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Modeling Production Risk with a Two-Step Procedure

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This study deals with modeling of production risk by means of a two-step procedure. In contrast to earlier studies of production risk, we do not immediately adopt restrictive functional forms for the risky production technology. We first test for the presence of production risk. If production risk is found to be present, the mean and risk functions are estimated separately. This allows the use of more flexible functional forms for both the mean and the risk functions than commonly found in the literature. An empirical application to Norwegian salmon farming, where restrictive specifications of the technology are rejected, demonstrates the validity of our approach. Presence of production risk in many primary production sectors implies that this approach should be considered in productivity studies.

Key words: flexible functional forms, production risk, salmon

Introduction

Output risk is an inherent part of the production process in most primary industries, e.g., agriculture, aquaculture, fisheries, mining, and oil extraction. In those developing countries where subsistence agriculture predominates, production risk is an issue of great concern. For producers in industrialized countries, output risk may not have the same grave consequences, but is nevertheless an important economic issue since the level of risk will, for example, determine insurance costs and interest costs on loans. In the worst-case scenario, an adverse production shock can lead to bankruptcy for the producer.

An important characteristic of risky production processes is that random production shocks can be observed only after input decisions have been made. This is in contrast to the standard certainty case, where the only determinants of optimal input demands are the structure of the production technology and input and output prices facing the producer. In the presence of risk, the producer's risk preference structure and expectation formations also will be important in determining optimal behavior. In particular, when it comes to relative input uses, a source of deviation from competitive levels is the input's marginal contribution to the level of output risk (Ramaswami). Some inputs may reduce the level of output risk (e.g., pesticides), while others may increase risk (e.g., fertilizers).

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Most studies dealing with production risk are based on the Just-Pope (J-P) postulates (Just and Pope 1978). However, specifications of risky production models which are in accordance with the J-P postulates are often difficult to work with empirically, since the optimization problem becomes nonlinear when popular functional forms such as the translog are used. Because of this, the functional forms used in empirical work are often highly restrictive. The most commonly used functional form in the J-P model framework is the Cobb-Douglas. The usual econometric translog specification has multiplicative interaction between the translog function and the error term. This specification violates the J-P postulates, but may be extended to allow for greater flexibility with respect to production risk (Kumbhakar).

Here, we exploit the fact that production uncertainty can be regarded as heteroskedasticity when the J-P postulates hold. In the J-P model, $y = f(\mathbf{x}; \alpha) + h(\mathbf{z}; \beta)\varepsilon$; inputs $\mathbf{x}$ influence both mean output and output risk through a mean production function $E[y] = f(\mathbf{x})$ and a variance (risk) production function $\text{var}(y) = h(\mathbf{z})^2\sigma_\varepsilon^2$, where $\mathbf{z}$ may contain some or all of the elements in $\mathbf{x}$ and/or additional variables. This model framework allows us to test for production risk (heteroskedasticity) and estimate the parameters of the mean and risk functions in separate steps. There are several reasons why this is useful.

First, since production uncertainty appears as heteroskedasticity in an econometric model, presence or absence of production uncertainty can be investigated using heteroskedasticity tests on the mean production function. Hence, when production uncertainty is an issue, tests against heteroskedasticity should always be the first step, as this might be interpreted as a test against production uncertainty in this context. This is in contrast to the current practice in the empirical literature on production risk, where a parametric model for the output variance is immediately specified when production risk is considered to be present.

Moreover, White (1982) and Gourieroux, Monfort, and Trognon show that for maximum likelihood estimators belonging to the linear exponential family, asymptotically normal estimators for the first-order moments can be obtained under a wide variety of distributional misspecifications. Heteroskedasticity is one type of distributional misspecification for which one can obtain parameter estimates with robust standard errors. Thus, production uncertainty can be regarded as a special case of the general problem of estimating models with standard errors that are robust to distributional misspecification. This allows us to divide the problem into two parts. If the main focus is on the mean function, a heteroskedasticity-consistent estimator will provide not only consistent estimates, but valid inference as well. Likewise, if one is also interested in the risk structure of the production technology, the variance function can be estimated separately. Since the nonlinearity inherent in the Just-Pope framework in general is due to the joint estimation of the mean and variance functions, this two-step procedure may greatly simplify estimation. It will also allow the use of more complex specifications for both the mean and the variance functions. Support for flexible functional forms and firm-specific effects is found in a large number of empirical productivity studies which do not deal with production risk, and restrictive functional forms like the Cobb-Douglas and firm homogeneity are generally rejected.

The approach is illustrated with an application to the Norwegian salmon farming industry. First, a flexible mean production function is estimated separately, and tests for heteroskedasticity are undertaken. Having found evidence of heteroskedasticity in inputs, a flexible parametric model of production risk is estimated. We then test for the validity of firm-specific levels of output risk and second-order terms in input levels. We find that a flexible specification of both the mean portion and the risk portion of the salmon production technology is appropriate.

Our analysis proceeds as follows. First, we discuss implications of the literature on production risk. An approach to testing for production risk is then detailed. Next, we present the two-step approach to empirical estimation of the parameters of a risky production technology, followed by an empirical application to a panel data set of Norwegian salmon farms. Our summary and conclusions are offered in the final section.

Production Risk

In their seminal paper, Just and Pope (1978) presented eight postulates for the stochastic production function which they argued were appropriate on the basis of a priori theorizing and empirical observations. Some of the J-P postulates impose restrictions on the mean function which are analogous to those that apply to the usual deterministic specification. However, there are also some additional flexibility requirements for output variance. An important requirement for the output variance function is that positive, zero, and negative marginal risk in input levels each should be possible. In other words, inputs are allowed to increase or reduce the level of output risk. This is a distinguishing feature from popular log-log production function specifications such as $y = f(\mathbf{x})e^{\varepsilon}$, where $f(\mathbf{x})$ is parameterized as a Cobb-Douglas or a translog, and ε is an exogenous stochastic shock. For such specifications, marginal risks are always positive. For the common additive specification $y = f(\mathbf{x}) + \varepsilon$, where the variance of ε is a constant, marginal risks are always zero.

J-P presented a production function which satisfies all eight postulates—the Just-Pope form—which has become the dominant form in subsequent theoretical and empirical research on production risk. In its most general form, the J-P production function is specified as

$$(1) \qquad y = f(\mathbf{x}; \alpha) + h(\mathbf{z}; \beta)\varepsilon,$$

where $f(\cdot)$ is the *mean function* and $h(\cdot)$ is the *variance function* (or *risk function*), $\mathbf{x}$ and $\mathbf{z}$ are vectors of inputs (with parameters α and β) which may be identical or have some unique elements. The exogenous stochastic disturbance (or production shock) is represented by ε, where $E[\varepsilon] = 0$ and $\text{var}(\varepsilon) = \sigma_{\varepsilon}^2$. A nice feature of the J-P form is the separation of the mean effect and the variance effect of changes in input levels. Mean output is given by $E[y] = f(\mathbf{x}; \alpha)$, while the variance of output is given by $\text{var}(y) = [h(\mathbf{z}; \beta)]^2\sigma_{\varepsilon}^2$. From an econometric viewpoint, this formulation is also useful because the variance function can be interpreted as a heteroskedastic disturbance term. This can be seen by reformulating the J-P form as $y = f(\mathbf{x}; \alpha) + u$, where u is the error term with variance $\text{var}(u) = [h(\mathbf{z}; \beta)]^2\sigma_{\varepsilon}^2$.

Since production risk may be modeled as heteroskedasticity, the parameters in the mean production function cannot be efficiently estimated if the production risk is not accounted for. In the empirical literature, this is done by estimating the production function and the variance function together, primarily by a feasible generalized least squares (FGLS) three-stage estimator.[1]

The Just-Pope form has been criticized for not accommodating the potential effects of input level changes on third and higher moments of the output distribution (Antle). Some researchers have estimated production models which allow for *heteroskewness* and possibly *heterokurtosis* in input levels (Antle; Nelson and Preckel; Saha, Havenner, and Talpaz). These studies have addressed interesting aspects of production risk, but the application of such models may be limited due to econometric intractability with respect to estimation and testing, and a belief among researchers that heteroskewness and heterokurtosis have limited significance for producer decisions. Producers generally face a very complex environment. It can be argued that they will at most be able to form expectations about the effects of inputs on output variance, while they have too little information available to form reasonable expectations on the shape of the output distribution or the effects of inputs on output skewness.

Testing Against Production Risk

The first issue to address when analyzing a production sector is to investigate whether any significant production risk is present. A test of production risk can draw on the theoretical framework of Just and Pope. Since production risk is specified as heteroskedasticity in the J-P framework, any test against heteroskedasticity can be used. If heteroskedasticity is not detected, this can be regarded as evidence against production risk, and the researcher can proceed within a conventional deterministic production model framework.

When testing for heteroskedasticity, both a general test such as White's (1980) test and tests against specific alternatives might be used. Tests where the alternative has a specific functional form ordinarily will have greater power against the chosen alternative than general tests (Godfrey). One might also argue that a test against heteroskedasticity may capture the effects of features of the data other than production risk. This is certainly true, but if these problems introduce heteroskedasticity, they should nevertheless be accounted for in the econometric specifications. Hence, while finding no heteroskedasticity leads to the conclusion of no production risk, detecting heteroskedasticity does not unambiguously imply production risk.

This leads us into a general problem with treating production uncertainty as heteroskedasticity. It is well known that most misspesification tests have power against a wide variety of alternatives (Godfrey; MacKinnon). It is therefore often recommended that an appropriately specified econometric model should pass a "battery" of misspecification tests (McGuirk, Driscoll, and Alwang). This is in general very reasonable, since a well-specified econometric model should not show any sign of misspecification. However, one should find heteroskedasticity when the production technology which generated the data exhibits production risk. Thus, heteroskedasticity is not a problem for the economic interpretation of the model when this is recognized.

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[1] See, for example, Just and Pope (1979); Griffiths and Anderson; McCarl and Rettig; Wan and Anderson; Hallam, Hassan, and D'Silva; Wan, Griffiths, and Anderson; Hurd; and Traxler et al. for empirical applications of the Just-Pope model.

When production risk is of interest, the analyst is then faced with two options. One can interpret evidence of heteroskedasticity as evidence of production risk and proceed to investigate the economic structure of interest. While this is the most reasonable approach when the economic structure is of interest, it creates an econometric problem since the battery of misspecification tests is no longer useful. In particular, when heteroskedasticity is present, a number of other misspecification tests are likely to indicate that there is a problem with the model since these tests also have power against this alternative. This makes it very difficult to evaluate the appropriateness of the econometric specification of the model when production uncertainty is present. The alternative option is to treat heteroskedasticity as an econometric problem, and attempt to correct or eliminate this problem from the model until it passes the battery of misspecification tests. However, with this approach it is difficult to find any justification for investigating the economic content of heteroskedasticity—in our case production uncertainty.

We think it is important to investigate production uncertainty and, accordingly, that this is a situation where the econometric misspecification tests cannot carry too much weight. However, this discussion clearly indicates that one should evaluate the estimated model very carefully. In this context, three more informal criteria for choosing functional form provided by Alston and Chalfant might be useful, as they are applicable also more generally. Alston and Chalfant indicate one should (*a*) use a relatively flexible functional form, (*b*) test for the sensitivity of the results to functional form (or more generally, to model specification), and (*c*) be conservative in drawing implications from results that might be sensitive to functional form or other joint hypotheses.

It also may be of interest to note that the possible existence of production uncertainty in the current literature is usually investigated using a fully specified model with an explicit parameterization of production uncertainty. One then checks whether production uncertainty is present by evaluating the parameters in the variance function, and even this procedure is often rather informal. For example, Wan and Anderson conclude that "most of the estimates determining the marginal risk effects lack statistical significance" (p. 85). A more appropriate first-step testing approach, if the risk function is specified according to Harvey, would have been to test whether all the parameters in the risk function except for the constant term were jointly zero. This would be a test with homoskedasticity or no production risk as the null hypothesis. Yet this approach requires estimating both the mean and the variance functions simultaneously.

A Two-Step Procedure to Modeling Production Risk

Provided that production risk is found to be present, there are two issues of interest—the mean production function $f(\cdot)$ and the risk function $h(\cdot)$. Although it is not necessary, these two functions are typically estimated jointly, thus complicating the estimation procedure. Further, it frequently will be the case that information from only one of the functions is of interest.

The fact that production risk appears as heteroskedasticity in the Just-Pope formulation implies that ordinary least squares (OLS) parameter estimates of the mean production function are inefficient, although they are still consistent. However, the standard errors of the parameters affecting the conditional mean can be consistently estimated by employing White's heteroskedasticity-consistent estimator of the covariance matrix, enabling valid inference without paying attention to the risk function. As long as the information of interest is related only to the production function, one need not be concerned with the risk function.

Furthermore, a heteroskedasticity-consistent covariance matrix does not assume any particular functional specification of the risk function. This might in many cases be an advantage, as the efficiency of both an FGLS and an ML estimator depends on the assumption that the chosen functional form for the risk function is correct. One is no better off by accounting for the risk function if it is incorrectly specified, since the parameters in the production function still will be inefficient. However, there are occasions when employing White's estimator instead of a parametric specification is not advantageous. As is the case for all semiparametric estimators, the heteroskedasticity-consistent covariance matrix will be less efficient than a correctly specified parametric model of heteroskedasticity in small samples.

It is well known that the parameters of the risk function may be estimated consistently separately from the mean function (Harvey). This is, in fact, the basis for all FGLS procedures used when estimating the production function in this kind of application. However, to provide valid inference for the risk function parameters, they must be estimated by a heteroskedasticity-consistent estimator (Saha, Havenner, and Talpaz).

An advantage of separating the estimation of the mean and the variance functions is that a more detailed specification is possible for each of the functions, since each of the functions in general can be modeled as linear in the parameters. This might be of importance, since the use of a too restrictive functional form can well be the source of apparent production risk. In particular, the literature without production risk usually rejects restrictive functional forms like the Cobb-Douglas in favor of more general alternatives. Furthermore, a growing body of empirical panel data studies provides strong support for the presence of producer heterogeneity in many sectors, including agriculture. When panel data are available, one should account for firm-specific and other group-specific effects, since the parameter estimates will be inconsistent if these effects are not controlled for.

An Empirical Application to Norwegian Salmon Aquaculture

Salmon aquaculture is one of the fastest growing sectors of biological production. From 1980 to 1997, global farmed salmon production expanded from 6,900 metric tons to 719,000 metric tons.[2] However, despite its impressive growth, the industry has experienced a high degree of turbulence and large cross-sectional variations in profitability, manifesting in a large number of bankruptcies and restructuring of the industry. The observed cross-sectional differences in economic performance can be partly attributed

[2] See Asche for a brief review of the industry.

Table 1. Overall Summary, Nonnormalized Variables Statistics Sample

Variable	Mean	Std. Dev.	Minimum	Maximum
Output (y) in kg	355,982.3	236,220.9	11,050	2,014,140
Materials (M) in real NOK[a]	998,381.0	907,280.9	9,657	9,636,125
Feed (F) in kg	340,299.1	242,741.0	2,358	2,479,452
Capital (K) in real NOK[a]	2,572,787.1	2,277,836.4	4,707	37,212,584
Labor (L) in hours worked	7,034.5	3,694.1	250	42,906
Fish input (I) in kg	150,379.0	109,897.5	50	1,015,800

Note: n = 1,953 observations.
[a] Deflated by Consumer Price Index (CPI).

to stochastic production shocks. It can be argued that salmon aquaculture is more risky than traditional land-based livestock production, because of salmon's high sensitivity to its marine environment.[3] In general, animals are less sensitive to their environment and farmers have a higher degree of control on important biophysical variables in land-based meat production. The high sensitivity of the salmon to its environment, together with the rough weather conditions under which farms are forced to operate, probably means that there will be a relatively high permanent level of output risk compared to other types of meat production.

The production function for the Norwegian salmon farmers is specified with five inputs: fish feed (denoted F), fish input (I), capital (K), labor (L), and materials (M).[4] It is estimated on an unbalanced panel with a total of 1,953 observations on 372 farms observed from three to nine years during the period 1985–93. The data were collected from the *Norwegian Directorate of Fisheries'* annual survey of fish farms. Summary statistics for the sample are provided in table 1, where NOK denotes Norwegian Kroner.

The most important input in salmon farming is fish feed, with a cost share of about 40%. Feed is expected to increase the level of output risk, ceteris paribus. Because the salmon is not able to digest all the feed, the excess is released into the environment as feed waste or feces. This organic waste consumes oxygen, and thus competes with the salmon for the limited amount of oxygen available in the cages. In addition, feed waste also leads to production of toxic by-products, such as ammonia. Furthermore, production risk is expected to increase with the quantity of fish released into the cages, due to the increased consumption of oxygen and production of ammonia. Labor input, on the other hand, is expected to have a risk-reducing effect, since the ability to monitor the fish, repair equipment, and time feeding will increase. There is a large body of research in other fields (e.g., marine biology) which implicitly supports the hypotheses on risk effects of feed and fish input, and also some evidence on labor input (Johannessen; Ervik et al.). In contrast, we do not have any strong a priori presumptions on the risk effects of capital and materials input.

[3] Important parameters for salmon are the oxygen concentration, sea temperature, salinity, and concentrations of disease bacterias/viruses and toxic algaes.

[4] Salvanes (1989, 1993) and Bjørndal and Salvanes estimate cost functions for the Norwegian salmon industry, but without taking account of uncertainty.

Several studies have documented that there is substantial heterogeneity in Norwegian salmon farming in terms of the quality of management and workers, production technology, and the biophysical conditions at the farm site.[5] Consequently, different production outcomes for two farms using the same vector of inputs may be due to firm-specific effects of a more permanent nature as well as stochastic biophysical shocks.

We assume that the salmon production technology has the general Just-Pope form:

$$y_{it} = f(\mathbf{x}_{it}; D_t, \alpha, \mu_i) + u_{it}, \quad \mathrm{var}(u_{it}) = h(\mathbf{x}_{it}; D_t, \beta, \lambda_i), \tag{2}$$

where i and t are firm and time subscripts, respectively; $\mathbf{x}_{it}$ is a K-vector of input levels; D_t is a time-specific dummy variable; α and β are vectors of parameters; μ_i is a firm-specific effect on mean output; and λ_i is a firm-specific effect on output risk. By implementing firm-specific effects both in the mean and risk portions of the technology, we are able to separate the effect of unobserved firm characteristics on the mean and variance of output. Similarly, the inclusion of time dummies in both $f(\cdot)$ and $h(\cdot)$ allows us to analyze the effects of technical change on E[y] and var(y) separately.

A linear quadratic (LQ) mean production function is used here (Driscoll, McGuirk, and Alwang):

$$\begin{aligned} y_{it} = {} & a_0 + \sum_k \alpha_k x_{k,it} + 0.5 \sum_j \sum_k \alpha_{jk} x_{j,it} x_{k,it} + \sum_{t=t_i}^{T_i} \alpha_t D_t \\ & + \sum_k \sum_{t=t_i}^{T_i} \alpha_{kt} D_t x_{k,it} + \mu_i + u_{it}. \end{aligned} \tag{3}$$

For the variance function, we employ a special case of Harvey's specification, var(u) = exp[$\mathbf{z}\beta$], where the $\mathbf{z}$'s are input levels or transformations of input levels, e.g., logarithms of inputs and second-order terms.[6] A nice property of the variance function in Harvey's formulation is that positive output variances are always ensured. Note that in the Just-Pope model, var(y) = var(u). In our specification, the argument of the exponent is a linear quadratic function with time- and firm-specific effects:

$$\mathrm{var}(u_{it}) = \exp\left(\sum_k \beta_k x_{k,it} + 0.5 \sum_k \sum_j \beta_{kj} x_{k,it} x_{j,it} + \sum_t \beta_t D_t + \lambda_i \right). \tag{4}$$

In the present analysis, we use only the linear quadratic functional form for the mean and variance functions. However, very similar results have been obtained for salmon aquaculture with other flexible functional forms. Hence, the conclusions drawn here based on the linear quadratic specification will not change much if $f(\mathbf{x})$ instead is specified as a Leontief, or $h(\mathbf{x})$ is specified with a translog or Leontief in the exponent.[7]

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[5] See, for example, Berge and Blakstad, and Johannessen for documentation of the producer heterogeneity in salmon farming. Surveys of a large number of farm sites have shown that there are considerable differences in the biophysical productivity.

[6] The first element of $\mathbf{z}$, z_0, is taken as unity. This implies that var(ε) = exp(β_0).

[7] Estimated elasticities derived from different functional specifications of the mean function and the variance function are provided in Tveterås.

Table 2. Goldfeld-Quandt *F*-Test Statistics

Central Observation Omitted[a]	Materials (M)	Feed (F)	Capital (K)	Labor (L)	Fish Input (I)
One	11.202	24.620	0.861	17.279	21.846
1/6	12.626	30.712	0.861	20.250	25.006

Note: The test statistic is F-distributed with $\{n_1 - K\}$ and $\{n_2 - K\}$ degrees of freedom, where n_1 and n_2 are the number of observations in the two subsamples and K is the number of parameters in the model.

[a] The two subsamples for which separate regressions were estimated for each input have 976 and 814 observations, respectively, in the two tests.

Testing for Production Risk

The LQ mean production function was first estimated by OLS on the salmon farm panel data set. Based on the OLS estimates, we tested for heteroskedasticity or the presence of significant marginal output risk in input levels using a number of heteroskedasticity tests.

A Breusch-Pagan (B-P) test was first undertaken (Breusch and Pagan).[8] The B-P test statistic is 1,864.04 (p-value 0.000), and is distributed as χ^2 with 440 degrees of freedom (df). Hence, the hypothesis of homoskedasticity can be rejected at all conventional significance levels.

Next, Goldfeld-Quandt (G-Q) tests were performed for all five inputs (Goldfeld and Quandt).[9] The LQ mean function was estimated under the assumption of firm homogeneity. The G-Q test involves sorting of data by right-hand variables, splitting the observations into two subsamples, and estimating separate regressions for each subsample. The G-Q test was not undertaken with firm-specific effects included since there is a risk of being left with only one observation on some firms. Two tests were run for each input; in the first test only the central observation was omitted, and in the second test the 1/6 central observations were omitted. Table 2 presents the F-distributed test statistics. For all inputs except capital, the G-Q test rejects the null hypothesis of homoskedasticity with wide margins at conventional significance levels. For capital, the homoskedasticity hypothesis is maintained.

Harvey tests were also performed for the specified model with a restricted specification of the variance function.[10] This test is based on the second-stage estimates. The null hypothesis of the Harvey test is that all coefficients of the multiplicative variance function, except the intercept β_0, are zero. The Harvey test statistic is RSS/4.9348, where RSS is the residual sum of squares of the estimated variance function, and

[8] The most general test of heteroskedasticity (and other possible misspecifications) is the White test (White 1980). Due to the large number of regressors required by the White test in the context of our model specification, it was not undertaken here.

[9] In the Goldfeld-Quandt test, the observations are ranked by the independent variable, of which the variance is assumed to be a function. The sample is then divided into two groups with n_1 and n_2 observations, and the model is estimated separately for these two sets of observations. The test statistic is $F = s_1^2/s_2^2$, where s^2 is the OLS estimator of the regression variance σ^2.

[10] The second-order input terms and firm-specific effects are omitted from the restricted specification.

is asymptotically distributed as χ^2 with degrees of freedom equal to the number of regressors. The estimated Harvey test statistic is 165.249 for the restricted variance function. This is much higher than the critical χ^2 value of 27.688 (13 df) at the 1% significance level. Hence, the null hypothesis of homoskedasticity is rejected.

In all, the tests provide substantial evidence of output heteroskedasticity in input levels, and accordingly indicate that output risk is present in salmon farming.

Modeling the Mean Function

Since production risk was found to be present, we reestimated the mean function using White's heteroskedasticity-consistent covariance estimator to provide valid inference. Parameter estimates are presented in table 3.

Statistical tests support the flexible specification in terms of second-order input terms and time dummies.[11] It is difficult to provide a meaningful interpretation of the estimated parameters, and empirical results are consequently presented in terms of elasticities. Table 4 reports overall sample average elasticity estimates.[12] We see that the output elasticity, E_k, is positive for all inputs. As expected, feed is found to be the most important output in terms of output elasticity, with a sample average value of 0.51, followed by fish input with a sample average output elasticity of 0.27. Returns to scale (*RTS*), which is the sum of the k output elasticities, is 0.89. In other words, the sample average farm has exhausted scale economies. The average annual rate of technical progress (*TC*) is 4.2%. This high rate of technical progress is not surprising, since our data set covers a period characterized by intensive learning and rapid introduction of new innovations in feed, salmon genetics, and medication technologies. The finding is supported by other studies of the industry (e.g., Asche).

Modeling the Risk Function

Next, the variance function was estimated in a separate step, using the predicted residuals from the estimated mean function. Variance function parameter estimates are provided in table 3, and derived elasticity estimates in table 4. The standard errors of the β's are White-adjusted.

Table 5 presents the results of Wald tests on the structure of the risk component of the production technology. First we tested for a pooled variance function, where a common intercept (β_0) is assumed instead of N firm-specific intercepts (λ_i), i.e., $\{\lambda_1 = \lambda_2 = \ldots = \lambda_N = \beta_0\}$. The pooled specification was clearly rejected by a Wald test at the 1% significance level, with a test statistic of 2,118.23 and 372 degrees of freedom (p-value 0.0000). Hence, salmon farms are heterogeneous also with respect to the level of production risk.

[11] The null hypothesis that all the coefficients of the second-order terms were jointly equal to zero was rejected by an F-test at the 1% significance level with an F-statistic of 4.23 (15 and 1,519 df). The null hypothesis of a common intercept was also rejected at the 1% level with an F-statistic of 3.03 (372 and 1,513 df).

[12] In this example, we have presented only elasticities derived from a generalized quadratic form. However, other flexible functional forms for the mean and variance functions, such as the generalized Leontief, provide very similar elasticity estimates. Results are available from the authors upon request.

Table 3. Parameter Estimates for Mean and Variance Functions

Mean Function OLS Estimates w/White-Adjusted Standard Errors

Parameter	Coefficient	Std. Error	t-Value	Parameter	Coefficient	Std. Error	t-Value
α_{M}	0.098	0.026	3.827	α_{F85}	-0.022	0.070	-0.313
α_{F}	0.429	0.059	7.333	α_{F86}	-0.039	0.069	-0.566
α_{K}	0.095	0.037	2.580	α_{F87}	0.024	0.064	0.368
α_{L}	0.098	0.055	1.769	α_{F88}	0.033	0.072	0.458
α_{I}	0.340	0.051	6.619	α_{F89}	0.098	0.063	1.568
α_{MM}	-0.001	0.003	-0.287	α_{F90}	-0.056	0.067	-0.832
α_{ML}	0.045	0.018	2.589	α_{F91}	0.053	0.069	0.763
α_{MI}	-0.014	0.014	-1.025	α_{F92}	0.020	0.054	0.373
α_{MF}	-0.037	0.016	-2.222	α_{L85}	0.063	0.054	1.163
α_{MK}	0.000	0.012	0.031	α_{L86}	-0.020	0.050	-0.396
α_{FF}	-0.008	0.019	-0.431	α_{L87}	-0.057	0.050	-1.134
α_{FK}	0.000	0.018	-0.001	α_{L88}	-0.025	0.049	-0.504
α_{LF}	0.013	0.037	0.355	α_{L89}	0.039	0.042	0.937
α_{IF}	0.077	0.029	2.617	α_{L90}	0.019	0.045	0.428
α_{KK}	0.004	0.005	0.905	α_{L91}	-0.012	0.058	-0.212
α_{LK}	-0.037	0.017	-2.225	α_{L92}	-0.010	0.043	-0.228
α_{IK}	0.003	0.016	0.204	α_{I85}	0.025	0.062	0.397
α_{LL}	-0.030	0.015	-1.951	α_{I86}	0.025	0.076	0.324
α_{LI}	-0.017	0.031	-0.542	α_{I87}	-0.058	0.055	-1.050
α_{II}	-0.020	0.015	-1.287	α_{I88}	-0.070	0.062	-1.126
α_{85}	-0.168	0.047	-3.562	α_{I89}	-0.165	0.059	-2.808
α_{86}	-0.117	0.047	-2.500	α_{I90}	-0.093	0.057	-1.619
α_{87}	-0.028	0.048	-0.574	α_{I91}	-0.112	0.057	-1.968
α_{88}	-0.026	0.056	-0.454	α_{I92}	-0.123	0.052	-2.354
α_{89}	-0.017	0.051	-0.344	α_{K85}	-0.052	0.038	-1.368
α_{90}	0.018	0.046	0.398	α_{K86}	-0.015	0.036	-0.402
α_{91}	-0.015	0.048	-0.308	α_{K87}	-0.056	0.036	-1.545
α_{92}	0.016	0.046	0.359	α_{K88}	-0.039	0.036	-1.085
α_{M85}	-0.094	0.050	-1.892	α_{K89}	-0.061	0.037	-1.664
α_{M86}	-0.109	0.037	-2.986	α_{K90}	-0.078	0.037	-2.128
α_{M87}	-0.084	0.034	-2.506	α_{K91}	-0.024	0.038	-0.621
α_{M88}	-0.055	0.029	-1.905	α_{K92}	-0.014	0.032	-0.445
α_{M89}	-0.052	0.029	-1.788				
α_{M90}	0.024	0.027	0.888				
α_{M91}	-0.020	0.026	-0.760				
α_{M92}	-0.014	0.023	-0.639				

Adjusted R^2 = 0.937, Log-likelihood function = 974.771

(continued)

Table 3. Continued

Variance Function Parameter Estimates							
Parameter	Coefficient	Std. Error	*t*-Value	Parameter	Coefficient	Std. Error	*t*-Value
β_M	0.095	0.212	0.449	β_{KK}	0.053	0.038	1.385
β_F	0.428	0.351	1.220	β_{LK}	-0.178	0.163	-1.095
β_K	-0.307	0.230	-1.339	β_{IK}	0.073	0.136	0.541
β_L	0.474	0.418	1.135	β_{LL}	-0.246	0.171	-1.441
β_I	0.717	0.320	2.242	β_{LI}	-0.517	0.300	-1.723
β_{MM}	-0.028	0.029	-0.963	β_{II}	-0.080	0.120	-0.664
β_{ML}	0.138	0.178	0.772	β_{85}	-0.409	0.288	-1.420
β_{MI}	0.079	0.122	0.649	β_{86}	-0.367	0.286	-1.281
β_{MF}	-0.262	0.151	-1.740	β_{87}	-0.304	0.267	-1.138
β_{MK}	-0.002	0.105	-0.019	β_{88}	-0.390	0.234	-1.667
β_{FF}	-0.092	0.106	-0.860	β_{89}	0.089	0.216	0.412
β_{FK}	0.070	0.133	0.526	β_{90}	-0.116	0.219	-0.528
β_{LF}	0.518	0.230	2.251	β_{91}	0.215	0.225	0.953
β_{IF}	0.056	0.189	0.299	β_{92}	0.174	0.207	0.842

Adjusted R^2 = 0.124, Log-likelihood function = -4,208.30

Table 4. Sample Average Elasticity Estimates

	Mean Function Elasticity Estimates[a]						
	E_L	E_F	E_I	E_K	E_M	*RTS*	*TC*
Mean	0.044	0.512	0.274	0.015	0.039	0.886	0.042
Std. Dev.	0.102	0.151	0.117	0.046	0.071	0.199	0.082

	Variance Function Elasticity Estimates[b]						
	VE_F	VE_I	VE_K	VE_L	VE_M	*TVE*	*TCV*
Mean	0.580	0.149	-0.162	-0.195	-0.093	0.279	0.080
Std. Dev.	0.580	0.411	0.566	0.865	0.505	0.906	0.268

[a] E_k = output elasticity with respect to input k; *RTS* = returns to scale (the sum of E_k's); *TC* = rate of technical change mean production function.

[b] $VE_k = (\partial \text{var}(y)/\partial x_k)(x_k/\text{var}(y))$ is the output variance elasticity with respect to input k; *TVE* = total output variance elasticity (the sum of VE_k's); *TCV* = rate of technical change variance production function.

Table 5. Results of Wald Tests of Risk Structure

Hypothesis	Wald Statistic	Degrees of Freedom	p-Value
Intercept equal for all firms (pooled model)	2,118.23	372	0.0000
No marginal risk effects of inputs	58.15	20	0.0000
No second-order risk effects of inputs	33.26	15	0.0043
No time-specific effects on output risk	12.51	8	0.1298
Only firm-specific effects	122.82	28	0.0000

Next, we tested the hypothesis that there are no marginal risk effects of inputs, i.e., that all the β_k's and β_{jk}'s are jointly equal to zero. This hypothesis was rejected at the 1% level with a Wald test statistic of 58.15 and 20 degrees of freedom (p-value 0.0000). We also proceeded to test the appropriateness of the second-order approximation in input levels. The null hypothesis that all β_{jk}'s are jointly equal to zero was rejected at the 1% level with a Wald test statistic of 33.26 and 15 degrees of freedom (p-value 0.0043).

The importance of time-specific effects on the level of output risk was also tested. A Wald test of the null hypothesis that all time-specific effects are equal was not rejected at conventional significance levels. The Wald test statistic was 12.51 (8 df), with an associated p-value of 0.1298. Finally, a test was conducted on the joint hypothesis that all parameters except the λ_i's are equal to zero, i.e., that only firm-specific effects explain different levels of output risk across farms. With a Wald test statistic of 122.82 and 28 degrees of freedom, this hypothesis was also rejected at conventional confidence levels (p-value 0.0000). Hence, it seems likely that time-invariant firm-specific effects are more important than time-specific effects for output risk differences between farm observations in our data set.

In summary, the Wald tests provide support for a flexible specification of the risk portion of the technology and show that farms are heterogeneous also with respect to the level of production risk. These findings are interesting, considering that earlier studies have used very restrictive specifications (e.g., Cobb-Douglas) and postulated that firms are homogeneous.

Let us examine what further information the estimated variance function provides about the structure of production risk in salmon farming. Derived elasticities from the estimated variance functions are presented in table 4. According to the output variance elasticities with respect to inputs (VE_k, where $k = F, K, I, L, M$), labor, capital, and material inputs all have a risk-decreasing effect, while both fish feed and fish input have a risk-increasing effect. Fish feed in particular seems to have a large effect on the level of output risk, with an elasticity of 58% for the sample average firm. This confirms our a priori expectation that an increase in the amount of feed released into a confined area will make the marine environment less hospitable for the farmed fish due to increased competition for oxygen and production of nontoxic by-products. The potential for loss will therefore increase in the event of adverse stochastic shocks—for example, disease outbreaks or high sea temperatures.

Following Ramaswami, the implications of the estimated variance function elasticities are that a risk-averse producer will employ more labor, capital, and materials input than the risk-neutral producer, but also will employ less fish feed and fish input. The total variance elasticity, which is the analog to the returns-to-scale measure derived from the mean production function, is 27.9%. In other words, an increase in the scale of operation through a proportional increase in input levels not only leads to an increase in mean output (as we found earlier), but also to an increase in the level of output risk for the average farm. The degree of risk aversion will thus determine whether such an expansion will provide a higher expected utility for the salmon farmer.

According to table 4, the average annual rate of technical change was 4.2% for the mean function and 8% for the variance function, implying that both mean output and output risk increased for the sample average salmon farm during the period 1985–93. The positive technical change for the mean production function is supported by other studies of the industry (e.g., Asche). An examination of the estimated parameters associated with the time dummies suggests that the statistical support for technical change is weaker for the variance function than for the mean function.[13] One should also be careful in interpreting the estimated time-specific effects only as being determined by technological change, since "global" biophysical shocks such as sea temperature changes and large-scale fish disease outbreaks also will be captured in these parameter estimates.[14]

The degree of risk aversion determines whether technical change leads to an increase in the expected utility of Norwegian salmon farmers. The more risk averse farmers are, the more weight they will assign to the increase in production risk relative to the increase in expected output (Ghosh, McGuckin, and Kumbhakar). According to theory, the rationale of the average salmon farmer for adopting a riskier production technology is that the increase in mean output associated with the new technology is sufficiently large to provide an increase in producer welfare (i.e., expected utility).

Summary and Conclusions

In this study, we have exploited the fact that production risk can be treated as heteroskedasticity when the Just-Pope postulates hold. This allows us to estimate separately the mean and the risk functions. The parameters in both functions and their standard errors then can be estimated consistently using White's heteroskedasticity-consistent covariance matrix. This is useful in applied work, since it allows the use of more flexible functional forms than is the current practice. In particular, instead of using the restrictive Cobb-Douglas form, we use a second-order approximation for both the mean and risk portions of the production function. We also introduce firm-specific effects in both the mean function and risk function.

We test for the presence of heteroskedasticity in an application on a panel data set of Norwegian salmon farms. After having detected heteroskedasticity, we estimate the

[13] It should be noted, however, that Tveterås estimates a positive rate of technical change for the variance function for all the flexible functional forms employed in that paper. The high consistency across different econometric specifications should at the minimum indicate that production risk did not decrease during the period.

[14] Unfortunately, we do not have additional information in the data set which would allow us to distinguish between such global (spatially correlated) biophysical shocks and technological change.

risk function in the next step. Extensive testing is undertaken on the risk structure of the salmon farming technology. We find that farms are heterogeneous with respect to production risk. In other words, farms employing the same input levels have different levels of output risk. Inputs are found to be risk-controlling instruments, and the second-order approximation is also supported by statistical tests.

The theory of firm behavior under risk shows that the structure of production risk plays an important role in production decisions of risk-averse producers, both with respect to optimal input levels and to adoption of new technologies. Since production risk is an inherent feature of the production process in most primary industries, the approach outlined in this study should be considered in empirical investigations of productivity for these industries.

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