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The Effect of Stochastic Irrigation Demands and Surface Water Supplies on On-Farm Water Management

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This study presents a procedure for simultaneously addressing stochastic input demands and resource supplies for irrigated agriculture within a linear modeling framework. Specifically, the effect of stochastic crop net irrigation requirements and streamflow supplies on irrigation water management is examined. Irrigators pay a self-protection cost, in terms of water management decisions, to increase the probability that stochastic crop water demand is satisfied and anticipated water supply is available. Self-protection cost is lower when increasing the probability that anticipated water supplies are delivered, ceteris paribus, than when increasing the probability that the crop receives full net irrigation requirement in the study region.

Key words: irrigation efficiency, risk, stochastic input demand, stochastic resource supply

Introduction

Water management policy must be sensitive to factors that motivate agricultural water use. Willis found that agricultural diversions (both ground and surface water) exceeded the quantity required to satisfy expected net irrigation requirement (NIR) for the irrigation systems and management levels employed in an irrigated river basin.[1] He also found that groundwater pumping capacity considerably exceeded the capacity required to satisfy crop NIR under average weather conditions, median streamflow supplies, and expected irrigation efficiencies. The perceived overapplication of irrigation water and excess pumping capacity is attributable to uncertain water demand and supply conditions confronting irrigated agriculture.

It is well known that a risk-averse individual will use a greater quantity of a risk-reducing input than a risk-neutral individual (Anderson, Dillon, and Hardaker; Pope and Kramer; Loehman and Nelson). Thus, it is likely risk-averse irrigators will develop water management plans that prescribe water applications exceeding the quantity required to satisfy expected NIR to protect against crop water stress and reduce yield variability. However, concavity of the utility function is not required to provide a rational explanation for the perceived overuse of irrigation water. Antle has shown that when dealing with dynamic models, production uncertainty affects productivity and expected income, which in turn affect optimal input levels whether a farmer is risk neutral or risk averse. More recently, Babcock reports that when yields have a linear

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[1] NIR is the consumptive use quantity of irrigation water required to prevent plant stress in any defined time period.

response and plateau functional form, with a stochastic yield plateau, it can be profitable for a risk-neutral farmer to apply fertilizer above the optimal level applied under production certainty. Babcock shows that under specific price and marginal productivity relationships for fertilizer, expected profits are greater if the farmer applies fertilizer above production certainty levels to make fertilizer nonlimiting when stochastic weather conditions are ideal for maximum crop yield. Letey, Vaux, and Feinerman found that at low water prices optimal irrigation application rates are 50% to 100% greater under application uncertainty than they are with application certainty when yield is a concave function of crop water use. Thus, production uncertainty in combination with a concave production technology can affect factor input levels regardless of the decision maker's risk preference.

The two water-related sources of uncertainty confronting irrigated agriculture are uncertain input demand and uncertain resource supply. Uncertain input demand consists of stochastic crop NIR and the potential for stressing the crop and affecting crop yield or quality, and thus profitability. NIR is influenced by temperature, precipitation, and the rate of crop development. In many years, an ex post evaluation of water use for a given irrigation technology would conclude that irrigated agriculture is inefficient in its use of water. However, the excess water applied ex ante is designed to assure at an acceptable risk level that the crop is not stressed between irrigations. This management strategy is especially likely in regions with unsophisticated irrigation technologies where system labor costs and irrigation setup times reduce management flexibility and constrain irrigation schedules to fixed time intervals.

Stochastic surface water supply is the resource supply risk confronting irrigated agriculture. Kramer, McSweeny, and Starvos report that risk-averse farmers alter their planned use of resource supplies when supplies are uncertain. Given that low streamflows can prevent irrigators from diverting their entire surface right, it is likely that the streamflow probability distribution influences cropping and water management decisions. Water supply variability is reduced when dependable groundwater is substituted for less reliable surface diversions, and can be driven to zero if sufficient well capacity exists to adequately irrigate all acreage. Hence, a risk-averse farmer may knowingly underestimate expected surface supplies to guarantee that anticipated surface diversions are available at an acceptable risk (probability) level.

Management decisions which reduce the probability of crop water stress have an economic cost commonly referred to as a self-protection cost (Heibert). Self-protection cost is a function of individual risk preferences and the cost of the actions taken to satisfy NIR at the specified risk level. Management options include a variety of actions such as increasing groundwater use, increasing irrigation efficiency through management and/or technology, adopting less water-intensive crops, and reducing irrigated acreage.

This study focuses on the influence of stochastic water demands and supplies on farm-level water allocation decisions. The modeling approach incorporates Wicks and Guise's minimization of total absolute deviations (MOTAD) with risky input-output coefficients (RINOCO) procedure within a linear chance constrained programming (CCP) modeling framework for dealing with right-hand-side (RHS) resource supply uncertainty (Charnes and Cooper). By defining risk in terms of stochastic crop NIR and surface water supply availability, uncertainty appears in the technical coefficients and RHS resource supply parameters. The stochastic analysis is limited to the constraint set parameters to focus on those risk aspects most relevant to on-farm water use.

Study Area and Data

The Walla Walla River Basin, located in southeastern Washington State (73% of basin) and northeastern Oregon (27% of basin), provides the empirical setting. Precipitation ranges from seven inches near the basin's western edge to over 40 inches in the Blue Mountains to the east. Irrigated agriculture is concentrated along the valley floor where annual precipitation averages less than 20 inches. Twenty-two irrigated crops are grown, with alfalfa seed, alfalfa hay, and wheat accounting for nearly 71% of the 61,819 irrigated acres in 1989 [U.S. Department of Agriculture (USDA)]. The remaining irrigated acreage includes a wide variety of vegetable crops, most notably onions and beans, tree fruits, and pasture.

Two aquifer systems underlie the basin. A basalt aquifer ranging in depth from 125 to 2,000 feet below the surface underlies the entire basin. An unconfined gravel aquifer overlays the basalt system on the central basin valley floor. This aquifer is primarily recharged by precipitation, irrigation return flows, and seepage from streams and irrigation canals, and has a storage capacity of five million acre-feet and an economically manageable reserve capacity of one million acre-feet (MacNish, Myers, and Barker). Rainfall and snowpack from the Blue Mountains is the primary source of streamflow. Monthly streamflows fluctuate dramatically between years. Flows are generally greatest in early spring, while dry stream beds regularly occur in late summer due to low precipitation and high irrigation demands (Willis).

In 1989, 49% of all irrigation water came from surface diversions, with groundwater accounting for the remaining 51% (Willis). Seventy-seven percent of all groundwater was diverted from the shallow gravel aquifer. Sprinkler irrigation covers 97% of irrigated acreage with gravity systems being limited to irrigated pasture (Willis). Side-roll and handline systems account for 95% of sprinkler acreage. USDA Agricultural Stabilization and Conservation Service (ASCS) personnel, local commodity groups, and farmers considered the 1989 irrigated crop mix and acreage to be representative of long-run basin cropping practices. Average monthly temperatures, precipitation, and stream-flow supplies were also near long-run averages in 1989.

Prior research linked an economic optimization model, incorporating 20 farming regions, to a detailed hydrology model that monitored monthly surface flow levels for 193 basin stream reaches, and regional and basinwide groundwater fluxes (Willis). Hydrologic characteristics, irrigation practices, and irrigated and dryland acreage were used to identify each region. The model was calibrated for a detailed 1989 data set. Monthly water diversion locations were identified to the nearest tenth of a mile. Each of the 1,745 diversion locations was identified as either a surface, gravel aquifer, or basalt aquifer diversion.[2]

A modified Blaney-Criddle approach was used to estimate monthly NIR for each crop grown in the basin for the 42-year period spanning October 1948 through September 1989 (James et al.; Willis). The NIR estimates were derived assuming that each crop

[2] A detailed discussion of all irrigated and dryland crop acreage and yields, irrigation systems and efficiencies, monthly diversions by region and water source, weather and hydrologic data, and the procedure used to link and calibrate the spatially and temporally disaggregated economic and hydrologic models is contained in Willis.

fully uses effective growing and dormant season precipitation. The monthly precipitation and temperature data required to estimate NIR were collected from National Oceanic and Atmospheric Administration historical records for nine basin weather stations. Forty-two years of data (1948–89) on monthly streamflows entering the basin were obtained from seven U.S. Geological Survey (USGS) gage stations.

This integrated planning model is used here to simulate 42 years of monthly surface flows available to one downstream region for agricultural diversion under the condition that the upstream regions, who hold senior water rights, divert their full appropriative surface right each month of the irrigation season. The downstream region has a diverse crop mix, practices conjunctive ground-surface water management, diverts 30% of its irrigation water from surface sources under median flow conditions, and has a surface water right junior to all upstream users.

Alfalfa hay, winter wheat, and alfalfa seed account for 69% of the 2,053 irrigated acres in the region, with the remainder in asparagus, green beans, onions, orchards, and pasture. The soils are a mix of silt loam and sandy loam. Average annual precipitation is 17 inches in the region, slightly less than the basin average of 20 inches. All irrigated acreage is under handline or side-roll sprinkler technology except for 65 acres of fruit trees using solid set sprinkler systems and 250 acres of flood irrigated pasture. All sprinkler systems are capable of achieving a 65% irrigation efficiency, and gravity systems have an irrigation efficiency of 45% (Hooker). Pump lifts for wells diverting from the gravel aquifer average 50 feet, and basalt aquifer lifts average 150 feet (Willis). Crop budget data are contained in Willis.

Chance Constrained Programming

Chance constrained programming (CCP) can be viewed as using the probability of satisfying stochastic constraints to provide appropriate safety margins (Sengupta). A typical CCP constraint must satisfy the following inequality:

$$\text{(1)} \qquad \text{Prob}\left(\sum_{j=1}^{n} a_{ij} X_j \le b_i \right) \ge \alpha_i,$$

where Prob is probability, α_i is a probability measure, a_{ij} are technical coefficients, b_i is resource availability, and X_j are the decision variables (Charnes and Cooper). A chance constraint explicitly accepts that feasibility is not always assured and restricts the probability of infeasibility while optimizing other policy goals. If we assume $0 \le \alpha_i \le 1$, then it is permissible to violate the probabilistic constraint with, at most, probability $(1 - \alpha_i)$ for any choices of the X_j decision variables. The next three subsections review how CCP programming can be used to control for the uncertainty of stochastic resource supplies and input demands, individually and jointly, in the context of water resource modeling.

Stochastic Resource Supply

When resource supply is the sole stochastic parameter, the appropriate value for b_i is the maximum value that allows the $\text{Prob}(\sum_j a_{ij} x_j \le b_i) \ge \alpha_i$ to be true. This maximum value, b_i^*, is the deterministic equivalent value that converts the stochastic programming problem into a deterministic programming problem and allows the constraint to be respecified as:

(2) $$\sum_{j=1}^{n} a_{ij} x_j \leq b_i^*.$$

When the resource supply parameter is normally distributed, b_i^* can be parameterized as $b_i^* = \mu_i - z_i^\alpha \sigma_i$, where μ_i is mean resource supply, σ_i is the standard deviation of resource supply, and z_i^α is the standard normal deviate that assures α percent of the supply outcomes are greater than b_i^*. Thus, when b_i is normally distributed, (2) can be respecified in the parametric form:

(3) $$\sum_{j=1}^{n} a_{ij} x_j \leq \mu_i - z_i^\alpha \sigma_i.$$

The higher the α percentage, the greater the downward safety margin adjustment $(-z_i^\alpha \sigma_i)$ to mean resource supply (μ_i). When resource supply is certain, a farmer will develop a farm plan where resource demands do not exceed resource supplies (the deterministic LP solution), and will set the critical z_i^α value to zero. In contrast, when resource supply is uncertain, a farmer wishing to satisfy the constraint with a high degree of confidence (say 95% of the time) will set the critical z_i^α value to 1.645. In irrigated agriculture, this could require that water demand be no greater than expected surface water supplies modified by a marginal downward risk adjustment for surface flow uncertainty, assuming no groundwater is available.

Stochastic Input Demand

Stochastic input demand can be accommodated within the linear CCP framework by using the procedure developed by Wicks and Guise. Their technique allows specifying a stochastic constraint with uncertain monthly NIR and known monthly surface diversion supplies as:

(4) $$\sum_{j=1}^{n} \frac{\mu_{ij}}{EFF_{js}} X_j + z_i^\alpha \sigma_i \leq b_i,$$

where μ_{ij} is mean per acre NIR for crop j in month i, X_j denotes acres in crop j, σ_i is the standard deviation of the quantity of applied water needed in month i to satisfy the selected crop mix NIR under the available irrigation technologies, z_i^α is the standard normal deviate that assures month i crop mix NIR is satisfied α percent of the time, EFF_{js} is irrigation efficiency of system s in crop j, and b_i is assumed known monthly surface diversion. Dividing each μ_{ij} by its respective irrigation efficiency converts the NIR measures into applied water measures, a unit commensurate with the assumed known surface diversion (b_i) value. A farmer would upwardly adjust the quantity of irrigation water applied to the selected crop mix by specifying the appropriate z_i^α value to satisfy monthly NIR at a given risk level. The adjustment is analogous to that involving stochastic resource supplies except the adjustment is upward and not downward.

A mean absolute deviation (MAD) procedure is used to derive endogenously an unbiased estimate of σ_i and preserve the linearity of the constraint set. The MAD procedure does not require individual crop NIR to be independent within a month, but does require that the monthly quantity of water applied to meet the NIR of the selected crop mix be normally distributed when standard normal variables are used to satisfy the monthly chance constraints at a specific probability level. The normality assumption is statistically tested in a subsequent section.

In maintaining linearity of the constraint set, a measure of statistical efficiency is sacrificed (Hazell and Norton). From a computational perspective, Boisvert and McCarl consider this to be an acceptable tradeoff because nonlinear algorithms often fail to achieve a global optimum in large models with more than a few nonlinear constraints. Such models arise in any realistic water policy analysis. Moreover, the ability to endogenously estimate the standard deviation of monthly water demand allows the deterministic equivalent value to be parameterized using fractiles from the Z distribution for alternative probability levels.

Stochastic Supply and Demand

Simultaneous satisfaction of input demand uncertainty and resource supply uncertainty at a specific probability level within a constraint transforms the problem into a nonlinear joint chance constrained programming (JCCP) problem. Nonlinearities also arise if two or more linear chance constraints are controlled at a joint probability level. However, linear CCP techniques can be used to approximate these nonlinear joint constraints when the stochastic parameters have a specific correlation structure. Consider the following stylized stochastic constraint set for a three-crop, two-month growing season in which all parameters are stochastic:

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + z_1^{\alpha}\sigma_1 \le b_1^{\delta}, \tag{5}$$

$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + z_2^{\alpha}\sigma_2 \le b_2^{\delta}, \tag{6}$$

where

a_{ij} = median crop j NIR per acre in month i divided by crop j irrigation efficiency (applied water requirement for crop j in month i),
b_i^{δ} = maximum month i water supply which is available δ percent of the time,
z_i^{α} = the standard normal variable for month i which assures the quantity of water applied in month i satisfies crop NIR α percent of the time,
σ_i = the endogenous month i standard deviation estimate of required irrigation water for the optimal crop mix, and
X_j = irrigated acreage of crop j (decision variables).

Four correlation patterns are of interest: (a) crop NIRs within a month, (b) NIRs among months, (c) streamflow supply and NIR within months, and (d) streamflow supplies among months.

Focusing on the input demand coefficients, crop water demands are likely to be positively correlated within a month. If crop 1 NIR is above (below) average, it is likely to be above (below) average for crop 2 and crop 3.[3] The linear MAD procedure used to

[3] Statistical tests on the 42-year data set of monthly crop NIRs found that individual crop NIRs are positively correlated within a month.

estimate σ_i can accommodate this positive correlation because independence of the technical coefficients is not required.

Statistical tests found crop NIR to be independent among months and independent of the streamflow level within a month. NIR is determined primarily by weather patterns and crop biomass in a given month, whereas monthly streamflow levels are primarily a function of dormant season precipitation (including snowpack). Monthly streamflow levels were found to be positively autocorrelated. Given these correlations, the impact of both sources of risk on ex ante water allocation can individually and jointly be accommodated within a linear CCP framework.

Independence of monthly crop NIR and streamflow supplies allows a conservative lower bound to be established that jointly controls both risk sources at or above a specified probability level in a month. For example, monthly stochastic NIR and surface water supply can be jointly controlled at the 0.95 probability level by satisfying each risk source at the 0.975 probability level, or any other combination of probabilities, such that the product of the probabilities equals 0.95. Given that individual probability levels must be specified a priori for each risk source, this approach lacks the flexibility of JCCP where the optimal probability levels needed to satisfy the joint probabilistic constraint are endogenously determined for each risk source.

Because crop input demands are independent across months, a similar approach can be used to assure all stochastic input demands are met over the entire growing season at a joint probability level. Under independence, the monthly NIR for a given crop mix can be jointly satisfied at the 0.95 probability level over a hypothetical two-month growing season by satisfying each monthly demand constraint at the 0.975 probability level, since $(0.975)^2$ roughly equals 0.95. This involves setting the z_i^α value in (4) equal to 1.96 for each month of the irrigation season.

When monthly resource supplies are independent, a similar procedure can be used to establish a joint probability level that anticipated monthly surface water supplies are delivered over the entire irrigation season. If monthly water supplies are not independent, as in our case, the Bonferroni inequality can be used to develop a conservative lower bound on the joint probability that anticipated surface diversions will be available for all months.[4] When resource supplies are positively correlated and distributed multivariate normal, the Slepian inequality can be used to improve the accuracy of the Bonferroni lower-bound approximation (Bawa). Even though these approximation techniques require the a priori specification of the individual monthly probabilities, they are a valuable alternative to JCCP because when the stochastic parameters are not independently distributed, calculating the joint probability level is often exceedingly difficult. If multivariate normality is assumed, enormous computational difficulties can still arise if the solution requires evaluating multivariate probability integrals (Balintfy; Jagannathan).

Each of the linear approximations to a joint chance constraint specification reduces the dimension of the feasible solution space from the situation where the constraints are

[4] The Bonferroni inequality states that the $\text{Prob}(b_1 \geq B_1, b_2 \geq B_2, ..., b_p \geq B_p) \geq \text{Prob}(b_1 \geq B_1) + \text{Prob}(b_2 \geq B_2) + ... + \text{Prob}(b_p \geq B_p) - (p-1)$, where b_i is a random outcome and B_i is some arbitrary value (Mittelhammer); that is, the probability that all p constraints are satisfied simultaneously is greater than or equal to the sum of the probabilities that each constraint is satisfied individually less the number of stochastic constraints minus one. Hence, a lower bound on the probability of simultaneously satisfying all p constraints at probability α can be derived by equating $\text{Prob}(b_1 \geq B_1) + ... + \text{Prob}(b_p \geq B_p) - (p-1)$ to α by appropriately selecting the B_i values. Selecting the B_i values in this manner (the deterministic equivalent values for each monthly streamflow level) guarantees that the p stochastic constraints are jointly satisfied at least at the α probability level.

treated individually, ceteris paribus, and hence increases the magnitude of on-farm self-protection cost at each probability level when the constraints are binding. Thus, the additional security provided by satisfying either all of the chance constraints or a subset of the constraints at a specified joint probability level comes at an additional cost.

Empirical Model

The CCP model used to analyze the effect of stochastic monthly NIRs and/or streamflow supplies on irrigation efficiency and crop selection is specified as follows:

$$\text{(7)} \qquad \max NR = \sum_{j=1}^{10}\sum_{s=1}^{5} C_{js}X_{js} + DR * A_d - \sum_{j=1}^{10}\sum_{i=1}^{12}\sum_{s=1}^{5} SEC_s * AW_{jis} - \sum_{p=1}^{2}\sum_{i=1}^{12} GWC_p * GWQ_{pi}$$

s.t.:

$$\text{(8)} \qquad X_{js} \leq AC_{js}, \quad j = 1, \ldots, 10;\ s = 1, \ldots, 5;$$

$$\text{(9)} \qquad \sum_{j=1}^{9}\sum_{s=1}^{5} X_{js} + A_d \leq TA;$$

$$\text{(10)} \qquad \mu_{ji}X_{js} - EFF_{js}AW_{jis} \leq 0, \quad j = 1, \ldots, 10;\ i = 1, \ldots, 12;\ s = 1, \ldots, 5;$$

$$\text{(11)} \qquad \sum_{j=1}^{10}\sum_{s=1}^{5} AW_{jis} - \sum_{p=1}^{2} GWQ_{pi} + z_i^{\alpha}\sigma_i \leq SD_i^{\delta}, \quad i = 1, \ldots, 12;$$

$$\text{(12)} \qquad GWQ_{pi} \leq CAP_p, \quad p = 1, 2;\ i = 1, \ldots, 12;$$

$$\text{(13)} \qquad \sum_{i=1}^{12} GWQ_{1,i} - \sum_{i=1}^{12} R * GWQ_{2,i} \leq 0;$$

$$\text{(14)} \qquad \sum_{j=1}^{10} DAW_{jisy}X_{js} - NDEV_{iy} \leq 0, \quad i = 1, \ldots, 12;\ s = 1, \ldots, 5;\ y = 1, \ldots, 42;$$

$$\text{(15)} \qquad \sum_{y=1}^{42} NDEV_{iy} - TND_i = 0, \quad i = 1, \ldots, 12;$$

$$\text{(16)} \qquad \Delta TND_i - \sigma_i = 0, \quad i = 1, \ldots, 12;$$

$$\text{(17)} \qquad X_{js}, A_d, GWQ_{pi}, AW_{jis}, NDEV_{iy}, TND_i, \sigma_i \geq 0,$$

where

NR = net revenue;
X_{js} = the number of acres planted to irrigated crop j under a given irrigation system s;
C_{js} = the per acre return from activity X_{js} excluding irrigation energy cost;
DR = per acre dryland return from a winter wheat-fallow rotation;
A_d = acres in dryland rotation;
SEC_s = the acre-inch energy cost to irrigate under system s;
AW_{jis} = acre-inches of water applied to crop j in month i under irrigation system s;
GWC_p = the cost of pumping an acre-inch of groundwater from the gravel aquifer (p = 1) or the basalt aquifer (p = 2);
GWQ_{pi} = the quantity of water (acre-inches) pumped from the gravel aquifer (p = 1) or the basalt aquifer (p = 2) in month i;
AC_j = baseline acreage in irrigated crop j;
TA = total irrigated acreage under the baseline less irrigated pasture acreage;
μ_{ji} = mean NIR for crop j in month i;
EFF_{js} = risk-neutral irrigation efficiency for crop j under irrigation system s;
z_i^{α} = the standard normal deviate for month i, the value of which is dependent on the selected α probability level;
σ_i = the standard deviation of the quantity of water applied to satisfy the crop mix NIR in month i;
SD_i^{δ} = the deterministic equivalent value for the stream diversion level which is realized δ percent of the time in month i;
CAP_p = monthly pumping capacity of all wells pumping from the gravel aquifer (p = 1) and the basalt aquifer (p = 2);
R = a ratio parameter that assures seasonal water use from the gravel aquifer to that of the basalt aquifer does not exceed the calibrated baseline level;
DAW_{jisy} = the signed deviation of the per acre quantity of applied water under irrigation system s required to satisfy crop j NIR in month i and year y from the mean application rate for crop j in month i under system s;
$NDEV_{iy}$ = the negative deviation of the quantity of applied water required to satisfy the crop mix NIR in month i and year y from mean applied quantity in month i;
TND_i = the sum of all negative deviations for the quantity of water applied to satisfy the selected crop mix NIR in month i; and
Δ = a constant equal to $[2\pi/(n(n-1))]^{-0.5}$.[5]

The objective function (7) computes expected regional net returns to land irrigated under the baseline condition at alternative probability levels of controlling each source of production risk. Equation (8) limits the number of irrigated acres in each crop. A dryland winter wheat-fallow rotation can be substituted for baseline irrigated acreage

[5] As presented in Boisvert and McCarl, the relationship between the standard deviation estimate (σ) and mean absolute deviation (MAD) for the normal distribution was established by R. A. Fisher. Fisher showed that for a sample of size n, σ can be approximated by the MAD multiplied by the constant $F^{0.5}$. F is generally referred to as Fisher's F, and equals $(\pi n)/(2(n-1))$. Since the MAD is computed as total absolute deviations (TAD) divided by n, (MAD = TAD/n), and the sum of the positive deviations equals the sum of the negative deviations, Fisher's relationship can be rewritten as $\sigma \approx F^{0.5}$(2TND). Solving this relationship for TND results in TND $\approx \Delta\sigma$, where $\Delta = [(2\pi)/(n(n-1))]^{-0.5}$.

as the probability of satisfying NIR and/or receiving a given surface diversion in any month is increased from risk-neutral levels. The degree to which a dryland rotation can be substituted for irrigated acreage is modeled by (9). Dryland crops cannot be substituted for irrigated pasture because irrigated pasture is found only on marginal, irregularly shaped fields with little potential to be profitably farmed without irrigation. Equation (10) requires crop NIR to be satisfied under average conditions and specified irrigation system efficiency for each crop j in month i. Equation (11) is the probabilistic monthly water balance constraint that jointly satisfies NIR and surface water supply availability at the various probability levels. Groundwater supplies can be used to supplement surface diversions and/or irrigate to higher ex ante NIR. The standard normal distribution is not used to parameterize the monthly surface supply deterministic equivalent values, SD_i^δ, because nonparametric statistical tests found the simulated flow levels to be gamma distributed. The empirical distribution was used to provide the SD_i^δ values.

Equation (12) constrains monthly groundwater use to current pumping capacity. Equation (13) assures the annual ratio of gravel aquifer use to basalt aquifer use does not exceed the baseline ratio, and prevents the model from exhausting gravel aquifer supplies before pumping from the deeper basalt aquifer. Not all farms in the region have access to both groundwater sources, and this constraint maintains the spatial integrity of the farm region without sacrificing overall model flexibility.

The negative deviation of the quantity of applied water needed to satisfy crop mix NIR in month i and year y is computed in (14). Equation (15) sums all month i negative deviations into a total negative deviation (TND) estimate. Fisher's constant, F, is used to translate the TND estimate into the monthly standard deviation estimate, σ_i, in (16). The σ_i value is transferred into the appropriate monthly water balance equation, where it influences the optimal irrigated crop mix and/or causes additional groundwater use when the z_i^α value in (11) is nonzero and the constraint is binding. The endogenous σ_i estimate provides the programming model with the ability to optimally trade off the marginal benefit of maintaining the current crop mix versus the cost of satisfying NIR at higher probability levels.

Risk-Free Baseline

In order to estimate the impact of water supply and demand uncertainty on water application rates and crop selection, a risk-free baseline situation was established by optimizing the CCP model for the baseline crop mix under average monthly NIRs, median monthly surface diversion levels, and expected irrigation efficiency for each crop and irrigation system. With complete certainty, regional net return over variable cost is $653,481 and average irrigation efficiency is 60%.

MAD Estimator of NIR Standard Deviation

Information on the accuracy of using the MAD estimator to linearize the standard deviation estimate for monthly NIR is presented in table 1 for the baseline crop mix. Given that fractiles from the standard normal distribution are used to establish the

Table 1. Standard Deviation and Normality Tests for Monthly Net Irrigation Requirement for the Baseline Risk-Neutral Crop Mix

		Standard Deviation Estimates		Normality Tests			
				Shapiro-Wilk Test		Lilliefors Test	
Month	Mean NIR (ac./in.)	MAD Approach[a] (ac./in.)	Classical Approach[b] (ac./in.)	Test Statistic	Probability Level[c]	Test Statistic	Probability Level[d]
March	129	148	132	0.843	0.000	0.196	0.000
April	1,434	378	397	0.978	0.691	0.124	0.101
May	5,986	1,348	1,249	0.956	0.215	0.122	0.116
June	9,303	1,651	1,611	0.959	0.206	0.105	0.276
July	11,071	1,232	1,231	0.953	0.128	0.083	0.646
August	8,852	1,349	1,247	0.967	0.363	0.106	0.269
September	4,687	942	927	0.968	0.382	0.086	0.588
October	776	744	694	0.889	0.001	0.179	0.002

[a] Endogenous MAD estimate of the standard deviation of monthly NIR for the optimal crop mix with complete certainty using 42 years of data.
[b] Calculated using the number of irrigated acres by crop in the optimal solution and actual monthly NIR for each crop over the 42-year period, 1948–89.
[c] Probability of a value less than or equal to the reported Shaprio-Wilk test statistic if the sample data are derived from a normally distributed population.
[d] Probability of a value greater than or equal to the reported Lilliefors test statistic if the sample data are derived from a normally distributed population.

probability the crop mix receives its full NIR, statistical tests of the normality assumption also are presented in table 1.

Column 2 reports the mean NIR for each month of the irrigation season. NIR is greatest in the months of June, July, and August, accounting for 69% of all crop demand. The linearized monthly standard deviation estimates are compared with monthly estimates derived from the conventional estimation procedure in columns 3 and 4. The conventional standard deviation estimates were calculated outside the programming model using crop acreage data provided by the programming model in combination with the calculated monthly crop NIR values for 1948–89. The standard deviation estimates of monthly NIR provided by each technique are approximately equal over the six months that account for 98% of all water use (April through September), never differing by more than 8%, and by less than 2.5% in three months—providing empirical support for the accuracy of using the linearized standard deviation estimator.

Normality of the monthly crop mix NIR is tested using two nonparametric statistical tests: the Shapiro-Wilk test and the Lilliefors test. Monte Carlo studies have found the Shapiro-Wilk test is more sensitive to departures from normality in the tails of the distribution, whereas the Lilliefors test is more sensitive to normality departures over the entire distribution (Stephens). The well-known central limit theorem states that a random sample mean from a nonnormal population tends to be normally distributed (Mittelhammer). Therefore, even though individual monthly crop NIR is not normally distributed, it does not follow that the monthly NIR for a given crop mix is nonnormally distributed. As reported in table 1, both statistical tests support normality of the crop mix NIR in those months when crop consumptive demand is greatest, April through September. Normality is rejected only in the first and last months of the irrigation season (March and October) when minimal irrigation occurs. Thus there is strong empirical evidence that crop mix NIR is normally distributed in the months that crop water demand is greatest, which validates using the Z distribution to establish the chance constraint probability levels.

Risk Analysis

With complete certainty, average regional irrigation efficiency is 60%, higher than the documented 55% efficiency derived from historical records for the baseline condition. The documented lower average efficiency level is consistent with the irrigation systems and schedules used in the region. The side-roll, handline, and gravity technologies employed restrict irrigators from irrigating on a scientifically based cumulative evapotranspiration schedule, and result in most crops being irrigated on a fixed schedule. Moreover, labor and setup time requirements for sprinkler systems prevent irrigators from adding additional irrigations in drought periods. Thus, irrigators irrigate to levels in excess of average NIR to minimize the likelihood the crop is stressed between irrigations. The excess monthly groundwater pumping capacity within the region is also consistent with profit maximization. Surface delivery uncertainty is reduced when dependable groundwater supplies are substituted for less certain surface supplies. In the two sections that follow, we analyze the consequences of stochastic NIR and streamflow supplies on water management decisions for two scenarios.

Scenario 1: Unrestricted Groundwater Use

Scenario 1 releases the monthly groundwater capacity constraint to allow a farmer the option of pumping sufficient groundwater to satisfy monthly NIR on all baseline acreage at each probability level when the marginal value of water exceeds the additional energy cost. The additional fixed cost incurred in increasing regional groundwater capacity beyond the current level is ignored in order to determine maximum application rates. When the pumping constraint is not released, irrigated acreage must be reduced to satisfy the stochastic constraints at higher probability levels. Scenario 2 (discussed in the next section) constrains monthly groundwater use to current capacity. Results for both scenarios are reported for four selected probability levels which satisfy the chance constraints as nonjoint events: 0.50, 0.70, 0.90, and 0.95.

The self-protection costs reported in table 2 for Scenario 1 were derived by parametrically varying the probability of satisfying crop NIR over the probability levels, holding the surface delivery probability constant. This process was repeated four times, once for each surface probability level, yielding the 16 reported estimates. It is more costly to increase the probability that monthly NIR is satisfied while maintaining anticipated surface diversions at median supply levels than to increase the probability that anticipated monthly surface diversions are delivered and irrigating to average NIR. For example, self-protection cost is $20,066 when satisfying NIR at the 0.95 probability level and streamflow supplies at the 0.50 probability level. This is over five times larger than the $3,725 cost of assuring that anticipated streamflow supplies are available at the 0.95 level and irrigating to average NIR.

Self-protection cost is lower when dealing with stochastic streamflows because the quantity of water applied remains constant over the irrigation season as dependable groundwater is substituted for unreliable surface diversions to increase the probability that anticipated surface supplies are delivered. Thus, the sole cost incurred in reducing surface supply variability is the energy cost associated with pumping additional groundwater. Self-protection cost is greater when satisfying NIR at higher probability levels because water use increases, affecting cost in two ways. First, energy cost for all nongravity irrigation technologies increases in response to higher water application levels. Second, groundwater energy cost also increases if groundwater use rises above baseline levels, which is always the case in this region.

Self-protection cost is $23,791 when controlling both sources of risk at the 0.95 probability level. This cost is the sum of individually controlling each risk source at the 0.95 level ($3,725 plus $20,066) because irrigated acreage is unchanged from the risk-neutral baseline level.[6] Self-protection cost is minimized by increasing groundwater use in this region, instead of substituting a dryland rotation for low-value irrigated acreage, due to low groundwater pump lifts and relatively low energy cost. Irrigators pay a modest self-insurance cost, in terms of higher irrigation energy costs, to irrigate to above-average NIR and reduce water supply variability. In other settings, higher pump lifts and/or higher energy prices would increase self-protection cost and could potentially force low-value irrigated acreage out of production.

[6] Recall that each source of risk is independently controlled for in this example. To jointly control both sources of risk at the 0.95 level in any month, higher individual probabilities must be specified such that the product of the individual probabilities equals 0.95, given the independence of streamflow supplies and NIR.

Table 2. Impact of Stochastic Input Demand and Supply on Net Income, Applied Irrigation Water, and Irrigation Efficiency: Unrestricted Groundwater Use (Scenario 1)

NIR Probability Level	Streamflow Probability Level	Self-Protection Cost ($)[a]	Average Consumptive Requirement (acre-feet)	Applied Irrigation Water (acre-feet)			Irrigated Acres (acres)	Irrigation Efficiency (%)
				Total	Surface	Groundwater		
Baseline (complete certainty)		0	3,520	5,864	1,750	4,114	2,052	60.02
			Change Relative to Baseline					
0.50	0.50	0	0	0	0	0	0	60.02
0.70	0.50	6,372	0	528	86	442	0	55.06
0.90	0.50	15,726	0	1,302	209	1,092	0	49.12
0.95	0.50	20,066	0	1,643	232	1,411	0	46.89
0.50	0.70	402	0	0	-60	60	0	60.02
0.70	0.70	6,774	0	528	26	502	0	55.06
0.90	0.70	16,128	0	1,302	149	1,152	0	49.12
0.95	0.70	20,468	0	1,643	172	1,471	0	46.89
0.50	0.90	2,338	0	0	-349	349	0	60.02
0.70	0.90	8,710	0	528	-261	789	0	55.06
0.90	0.90	18,064	0	1,302	-140	1,441	0	49.12
0.95	0.90	22,404	0	1,643	-117	1,760	0	46.89
0.50	0.95	3,725	0	0	-556	556	0	60.02
0.70	0.95	10,097	0	528	-469	997	0	55.06
0.90	0.95	19,451	0	1,302	-346	1,648	0	49.12
0.95	0.95	23,791	0	1,643	-320	1,967	0	46.89

[a] A self-protection cost is incurred when satisfying monthly NIR and/or assuring surface supply availability at given probability levels. This cost is calculated as the difference between regional net returns under input demand and supply certainty ($653,481) and net returns at the given probability levels.

Surface supply uncertainty does not affect average irrigation efficiency in areas with conjunctive water management capability because the quantity of water applied remains constant as dependable groundwater is substituted for a portion of the uncertain surface water supplies. In contrast, stochastic input demand decreases average irrigation efficiency when monthly NIR is satisfied at higher probability levels since additional water is applied at each irrigation to reduce the probability of water stress between irrigations. Average irrigation efficiency decreases from 60% under certainty to 47% under uncertainty when the stochastic monthly NIR is satisfied at the 0.95 probability level. Total water use increases by 28% and groundwater use is increased by an even greater percentage (48%) even though average consumptive crop requirement remains constant at all probability levels. This result indicates that water policy analysts need to consider water input demand risk when predicting irrigation efficiency under a given technology.

Annual surface diversions generally exceed the certain baseline level when monthly NIR is satisfied at higher probability levels, and the probability of receiving anticipated surface diversions is maintained at the 0.50 or 0.70 level. This occurs because March, April, and May surface supplies exceed baseline demand requirements and are used to minimize groundwater costs early in the irrigation season when irrigating to above-baseline NIR. Annual surface diversions fall below the baseline level when controlling surface supply risk above the 0.70 probability level because the deterministic equivalents for late spring and summer surface supplies are reduced by a greater quantity than is offset by the additional early spring surface diversions.

Scenario 2: Restricted Groundwater Use

Self-protection cost is up to 56% greater when monthly groundwater use is restricted to current capacity. As reported in table 3, self-protection cost is $37,003 when both risk sources are satisfied at the 0.95 probability level, $13,212 more than when groundwater use is unrestricted. With limited groundwater use, self-protection cost is greater because the stochastic constraints can only be satisfied at the higher probability levels by reducing irrigated acreage. Thirty-one acres of irrigated wheat and 250 acres of irrigated pasture are taken out of production when each risk source is controlled at the 0.95 probability level. A dryland winter wheat-fallow rotation is substituted for the lost irrigated wheat acreage to minimize self-protection cost. Similar to Scenario 1, self-protection cost is greater when satisfying monthly NIR at higher probability levels and maintaining anticipated monthly surface supplies at baseline levels than when increasing the probability that anticipated monthly surface supplies will be delivered and satisfying monthly NIR at baseline levels.

Average irrigation efficiency is generally higher when groundwater use is restricted than in Scenario 1 because an increasing quantity of rill irrigated pasture acreage is taken out of production to satisfy the stochastic monthly water balance constraints when irrigating above average NIR. However, average irrigation efficiency still declines by 10 percentage points from the baseline level when each risk source is controlled at the 0.95 probability level.

Unlike Scenario 1, stochastic streamflow supplies affect average irrigation efficiency at higher probability levels. Average irrigation efficiency increases because groundwater use can no longer completely substitute for unreliable surface diversions and low-value

Table 3. Impact of Stochastic Input Demand and Supply on Net Income, Applied Irrigation Water, and Irrigation Efficiency: Monthly Groundwater Use Constrained to Existing Capacity (Scenario 2)

NIR Probability Level	Streamflow Probability Level	Self-Protection Cost ($)[a]	Average Consumptive Requirement (acre-feet)	Applied Irrigation Water (acre-feet)			Irrigated Acres (acres)	Irrigation Efficiency (%)
				Total	Surface	Groundwater		
Baseline (complete certainty)		0	3,520	5,864	1,750	4,114	2,052	60.02
			Change Relative to Baseline					
0.50	0.50	0	0	0	0	0	0	60.02
0.70	0.50	6,372	0	528	86	442	0	55.06
0.90	0.50	18,629	-225	733	193	540	-86	50.70
0.95	0.50	24,447	-361	703	232	471	-137	48.82
0.50	0.70	402	0	0	-60	60	0	60.02
0.70	0.70	6,990	-16	492	25	467	-6	55.09
0.90	0.70	20,595	-347	425	122	303	-132	51.29
0.95	0.70	26,407	-479	395	162	233	-182	49.38
0.50	0.90	2,338	0	0	-349	349	0	60.91
0.70	0.90	11,221	-193	77	-275	352	-73	57.10
0.90	0.90	24,745	-512	9	-181	190	-195	51.96
0.95	0.90	30,534	-639	-21	-142	122	-243	49.28
0.50	0.95	4,031	-24	-54	-557	503	-9	61.43
0.70	0.95	13,634	-267	-99	-487	388	-102	57.59
0.90	0.95	27,035	-581	-166	-394	228	-221	51.93
0.95	0.95	37,003	-722	-196	-356	161	-281	49.22

[a] A self-protection cost is incurred when satisfying monthly NIR and/or assuring surface supply availability at given probability levels. This cost is calculated as the difference between regional net returns under input demand and supply certainty ($653,481) and net returns at the given probability levels.

irrigated pasture acreage is taken out of production. Rill irrigated pasture is 45% efficient under baseline conditions, and any reduction in irrigated pasture acreage increases average irrigation efficiency.

The empirical evidence suggests study area farmers irrigate to slightly above-average NIR to protect against crop water stress. Observed regional water use and irrigation efficiency values for the study region in 1989 closely parallel the situation where NIR is satisfied at the 0.70 probability level and expected surface diversion supplies are set to their median level. Model simulations found that, except when satisfying both sources of risk at the 0.95 probability level, the stochastic constraints are maintained at higher probability levels by using additional groundwater and/or taking irrigated pasture acreage out of production. Irrigators confirmed they reduce irrigated pasture acreage and increase groundwater use in drought periods and/or low flow years as a low-cost buffer to shield their more valuable irrigated acreage from the impact of stochastic NIR and/or streamflow supplies (Willis).

Summary and Conclusions

This study presents a technique for simultaneously addressing stochastic input demands and resource supplies within a linear modeling framework. The technique is a useful alternative to nonlinear programming for researchers developing large programming models that contain more than a few stochastic parameters in the constraint set. Modern nonlinear algorithms still often fail to attain a global optimum in large models with more than a few nonlinear constraints.

The modeling procedure was used to determine how stochastic NIR and surface water supplies affect on-farm water management under input demand and resource supply uncertainty. Irrigators apply 28% more water when both sources of uncertainty are controlled at the 0.95 probability level than is applied under production certainty. The additional water application reduces average regional irrigation efficiency from 60% under certainty to 47%. Groundwater use is 48% greater than it is under production certainty when both risk sources are controlled at the 0.95 probability level. Groundwater use increases for two reasons. First, groundwater is blended with available surface supplies to irrigate to higher-than-average NIR. Second, dependable groundwater supplies are substituted for less dependable surface water supplies in the later months of the irrigation season to reduce the variability of monthly water supplies.

From a water policy perspective, the ability to model the effect of stochastic water demand and supply on on-farm water management is critical to accurately anticipating the response by irrigated agriculture to a basinwide in-stream flow policy. A policy designed to increase streamflows in low flow months to facilitate fish migration could fall short of policy expectations if irrigators substitute considerably more groundwater from an unconfined aquifer than anticipated due to production uncertainty. Groundwater diversions in excess of the production certainty level will lower the static level of groundwater, increase stream seepage, and/or decrease aquifer spring discharge, and eventually cause surface flows to fall below the policy expected level.

Low water prices encourage using water as an inexpensive form of insurance. When groundwater use is not constrained, the per acre cost of controlling each source of uncertainty, individually and collectively, ranged from $0.40 to $11.59, depending on the probability level considered. Though not addressed in this study, the expected costs of reducing production uncertainty are small relative to the potential economic losses associated with inadequate water application. The incentive to overirrigate, on average, could be reduced by encouraging the adoption of modern irrigation technologies that eliminate the need to irrigate on a fixed time schedule. Older technologies inhibit frequent irrigations because labor availability and setup time per move force irrigators to schedule irrigations at fixed time intervals and cause irrigators to irrigate above average NIR to reduce the likelihood of crop water stress between irrigations.

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