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# On the Aggregation of Money Measures of Well-Being in Applied Welfare Economics

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This article investigates the properties, good and bad, of social evaluations based on four money measures of well-being or changes in well-being: compensating variations, money metrics, extended money metrics, and welfare ratios. Consistency of social rankings (transitivity, asymmetry of preference), the possibility of incorporating inequality aversion, independence of the choice of reference prices, and the ethics implicit in the evaluations are considered. In addition, these procedures are contrasted with utility aggregation using equivalence scales.

*Key words:* consumer's surplus, money metrics, willingness to pay.

Cost-benefit analysis and other applications of welfare economics employ money measures of well-being or changes in well-being and evaluate economic states of affairs with aggregates of these measures—usually simple sums. The most favored measure is the compensating variation or willingness to pay. It is an index of welfare change for a household or individual. The equivalent variation (another Hicksian consumer's surplus) "measures" welfare change as well. The Marshallian consumer's surplus, with all its difficulties, is still used, usually as an approximation to the compensating variation (Willig). Other money measures of the level of well-being are money metric utilities—in both standard and extended forms—and welfare ratios. Compensating and equivalent variations and ordinary money metrics are determined by preferences alone, while extended money metrics and welfare ratios require interpersonal comparisons of utilities.

This article surveys the literature on the properties, good and bad, of these measures, using as criteria various aspects of their performance in performing social evaluations. In addition, their performance is contrasted with the use of aggregates of utilities—direct indexes of well-being—implemented with an equivalence-scale methodology.

Performance in social evaluation is viewed from several standpoints. First, rationality properties (transitivity, asymmetry of preference) of social binary relations based on the various measures are discussed. Second, given that states of affairs are ordered consistently, the possibility of incorporating inequality aversion is examined. Because concavity or nonconcavity of the utility functions that money measures represent has consequences for inequality aversion, concavity is the third area of investigation. Fourth, when indexes depend on a reference price or quantity vector or a reference household type—as money metrics and extended money metrics do—conditions for independence of the social ordering of this choice are investigated. Fifth, the ethics implicit in the evaluations produced with these aggregates are examined.

The possibility of incorporating inequality aversion is included because procedures that are restricted to zero inequality aversion are of limited usefulness in societies whose prevailing ethical judgments include a concern (even a small one) for distributive justice. Further, ethically flexible evaluation procedures can themselves contribute to a sensible discussion of the idea of distributional equity and the incentive trade-off.

The setting for these exercises is a general equilibrium one, with marketed private goods and unpriced public or semipublic goods. Two domains for prices and unpriced goods are employed: the first allows prices and the consumption of unpriced goods to differ among households, and the second requires them to be the same for all.

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Money measures that do not require interpersonal comparisons—consumers' surpluses and money metrics—are discussed first. Then extended money metrics and welfare ratios—that require interpersonal comparisons—are considered, and, finally, these procedures are contrasted with utility aggregation using equivalence scales.

No methodology in applied welfare economics is perfect. Practical work is always limited by the availability of data and the problem of estimating the economic consequences of projects. Different evaluation procedures are, therefore, bound to be differentially useful in different situations. One of the aims of this essay is to acquaint readers of this journal with the theoretical strengths and weaknesses of the various procedures in the hope that their ability to make good practical choices will be enhanced.

## Aggregation without Interpersonal Comparisons

### *Compensating Variations*

A single household or individual's compensating variation or willingness to pay for an economic change is the maximum amount the household would be willing to pay to secure the change. This definition can be extended to cover changes that make the household worse off, according to its preferences, and copes well with price and income changes, public or semipublic goods, externalities, and work and leisure.

If the household consumes private goods only and faces positive prices $p = (p_1, \ldots, p_m)$ with income (consumption expenditure) $y$, the compensating variation, $c$, for the change from $(p^b, y^b)$ ($b$ is for "before") to $(p^a, y^a)$ ($a$ is for "after") is given by

$$(1) \qquad V(p^a, y^a - c) = V(p^b, y^b) = u^b,$$

where $V$ is the household's indirect utility function, and $u^b$ and $u^a$ its (ordinal) utility levels before and after the change. Because

$$(2) \qquad V(p, y) = u \leftrightarrow y = E(u, p),$$

where $E$ is the expenditure function corresponding to $V$ [$E(u, p)$ is the minimum expenditure needed to reach the indifference surface indexed by $u$ at prices $p$], equation (1) may be solved for $c$, and

$$(3) \qquad c = y^a - E(u^b, p^a) = E(u^a, p^a) - E(u^b, p^a).$$

Equation (3) shows that $c$ is an exact measure of welfare change for the household: positive when the change makes the household better off, and negative when the household is made worse off. Equation (3) can be rewritten as

$$(4) \qquad c = (y^a - y^b) + (E(u^b, p^b) - E(u^b, p^a)).$$

Equation (4) makes it clear that if prices do not change, the compensating variation is just equal to income change. If a particular project is to be evaluated, then its costs may be assigned to particular households. These costs correspond to the term $(y^a - y^b)$ in equation (4). If the simple sum of compensating variations is used, the aggregate project cost may be subtracted from the aggregate surplus which considers only other effects of the project.

Economic states of affairs can be ranked by summing compensating variations across (at least two) households. Staying within the private-good framework for the moment, and allowing prices to vary across households (because of regional price variations and differing wage rates), an economic change moves the economy from $B = (\pi^b, Y^b)$ to $A = (\pi^a, Y^a)$, where $\pi = (p^1, \ldots, p^h, \ldots, p^H)$ is the vector of prices facing the $H$ households ($p^h$ is an $m$-vector, $m \geq 2$), and $Y = (y_1, \ldots, y_h, \ldots, y_H)$ is the vector of their incomes.

Two price domains are considered. The first allows prices to be household-specific, and it is called Price Domain I ($PD_\text{I}$):

$$(5) \qquad PD_\text{I} = \{\pi = (p^1, \ldots, p^H) \mid p^h \in E^m_{++} \quad \text{for all } h \in \{1, \ldots, H\}\}.$$

Price Domain II ($PD_\text{II}$) restricts the domain by requiring prices to be the same for all households:

$$(6) \qquad PD_\text{II} = \{\pi \in PD_\text{I} \mid p^h = p^k \quad \text{for all } h, k \in \{1, \ldots, H\}\}.$$

A social binary relation, $R$—meaning "is at least as good as"—may be defined using the compensating variations $(c^1, \ldots, c^h, \ldots, c^H)$, with

$$(7) \qquad ARB \leftrightarrow \sum_{h=1}^{H} c_h \geq 0,$$

a strict inequality for strict preference ($P$), and an equality for indifference (I).

This rule is the one most commonly used in cost-benefit analysis and applied welfare economics. In the former, costs are opportunity costs—reductions in total income caused by diversion of productive resources—and producers' surpluses are changes in incomes due to changes in profit. The rule is consistent with the Pareto principle, but does not satisfy Arrow's independence axiom of irrelevant alternatives.[1]

$R$ is not always a complete, reflexive, and transitive ordering,[2] and strict preference may be symmetric, allowing both $APB$ and $BPA$.[3] In Price Domain I ($PD_{\text{I}}$), there is no profile of preferences that results in a social ordering. In $PD_{\text{II}}$, however, with all households facing the same prices, a necessary and sufficient condition on household preferences for $R$ to be an ordering (Roberts) is the existence of an "aggregate consumer" (Gorman) with indirect utility functions

$$V^h(p, y_h) = \gamma(p)y_h + \beta^h(p), \tag{8}$$

$h = 1, \ldots, H$. Each household's preferences must be quasihomothetic (straight line Engel curves) and, for each price vector, all households' Engel curves must be parallel, a condition that is not met by real households.

In addition, Boadway showed that the compensating variation test is not the same as the compensation test.[4] For example, moves with lump-sum transfers between different Pareto-efficient states in a private goods economy typically result in a positive sum of compensating variations. This invalidates the standard efficiency argument for the compensating-variation test because no efficiency gain has occurred.

A more general test for project evaluation is given by

$$ARB \leftrightarrow F(c_1, \ldots, c_H) \geq 0, \tag{9}$$

with a strict inequality for social preference. $R$ is Pareto-inclusive if and only if $F$ is increasing in each of its arguments. $F$ could be chosen to exhibit inequality aversion, for example, with higher weights on the compensating variations of low-income households.

It has been shown, however (Blackorby and Donaldson 1985), that in $PD_{\text{I}}$, there is no choice for $F$ and no profile of household preferences that results in an $R$ that is an ordering. On the restricted price domain $PD_{\text{I}}$, $R$ is an ordering if and only if there is an aggregate consumer and $F$ is a weighted sum of the household surpluses, with

$$F(c_1, \ldots, c_H) = \sum_{h=1}^{H} a_h c_h, \tag{10}$$

$a_h > 0$, $h = 1, \ldots, H$, or any increasing transform. The weights must be attached to household names rather than, say, position in the income distribution. The reason for this is that compensating variations measure welfare change rather than levels—there is no way to distinguish between increases in income from \$10,000 to \$11,000 and from \$110,000 to \$111,000.

It may be possible to justify the use of distributional weights, however. If households can be assigned to income groups such as quintiles and if projects produce little or no changes in group membership, then weights may reflect income-group membership. Further, weights may be assigned using a single-parameter social-welfare function such as the $S$-Gini (Donaldson and Weymark). Unfortunately, an aggregate consumer is still required for consistency.

Turning to the ethical properties of tests based on compensating variations, if prices do not change, compensating variations are equal to income differences [equation (3)], and, when the sum is used,

$$ARB \leftrightarrow \sum_h y_h^a \geq \sum_h y_h^b. \tag{11}$$

This commits investigators to be indifferent to changes in the distribution of income, whatever prices might be. It is possible to ask whether this can be consistent with a Pareto-inclusive Bergson–Samuelson social welfare function, $W_D$. If it is, then the indirect Bergson–Samuelson function, $W_I$, can be written as

$$W_I(\pi, Y) = W_D(V^1(p^1, y_1), \ldots, V^H(p^H, y_H)) = \tilde{W}_I(\pi, \sum_h y_h). \tag{12}$$

This is a special case of a "price independent welfare prescription" (Roberts). Equation (12) cannot be satisfied on $PD_{\text{I}}$, and on $PD_{\text{II}}$, equation (12) can be true if and only if there is an aggregate consumer. In the more general case, with $R$ given by equation (9), consistency with a Pareto-inclusive social welfare function is impossible on $PD_{\text{I}}$ and requires equation (10) and an aggregate consumer on $PD_{\text{II}}$ (Blackorby and Donaldson 1985).

This result contains an important truth: that indifference to the distribution of income is inconsistent with every Pareto-inclusive social welfare function and, therefore, with the Paretian value judgment.[5]

This inconsistency is true in the same-price case ($PD_{II}$) as well as the more general case because real household behavior is not consistent with the aggregate-consumer condition. This result highlights the most important weakness of the compensating variation methodology as well: It cannot take account of the social concern for economic inequality.

Compensating variation tests are often used in situations where some goods—such as parks, highways, and other public and semipublic goods—are unpriced. If a single household's consumption of these goods is the vector $z$, then the compensating variation for a project which moves the household from $(p^b, y^b, z^b)$ to $(p^a, y^a, z^a)$ is

$$c = (y^a - y^b) + [\mathring{E}(u^b, p^b, z^b) - \mathring{E}(u^b, p^a, z^a)], \tag{13}$$

where $\mathring{E}$ is the conditional expenditure function corresponding to the direct utility function $U(x, z)$ (where $x$ is the household's consumption of private goods).

Writing $Z = (z^1, \dots, z^H)$, a project which moves the economy from $B = (\pi^b, Y^b, Z^b)$ to $A = (\pi^a, Y^a, Z^a)$ may be tested with the sum of compensating variations [equation (7)] or a more general function of them [equation (9)]. Again, the resulting social ranking is never an ordering of the alternatives when $PD_I$ is the price domain. In $PD_{II}$, consistency requires household preferences to satisfy

$$\mathring{V}^h(p, y_h, z^h) = \gamma(p)y_h + \beta^h(p, z^h) \tag{14}$$

if consumption of the unpriced goods can be different across households, and

$$\mathring{V}^h(p, y_h, z) = \gamma(p, z)y_h + \beta^h(p, z) \tag{15}$$

if consumption is the same for all—the pure public goods case.[6] Equation (14) is extremely restrictive, implying that consumption of $z$ is independent of income. In addition, because the less restrictive equation (15) implies an aggregate consumer for every value of $z$, it cannot describe real preferences.

A word about equivalent variations is in order. It is true that the equivalent variation performs better than the compensating variation for a single consumer. If the equivalent variation for one project is greater than the equivalent variation for another, the end state of the first project is preferred to the end state of the second, a property that is not shared by the compensating variation (Hause; Pauwels). This property is independent of the equivalent variation's aggregation consistency, however. All the above results for the compensating variation carry over (Blackorby and Donaldson 1985).

So far, the questions of aggregation consistency of compensating variations in intertemporal and uncertain economic environments have been ignored, in the belief that the case against their use in the certainty case is strong enough. There are, however, some difficulties in the single-consumer case in these environments that parallel the many-person certainty case. For example, given perfect capital markets, the discounted sum of consumers' surpluses cannot provide an exact index of welfare change (Blackorby, Donaldson, and Moloney).

The Marshallian consumer's surplus has been excluded from this discussion for two reasons. The first is its well-known problem of path-dependence (Silberberg), and the second is that the standard justification for its use is that it can approximate the compensating variation (Willig). In addition, because aggregate demand curves are used in most cases for the calculation of these surpluses, consistency requires that the necessary and sufficient condition for the existence of an aggregate demand curve—an aggregate consumer—must be satisfied.

## *Money Metric Utility*

Money metric utility (or, simply, the money metric) for a given commodity vector is the minimum income needed at fixed reference prices to pay for a commodity bundle that is at least as good as the one actually consumed.[7] In the private goods case, it is defined by

$$m = M_D(x, p^r) = E(U(x), p^r), \tag{16}$$

where $M_D$ stands for the direct money metric, $x$ is the commodity bundle consumed, $p^r$ is the vector of reference prices, and $U$ is the direct utility function. $M_D$ is invariant to increasing transforms of $U$ (since $E$ is transformed in an "opposite" way). Because $E$ is increasing in $u$, the money metric is a particular normalization of the utility function.[8] In contrast to the compensating variation, it provides an index of the utility level of the members of the household. No attention is paid to household composition. The indirect money metric utility function, $M_I$, is the indirect corresponding to $M_D$, and can be written as

$$m = M_I(p, y, p^r) = E(V(p, y), p^r). \tag{17}$$

Money metrics can be aggregated to evaluate the economic states $B = (\pi^b, Y^b)$ and $A = (\pi^a, Y^a)$ discussed above. Costs of a particular project are included in the vector $Y^a$. A function ($G$) like a social welfare function is used and

$$ARB \leftrightarrow G(m_1^a, \ldots, m_H^a) \geq G(m_1^b, \ldots, m_H^b), \tag{18}$$

where $m_h^b$ and $m_h^a$ are household $h$'s money metric values before and after the project. The simple sum of money metrics is a special case, with

$$ARB \leftrightarrow \sum_{h=1}^{H} m_h^a \geq \sum_{h=1}^{H} m_h^b. \tag{19}$$

Reference prices and actual prices are assumed to be in the same price domain ($PD_{\text{I}}$ or $PD_{\text{II}}$).

$R$ in equation (18) or equation (19) is always an ordering. There are never problems of preference reversals, even with household-specific prices. Further, the function $G$ may exhibit inequality aversion, and different degrees of inequality aversion can be incorporated by employing a single-parameter family of functions such as the $S$-Ginis. This is a great improvement over the compensating-variation test. The ordering $R$, in general, does depend on $\pi^r$, of course, and $\pi^r$ must be fixed once and for all, independently of $\pi^b$ and $\pi^a$. Rules based on money metrics do not satisfy independence of irrelevant alternatives.[9]

But money metric utilities are not without difficulties. In order for rules such as those in equations (18) and (19) to be consistent with usual distributional judgments, direct money metrics must be concave in $x$ and, equivalently (Diewert 1980), indirect money metrics must be concave in $y$ for all $p$. Without concavity, the rules in equations (18) and (19) may recommend that a fixed aggregate bundle of goods or a fixed total income should be given entirely to one person, even when $G$ in equation (18) is quasiconcave.

Quasihomothetic and, therefore, homothetic preferences have concave money metrics. If

$$V(p, y) = \gamma(p)y + \beta(p), \tag{20}$$

the expenditure function is

$$E(u, p) = \frac{u - \beta(p)}{\gamma(p)}, \tag{21}$$

and the money metric is

$$M_I(p, y, p^r) = \frac{\gamma(p)y + \beta(p) - \beta(p^r)}{\gamma(p^r)}, \tag{22}$$

which is affine and, therefore, concave in $y$ for all $p$ and for all $p^r$. The indirect money metric is concave in $y$ for all $p$—or, equivalently, the direct money metric is concave in $x$—for all reference prices, $p^r$, if and only if each household's preferences are quasihomothetic[10] (Blackorby and Donaldson 1988). These preferences may be different for each household, so that an aggregate consumer is not required. There are, however, well-behaved utility functions with concave representations whose money metrics are nonconcave for all choices of $p^r$ (see Blackorby and Donaldson 1988, p. 125). Concavity of money metrics is, therefore, a strong restriction—one that does not correspond to real household preferences.

If the aggregator function $G$ in equation (18) [the simple sum in equation (19)] is chosen independently of reference prices, $\pi^r$ ($p^r$ when prices are the same for all), then the social ordering, $R$, depends on $\pi^r$. It is possible, of course, to choose a different aggregator for each $\pi^r$ so that $R$ is the same for all $p^r$, but in practice this is not done. Suppose that, in a two-household society with the same prices for both households but different preferences,

$$u_1 = V^1(p, y_1) = \frac{y_1}{p_1} \tag{23}$$

and

$$u_2 = V^2(p, y_2) = \frac{y_2}{p_2}. \tag{24}$$

Then, indirect money metrics are

$$M_I(p, y, p^r) = p_1^r\left(\frac{y_1}{p_1}\right) = p_1^r u_1 \tag{25}$$

and

$$M_I(p, y, p^r) = p_2^r\left(\frac{y_2}{p_2}\right) = p_2^r u_2. \tag{26}$$

If the sum is used for social evaluation, the "social welfare function" is

$$p_1^r u_1 + p_2^r u_2, \tag{27}$$

the weighted sum of utilities with the weights equal to reference prices! Since it is unlikely that investigators intend to adopt such arbitrary and capricious ethics, it is useful to find the conditions under which the social ordering, $R$, in equations (18) or (19) is independent of the choice of $\pi^r$.

The general condition, whether prices are allowed to be household-specific or not, is that $R$ is independent of $\pi^r$ if and only if it is consistent with a Pareto-inclusive social welfare function, $W_D$, which in turn must, along with household preferences, be a price-independent welfare prescription (Roberts). That is,

$$\text{BRA} \leftrightarrow W_D(u_1^a, \ldots, u_H^a) \geq W_D(u_1^b, \ldots, u_H^b), \tag{28}$$

where

$$W_D(u_1, \ldots, u_H) = W_D(V^1(p^1, y_1), \ldots, V^H(p^H, y_H)) = \tilde{W}_I(\pi, G(y_1, \ldots, y_H)). \tag{29}$$

A proof of this result is provided in the appendix (theorem 1). It was discovered independently by Laisney and Schmachtenberg.

In the case that prices are the same for all households, equation (28) holds on the smaller price domain $PD_{\mathrm{II}}$. Note that the function $G$ is the same as $G$ in equation (18) so that an aggregate consumer [equation (8)] must exist when the sum of money metrics is employed.

Slivinski has shown that equation (29) holds on the larger domain of household-specific prices, $PD_{\mathrm{I}}$, if and only if: (*a*) households have preferences satisfying

$$V^h(p^h, y_h) = \gamma(p^h)\ln(y_h) + \beta^h(p^h). \tag{30}$$

$h = 1, \ldots, H$ ($\gamma$ is common to all) or any increasing transform; and (*b*) the aggregator function, $G$, is Cobb–Douglas, with

$$G(m_1, \ldots, m_H) = m_1^{\delta_1} \ldots m_H^{\delta_H}, \tag{31}$$

where $\delta_h > 0$ for all $h$. If preferences are required to be globally regular, then $\gamma$ in equation (30) must be independent of $p$ (Blackorby and Donaldson 1989), and preferences must therefore be homothetic (but can differ among households). This result has the immediate consequence that, if the sum of money metrics is employed, and if prices can be household-specific, $R$ cannot be independent of the choice of $\pi^r$.

If the price domain is $PD_{\mathrm{II}}$ so that prices cannot be household-specific, $R$ in equation (19) (when the sum of money metrics is used) is independent of $p^r$ if and only if an aggregate consumer exists [equation (8)]. If $G$ is additively separable, then it is ordinally equivalent to

$$G(m_1, \ldots, m_H) = \sum_h \phi_h(m_h), \tag{32}$$

and, in this case, satisfaction of equation (29) requires

$$\phi_h(m_h) = a_h m_h^\sigma, \tag{33}$$

where $\sigma \neq 0$, or

$$\phi_h(m_h) = a_h \ln(m_h), \tag{34}$$

with

$$V^h(p, y_h) = \gamma(p) y_h^\sigma + \beta^h(p) \tag{35}$$

or any increasing transform of equation (35), or

$$V^h(p, y_h) = \gamma(p)\ln(y_h) + \beta^h(p) \tag{36}$$

or any increasing transform (Roberts). This restriction is weaker than the aggregate-consumer requirement, but it is worth noting that the form of individual utility functions embodies the ethics of $G$.

In the case of unpriced goods, these results are easily extended. If a single household consumes $(x, z)$, where $x$ is a vector of private goods and $z$ is a vector of unpriced goods, the direct and indirect money metrics are

$$m = \mathring{E}(U(x, z), p^r, z^r) \tag{37}$$

and

$$m = \mathring{E}(\mathring{V}(p, y, z), p^r, z^r), \tag{38}$$

where $U$ is the directed utility function, $\mathring{V}$ is the conditional indirect, $\mathring{E}$ is the conditional expenditure function, and $z^r$ is a reference vector of unpriced goods.

Social states are ranked with equation (18) or equation (19), but, in general, the social ordering depends on the choices of reference prices and reference unpriced goods. Two domains for $Z$ and $Z^r$ are possible: $ZD_{\mathrm{I}}$ is the case in which both $z$ and $z^r$ can vary across households; and $ZD_{\mathrm{II}}$ makes $z$ and $z^r$ the same for each household.

Four theorems are possible, but only two are considered—on the domains $PD_{\mathrm{I}}/ZD_{\mathrm{I}}$ and $PD_{\mathrm{II}}/ZD_{\mathrm{II}}$. On both domains, $R$, defined by equation (18), is independent of reference prices and quantities if and only if: (*a*) it is consistent with a Pareto-inclusive social welfare function, $W$; and (*b*) $W$ and household preferences provide a price-independent welfare prescription, with

$$(39)\qquad \begin{aligned} W_D(u_1, \ldots, u_H) &= W_D(\mathring{V}^1(p^1, y^1, z^1), \ldots, \mathring{V}^H(p^H, y^H, z^H)) \\ &= \tilde{W}_I(\pi, G(y_1, \ldots, y_H), Z), \end{aligned}$$

where $Z = (z^1, \ldots, z^H)$. On the unrestricted domain $PD_{\mathrm{I}}/ZD_{\mathrm{I}}$, the conditional indirects must satisfy

$$(40)\qquad \mathring{V}^h(p^h, y^h, z^h) = \gamma(p^h, z^h)\ln(y_h) + \beta^h(p^h, z^h),$$

$h = 1, \ldots, H$, and $G$ must be Cobb–Douglas [equation (31)].

On the restricted domain $PD_{\mathrm{II}}/ZD_{\mathrm{II}}$, additive separability of $G$ [equation (32)] and independence of $R$ from reference prices and quantities hold if and only if equations (33) and (34) are satisfied, and

$$(41)\qquad \mathring{V}^h(p, y_h, z) = \gamma(p, z)^{\sigma}_{h} + \beta^h(p, z)$$

or

$$(42)\qquad \mathring{V}^h(p, y_h, z) = \gamma(p, z)\ln(y_h) + \beta^h(p, z)$$

for all $h$.

These results suggest that although money metric utilities are an improvement over consumers' surpluses, they are still seriously deficient as measures of well-being for social evaluation.

## Aggregation with Interpersonal Comparisons

Compensating variations and money metric utilities are based on preferences alone. If two households with different numbers of people have the same preferences and experience the same change in income and prices, the compensating variation and money metrics are identical.

Measures of well-being that require interpersonal comparisons of well-being between people in different households attempt to account for differences in needs of different types of people (adults, children, disabled people) and to take account of economies of scale in consumption.

In this section, households are described by vectors of characteristics such as age, sex, and special needs of household members. $\alpha_h$ describes household $h$, $n_h$ is the number of people in household $h$, and $n = \sum_h n_h$ is the number of people in the economy.

The utility of each member of a household with characteristics $\alpha$ is

$$(43)\qquad u = U(x, \alpha)$$

when household consumption of marketed private goods is $x$. Because the names of household members may appear, in principle, in $\alpha$, equation (43) is a generalization of the usual model. In practice, however, households are grouped by a smaller set of possible characteristics, and equation (43) represents a significant restriction (see Blackorby and Donaldson 1989, 1991 for a discussion). This model ignores intrahousehold inequality.

The indirect and expenditure functions associated with $U$ are $V$ and $E$, where $u = V(p, y, \alpha)$ is the utility level of a household with characteristics $\alpha$ and household income $y$ facing prices $p$, and $E(u, p, \alpha)$ is the minimum expenditure needed at prices $p$ to bring each member of a household with characteristics $\alpha$ to utility level $u$.

It is assumed that levels of well-being (utility) are comparable interpersonally. That is, statements like

$$(44)\qquad U(\bar{x}, \bar{\alpha}) = U(\hat{x}, \hat{\alpha}),$$

or similar statements with inequalities are assumed to be meaningful when $\bar{\alpha} \neq \hat{\alpha}$. Utilities may be ordinally measurable as long as increasing transforms of $U$ are not $\alpha$-specific. Other measurability properties are possible (see Blackorby and Donaldson 1991 for a discussion).

This model permits social evaluations in which each person in the economy counts. Using a social welfare function defined on all $n$ people, states are compared using

$$W(\underbrace{u_1, \ldots, u_1}_{n_1};\ \underbrace{u_2, \ldots, u_2}_{n_2};\ \ldots;\ \underbrace{u_h, \ldots, u_h}_{n_h}), \tag{45}$$

where $u_h$ is the utility level of each person in household $h$.

The inclusion of unpriced goods ($Z$) in the model is straightforward and results in the direct utility function $\mathring{U}(x, z, \alpha)$, conditional indirect $\mathring{V}(p, y, z, \alpha)$, and conditional expenditure function $\mathring{E}(p, y, z, \alpha)$.

*Extended Money Metrics*

Extended money metrics or equivalent incomes (King) are defined using a reference household with characteristics $\alpha^r$. This is usually chosen to be a single-adult household.

The extended money metric is the minimum expenditure a reference individual must have at reference prices to achieve the level of utility that each member of the household in question has. The direct extended metric in the private goods case is

$$m = \tilde{M}_D\,(x, \alpha, p^r, \alpha^r) = E(U(x, \alpha), p^r, \alpha^r), \tag{46}$$

and the indirect is

$$m = \tilde{M}_I\,(p, y, \alpha, p^r, \alpha^r) = E(V(p, y, \alpha), p^r, \alpha^r). \tag{47}$$

Following King, the same reference prices are used for every household. The indirect money metric is sometimes called equivalent income ($y_e$) because it solves the equation

$$V(p^r, y_e, \alpha^r) = V(p, y, \alpha).[11] \tag{48}$$

The extended money metrics need interpersonal comparisons because $\alpha^r$ and $\alpha$ are different in equation (48).

Extended money metrics extend the advantages of money metrics by allowing investigators to take account of the well-being of each person in the economy [as in equation (45)], economies of scale in household consumption, and differential household needs.

They inherit the problems of ordinary money metrics, however. They are not, in general, concave (because the extended money metric for the reference household is its ordinary money metric), and the social ordering $R$ [determined by equation (18) or equation (19)] is not, in general, independent of $p^r$. Further, the ordering $R$ is not normally independent of the choice of a reference household.

Suppose, for example, that there are two single-person household types facing the same prices, with utility functions

$$V(p, y_1, \alpha_1) = \left(\frac{y_1}{p_1}\right)^{1/2}, \qquad V(p, y_2, \alpha_2) = \left(\frac{y_2}{p_2}\right)^{1/3}. \tag{49}$$

Choosing $\alpha^r = \alpha_1$ yields extended money metrics

$$\tilde{M}_I(p, y_1, \alpha_1, p^r, \alpha_1) = p_1^r\left(\frac{y_1}{p_1}\right), \qquad \tilde{M}_I(p, y_2, \alpha_2, p^r, \alpha_1) = p_1^r\left(\frac{y_2}{p_2}\right)^{2/3}. \tag{50}$$

Choosing $\alpha^r = \alpha_2$ yields

$$\tilde{M}_I(p, y_1, \alpha_1, p^r, \alpha_2) = p_2^r\left(\frac{y_1}{p_1}\right)^{3/2}, \qquad \tilde{M}_I(p, y_2, \alpha_2, p^r, \alpha_2) = p_2^r\left(\frac{y_2}{p_2}\right). \tag{51}$$

Household 1's money metric with $\alpha^r = \alpha_2$ is not concave in $y$ for any $p$, $p^r$. This nonconcavity occurs even though both households have concave, homothetic utility functions. Aggregation in the two cases clearly results in different social orderings.

The conditions for independence of the social ordering of reference prices $p^r$ is less restrictive than in the ordinary money metric case (the above example is independent of $p^r$ when the sum is used). Conditions can be found for the additively separable case, however (see theorems 2 and 3 in the appendix). In that situation, independence requires reference preferences only to satisfy equation (35) or equation (36) with corresponding functional forms for the aggregator.

In the case of unpriced goods, the extended money metrics are

(52) $$m = \tilde{M}_D(x, z, \alpha, p^r, z^r, \alpha^r) = \mathring{E}(\mathring{U}(x, z, \alpha), p^r, z^r, \alpha^r)$$

and

(53) $$m = \tilde{M}_I(p, y, z, \alpha, p^r, z^r, \alpha^r) = \mathring{E}(\mathring{V}(p, y, z, \alpha), p^r, z^r, \alpha^r).$$

Independence of $R$ from $(p^r, z^r)$ requires reference preferences to satisfy equation (41) or equation (42).

Where do the interpersonal comparisons needed for extended money metrics come from?[12] They can be provided formally by choosing a particular utility function for each possible $\alpha$, but this requires a set of judgments that are separate from knowledge of preferences. One possibility is to choose consumption vectors for every utility level and household type such that utilities are believed to be equal. That is, for utility level $u$ and characteristics $\alpha$, $X(u, \alpha)$ makes

(54) $$U(X(u, \alpha), \alpha) = u,$$

for all $\alpha$ and $u$. This provides a way to choose an appropriate normalization of $U$ and, therefore, $V$ and $E$ (or $\mathring{V}$ and $\mathring{E}$).

### *Welfare Ratios*

The welfare ratio for a household with characteristics $\alpha$ is the ratio of household income to the minimum expenditure needed for a reference level of utility $u^r$, or, in the private goods case,

(55) $$T(p, y, \alpha, u^r) = \frac{y}{E(u^r, p, \alpha)}.$$

If $u^r$ is the poverty level of utility, $E(u^r, p, \alpha)$ is the poverty line at prices $p$ for a household with characteristics $\alpha$. The welfare ratio is measured in "poverty lines" and is an index of the well-being of each household member.[13] Welfare ratios can be aggregated to make social evaluations with a social welfare function [see equation (45)].

The welfare ratio is an index of well-being that is not exact—the function $T(\cdot, \cdot, \cdot, u^r)$ is not ordinally equivalent to $V$ unless full homotheticity is satisfied. Full homotheticity requires that for all $\tilde{x}$, $\hat{x}$, $\tilde{\alpha}$, and $\hat{\alpha}$,

(56) $$U(\tilde{x}, \tilde{\alpha}) = U(\hat{x}, \hat{\alpha}) \leftrightarrow U(\lambda\tilde{x}, \tilde{\alpha}) = U(\lambda\hat{x}, \hat{\alpha})$$

for all $\lambda > 0$. For $\tilde{\alpha} = \hat{\alpha}$, equation (56) implies that individual preferences are homothetic and, when $\tilde{\alpha} \neq \hat{\alpha}$, equation (56) requires that a common scaling of consumption bundles preserves utility equality. In this special case, welfare ratios are proportional to extended money metrics.

Since full homotheticity is not normally satisfied, welfare ratios must be regarded as inexact indexes of well-being: approximations based on homothetic preferences which, in turn, are based on reference indifference surfaces.

Although they are inexact, welfare ratios have some attractive properties: first, they take account of family size and economies of scale through $E(u^r, p, \alpha)$; second, they are always concave ($T$ is linear in $y$ for all $p, \alpha, u^r$); third, they are sensitive to (possibly household-specific) price changes through $E(u^r, \cdot, \alpha)$; and fourth, they economize on interpersonal comparisons—all that is needed is normalization of $U$ at the reference level of utility. The requisite interhousehold normalization can be done by finding consumption vectors $X(u^r, \alpha)$, one for each household type, such that equation (54) is satisfied with $u = u^r$ only. These commodity bundles could be provided by poverty researchers.

When preferences are homothetic and, as a consequence, welfare ratios are exact, the social ordering is independent of $u^r$ if and only if $W$ in equation (45) is homothetic (Blackorby and Donaldson 1987). For that reason, a homothetic social welfare function is a reasonable choice.

Welfare ratios can be extended to include unpriced goods, with

(57) $$T(p, y, z, \alpha, u^r) = \frac{y}{\mathring{E}(u^r, p, z, \alpha)}.$$

If $u^r$ is the poverty utility, then the denominator is the poverty line, and it depends on $z$ as well as $p$.

Although welfare ratios are a little rough, they are fairly easily calculated, and their linearity in income permits a clear interpretation of the distributional judgments in social evaluations. Given the inadequacy of available data in so many applications of welfare economics, they might prove attractive.

## Utility Aggregation Using Equivalence Scales

None of the money measures of well-being described above are very satisfactory. Compensating (and equivalent) variations are the worst of all, but, although money metrics and welfare ratios have advantages, they retain some real problems. Of course, some simplification of reality is inevitable in applications of welfare economics, but it is important to have a methodology that can deal with the essential elements—price change, preference diversity, differences in household types, and flexible ethics.

Equivalence scales may be able to provide a useful methodology for making interpersonal comparisons and can be used to estimate poverty lines, compute the denominators of welfare ratios, and provide a methodology for utility-based welfare analysis. Equivalence scales are either commodity-independent or commodity-specific.[14] The discussion here is confined to the commodity-independent case.

In a private-goods economy, the number of adult equivalents in a household with characteristics $\alpha$ facing prices $p$ with income $y$ is $d$, where

$$u = V(p, y, \alpha) = V\left(p, \frac{y}{d}, \alpha^r\right). \tag{58}$$

If $y$ is \$60,000 and $d = 3$ for a family of four, then the household is equivalent, in terms of well-being, to four reference single adults with \$20,000 each. The magnitude of $d$ reflects both economies of scale in consumption and special needs.

Equation (58) can be solved for $d$, with

$$d = D(u, p, \alpha) = \frac{E(u, p, \alpha)}{E(u, p, \alpha^r)}, \tag{59}$$

where $d$ is the ratio of the cost to the household of the actual utility level to the cost for a reference person. Because $u$ in equation (59) is normally unobservable, empirical tractability demands that $D$ be independent of $u$. It is independent of $u$ if and only if a condition called equivalence-scale exactness (ESE) is satisfied, in which case $E$ is multiplicatively separable into a function of $u$ and $p$, and a function of $p$ and $\alpha$ (Blackorby and Donaldson 1989, 1991; Lewbel 1988a, b). Given ESE, the utility-independent equivalence scale $\Delta$ is

$$d = \Delta(p, \alpha) = \frac{E^1(u, p)E^2(p, \alpha)}{E^1(u, p)E^2(p, \alpha^r)} = \frac{E^2(p, \alpha)}{E^2(p, \alpha^r)}. \tag{60}$$

In addition, $E$ can be written as

$$E(u, p, \alpha) = E^r(u, p)\Delta(p, \alpha), \tag{61}$$

where $E^r$ is the reference person's expenditure function $E(\cdot, \cdot, \alpha^r)$.

Equivalence-scale exactness is equivalent to a condition on the indirect utility function $V$ called income-ratio comparability (IRC) (Blackorby and Donaldson 1989, 1991). It requires that if any household and the reference household experience utility equality at the same prices, then common income scaling preserves that equality. Formally, IRC requires (for all $\alpha$, $p$, $\tilde{y}$, and $\hat{y}$) that

$$V(p, \tilde{y}, \alpha) = V(p, \hat{y}, \alpha^r) \leftrightarrow V(p, \lambda\tilde{y}, \alpha) = V(p, \lambda\hat{y}, \alpha^r), \tag{62}$$

for all $\lambda > 0$. IRC is weaker than full homotheticity.

IRC can be used as an axiom to justify ESE. IRC/ESE has consequences for preferences—reference preferences can be chosen freely, but $V^r$ and $\Delta$ together determine $V$. As long as reference preferences do not satisfy the log-linear form

$$V^r(p, y) = \gamma(p)\ln(y) + \beta(p) \tag{63}$$

or any increasing transform, $\Delta$ can be identified from behavior alone (theorems 6.1 and 6.2, Blackorby and Donaldson 1989). If $V$ is required to be globally regular, then reference preferences need only be nonhomothetic.

Phipps has estimated equivalence scales for Canada from demand behavior. She specified a translog functional form for reference preferences and a Cobb–Douglas form for $\Delta$ (necessarily homogeneous of degree zero) and estimated both $V^r$ and $\Delta$.[15]

Because the utility of each member of household $h$ is

**Table 1. Summary of Aggregation Results**

| | Test Statistic Based on | | | |
|---|---|---|---|---|
| | Willingness to Pay (Compensating Variations) | Money Metrics | Extended Money Metrics | Welfare Ratios |
| Consistent social preferences in $PD_I/ZD_I$ | No | Yes | Yes | Yes |
| Consistent social preferences in $PD_{II}/ZD_{II}$ | Only in special cases | Yes | Yes | Yes |
| Inequality aversion possible with consistency | No | Yes | Yes | Yes |
| Concave representation of preferences | N.A. | Only in special cases | Only in special cases | Concave but not exact |
| Ordering independent of reference prices and quantities | N.A. | Only in special cases | Only in special cases | N.A. |

$$u_h = V(p^h, y_h, \alpha_h) = V^r\left(p^h, \frac{y_h}{\Delta(p^h, \alpha_h)}\right), \tag{64}$$

social evaluations can be made using equation (45).

In general, the social ordering is not independent of the representation of reference preferences chosen—increasing transforms of $V^r$ will normally change the social orderings if $W$ is unchanged. That means that some way to normalize $V^r$ must be found. This has, however, been done—by Phipps in the commodity-independent case, and by Jorgenson and Slesnick (1984, 1987) in the commodity-specific case.

This methodology has many advantages over the aggregation of money measures of well-being: there are no reference prices to affect the social ordering; the utilities in equation (64) (and, therefore, the social ordering) are unaffected by the choice of a reference household; there are no problems of concavity as long as the utility function $U$ is concave in $x$ for all $\alpha$; inequality aversion is easily incorporated into social evaluations; and household-specific prices are easily handled.[16] The only restriction necessary is that IRC, and its consequences for preferences [equation (61)], must be assumed as a maintained hypothesis.

The model can be extended to cover unpriced goods ($z$); $z$ must, in the general case, appear in the reference utility function $V^r$ and in the equivalence scale $\Delta$. It is possible, however, for $\Delta$ to be independent of $z$, and that restriction might make implementation easier.

## Conclusion

Table 1 summarizes the main results for money measures of well-being. Although any practical procedure for social evaluation is necessarily theoretically imperfect, several judgments can be made. Social evaluations based on aggregates of household willingness to pay (compensating and equivalent variations) are, from a theoretical standpoint, the least satisfactory of all the methods discussed above. Because the social binary relations lack ordinary rationality properties (asymmetric preference, for example), and because these tests are not efficiency tests (even in first-best environments), it is difficult to claim that applied welfare economics of this sort is a sensible exercise. This is true even if income effects are small and social preference reversals or intransitivities rarely occur. What the theoretical results show is that there is no consistent ranking of alternative states of affairs based on these statistics. It follows, therefore, that all particular rankings are meaningless. They cannot be regarded as approximating an underlying ordering because there is nothing to approximate. Further, because these procedures cannot accommodate inequality aversion in a sensible way, the case against them is strengthened.

Such a strong case cannot be made against the use of money metrics and extended money metrics. There are problems of nonconcavity and sensitivity to reference prices to be sure, but economic states are always ordered, and ethical flexibility is possible. Investigators should be aware, however, that the choice of reference prices may have nontrivial ethical consequences. Welfare ratios and their aggregates avoid the problems of money metrics by approximating preferences as homothetic. This may be an acceptable price to pay, especially if data and economic projections are less than perfect.

A good case can be made for the view that aggregating utilities is, when possible, better than aggregating money measures of well-being. However, as the section on equivalence scales makes clear, practical procedures demand models that make somewhat unrealistic abstractions from the real world.

The best way to test the usefulness of different social-evaluation methods is, then, in the laboratory of real-world applications of welfare economics. Because the prevailing standard methodology is so defective from a theoretical point of view, other methods or combinations of methods deserve the serious attention of practitioners.

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## Notes

[1] Suppose two households face common prices that a project changes from $p^b = (10, 10)$ to $p^a = (5, 20)$ leaving incomes unchanged at (20, 20). If preferences are represented by $V^1(p, y_1) = y_1/p_1$ and $V_2(p, y_2) = y_2/p_2$, compensating variations are $c_1 = 10$ and $c_2 = -20$, so $BPA$. Suppose $V^2$ is replaced with $\hat{V}^2$, where $\hat{V}^2(p, y_2) = y_2/(p_1 + p_2)$. Household 2 is still made worse off by the change, but now $\hat{c}_2 = -5$, changing the sum to $+5$ and social preference to $APB$. Independence requires the ordering of any pair of alternatives to be the same for all preference profiles in which the individual orderings of the pair are the same.

[2] The term "ordering" is used to indicate a binary no-worse-than relation that is reflexive, complete, and transitive.

[3] Consider two households in a one-good economy. Each household's indirect utility function is $V(p, y) = y/p$. If household 1's price and income moves from (5, 5) to (20, 40), its compensating variation is $c_1 = 20$ with $\hat{c}_1 = -5$ for the reverse move. If household 2 moves from (20, 40) to (10, 10), its compensating variations are $c_2 = -10$ with $\hat{c}_2 = 20$ for the reverse move, yielding $c_1 + c_2 = 10$ and $\hat{c}_1 + \hat{c}_2 = 15$, a preference reversal.

[4] The test based on the sum of equivalent variations has a different relationship to the compensation test but, again, the two are not equivalent (Blackorby and Donaldson 1990). The compensation test for a potential Pareto improvement ranks social states with a counterfactual—compensation that could be paid but is not. If compensation is paid, there is, of course, no problem.

[5] The Paretian value judgment is just the ordinary Pareto principle, which makes social changes to which each household is indifferent a matter of social indifference, and changes which at least one household prefers and all weakly prefer socially preferred. This is the common element in the set of all Pareto-inclusive social welfare functions.

[6] This result follows from a trivial generalization of theorems 1 and 2 in Blackorby and Donaldson (1985).

[7] See Weymark for a theoretical discussion. Both McKenzie and Samuelson introduced money metric utilities, and their use in applied welfare economics has been advocated by Diewert (1983) and King, among others.

[8] The household's utility function may be defined by maximizing a "social welfare function" of its members' utilities with lump-sum transfers of goods and services. An aggregate consumer is not needed.

[9] Suppose $V^1(p, y_1) = y_1/p_1$, $V^2(p, y_2) = y_2/p_2$, incomes are 10 for each household, $p^r = (1, 3)$ for both, and a project changes common prices from $p^b = (10, 10)$ to $p^a = (5, 20)$. The money metrics are 1 and 3 for $B$, and 2 and 3/2 for $A$. Using equation (19), $BPA$. If $V^2$ is changed to $\tilde{V}^2(p, y_2) = y_2/(p_1 + p_2)$, 2 still prefers $B$ to $A$, but 2's money metrics are now 2 for $B$ and 8/5 for $A$, so the sums become 3 for $B$ and 18/5 for $A$, reversing social preference to $APB$.

[10] Quasihomothetic preferences are not globally regular. For global regularity, preferences must be homothetic.

[11] It is assumed that a solution to this equation always exists.

[12] Buccola discusses this question.

[13] See Blackorby and Donaldson (1987); Lampman; Morgan, Meyers, and Baldwin; Watts; and Wolfson.

[14] See Barten; Blackorby and Donaldson (1989, 1991); Blundell and Lewbel; Browning; Deaton and Muellbauer; Fisher; Grunau; Jorgenson and Slesnick (1984, 1987); Lewbel (1985, 1988a, b); Muellbauer (1974, 1977); Nelson; Pollak and Wales (1979, 1981); and Ray.

[15] Phipps' estimates are reasonable: they lie in the right ranges and are sensitive to prices in the expected way.

[16] The version of independence of irrelevant alternatives that allows for interpersonal comparisons is satisfied: the ranking of any two states depends only on utilities in those states.

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## Appendix

This appendix provides proofs of several results about the aggregation of money metrics. Utility functions are assumed to be continuous in $x$ or in $(p, y)$ and the function $G$ in equation (18) is assumed to be continuous and increasing. Only the private-goods case is considered, and results are proved both for the domain of household-specific prices ($PD_{\mathrm{I}}$) and for the domain in which prices are the same for all households ($PD_{\mathrm{II}}$). For the aggregation of ordinary money metrics, $\pi$ and $\pi^r$ are assumed to belong to the same price domain, and all incomes are positive.

*Theorem 1*

The ordering $R$, given by equation (18), is independent of $\pi^r$ for all $\pi^r \in PD_{\mathrm{I}}$ (resp. $PD_{\mathrm{II}}$): (*a*) if and only if $R$ is consistent with a Pareto-inclusive (increasing) continuous social welfare function $W_D$, with

$$ARB \leftrightarrow W_D(u_1^a, \ldots, u_H^a) \geq W_D(u_1^b, \ldots, u_H^b); \tag{A1}$$

and (*b*) if and only if $W_D$ and individual preferences provide a price-independent welfare prescription (Roberts), with

$$W_D(V^1(p^1, y_1), \ldots, V^H(p^H, y_H)) = \tilde{W}_I(\pi, G(y_1, \ldots, y_H)), \tag{A2}$$

with $\tilde{W}_I$ increasing in its second argument for all $\pi \in PD_{\mathrm{I}}$ ($PD_{\mathrm{II}}$) and $G$ homothetic.

*Proof.* (*a*) Equation (18) may be rewritten as

$$ARB \leftrightarrow G(E^1(u_1^a, p^{1r}), \ldots, E^H(u_H^a, p^{Hr})) \geq G(E^1(u_1^b, p^{1r}), \ldots, E^H(u_H^b, p^{Hr})). \tag{A3}$$

If $R$ is independent of $\pi^r$, $\pi^r$ may be set arbitrarily, say to $((1, \ldots, 1), \ldots, (1, \ldots, 1)) = (1_m, \ldots, 1_m)$, so that

$$ARB \leftrightarrow W_D(u_1^a, \ldots, u_H^a) \geq W_D(u_1^b, \ldots, u_H^b), \tag{A4}$$

where

$$W(u_1, \ldots, u_H) := G(E^1(u_1, 1_m), \ldots, E^H(u_H, 1_m)). \tag{A5}$$

Increasingness and continuity of $W$ follow from the properties of $G$ and $(E^1, \ldots, E^H)$. Satisfaction of equation (A1) is clearly sufficient for independence of $R$ from $\pi^r$.

(*b*) Now let $\pi = \pi^a = \pi^b$. If $R$ is independent of $\pi^r$, $\pi^r$ may be set equal to $\pi$ (on either domain). This makes $M_I^h(p^h, y_h, p^{hr}) = y_h$, and equation (18) becomes

$$(\pi, y_1^a, \ldots, y_H^a)R(\pi, y_1^b, \ldots, y_H^b) \leftrightarrow G(y_1^a, \ldots, y_H^a) \geq G(y_1^b, \ldots, y_H^b), \tag{A6}$$

which means that $y$ is strictly separable from $\pi$ in $W(V^1(p^1, y_1), \ldots, V^H(p^H, y_H))$ ($\pi \in PD_{\mathrm{I}}$ or $PD_{\mathrm{II}}$), and equation (A2) results. Roberts showed that, because of homogeneity of degree zero of each $V^h$ in $(p, y)$, $G$ must be homothetic.

Sufficiency can be shown as follows. Suppose equation (A2) is satisfied. Then

$$\begin{aligned} W_D(u) = W_D(u_1, \ldots, u_H) &= W_D(V^1(p^1, y_1), \ldots, V^H(p^H, y_H)) \\ &= \tilde{W}_I(\pi, G(E^1(u_1, p^1), \ldots, E^H(u_H, p^H))), \end{aligned} \tag{A7}$$

for all $\pi \in PD_{\mathrm{I}}$ ($PD_{\mathrm{II}}$). Equation (A3) implies that

$$\begin{aligned} ARB &\leftrightarrow G(E^1(u_1^a, p^{1r}), \ldots, E^H(u_H^a, p^{Hr})) \geq G(E^1(u_1^b, p^{1r}), \ldots, E^H(u_H^b, p^{Hr})) \\ &\leftrightarrow \tilde{W}_I(\pi^r, G(E^1(u_1^a, p^{1r}), \ldots, E^H(u_H^a, p^{Hr})) \geq \tilde{W}_I(\pi^r, G(E^1(u_1^b, p^{1r}), \ldots, E^H(u_H^b, p^{Hr})) \\ &\leftrightarrow W_D(u_1^a, \ldots, u_H^a) \geq W_D(u_1^b, \ldots, u_H^b), \end{aligned} \tag{A8}$$

which makes $R$ independent of $\pi^r$.

In theorems 2 and 3, the aggregation of extended money metrics is considered. There are two domains for actual prices, $PD_{\mathrm{I}}$ and $PD_{\mathrm{II}}$, but reference prices are restricted to $PD_{\mathrm{II}}$. This is reasonable because extended money metrics are measured in dollars needed by the reference single adult.

*Theorem 2*

$R$, defined by

$$\begin{aligned} ARB \leftrightarrow\ & G(\tilde{M}_I(p^{1a}, y_1^a, \alpha_1, p^r, \alpha^r), \ldots, \tilde{M}_I(p^{Ha}, y_H^a, \alpha_H, p^r, \alpha^r)) \\ \geq\ & G(\tilde{M}_I(p^{1b}, y_1^b, \alpha_1, p^r, \alpha^r), \ldots, \tilde{M}_I(p^{Hb}, y_H^b, \alpha_H, p^r, \alpha^r)), \end{aligned} \tag{A9}$$

is independent of $p^r$ for all $\pi$ in $PD_{\mathrm{I}}$ ($PD_{\mathrm{II}}$) if and only if $R$ is consistent with a continuous, Pareto-inclusive (increasing) social welfare function $W$ [equation (A1)].

The proof of theorem 2 is identical to the proof of theorem 1(a), and is omitted.

If $G$, the aggregator function for extended money metrics, is symmetric and additively separable, then necessary and sufficient conditions for $R$ to be independent of $p^r$ are given by theorem 3.

*Theorem 3*

If there is one person in each household, the ordering $R$, given by

(A10) $$ARB \leftrightarrow \sum_h \phi(\tilde{M}_I(p^{ha}, y_h^a, p^r, \alpha^r)) \geq \sum_h \phi(\tilde{M}_I(p^{hb}, y_h^b, p^r, \alpha^r)),$$

with $\phi$ continuous and increasing, is independent of $p^r$ on price domains $PD_I$ and $PD_{II}$ if and only if reference preferences are

(A11) $$V(p, y, \alpha^r) = \gamma(p)y^\sigma + \beta(p),$$

where $\sigma \neq 0$, or

(A12) $$V(p, y, \alpha^r) = \gamma(p)\ln(y) + \beta(p),$$

or any increasing transform. When equation (A11) is satisfied, $\phi$ is

(A13) $$\phi(t) = at^\sigma + b,$$

($a > 0$ for $\sigma > 0$, $a < 0$ for $\sigma < 0$), and when equation (A12) is satisfied, $\phi$ is

(A14) $$\phi(t) = a\ln(t) + b.$$

*Proof.* Equation (A10) implies that

(A15) $$ARB \leftrightarrow \sum \phi(E^r(u_h^a, p^r)) \geq \sum \phi(E^r(u_h^b, p^r)),$$

where $E^r(u, p) := E(u, p, \alpha^r)$. If $R$ is independent of $p^r$, $p^r$ can be chosen arbitrarily to get

(A16) $$ARB \leftrightarrow \sum \phi(u_h^a) \geq \sum \phi(u_h^b),$$

with $\phi$ increasing and continuous. It follows that

(A17) $$\sum \phi(E^r(u_h, p^r)) = F(\sum \phi(u_h), p^r),$$

where $F$ is continuous and increasing in its first argument. Defining $z_h := \phi(u_h)$ and $f(z_h, p^r) := \phi(E^r(\phi^{-1}(z_h), p^r) = \phi(E^r(u_h, p^r))$, equation (A17) can be written as

(A18) $$F(\sum z_h, p^r) = \sum f(z_h, p^r).$$

For each $p^r$, this is a Pexider equation (Eichhorn) whose solution is

(A19) $$f(z_h, p^r) = \bar{\gamma}(p^r)z_h + \bar{\beta}(p^r).$$

It follows that

(A20) $$E^r(u, p) = \phi^{-1}(\bar{\gamma}(p)\gamma(u) + \bar{\beta}(p)),$$

and

(A21) $$V^r(p, y) = \phi^{-1}(\gamma(p)\phi(y) + \beta(p)),$$

where $\gamma(p) := 1/\bar{\gamma}(p)$ and $\beta(p) := -\bar{\beta}(p)/\bar{\gamma}(p)$. $V^r$ must be homogeneous of degree zero in $(p, y)$ and that requires, for all $\lambda > 0$,

(A22) $$\gamma(\lambda p)\phi(\lambda y) + \beta(\lambda p) = \gamma(p)\phi(y) + \beta(p),$$

or

(A23) $$\phi(\lambda y) = \frac{\gamma(p)}{\phi(\lambda p)}\phi(y) + \frac{\beta(p)}{\gamma(\lambda, p)} = \hat{\gamma}(\lambda, p)\phi(y) + \hat{\beta}(\lambda, p).$$

For each $p$, this is a functional equation whose solution is (Eichhorn, theorem 2.7.3, p. 42) equations (A13) or (A14), which, using equation (A21), imply equations (A11) and (A12). Sufficiency is easily shown.

It makes little ethical sense to allow $G$ to be nonsymmetric. Theorem 3, however, can be extended to this case, with the same symmetric result. Theorem 3 can easily be specialized to the case where $G$ is the simple sum of extended money metrics. In that case, reference preferences must be

(A24) $$V(p, y, \alpha^r) = \gamma(p)y + \beta(p)$$

or any increasing transform.

If the price domain for $\pi^r$ is extended to $PD_I$, then $\bar{\gamma}(p^{hr})$ in equation (A19) must be independent of $h$, and so $\gamma$ in equation (A11) cannot depend on $p$. This leaves only equation (A12) which requires a Cobb-Douglas $G$. If $G$ is the simple sum, an impossibility results.

If, in the case that all households contain one member, $R$ is required to be independent of the choice of $\alpha^r$, then each household's preferences must satisfy equation (A11) or equation (A12), but preferences may differ among households.