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The Robustness of Single Index Models in Crop Markets: A Multiple Index Model Test: Comment

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A recent article by Steven C. Blank purports to evaluate the assumptions underlying the single index model (SIM) when it is applied in the farm planning context. Instead, it evaluates the extent to which the capital asset pricing model (CAPM) equilibrium conclusions arise in crop returns. While these issues have been dealt with previously (Hutchinson and McKillop; Collins), the important differences between the CAPM and the SIM apparently are not yet well understood by many. The SIM regresses each individual set of crop returns (CR_i) on an index I, $CR_i = \alpha_i + \beta_i I + \epsilon_i$, and uses the estimates of the three parameters to approximate a full covariance matrix in a normative farm planning model as an alternative to the more complex Markowitz model. The CAPM, on the other hand, is a model of equilibrium returns to risky assets which states that if a long list of asset homogeneity and market efficiency assumptions are met, the equilibrium expected return on a risky asset (R_i) should be a linear function of its beta coefficient, $R_i = R_f + \beta(R_M - R_f)$, where R_f is the riskless rate of return and R_M is the expected return to a market index of all risky assets. The SIM simply is an approximation technique. The CAPM is a model of equilibrium returns for risky assets. While the SIM may or may not be useful as an approximation to the Markowitz portfolio model, it is clear that there is no reason for crop returns in a particular region (on their excess returns over the rental rate) to be precisely related to their systematic risk. Therefore, Blank's analysis is fundamentally flawed because it does not test the SIM in any way, but instead tests and rejects CAPM hypotheses that should be rejected a priori.

Sharpe (1963) proposed the SIM as a normative tool which could be used to approximate the Markowitz portfolio model. His primary objectives were conservation of computer time and dealing with large portfolio problems on small computers. However, an additional feature was that the beta coefficient obtained from regressing the individual activity returns on an index could be regarded as a measure of the risk that a particular activity would add to a well diversified portfolio. No such measure exists for the Markowitz model. For a model that considers n potential activities, one must consider the combined effects of an activity's variance along with the effects of $n - 1$ covariances. In this spirit, Collins and Barry proposed using the SIM for farm planning as an approximation of a full covariance model for farm planning. The SIM has several possible advantages for farm planning in the context of diversified farming. It allows large problems to be quickly solved on very small computers (or even hand calculators) and the betas provide a basis for comparing the risk contributions of alternative crops. In addition, only $3n$ parameters must be estimated to evaluate the risk of a SIM portfolio, while $(n^2 + n)/2$ parameters must be estimated for the full covariance model. This reduces the estimation required from limited data when $n > 5$. Most of all, they thought the SIM might have more intuitive appeal to farmers than the Markowitz model. But the important point is that the SIM is intended to have a normative purpose, i.e., provide a simple approximation of a Markowitz model for formulating approximately risk efficient farm plans. Tests of the robustness should evaluate this objective, i.e., the SIM is robust when it approximates

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a Markowitz result well (Turvey, Driver, and Baker). The SIM is not a model of equilibrium crop returns.

In the securities literature, Sharpe (1964) and Lintner quickly realized that since the SIM beta measured the contribution that an individual stock made to the riskiness of a well diversified portfolio, the risk premium required for a particular stock would be proportional to its beta. Therefore, the return an investor would require to hold stock i (RR_i) is the riskless rate plus the risk premium: $RR_i = R_f + \beta(R_M - R_f)$. Given a constant expected dividend, the rate of return an investor *expects* to earn depends on the price (P_i) of the stock. Specifically, where D_i is the expected dividend, the rate of return an investor expects to earn on stock i (ER_i) is $ER_i = D_i/P_i$. If the expected return on stock i exceeds the required return, and if markets are efficient, and investors and securities are homogeneous, investors will buy the stock driving the price up and expected return down until an equilibrium is reached, where $ER_i = RR_i = R_f + \beta(R_M - R_f)$, or the expected return equals the required return. This equilibrium result is the *capital asset pricing model,* not the *single index model.* The capital asset pricing model says that prices of capital assets will adjust such that the expected rate of return on a security will, in equilibrium, equal the rate of return that an investor requires to hold a security, which is a function of the beta coefficient from the single index portfolio model. The validity of the CAPM equilibrium has been tested frequently in the securities context by regressing stock returns on their betas producing the well-known results. However, it is also well known that these tests are simultaneous tests of the assumptions of the CAPM and the efficiency of financial markets. Given the heterogeneity and market inefficiencies in the production of crops, it is clear that one would not expect crop returns for a particular farmer or region to depend only on their systematic risk.

It is clear that there are valid empirical questions in agricultural economics that legitimately may be addressed with financial market models such as CAPM or APT. For example, one might wish to examine agricultural assets to see if their market prices adjust to create financial rates of return comparable to nonagricultural investments of equivalent risk. Several good examples of such studies may be found between Barry's pioneering application of the CAPM to farm land and the very recent APT study by Bjornson and Innes. While the heterogeneity of agricultural land and inefficiency of land markets do not match their financial counterparts, these problems do not create a fatal flaw for these studies because the basic market mechanisms necessary to create such an equilibrium do exist, if imperfectly.

However, there is no reason to expect that the CAPM equilibrium will occur for crop returns. In order for the CAPM equilibrium to apply to crops, returns to each crop grown in each micro location would have to adjust such that the return earned by the farmer on each crop would be just enough to compensate the farmer for the systematic risk that the crop adds to the farm plan in that location. If this were so, in regions where a crop has low systematic risk, yields or prices would have to somehow fall or costs would have to rise until the CAPM equilibrium was reached. On the other hand, the same crop grown in another area where it had high systematic risk would have to have its returns somehow increase. This is clearly nonsense. While a share of IBM stock has the same risk and return characteristics in Minot, North Dakota as it does in San Diego, California, the risks and returns from growing avocados will be somewhat different in the two locations, and no market mechanism exists to make them equalize.[1] Since there is no mechanism to cause returns in each growing region to adjust according to the extent that crop adds risk to a well diversified farm plan in that region, there is no reason to expect actual crop returns in a region to be closely related to their SIM beta coefficients.

Given that different areas are better suited to some crops and less well suited for others, this will mean that there will be opportunities for farmers to choose crops that have high returns relative to the risk that those crops add to their farm. This, of course, is the purpose of normative farm planning models like the Markowitz portfolio model, and the SIM approximation. There is reason to expect that farmers in Minot will not choose to grow avocados; therefore, it is not surprising that one finds a weak relationship between actual crop returns in an area and their betas for that area. But there is no mechanism to create a precise CAPM equilibrium.

Blank recognizes that the CAPM result will not occur across regions, but fails to recognize that there is also no reason for it to occur within a region. Given that different regions will have different systematic risks and expected returns for a particular crop, one would expect a Ricardian result rather than a CAPM equilibrium. In other words, a crop would only be added to a farm plan in a particular region if the expected return from the crop is greater than or equal to the required return. Therefore, only in the marginal region where the expected and required returns are equal would the CAPM result be observed. As a result, one would not expect to find a CAPM equilibrium even for crop returns in a given area. Blank's statistical results clearly support this assertion. The SIM is a planning model, not a model of equilibrium crop returns. Equilibrium crop returns for a specific region or farm depend on many factors other than systematic risk.

Blank's hypothesis (*a*) states, "The relationship between returns, R_i, and risk, β_i, is linear." It is clear that R_i is actual returns rather than required returns and, therefore, a test of this hypothesis is clearly a test of a CAPM equilibrium result, not a SIM assumption. While one would not be surprised to find that farmers have a tendency to choose high beta crops only if their returns are high, one would not expect to find a great deal of precision in this relationship. The correct SIM assumption is that the relationship between a crop's return, R_i, and the index of returns for possible crops for the local area is linear. Since there is no equilibrating mechanism, no precise relationship is expected between R_i and β_i.

Blank's hypothesis (*b*) states, "The intercept of the SIM . . . equals the risk-free (cash-leasing) return." The SIM regresses individual crop returns on an index of local crop returns, not on beta. The SIM intercept is the expected return for the crop when the index equals zero, not when beta equals zero. There is no reason to expect any particular value for this parameter, but it is very clear that it should bear no relationship to the cash lease rate for land. While land rents are complex, it seems clear they would not be based on the returns realized when the average crop in the region earns a zero return. Only if a CAPM equilibrium existed would a zero beta crop be expected to earn the rental rate. Therefore, hypothesis (*b*) is also a test of a CAPM conclusion, not a SIM assumption.

His hypothesis (*c*) states, "An enterprise's residual (diversifiable) variability, ϵ_i, does not affect its ranking among alternative investments." While it is true that beta measures the contribution to the risk of a well diversified portfolio (with hundreds of stocks), it is only approximately true for small portfolios such as farm plans, but in any case, it is not an assumption of the SIM. The SIM approximation of portfolio risk considers both systematic and nonsystematic risk, i.e., both the betas and the residual variances are included. Specifically, the SIM estimate of a portfolio's variance is $\sigma_I^2 X'\beta\beta'X + X'\sigma X$, where X is the vector of portfolio proportions, σ_I^2 is the variance of the index and σ is a diagonal matrix of residual variances from the SIM regressions. [See equations (15) and (16), Collins and Barry, p. 157.] Inclusion of residual variance may be important when portfolios are small, as in the typical farm planning problem. Clearly, Blank's hypothesis (*c*) has nothing to do with the SIM.

Blank's hypotheses (*d*) and (*e*) are ambiguous at best, but they cannot be characterized as important assumptions of the SIM. The important assumption that is required for the SIM to work well is that the errors not be correlated across the SIM regression equations. This happens frequently in the securities context and is referred to as an industry effect. This undoubtedly also occurs in the farm planning context, but Blank does not adequately address this crucial assumption. The existence of these industry effects is the rationale for the use of multiple index models.

Blank also asserts that an opportunity set that is linear in β will arise only when land leasing markets are highly competitive and efficient. Linearity with respect to beta is clearly a CAPM result, the analog of the security market line. Collins and Barry show that linearity occurs in portfolio variance. Neither efficiency nor competition is necessary for a particular farmer to have a linear opportunity set, only that leasing opportunities be available. If the cost of leasing land from others is not equal to the return from leasing land to others, the opportunity set will have two linear sections with different slopes.

Therefore, Blank's paper does not adequately address the crucial assumption of the SIM and instead tests CAPM equilibrium results that are not relevant to the crop returns that he studies. Appropriate testing of the SIM would evaluate how well it approximates a Markowitz result in terms of the composition of the tangency portfolio. It is well known that the SIM may not be very accurate in measuring the variance of a portfolio, but it errs in a consistent fashion so that it appears to do well in estimating the *composition* of the tangency portfolio. That is, the SIM is robust if the true variance (calculated with the full covariance matrix) of the SIM tangency portfolio is close to the variance of the tangency portfolio calculated with a Markowitz full covariance model for a broad variety of circumstances.

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Endnote

[1] The only conceivable mechanism that could accomplish this would be the development of highly crop-specific land rents. However, it is difficult to imagine such a mechanism for field crops grown on owned land.

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