21.25	45	1,094	44.33	1,094	.00	850	.06
46	21.25	46	851	45.50	851	.00	851	.00
47	21.25	48	1,499	49.38	1,499	.00	1,471	.62
51	23	40	712	47.70	712	.00	676	.25
52	23	42	607	44.31	607	.00	607	.00
53	23	43	365	43.66	365	.00	365	.00
54	23	44	365	43.65	365	.00	365	.00
55	23	45	1,094	44.21	1,094	.00	1,094	.00
56	23	46	1,094	45.18	1,094	.00	1,094	.00
57	23	48	1,580	48.42	1,580	.00	743	.69
61	24.75	40	713	47.35	715	.00	679	.29
62	24.75	42	689	44.36	689	.00	446	.13
63	24.75	43	608	43.80	608	.00	608	.00
64	24.75	44	365	43.64	365	.00	365	.00
65	24.75	45	1,337	44.12	1,337	.00	1,337	.00
66	24.75	46	1,094	45.05	1,094	.00	1,094	.00
67	24.75	48	1,823	47.87	1,836	2.43	1,823	.00
71	26.5	40	715	47.25	716	.00	680	.33
72	26.5	42	689	44.63	689	.00	689	.00
73	26.5	43	608	43.95	608	.00	608	.00
74	26.5	44	1,418	43.85	1,418	.00	1,418	.00
75	26.5	45	1,337	44.24	1,337	.00	1,337	.00
76	26.5	46	1,094	44.92	1,094	.00	1,094	.00
77	26.5	48	1,097	47.42	2,097	2.38	1,095	.00
81	30	40	716	47.68	716	.00	716	.00
82	30	42	717	45.17	1,445	.00	1,445	.00
83	30	43	1,445	44.76	1,445	.00	1,445	.00
84	30	44	1,418	44.53	1,418	.00	1,418	.00
85	30	45	1,419	44.77	1,419	.00	1,418	.03
86	30	46	1,419	45.35	2,187	3.06	1,337	.27
87	30	48	1,341	47.28	2,160	2.29	1,094	.77

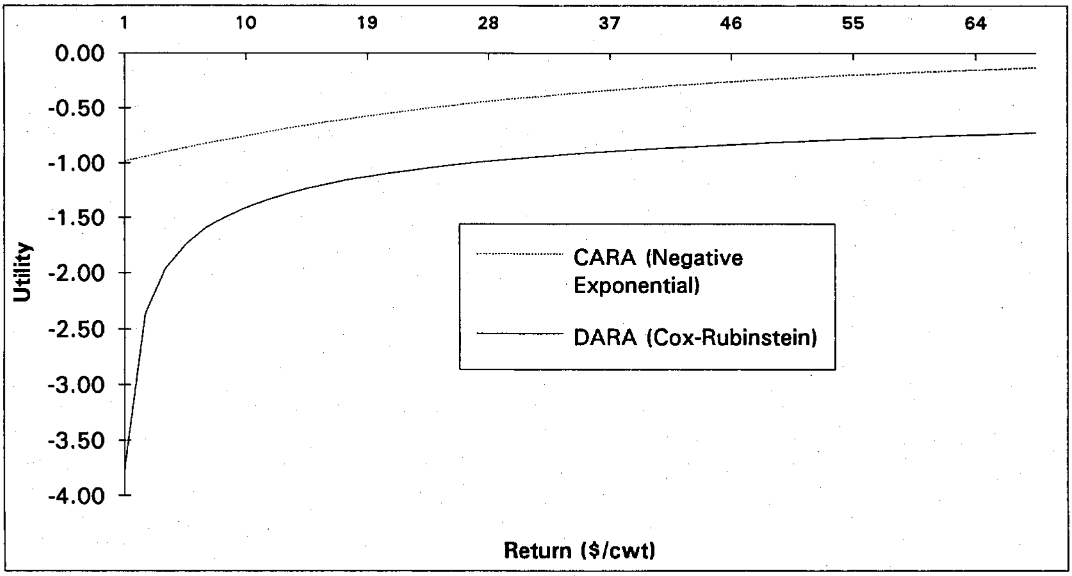

Figure 1. Negative exponential (CARA) and Cox-Rubinstein (DARA) utility specifications

familiarity, we focus on differences between the negative exponential utility function and the MV and MV3 procedures. The only difference between these results and the DARA specification is that the *CE* loss from differences in the optimal strategies is modestly larger with the DARA specification.

Summary results of the simulations are provided in tables 1 and 2. Each row of a table represents an alternative scenario of the ending price distribution. For each price scenario, the strategy selected as "best" under the utility function and approximating approaches is identified.[10] The *CE* is presented for the optimal strategy under the negative exponential utility specification. The losses (differences) in *CE* from choosing a strategy using the MV3 and MV approximations when the negative exponential is assumed to be correct also are identified.[11] For example, for the risk averse producer under scenario 57 (i.e., expectations of the volatility and mean equal to 23 and 48, respectively), the best strategy is #1,580; with the MV3 approximation, it is also #1,580; and with the MV, it is #743. Calculated using the negative exponential specification, the *CE* of strategy #1,580 is $48.42/cwt. Because identical strategies are selected, the use of the MV3 approximation results in no loss. The loss in *CE* from using the MV approximation is $.69/cwt. The results for a slightly risk averse producer are interpreted similarly, with the *CE* and the loss calculated under the slightly risk averse negative exponential specification.

In general, the results suggest that the approximating procedures work rather well, particularly at or around market expectations and at low levels of volatility. Under market expectations (volume = 23, mean = 44), the same strategies are selected by all three specifications and do not involve options. For the risk averse producer, this strategy is #365, a traditional hedge using a short futures position. For the slightly risk averse producer, the strategy selected is #1,094, a long cash-only position. Under market expectations, no loss in *CE* exists by selecting a marketing strategy that considers only mean and variance. The absence of options in the strategy mix is consistent with Lapan, Moschini,

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Notes: The utility specifications are defined in the text. "Volatility" and "Mean" reflect expected annualized volatility and mean in $/cwt of the second period price distribution. See text for a discussion of the selection of "best" strategy and differences in certainty equivalence (*CE*).

Table 2. Best Strategies and *CE* Loss in $/cwt when Approximations Are Used Rather than Negative Exponential: Slightly Risk Averse Producer

Price Expectations			Neg. Exponential		3rd-Order Approximation (MV3)		Mean-Variance (MV)	
Scenario	Volatility	Mean	Best	*CE*	Best	Loss	Best	Loss
11	9	40	703	53.96	703	.00	703	.00
12	9	42	28	51.43	28	.00	28	.00
13	9	43	1	51.98	1	.00	1	.00
14	9	44	730	52.52	730	.00	730	.00
15	9	45	1,459	53.77	1,459	.00	1,459	.00
16	9	46	1,468	55.16	1,468	.00	1,468	.00
17	9	48	1,484	61.25	1,484	.00	1,484	.00
21	16	40	703	53.16	703	.00	703	.00
22	16	42	352	48.07	352	.00	352	.00
23	16	43	28	47.16	28	.00	28	.00
24	16	44	730	47.18	730	.00	730	.00
25	16	45	1,468	48.40	1,468	.00	1,468	.00
26	16	46	1,471	50.87	1,471	.00	1,471	.00
27	16	48	1,485	58.62	1,485	.00	1,485	.00
31	19.5	40	703	52.57	703	.00	703	.00
32	19.5	42	379	46.75	379	.00	379	.00
33	19.5	43	352	45.14	352	.00	352	.00
34	19.5	44	847	44.53	847	.00	847	.00
35	19.5	45	742	45.85	742	.00	742	.00
36	19.5	46	1,472	48.77	1,472	.00	1,472	.00
37	19.5	48	1,485	56.88	1,485	.00	1,485	.00
41	21.25	40	703	52.24	703	.00	703	.00
42	21.25	42	676	46.17	676	.00	676	.00
43	21.25	43	361	44.34	361	.00	361	.00
44	21.25	44	1,093	43.95	1,093	.00	1,093	.00
45	21.25	45	770	45.07	770	.00	770	.00
46	21.25	46	1,472	47.71	1,472	.00	1,472	.00
47	21.25	48	1,485	55.90	1,485	.00	1,484	.31
51	23	40	703	51.87	703	.00	703	.00
52	23	42	703	45.68	676	.01	676	.01
53	23	43	607	43.92	607	.00	607	.00
54	23	44	1,094	43.74	1,094	.00	1,094	.00
55	23	45	1,094	44.73	1,094	.00	1,094	.00
56	23	46	1,581	46.83	1,499	.01	1,499	.01
57	23	48	1,485	54.85	1,485	.00	1,484	.33
61	24.75	40	703	51.46	703	.00	703	.00
62	24.75	42	713	45.63	713	.00	713	.00
63	24.75	43	689	44.13	689	.00	689	.00
64	24.75	44	1,337	43.81	1,337	.00	1,337	.00
65	24.75	45	1,095	44.73	1,095	.00	1,094	.05
66	24.75	46	1,824	46.66	1,824	.00	1,824	.00
67	24.75	48	1,485	53.73	1,485	.00	1,484	.34
71	26.5	40	703	51.02	703	.00	703	.00
72	26.5	42	716	45.96	716	.00	716	.00
73	26.5	43	716	44.52	689	.04	689	.04
74	26.5	44	1,418	44.13	1,418	.00	1,418	.00
75	26.5	45	1,338	44.88	1,338	.00	1,338	.00
76	26.5	46	1,107	46.67	1,098	.04	1,098	.04
77	26.5	48	1,593	53.04	1,836	.01	1,754	.52
81	30	40	707	50.49	707	.00	707	.00
82	30	42	716	46.90	717	.02	717	.02
83	30	43	720	45.88	720	.00	720	.00
84	30	44	1,449	45.53	1,449	.00	1,446	.04
85	30	45	1,458	46.17	1,458	.00	1,449	.04
86	30	46	1,431	47.61	1,431	.00	1,422	.34
87	30	48	2,079	52.76	2,079	.00	1,107	.70

Refer to notes to table 1.

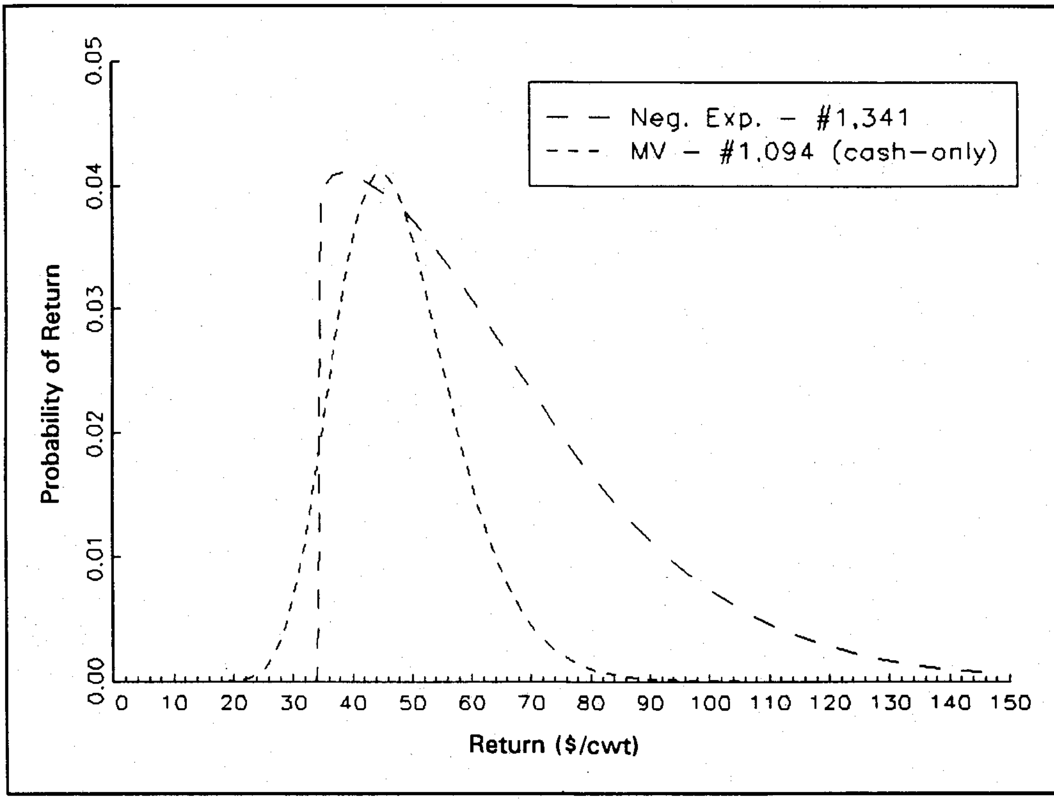
and Hanson who demonstrate that options are not used as hedging instruments when futures and options prices are perceived to be unbiased.

The use of the MV3 leads to the same optimal strategy as the negative exponential specification in 88% of the cases. For the slightly risk averse producer, when the expected volatility is below the market's expectation (i.e., scenarios 11–47), the strategies are identical. For higher volatilities, a small number of strategy differences occur, but their appearance seems to be random and their *CE* losses never exceed $.04/cwt. For the risk averse producer, differences in strategy selection do not occur when the expected volatility is equal to or less than the market's expectations (i.e., scenarios 11–57). However, above this point, several large differences appear at combinations of high expected mean and volatility (scenarios 67, 77, 86, and 87).

The similarity of the optimal strategies under most price scenarios is not surprising since the MV3 is a third-order approximation to the same underlying utility function, which permits skewness to enter into the approximating function. However, the large differences in *CE* at high expected mean and volatility are unexpected, but are attributable to the increased skewness of the returns distribution at high means and levels of volatility, the higher level of risk aversion, and the nature of the third-order approximation.[12] Higher levels of mean and volatility allow the producer the opportunity to choose futures and options positions to achieve higher levels of positive skewness. Unfortunately, the MV3 approximation breaks down at high levels of variance and skewness in the returns distribution, and high levels of risk aversion. The third term in expression (5) becomes large as skewness and variance increase, causing the approximation to become an increasing function of variance. Examining the first and second partials of (5) with respect to variance (v) indicates that, for skewness (s) greater than zero, utility is a decreasing and then increasing function of variance. Expected utility is at a minimum where $v = (2/qs)^2$, meaning that as skewness (s) and risk aversion (q) increase, utility begins to increase with increasing variance at smaller levels of variance. The use of this approximation in these circumstances results in the selection of strategies associated with increasing variance, generates large differences in *CE*, and is inconsistent with utility maximization for the risk averse producer.

The use of the MV procedure identifies the same optimal strategy as the negative exponential specification in 73% of the cases. The pattern of *CE* losses is much less pronounced than under the MV3 approximation. Also, it seems to be slightly more symmetric, occurring more often below the market's mean expectation than under the MV3 approximation. In figure 2, a representation is provided of the return density functions for strategies chosen under the negative exponential and the MV for the risk averse producer under price scenario 87. In this most extreme case, where *CE* differs by $.77/cwt, the return density functions differ substantially. The return density function for the MV cash-only position reflects the lognormal distribution of prices because no options positions are included. In contrast, under the negative exponential, the effect of multiple-options positions is demonstrated by the truncated and positively skewed shape of the return density function. Evaluated under the same ending distribution of prices, the difference in the density functions highlights the effects of options on the return distribution and the difficulty of the MV procedure in capturing the full range of risk preferences in this extreme case. Nevertheless, within $1 of the market's mean expectation, the mean-variance procedure correctly identifies the optimal strategy in 85% of the cases, with the largest *CE* differences being only $.05/cwt. Within $2 of the market's mean expectation, the MV procedure correctly identifies the optimal strategy in 80% of the cases, with the largest difference being $.34/cwt. The MV approximation also yields considerably smaller *CE* differences than MV3 in those cases where the MV3 differences are relatively large.

Finally, a slight asymmetry exists in the pattern of losses for both approximating functions. When the expected volatility is well below market expectations (scenarios 11–27), optimal strategies under the negative exponential function and the two approximating procedures are identical. Above this point, differences in the strategies selected and *CE* losses occur. The exact reason for this occurrence is difficult to identify, but may be related



Negative Exponential
Strategy #1,341; p42, c44, c46
Expected Value = 52.97
Variance = 557.99
Skewness = 1.58
Kurtosis = 5.56

Mean-Variance (MV)
Strategy #1,094; cash-only
Expected Value = 48.00
Variance = 106.07
Skewness = .64
Kurtosis = 3.68

Figure 2. Return density functions for optimal strategies chosen under negative exponential and MV utility specifications with expected mean = 48 and expected volatility = 30

to the low variance which implies reduced uncertainty about the returns from alternative strategies. At extremely low levels of volatility, the mean of the return distribution takes on an increased importance. Selection of the optimal strategy is based more on the expected utility from higher mean returns. This can be seen clearly in the context of equations (4) and (5). To provide insight into this proposition, additional experiments were performed for the MV under price scenarios 11–27 assuming risk neutrality ($q = 0$), which effectively eliminates the importance of uncertainty in the decision process. For both risk averse and slightly risk averse producers, the optimal strategies and *CE* were almost identical to those identified by the negative exponential and the MV in tables 1 and 2. Hence, at very low levels of volatility and reduced uncertainty about the return distribution from alternative strategies, all three specifications identify basically the same strategies which provide the highest mean returns.

Summary and Conclusions

The availability of options on agricultural futures has raised some concern about the usefulness of the mean-variance framework in risk-management analyses since options can truncate and highly skew the return distributions of marketing strategies. Here, a two-period simulation model of a hog producer's hedging decisions was used to investigate differences in optimal strategies and in ex ante utility under alternative utility specifications in the presence of options and futures contracts. The results from two approximating procedures, mean-variance and a third-order Taylor series expansion to a negative exponential function, were contrasted with those generated by the negative exponential utility function (a CARA specification) and the Cox–Rubinstein utility function (a DARA specification). The third-order approximation permits the skewness of the return distribution often imparted by option positions to influence the selection of the optimal strategy.

Over the simulation values considered here, the marketing strategies selected under the CARA and the DARA specifications were identical. Limited differences in the shape of the utility functions existed except at very low levels of wealth. The similarity in the results also may be attributable to the integer marketing constraints which do not permit fractions of futures, put, or call contracts to be used.

The findings suggest that the third-order approximation and the mean-variance framework provide rather good approximations, particularly at or around market expectations. Overall, both approximating procedures accurately identify optimal strategies in a high percentage of cases. When errors occur, often the magnitude of the *CE* differences is small on a $/cwt basis, except for the third-order approximation, which is particularly sensitive to positive skewness at higher levels of risk aversion and volatility. While the third-order approximation is the most accurate at identifying the optimal strategies, the performance of the mean-variance formulation also is attractive, particularly in light of its ease of understanding and use, and because it is less susceptible to the large errors encountered in the MV3 formulation.

In addition, the results suggest that when producers have low volatility expectations relative to the market, the MV3 and MV formulations also work well. For the risk averse and slightly risk averse producers, appropriate strategies are identified in most cases when volatility expectations were below the market's. This indicates that if a producer's volatility expectations are low relative to the market, as some research has suggested, then the use of these approximations may identify utility maximizing strategies in a consistent manner.

In general, the results regarding the usefulness of the mean-variance framework in the presence of options are consistent with those of Hanson and Ladd who examined this question in a more simplified framework which assumed continuous (non-discrete) positions in only futures and a put option with a single strike price, normally distributed output price, no transactions costs, and no basis uncertainty. The findings from our simulation, which considers discrete contracts, basis risk, lognormality in prices, transactions costs, and alternative utility specifications, do not change the general conclusion that the mean-variance criterion is a good evaluation tool.

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Notes

[1] The use of the expected utility framework to analyze decision making in a risky environment has been criticized (Machina). Similarly, it is possible to generate examples where a mean-variance analysis leads to results which are inconsistent with expected utility theory. Nevertheless, the use of expected utility and mean-variance in theoretical and applied decision making suggests the importance of our analysis.

[2] To make the empirical analysis manageable, the number of strike prices for puts and calls and the number of contracts and options are limited. This is discussed below in the empirical specification section.

[3] Our findings suggest that with larger production units, and less likelihood of the substitution of options for futures, MV also may work well.

[4] Margin requirements are not explicitly considered here since they can be satisfied by pledging U.S. Treasury Bills. Also, margin calls are not explicitly considered in the structure of the model.

[5] Examination of situations where multiple futures contracts or multiple options contracts at the same strike price were most likely to occur (i.e., where producer expectations of price mean and volatility differ most from the market) indicated that these one-contract restrictions were not binding.

[6] Examination of tables 1 and 2 may facilitate an understanding of the structure of the simulations. These tables provide results for the negative exponential, MV, and the MV3 specifications for the risk averse and slightly risk averse producers under the 56 price scenarios. Similar calculations for the Cox–Rubinstein specification were made, but, as discussed later, were not presented.

[7] Daily closing futures prices from the Chicago Mercantile Exchange and daily high and low prices from Interior Iowa, Omaha, and Sioux City livestock markets were provided by G. Futrell and D. O'Brien, Department of Economics, Iowa State University.

[8] For the period 1983–87, errors in the futures price forecasts of the price at contract expiration six months later ranged from \$.04/cwt to \$18.99/cwt, with an average error of \$5.66/cwt. For the period 1980–88, the annualized volatilities of the hog futures closing prices ranged from 16 to 30. Eales et al. found that soybean producers had annualized volatility expectations of prices as low as 9.45, even when the market's implied volatility was 22.

[9] The double integrals in equations (7)–(10) are computed using Gaussian adaptive composite quadrature. Gaussian quadrature is performed by choosing N cash and N futures prices to interpolate the continuous integrand. To complete the inner integral, R is calculated for each cash price, given the set of futures prices. These values of R are multiplied by the joint density function evaluated at the combinations of cash and future prices used to calculate R. These values are in turn multiplied by standard quadrature interpolating values (weights) which depend on the order of integration and are taken from tables. To increase precision, composite quadrature divides the intervals LF^2 to UF^2 and LY to UY into several subintervals. Adaptive quadrature adapts the length of each of these subintervals to increase the precision where the function changes most rapidly. For further discussion, see Conte and de Boor. The GAUSS computer programs used are available from the authors. An alternative procedure to optimize (7) involves iterating between a numerical integration routine and a nonlinear optimization routine (Kaylen, Preckel, and Loehman). This approach permits non-integer solutions, ignoring marketing constraints, and removes part of the attractiveness of options positions which can be used to achieve intermediate trading positions (Hauser and Andersen).

[10] A description of the "best" strategies is omitted for brevity, but is available from the authors upon request.

[11] In several cases, the optimal strategies were different, but the loss in CE was less than 1¢ per cwt.

[12] The skewness of the returns distribution under the negative exponential function for scenarios 67, 77, 86, and 87 are .53, 1.04, 1.92, and 1.58, respectively.

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Appendix

The Cox–Rubinstein (DARA) function is more likely to produce different rankings than the negative exponential (CARA) function when the initial wealth is small. To verify this, we show that the relative percentage difference in utility between the two functions resulting from incremental changes in wealth is very large when initial wealth is small, but is small when initial wealth is large.

Let $U_1 = -\exp(-qR)$, the CARA function, and $U_2 = (1/(1-d))R^{1-d}$, the DARA function, where the terms are defined in the text. Then,

$$\frac{dU_1}{dR} = q\exp(-qR), \qquad \text{and}$$

$$\frac{dU_2}{dR} = R^{-d}.$$

Expressing changes in utility as percentage changes resulting from incremental changes in R leads to

$$\frac{dU_1}{U_1} = \frac{q\exp(-qR)dR}{-\exp(-qR)} = -qdR, \qquad \text{and}$$

$$\frac{dU_2}{U_2} = \frac{R^{-d}dR}{(1/(1-d))R^{1-d}} = \frac{(1-d)dR}{R}.$$

The relative percentage change in utility between U_2 and U_1 for incremental changes in wealth can then be expressed as

$$\frac{\dfrac{dU_2}{U_2}}{\dfrac{dU_1}{U_1}} = \frac{\dfrac{(1-d)dR}{R}}{-qdR},$$

which after normalizing to make the risk aversion coefficients comparable, $d = R\cdot q$ (see text), can be written as

$$\frac{\dfrac{dU_2}{U_2}}{\dfrac{dU_1}{U_1}} = \frac{\dfrac{(1-d)dR}{R}}{-qdR} = \frac{-(1-qR)dR}{qR}.$$

As R gets smaller,

$$\lim_{R\to 0^+} \frac{(1-qR)dR}{qR} = \infty,$$

the relative difference between the two functions gets larger for smaller R. However, as R gets larger,

$$\lim_{R\to\infty} \frac{-(1-qR)dR}{qR} = \lim_{R\to\infty} \frac{q}{q} = 1 \text{ (by L'Hopital's Rule)}.$$

Hence, as R gets larger, the difference between the two functions approaches a positive finite constant, showing that differences between the two functions will be largest for small values of R.