Journal of Agricultural and Resource Economics, 18(1): 135–140

The Robustness of Single Index Models in Crop Markets: A Multiple Index Model Test: Reply

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Introduction

I am happy that Robert Collins' inability to distinguish between my (Blank 1991) empirical analysis of the single index model (SIM) and the evaluation of the capital asset pricing model (CAPM) that he asserts my article to be gives me an opportunity to further highlight some of the limitations of the SIM when it is applied in a farm planning context. As noted by Frankfurter, by McDonald and Lee, and by Stambaugh, there are empirical shortcomings of the SIM when it is applied in securities analysis, but as my paper noted, along with papers by Gempesaw et al., Blank (1990), and Amegbeto and Featherstone, empirical problems become even more troublesome when the SIM is used in crop selection decision making. My paper empirically tests some hypotheses derived by mathematical manipulation of the SIM equation or taken from the securities market literature, as explained below. The paper ". . . does not claim to test the validity of the SIM generally, it considers only the robustness of results generated when the SIM is applied in geographically disaggregated agricultural markets" (Blank 1991, p. 261). The SIM hypothesis test results indicate that in some cases ". . . an alternate model, such as a MIM [multiple index model], may be needed to deal with the heterogeneous nature of crop markets" (Blank 1991, p. 266). Before I discuss the hypotheses and the flaws in Collins' comments, I first offer a conceptual overview which may explain why there is so much confusion about the SIM.

The SIM Out of Context

The SIM, by design, is a tool for empirical work in portfolio selection. Unfortunately, we take the SIM out of its intended context when using it in farm planning (crop selection); therefore, we must recognize that its interpretation changes. The SIM is a simple, linear regression model developed in a securities market context that should not be expected to work as well in agricultural product markets. Returns from individual crop markets and physical/financial asset returns are two different concepts. Investment returns are a function of a company's current and expected future overall financial performance and, in the case of stocks, the company's dividend policy. Securities are traded internationally through efficient electronic markets by investors who compare returns across investment types. Also, most securities are not consumable;[1] the same assets can be traded between investors indefinitely. Crop returns, on the other hand, are a function of current local supply and demand conditions for that consumable product only and the grower's financial structure. It is inappropriate to compare returns from investments in companies or assets to the gross profits from a company's individual products, yet this is what is being done if we

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This is Giannini Foundation Research Paper No. 1041.

expect SIM results from the crop markets to conform to the SIM's interpretation derived from its use in securities analysis.

Does being out of context make the SIM useless as a farm planning tool? No, not if the empirical estimation and interpretation is still valid in the specific case. This means that model specification and the assumptions implied by that specification must lead to results which are a reasonably good approximation of "optimal" results for the portfolio being considered.

The two primary assumptions of the SIM are suspect, a priori, when empirically assessing agricultural product markets. Haugen (p. 153) states the main assumption as:

> Essentially, the single-index model assumes security returns are correlated for only one reason. Each security is assumed to respond, in some cases more and in other cases less, to the pull of a single index which is usually taken to be the market portfolio.

This gives the standard SIM a simple, linear regression specification:

$$R_i = \alpha_i + \beta_i(R_m) + \epsilon_i, \tag{1}$$

where R_i is the return on asset i, R_m is the return on the market index, α_i is a constant, and ϵ_i is an error term. Beta, β, is a standard measure used to indicate the relationship between an asset (or portfolio) and the index, R_m. Beta is also referred to as a measure of an asset's systematic risk relative to the index. The SIM is estimated using time series techniques with historical data. Whereas "macro events" affect the entire market and the asset (to the extent measured by beta), "micro events" affect individual firms only, causing residuals, ϵ_i, in the SIM output. The second major assumption of the SIM is that the residuals of different assets are not correlated, meaning that no other factors significantly influence the variance in asset (or portfolio) returns. Therefore, the total variance in returns from equation (1) can be expressed as

$$\sigma^2(R_i) = [\beta_i\sigma(R_m)]^2 + \sigma^2(\epsilon_i), \tag{2}$$

where the first component is systematic risk, and the asset's unsystematic (diversifiable) risk is the second component $[\sigma^2(\epsilon_i)]$.

What Haugen (pp. 158–61) calls "industry-type events" cause the assumption of *no* correlation in the residuals of different securities to be invalid. In agriculture, these events are "product" or "market" events which are unique to each product, causing the *extent* and the *sign* (+/−) of residual correlations to vary. Thus, we cannot further assume that all SIM error in estimates of residual variance are biased in the same direction such that portfolio composition is not significantly affected. It has been found in securities markets that correlation in residuals is almost always positive (Black, Jensen, and Scholes), indicating that residual variance is underestimated by the SIM. Due to the consistently positive sign of securities correlations, this SIM bias has not been found to significantly alter the composition of recommended portfolios. However, using a MIM may significantly reduce the correlation in residuals and, in applications involving agricultural products where negative residual correlations are common, generate betas that could change in either direction, compared to the SIM beta, thus changing the composition of the "optimal" portfolio.

Collins' View

Two areas of misunderstanding are evident in Collins' comments: the relationship between crop returns and the SIM beta, and that MIM betas may be a better normative tool than SIM betas. Collins' summary of the SIM and CAPM story focuses on the SIM's role as an approximation of Markowitz's full covariance model. He dwells on risk, not giving return the attention it deserves as a normative criterion. Also, he gives little mention to the empirics of MIMs. As I note below, both of these issues must be understood to judge the contribution of my paper.

To begin, Collins makes the startling assertion that there is no reason for crop returns to be related to their degree of risk (as measured by the SIM beta). Of course the relationship will not be precise for individual crops, but if the SIM (or any other planning tool) does not produce measures of risk (betas) that, in the whole, are positively related to returns, it is useless to decision makers. All risk-averse managers consider both *risk* and *return* when making investment decisions. The tradeoff between risk/return and its importance in decision making underlies the expected value-variance (E-V) analysis used throughout agriculture (see Robison and Barry, for example). Therefore, testing the hypothesis that such a risk/return relationship exists in a tool's output would logically be a first step in any assessment of a tool's value to decision makers. Arguing, as Collins does, that betas and returns are not related means that market efficiency over time is irrelevant; crops will be plotted randomly in E-V space. In fact, market efficiency is a necessary, although not sufficient, condition for the use of the SIM (or any planning tool based on historical data) in making normative rankings of future alternatives.

Collins repeatedly overlooks the all-important risk/return relationship necessary in decision making. Farmers must consider *both* factors in the tradeoff, not just risk. Yet, an accurate beta is necessary for selection of an efficient crop portfolio. Collins notes that the SIM is an "... approximation of a Markowitz model for formulating approximately risk efficient farm plans"; however, the point that "risk efficiency" refers to the least risky portfolio which provides *a given level of return* is never made. This omission is significant. The SIM is a means of quantifying risk, but that information is useful in a normative sense only if it is used with information on crop returns. In my paper, the focus on risk/return relationships is an effort to assess whether SIM output can, in fact, be used with confidence to identify efficient farm plans and should in no way be interpreted as an analysis of CAPM equilibrium crop returns. Such an interpretation is nonsense.

I agree with Collins that CAPM equilibrium should not be expected in crop returns, although he appears confused about our agreement. Collins notes that my "... statistical results clearly support ..." the assertion that CAPM equilibrium should not be expected in crop returns in a given area, even though his paragraph begins by charging that my paper "... fails to recognize ..." that point.

Also, my study reports on empirical tests of the usefulness of the SIM as an approximation of the Markowitz portfolio model. In my experience, the most common empirical use of the SIM in agriculture is in evaluating marginal changes to a grower's current crop rotation. In this case, the beta and returns from alternate crops are compared to those of individual crops currently being produced. Thus, the absolute and relative sizes of individual crop betas are very relevant in decision making.

The second point of confusion is evident when Collins says that we should "... not expect crop returns for a particular farmer or region to depend only on their systematic risk." This is exactly my point! I argue that it may be more appropriate to expand the SIM into a MIM for use in some agricultural markets. My paper illustrates this point with empirical results.

To serve as a decision tool, SIM betas must be found to give a reasonably close approximation of a crop's risk, both in absolute and relative terms. Unfortunately, SIM betas for crops are often statistically insignificant, as shown in the tables of results in my article. If MIM betas are significantly different than SIM betas for a group of crops or if they change the relative ranking of crops in terms of risk, then farmers would make better decisions using a MIM.

There is reason to believe that the SIM may not lead to an efficient approximation of the optimal portfolio in agriculture, even though it works well in securities markets. This position is based on the discussion, presented earlier, of variation in the extent and sign of correlation in residuals of different crops. The hypotheses I tested, discussed next, offer empirical insights into the nature and extent of differences between SIM analyses of gross profits on particular crops and the returns on securities.

The Hypotheses

In my paper, I asked some questions which arose in my empirical work while trying to apply the SIM on farms. The net effect of these hypothesis tests is the implication that a MIM may be required in some agricultural product analyses to overcome empirical shortcomings of the SIM in those cases.

Hypothesis (*a*) comes from the discussion earlier. Rational decision making requires that farmers balance risk and return. To test whether there is a relationship between returns and the SIM measure of risk (beta), this hypothesis was derived from the basic SIM equation. As noted earlier, without some positive relationship between R_i and β_i, the SIM would be useless as a planning tool. The nature of the relationship could have been hypothesized as being either linear, as implied by the simple SIM regression model, or nonlinear, like the E-V literature suggests. I made the linear specification my "straw man," but tested for a concave relationship as well. The empirical results showed that a statistically significant positive relationship did exist between R_i and β_i and that the relationship may be nonlinear [equation (8), Blank 1991, p. 264].

Hypothesis (*b*) was designed to find what R_i would be for a crop with no risk (according to its beta), regardless of what returns were generated by the market. In my study, the β coefficients were adjusted to reflect risk in required returns ($/acre) by subtracting the risk-free rate from equation (1), giving

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i, \quad (3)$$

which is equation (4) in my article and the specification used in the empirical estimation.[2] The SIM in equation (3) appears similar to, but is quite different than, the standard CAPM.[3] In this equation, if $\beta_i = 0$ and $E(\alpha_i, \epsilon_i) = 0$, then $E(R_i) = R_f$ by simple mathematical manipulation of this version of the SIM. Obviously, this is a joint test of the linearity of the county E-V and of the $E(\alpha_i, \epsilon_i) = 0$ conditions of the SIM. In this specification of the SIM, the hypothesis test enables estimation of the intercept value. Collins and Barry implicitly propose this hypothesis in their figure 2, which shows that the efficient frontier would have an intercept of R_f if risk-free leasing transforms the nonlinear frontier into a linear opportunity set. In an article appearing after mine, Turvey, Baker, and Weersink present various separation theorems which support this hypothesis and empirical evidence which shows that operating risk and cash-rent determination are related. I note in my article that leasing will probably occur at many rates, not just R_f, due to information asymmetry among farmers. And finally, it is not uncommon for crops to have negative covariances with one another; thus betas of zero or less are possible (see Blank 1991, and Amegbeto and Featherstone for examples of negative betas). Therefore, it is important to evaluate whether the risk/return relationship becomes distorted at what a model says are supposed to be low or negative levels of risk. Hypothesis (*b*) is part of such an evaluation.

In his comments about hypothesis (*c*), Collins appears confused again about what is being tested and, as a result, agrees with the arguments I put forward. I am not concerned with risk totals, as Collins appears to be; I am concerned with the validity of SIM betas as a normative decision tool. Hypothesis (*c*) leads to a case for using a MIM to reduce correlation between residuals so to better explain the relationship between the crop and the market index. If residual variability is high (as Turvey and Driver show it often is in agriculture), a SIM beta is measured poorly, which could lead to poor portfolio choices. As Collins states, "Inclusion of residual variance may be important when portfolios are small, as in the typical farm planning problem." As my results for hypothesis (*c*) show, adding residual variance (and possibly other factors) as an explanatory variable to create a MIM may lead to a beta for the market index which is significantly different than its SIM beta. This difference in betas could lead to different rankings of alternate crops under consideration by a farmer, thus calling into question the robustness of the SIM in that case.

Collins' complaints about hypotheses (*d*) and (*e*) are simply that they are not "important assumptions of the SIM." I believe that it is important to note weaknesses of decision tools, which is what the two hypotheses do. Results from both hypothesis tests provide insight concerning empirical application issues. A long list of other hypotheses could have been posed, but (*d*) and (*e*) were chosen as examples derived from the securities literature simply to make the case for consideration of MIMs in agricultural product analyses.

Summary

My original paper is an empirical analysis of SIM performance in agricultural product markets (not a condemnation of the SIM) which raises questions about the need for a MIM rather than a simple, linear regression equation in some cases. A multivariate approach may be a better empirical fit in some (although not all) situations. My article demonstrates that SIM results from agricultural product analyses indicate potential for error in portfolio decision making. In this reply, I note some of the sources for that error: using the SIM out of its intended context, and the inconsistent nature of errors. Differences between covariance in crop return residuals compared to residuals in security returns mean that agricultural analysts cannot assume, as do security analysts, that the SIM errs in a consistent fashion. This means that the composition of the Markowitz tangency portfolio may be better approximated using a MIM, rather than the SIM. Also, it means that decisions regarding marginal changes to existing crop rotations may require the more accurate betas provided by a MIM.

In this reply to Collins' misinterpretation of my article, I seek to make clear that the SIM remains a useful decision tool for agricultural producers in many situations. As explained above, the hypotheses I tested in my article were derived by simple mathematical manipulation of the SIM equation or from the literature concerning SIM applications in securities markets; they clearly have nothing to do with the CAPM. Although the empirical results illustrate weaknesses with the SIM in agriculture, determination of whether those weaknesses are fatal must be made for each application undertaken. My goal was to alert others to the empirical shortcomings of the SIM which I discovered while attempting to use it as a farm planning tool in the agricultural product markets of California.

[*Received December 1992.*]

Notes

[1] Bonds, futures, and options contracts are types of financial securities which expire at a specific date, but the underlying assets are not "consumed"—the assets still exist in the same form after the security's maturity date.

[2] The adjustment was made to get "returns to risk" because it enables a direct comparison of risk and return and is a better basis for decision making. Collins and Barry also state that a farmer should maximize $(R_p - R_f)/\sigma_p$ to find the optimal portfolio. This is their equation (13).

[3] The CAPM is an equilibrium, one-period model; the SIM is not an equilibrium model and it is estimated using data from multiple historical periods. Although the two equations have some common parameters, the meaning and significance of those parameters differ dramatically.

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