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The Impact of Pollution Controls on Livestock-Crop Producers

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A discrete-time, continuous-space model of a livestock–crop producer is used to examine the long-run effects of phosphorus runoff controls on optimal livestock production and manure application practices. Quantity restrictions and taxes on phosphorus application are shown to reduce livestock supply and impose greater costs on livestock–crop producers than on crop-only producers. Restrictions on manure application, without accompanying restrictions on commercial fertilizer application, will have only a limited effect on phosphorus runoff levels.

Key words: environmental policy, nonpoint source pollution, phosphorus runoff.

Introduction

Phosphorus runoff from land receiving livestock manure applications is a growing national concern. The Environmental Protection Agency and many state legislatures are considering means of regulating manure applications. In this article, a discrete-time, continuous-space model of a livestock producing farm with an agricultural land base is developed and analyzed. The model is first used to examine optimal manure application patterns and the resulting soil phosphorus levels in the absence of phosphorus runoff controls; in this context, the model explains why many livestock producing farms have soil phosphorus levels that exceed agronomic recommendations. The model then is used to examine optimal livestock numbers, manure application patterns, soil phosphorus levels, and net returns under two alternative phosphorus pollution control policies: (*a*) a policy which limits soil phosphorus levels at every point on a farm, and (*b*) a policy which limits average soil phosphorus levels throughout a farm.

Background

During the 1970s and 1980s, the primary environmental concern was direct manure runoff into streams and lakes. Many federal and state laws were enacted during the period to regulate storage facilities and manure application practices of large livestock producers (Bock et al.). Agricultural economists analyzed the costs and effectiveness of many of these regulations. For example, Ashraf and Christensen examined the effects of a limit on everyday manure spreading on dairy farms, and Forster (1975) examined the impacts of Environmental Protection Agency (EPA) regulations on beef feedlot costs and optimal size.

Recently, however, concern has shifted from direct manure runoff to manure affluents which enter the environment indirectly. Phosphorus runoff has received considerable attention because it is a major affluent in several parts of the country, including the

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Chesapeake Bay and Great Lakes watersheds. Phosphorus runoff predominantly occurs through soil erosion and is thus directly related to soil phosphorus levels. Soil phosphorus levels build up whenever manure or commercial fertilizer applications provide more phosphorus to the soil than can be used by the crop.

Manure applications on many livestock producing farms are believed to exceed recommended levels based on purely agronomic considerations and have been blamed for causing "excessively high" soil phosphorus levels. As a result, policies currently are being considered to limit the rate at which manure can be applied to a parcel of land. For example, the EPA may require livestock producers with more than 1,000 animal units to adopt manure application practices which do not raise soil phosphorus levels beyond agronomic recommendations. Most eastern and midwestern states also are considering legislation that would apply to a wider range of livestock producers. For instance, Ohio has passed legislation that requires all expanding livestock operations to adopt manure management plans that would limit increases in soil phosphorus levels (Ohio General Assembly). Both the EPA and state regulations would limit the sustained application of manure at every point on a farm.

Phosphorus runoff control proposals have generated considerable debate. Livestock producers are concerned with the impacts of policies on production costs and net returns. Also, they perceive an inequity from having to observe phosphorus application restrictions while crop-only producers do not. Environmental groups question how effectively the proposed policies would control phosphorus runoff. And economists question the wisdom of having a set limit on manure applications based solely on agronomic considerations. Other policies, such as total farm runoff limits, tax/subsidy schemes, and pollution licenses, may reduce phosphorus runoff at lower costs (Baumol and Oates; Jacobs and Timmons; Moffitt, Zilberman, and Just).

A model designed to examine phosphorus runoff policies must contain both dynamic and spatial dimensions. Unlike examinations of nitrate runoff issues (e.g., Horner; Nanley; Taylor and Swanson), an examination of phosphorus runoff issues requires dynamic considerations because phosphorus soil levels are related to past phosphorus levels and applications. Moreover, a spatial framework is necessary to account for increases in hauling costs as manure is applied further from the livestock facility. In the following section, we develop a model possessing both of these features.

A Livestock-Crop Production Model

Consider a livestock-crop producer who owns a single livestock facility and uses the manure generated by the livestock to supplement commercial fertilizer applications on the crop. The producer's objective is to maximize the discounted sum of current and future annual profits from the livestock and crop enterprises:

$$(1) \qquad \sum_{t=0}^{\infty} \delta^t \left[p_t^l q_t + \int_A \{p_t^c f_t(x_t(a) + y_t(a), z_t(a), a) - k_t x_t(a) - h_t(r(a)) y_t(a)\} \, da \right],$$

subject to

$$(2) \qquad z_{t+1}(a) = g_t(x_t(a) + y_t(a), z_t(a), a), \quad \text{and}$$

$$(3) \qquad \eta q_t = \int_A y_t(a) \, da.$$

Here, $A \subset R^2$ denotes the crop production area, $a \in A$ denotes a point in the crop production area, and $r(a)$ denotes the distance from the livestock facility to point a. The endogenous variables are livestock produced, q_t; commercial fertilizer application rate at point a, $x_t(a)$; manure application rate at point a, $y_t(a)$; and carryover rate at point a, $z_t(a)$. The exogenous parameters are the annual discount factor, δ; unit profit contribution of livestock, p_t^l; unit price of crop, p_t^c; unit price of commercial fertilizer, k_t; unit cost of applying manure at a distance r, $h_t(r)$; manure produced per unit of livestock, η; the yield response function, $f_t(\cdot)$; and the phosphorus carryover function, $g_t(\cdot)$.

Equation (2) expresses phosphorus carryover as a function of phosphorus carryin and current commercial fertilizer and manure application. Equation (3) requires that all the manure generated by the livestock be applied on crop acreage. Commercial fertilizer and manure application rates and phosphorus carryover rates are measured conformably in pounds of phosphorus per acre. That is, one unit of x_t represents one pound of elemental phosphorus applied in the form of manure, one unit of y_t represents one pound of elemental phosphorus applied in the form of commercial fertilizer, and one unit of z_t represents a test level of one pound of elemental phosphorus per acre.[1]

The yield response and phosphorus carryover functions, f_t and g_t, are assumed to be twice continuously differentiable and subject to the following curvature conditions: $f_{t,2} \geq 0, f_{t,11} < 0, f_{t,22} \leq 0, f_{t,12} \leq 0$, $g_{t,1} \geq 0$, $1 \geq g_{t,2} \geq 0$, $g_{t,11} \leq 0$, $g_{t,22} \leq 0$, and $g_{t,12} \leq 0$. Since more time and power are required the further manure is hauled, the unit cost $h_t(r)$ of spreading manure at a distance r from the livestock facility is assumed to be a positive, continuous, and strictly increasing. We further assume that there is a minimum level of soil carryover $\bar{z} > 0$, and that the discount factor is less than one.

For the farmer's dynamic profit maximization problem (1)–(3), the optimal level of livestock production, q_t, the optimal rates of commercial fertilizer and manure application, $x_t(a)$ and $y_t(a)$, and the optimal rate of carryover, $z_t(a)$, are characterized by the constraints (2)–(3) and the Karush–Kuhn–Tucker complementarity slackness conditions:

(4) $$q_t \geq 0 : p_t^l + \eta\lambda_t \leq 0,$$

(5) $$x_t(a) \geq 0 : p_t^c f_{t,1}(a) + \mu_t(a) g_{t,1}(a) - k_t \leq 0,$$

(6) $$y_t(a) \geq 0 : p_t^c f_{t,1}(a) + \mu_t(a) g_{t,1}(a) - h_t(r(a)) - \lambda_t \leq 0, \quad \text{and}$$

(7) $$z_t(a) \geq 0 : p_t^c f_{t,2}(a) + \mu_t(a) g_{t,2}(a) - \delta^{-1}\mu_{t-1}(a) \leq 0.$$

Here, $f_{t,1}(a)$, $f_{t,2}(a)$, $g_{t,1}(a)$, and $g_{t,2}(a)$ are the partial derivatives of the yield and carryover functions evaluated at $(x_t(a) + y_t(a), z_t(a), a)$; λ_t is the current-valued shadow price of manure; and $\mu_t(a)$ is the current-valued shadow price of carryover at point a.

To simplify the analysis, we assume that the prices and costs and the yield response and carryover functions do not change over time. Under this assumption, the producer's dynamic profit maximization problem (1)–(3) has a unique optimal solution with a well-defined steady state. Dropping the time subscript t and denoting the steady-state solutions by q^*, $x^*(a)$, $y^*(a)$, $z^*(a)$, λ^*, and $\mu^*(a)$, it follows from (4)–(7) that:

(8) $$q^* \geq 0 : p^l + \eta\lambda^* \leq 0,$$

(9) $$x^*(a) \geq 0 : p^c f_1(a) + \mu^*(a) g_1(a) - k \leq 0,$$

(10) $$y^*(a) \geq 0 : p^c f_1(a) + \mu^*(a) g_1(a) - h(r(a)) - \lambda^* \leq 0, \quad \text{and}$$

(11) $$z^*(a) \geq 0 : p^c f_2(a) + \mu^*(a) g_2(a) - \delta^{-1}\mu^*(a) \leq 0,$$

where $f_1(a)$, $f_2(a)$, $g_1(a)$, and $g_2(a)$ are the steady-state partial derivatives of the yield and carryover functions at point a. These complementarity conditions, together with the constraints

(12) $$z^*(a) = g^*(x^*(a) + y^*(a), z^*(a), a) \text{ and}$$

(13) $$\eta q^* = \int_A y^*(a)\, da$$

completely characterize the steady-state solution to the farmer's dynamic profit maximization problem.

Since carryover levels are always positive, (11) may be solved explicitly for the steady-state shadow price of carryover,

(14) $$\mu^*(a) = p^c f_2(a)/(\delta^{-1} - g_2(a)),$$

allowing us to rewrite the commercial fertilizer and manure optimality conditions (9)–(10) as follows:

(15) $$x^*(a) \geq 0 : p^c\left[f_1(a) + \frac{f_2(a)g_1(a)}{\delta^{-1} - g_2(a)}\right] - k \leq 0;$$

(16) $$y^*(a) \geq 0 : p^c\left[f_1(a) + \frac{f_2(a)g_1(a)}{\delta^{-1} - g_2(a)}\right] - h(r(a)) - \lambda^* \leq 0.$$

As shown in the appendix, there exists a differentiable function ϕ such that for every $x \geq 0$ and $a \in A$, there is a $z > 0$ such that:

(17) $$\phi(x, a) = f_1(x, z, a) + \frac{f_2(x, z, a)g_1(x, z, a)}{\delta^{-1} - g_2(x, z, a)};$$

(18) $$z = g(x, z, a).$$

The function ϕ is strictly decreasing in x and may be interpreted, with qualification, as the long-run marginal product of phosphorus input.[2] Once derived, ϕ may be used to compactly rewrite the complementarity conditions (8)–(10) as follows:

(19) $$q^* \geq 0 : p^l + \eta\lambda^* \leq 0,$$

(20) $$x^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - k \leq 0, \quad \text{and}$$

(21) $$y^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - h(r(a)) - \lambda^* \leq 0.$$

These conditions, together with the manure balance constraint (13), completely characterize the steady-state solution to the farmer's profit maximization problem.

Under the mild assumption that $\min\{p^c\phi(0, 0), k\} > h(0) - p^l/\eta$, livestock production will be profitable, $q^* > 0$.[3] From (19) it then follows that the shadow price of manure,

(22) $$\lambda^* = -p^l/\eta,$$

will be negative and fully determined by the profit contribution and manure generated per unit of livestock.[4] The farmer optimally produces livestock until, on the margin, the explicit profit contribution of one unit of livestock just equals the implicit cost of disposing of the manure generated by the livestock.

From (20)–(22), the optimal steady-state manure–fertilizer application pattern is easy to visualize. There is a critical radius r^*, characterized by

(23) $$h(r^*) = k - \lambda^* = k + p^l/\eta,$$

at which the economic cost of applying manure and the cost of applying commercial fertilizer are equal. Out to a radius of r^*, manure is more economical to apply than commercial fertilizer; beyond a radius of r^*, commercial fertilizer is more economical. Inside the critical radius r^*, manure is applied at a rate that equates its long-run value marginal product to its shadow price plus cost of application:

(24) $$y^*(a) = \phi^{-1}\left(\frac{h(r(a)) + \lambda^*}{p^c}, a\right) = \phi^{-1}\left(\frac{h(r(a)) - p^l/\eta}{p^c}, a\right).$$

(Here, ϕ^{-1} refers to the inverse of ϕ with respect to its first argument.) Outside the critical radius, only commercial fertilizer is applied at a rate that equates its long-run value marginal product to its cost of application:

(25) $$x^*(a) = \phi^{-1}\left(\frac{k}{p^c}, a\right).$$

Of course, if the critical radius r^* exceeds the maximum radial extent of the farm, commercial fertilizer is uneconomical throughout the farm and manure is used exclusively.

Since ϕ^{-1} is strictly decreasing in its first argument and manure hauling costs rise with distance from the livestock facility, the rate of manure application exceeds the rate of commercial fertilizer application everywhere except at the critical radius, where they are equal. Thus, inside the critical radius, a livestock–crop producer applies more phosphorus (in the form of manure) than does a crop producer without a livestock operation. Indeed, a livestock–crop producer may actually find it economical to apply manure even though its marginal product is negative. In the special case that the yield response and carryover functions are spatially uniform, the optimal rate of manure application is a decreasing function of the distance from the livestock facility and the optimal rate of commercial fertilizer application will be constant.

From (22)–(25) it also follows that an increase in the profit contribution of livestock, p^l, raises the critical radius, livestock production, and total manure application, but reduces total commercial fertilizer application. An increase in the crop price, p^c, has no effect on the critical radius, raises total commercial fertilizer application, and has an indeterminate effect on livestock production and total manure application. An increase in the cost of commercial fertilizer, k, raises the critical radius, livestock production, and total manure application, but reduces total commercial fertilizer application. A uniform increase in hauling costs, $h(\cdot)$, reduces the critical radius, livestock production, and total manure application, but raises total commercial fertilizer application.

The Effects of Pollution Controls

We now examine the long-run pattern of livestock–crop production under two types of pollution controls that limit soil phosphorus levels. Point-wise controls would limit soil phosphorus levels at every point of the farm and are similar to regulations currently being considered by the Environmental Protection Agency and state legislatures. Whole-farm controls would limit the average soil phosphorus level throughout the farm, thus affording the producer some latitude in determining phosphorus applications at specific points on the farm.[5]

Since soil phosphorus levels are, in the long run, determined by the sustained levels of phosphorus application, point-wise controls can be equivalently stated as limits on the sustained levels of manure and commercial fertilizer applications at every point:

(26) $$x^*(a) + y^*(a) \leq m \qquad a \in A.$$

Similarly, whole-farm controls can be equivalently stated as limits on the sustained levels of total manure and commercial fertilizer applications throughout the farm:

(27) $$\int_A (x^*(a) + y^*(a))\, da \leq M.$$

Point-Wise Pollution Controls

Under point-wise pollution controls, the optimal steady-state rates of manure and commercial fertilizer application are characterized by the following complementarity slackness conditions:

(28) $$x^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - k - \tau^*(a) \leq 0,$$

(29) $y^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - h(r(a)) - \lambda^* - \tau^*(a) \leq 0,$ and

(30) $\tau^*(a) \geq 0 : x^*(a) + y^*(a) \leq m.$

Here, $\tau^*(a)$, the shadow price associated with the point-wise pollution constraint, represents an implicit pollution tax on phosphorus application at the point a.

From (28)–(30), the critical radius r^* at which commercial fertilizer becomes more economical than manure is not affected by the introduction of point-wise controls and continues to be characterized by (23). Inside the critical radius, the point-wise control is binding if and only if $p^c\phi(m, a) - h(r(a)) + p^l/\eta > 0$, in which case the implicit tax on manure application is:

(31) $$\tau^*(a) = p^c\phi(m, a) - h(r(a)) + p^l/\eta.$$

Outside the critical radius, the point-wise control is binding if and only if $p^c\phi(m, a) - k > 0$, in which case the implicit tax on commercial fertilizer application is:

(32) $$\tau^*(a) = p^c\phi(m, a) - k.$$

Clearly, if $h(0) - p^l/\eta > p^c\phi(m, a)$ for all $a \in A$, point-wise pollution controls will be nowhere binding and it will be optimal in the long run to apply manure and commercial fertilizer at the pre-controls levels (24) and (25). Conversely, if $p^c\phi(m, a) > k$ for all $a \in A$, point-wise pollution controls will be binding everywhere and it will be optimal in the long run to apply manure and commercial fertilizer at the maximum allowable rate m throughout the farm.

If the yield response and carryover functions are uniform throughout the farm, then, inside the critical radius, the implicit tax on manure application is positive and declining with distance from the livestock facility; outside the critical radius, the implicit tax on commercial fertilizer application is positive and constant. In the intermediate case in which $k > p^c\phi(m, 0) \geq h(0) - p^l/\eta$, there will be a radius r', characterized by

(33) $$h(r') = p^c\phi(m, 0) + p^l/\eta,$$

within which point-wise pollution controls will be binding. Within this radius, it is optimal in the long run to apply manure at the maximum allowable rate m; outside this radius, the point-wise pollution constraint is nonbinding and it is optimal in the long run to apply manure and commercial fertilizer at the pre-controls levels (24) and (25).

Thus, point-wise pollution controls are, in the long run, equivalent to placing a tax on phosphorus application that typically declines as the distance from the livestock operation to the point of application rises. At binding points, the pollution tax increases if either the crop price rises, the profit contribution of livestock rises, the cost of hauling manure falls, or the cost of commercial fertilizer falls.

Whole-Farm Pollution Controls

Under whole-farm pollution controls, the optimal steady-state rates of manure and commercial fertilizer application are characterized by the following complementarity slackness conditions:

(34) $x^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - k - \tau^* \leq 0,$

(35) $y^*(a) \geq 0 : p^c\phi(x^*(a) + y^*(a), a) - h(r(a)) - \lambda^* - \tau^* \leq 0,$ and

(36) $\tau^* \geq 0 : \int_A (x^*(a) + y^*(a))\, da \leq M.$

Here, τ^*, the shadow price associated with the whole-farm pollution constraint, represents an implicit pollution tax on phosphorus application, which is uniform across the farm.

The steady-state pattern of manure and commercial fertilizer application in the presence of whole-farm pollution controls is similar to that in the absence of controls. From (34)–(36), the critical radius r^* at which commercial fertilizer becomes more economical than manure application is not affected by the introduction of whole-farm controls and continues to be characterized by (23). Within the critical radius, manure is applied at a rate given by

$$(37)\qquad y^*(a) = \phi^{-1}\left(\frac{h(r(a)) + \lambda^* + \tau}{p^c}, a\right) = \phi^{-1}\left(\frac{h(r(a)) - p^l/\eta + \tau}{p^c}, a\right).$$

Outside the critical radius, commercial fertilizer is applied at a rate given by

$$(38)\qquad x^*(a) = \phi^{-1}\left(\frac{k + \tau}{p^c}, a\right).$$

In the special case that the yield response and carryover functions are spatially uniform, the optimal rate of manure application will fall with the distance from the livestock facility and the optimal rate of commercial fertilizer application will be constant.

Thus, whole-farm pollution controls are, in the long run, equivalent to placing a tax on phosphorus application that is uniform throughout the farm. In general, the pollution tax increases if the profit contribution of livestock rises, the cost of hauling manure falls, or the cost of commercial fertilizer falls.

An Empirical Illustration

As an empirical illustration, we now assess the long-run effects of point-wise and whole-farm pollution controls on a representative midwestern finishing hog producer. The producer purchases feeder pigs at 40 pounds and sells hogs at a weight of 230 pounds at a net profit contribution of $5 per hog. The hogs are fed in an open-front finishing facility with a pit manure storage. Based on these weights and facility, each hog is assumed to generate manure containing 2.3 pounds of phosphorus (Midwest Plan Service). The producer also raises corn on 1,200 tillable acres that are uniformly distributed radially from the finishing facility. It is assumed that the price of corn is $2.50 per bushel and the cost of commercially purchased fertilizer is $.50 per pound of phosphorus.

The yield response, carryover, and hauling cost functions are assumed to be spatially uniform over the entire farm and to have the following forms:

$$(39)\qquad f(x, z) = \alpha_0 + \alpha_1 x + \alpha_2 z - \frac{1}{2}\alpha_{11}x^2 - \alpha_{12}xz - \frac{1}{2}\alpha_{22}z^2,$$

$$(40)\qquad g(x, z) = \beta_0 + \beta_1 x + \beta_2 z, \quad \text{and}$$

$$(41)\qquad h(r) = \gamma_0 + \gamma_1 r,$$

where x is the amount of phosphorus applied (in either manure or commercial fertilizer form), z is phosphorus carryover, and r is the distance from the livestock facility. For these specifications, the long-run marginal product function takes the linear form

$$(42)\qquad \phi(x) = \phi_0 - \phi_1 x,$$

where

$$\phi_0 = \alpha_1 + \frac{\beta_1}{\delta^{-1} - \beta_2}\alpha_2 - \frac{\beta_0}{1 - \beta_2}\alpha_{12} - \frac{\beta_0\beta_1}{(\delta^{-1} - \beta_2)(1 - \beta_2)}\alpha_{22};$$

$$\phi_1 = \alpha_{11} + \left[\frac{\beta_1}{\delta^{-1} - \beta_2} + \frac{\beta_1}{1 - \beta_2}\right]\alpha_{12} + \frac{\beta_1^2}{(\delta^{-1} - \beta_2)(1 - \beta_2)}\alpha_{22}.$$

Note that $\phi_1 > 0$. Technical parameters of the yield response, carryover, hauling cost, and long-run marginal product functions are given in table 1.

In the absence of pollution controls, the 1,200 acres receive an average of 27.9 pounds

Table 1. Coefficients of the Yield, Long-Run Marginal Product, Carryover, and Hauling Cost Functions for a Representative Midwestern Hog-Corn Farm

Function	Parameter	Value
Yield	α_0	132.16500
	α_1	.35064
	α_2	.93185
	α_{11}	.00276
	α_{12}	.00461
	α_{22}	.02104
Marginal Product	ϕ_0	.81463
	ϕ_1	.02340
Carryover	β_0	2.01480
	β_1	.17967
	β_2	.79852
Hauling Costs	γ_0	2.20000
	γ_1	.80000

Note: Coefficients of the yield, marginal product, and carryover functions are adapted from Forster (1983).

of phosphorus per acre, with 61% of the phosphorus coming from manure applications (see table 2). Manure alone is applied out to the critical radius of .59 miles from the livestock facility. At a distance of .05 miles, manure is applied at a rate of 33.7 pounds of phosphorus per acre, yielding a carryover of 40 pounds of phosphorus per acre. Manure application rates and carryovers, respectively, decline to 26.3 and 33.4 pounds per acre at the critical radius. Beyond the critical radius, commercially purchased phosphorus is applied at a constant rate of 26.3 pounds per acre, yielding a carryover of 33.4 pounds per acre.

Assuming that the rate of erosion is uniform throughout the farm, total phosphorus runoff from the farm will decrease by 15% in the long run if phosphorus application is reduced to an average of 23.7 pounds per acre. Under a point-wise policy yielding a 15% runoff reduction, manure and commercial fertilizer will be optimally applied uniformly throughout the farm at the maximum allowable rate (see table 2). Manure alone is applied within the .59 critical radius and commercial fertilizer alone is applied at the maximum rate beyond. Manure, as a proportion of total phosphorus applied, drops to 59%.

On the other hand, under a whole-farm policy yielding a 15% runoff reduction, phosphorus will not be optimally applied at a uniform rate throughout the farm. Under whole-farm controls, manure will be applied above the average rate of 23.7 pounds of phosphorus per acre near the livestock facility where hauling costs are lower; to compensate, manure and commercial fertilizer are applied below the average rate further from the facility. At .05 miles from the facility, manure is applied at a rate of 29.5 pounds of phosphorus per acre, yielding a carryover of 36.3 pounds per acre. Manure application rates and carryovers then decline to 22.1 and 29.7 pounds of phosphorus per acre at the .59 critical radius. Outside the critical radius, commercially purchased fertilizer is applied at a rate of 22.1 pounds of phosphorus per acre. Manure, as a proportion of total phosphorus applied, rises to 62%.

As discussed earlier, the effects of point-wise and whole-farm pollution controls can be equivalently achieved through taxes on phosphorus application. As seen in table 2, the tax associated with a point-wise policy varies with the distance from the livestock facility. At .05 miles, the tax is $.59 per pound of phosphorus per acre; this declines to $.15 at the critical radius of .59 miles and remains at that level thereafter. The tax associated with a whole-farm control policy, on the other hand, is uniform throughout the farm at $.24 per pound of phosphorus per acre.

As seen in table 3, runoff controls will reduce the optimal number of hogs finished on

Table 2. Optimal Steady-State Rates of Phosphorus Application and Carryover and Implicit Pollution Tax at Differing Distances from the Livestock Facility, Under Alternative Policies Resulting in a 15% Reduction in Phosphorus Runoff for a Representative Hog-Corn Farm

	Distance (miles)	No Policy	Point-wise Policy	Whole-farm Policy
		(pounds/acre)		
Phosphorus Application	.05	33.7	23.7	29.5
	.15	32.3	23.7	28.1
	.25	31.0	23.7	26.8
	.35	29.6	23.7	25.4
	.45	28.2	23.7	24.0
	.55	26.9	23.7	22.7
	.65	26.3	23.7	22.1
	.75	26.3	23.7	22.1
Carryover	.05	40.0	31.1	36.3
	.15	38.8	31.1	35.1
	.25	37.6	31.1	33.9
	.35	36.4	31.1	32.7
	.45	35.2	31.1	31.4
	.55	33.9	31.1	30.2
	.65	33.4	31.1	29.7
	.75	33.4	31.1	29.7
		($/pound of phosphorus/acre)		
Pollution Tax	.05	.00	.59	.24
	.15	.00	.51	.24
	.25	.00	.43	.24
	.35	.00	.35	.24
	.45	.00	.27	.24
	.55	.00	.19	.24
	.65	.00	.15	.24
	.75	.00	.15	.24

Note: Phosphorus application within the critical radius of .59 miles is in the form of manure; outside the radius it is in the form of commercial fertilizer. The maximum radial extent of the farm is .77 miles.

the farm. Without controls, a total of 8,888 hogs are raised. In achieving a 15% runoff reduction, hog numbers will decline 18% under the point-wise policy and 14% under the whole-farm policy. In general, the percentage reduction in hog numbers is greater than the percentage reduction in runoff under the point-wise policy, while the percentage reduction in hog numbers is less than the percentage reduction in runoff under the whole-

Table 3. Steady-State Annual Hog Production Under Alternative Runoff Control Policies for a Representative Hog-Corn Farm

Percent Runoff Reduction	Hogs Produced		Percent Reduction	
	Point-wise Policy	Whole-farm Policy	Point-wise Policy	Whole-farm Policy
0	8,888	8,888	0	0
5	8,161	8,460	8	4
10	7,692	8,033	13	9
15	7,264	7,606	18	14
20	6,837	7,178	23	19
25	6,410	6,751	28	24
30	5,982	6,324	33	29

Table 4. Steady-State Annual Cost to Producers of Alternative Runoff Control Policies for a Representative Hog-Corn Farm

Percent Runoff Reduction	Point-wise Policy	Whole-farm Policy
	($)	
0	0	0
5	231	135
10	543	431
15	1,001	889
20	1,619	1,508
25	2,400	2,288
30	3,341	3,229

farm policy. Hog numbers are higher under the whole-farm policy because the whole-farm control allows for higher manure applications.

The effects of point-wise and whole-farm pollution controls on net producer revenues are shown in table 4. In general, the cost borne by producers rises with the required reduction in runoff. For example, under a point-wise policy, long-run producer profits fall $1,001 per year under a 15% runoff reduction and $3,341 a year under a 30% reduction; under a whole-farm policy, long-run producer profits fall $889 per year under a 15% runoff reduction and $3,229 per year under a 30% reduction. Because of the greater flexibility that they offer in achieving a stated runoff reduction, whole-farm controls impose lower costs on livestock–crop producers than point-wise controls.

The costs of point-wise phosphorus application controls on otherwise comparable 1,200-acre corn farms, one with hogs and one without, are shown in table 5. The corn-only farm faces no costs until phosphorus application is capped below the optimal uncontrolled rate of 26.3 pounds per acre. Because hog–corn producers apply manure at higher rates, the impacts of point-wise controls are felt at higher caps. In general, point-wise controls impose substantially greater costs on livestock–crop producers than on corn-only producers.

Finally, it should be noted that prohibiting "excess" manure application would have only a limited effect on total phosphorus runoff. Specifically, requiring manure applications to be no greater than the optimal uncontrolled level of commercial fertilizer application (26.3 pounds of phosphorus per acre) will reduce total farm runoff by only 6%. To achieve runoff reductions on the order of 20% or more, restrictions would have to be placed on both manure and commercial fertilizer application rates.

Table 5. Steady-State Annual Cost to Producers of a Point-Wise Control Policy for Representative Hog-Corn and Corn-Only Farms

Maximum Phosphorus Application	Hog–Corn	Corn-only
(pounds/acre)	($)	
30	18	0
29	47	0
28	99	0
27	186	0
26	328	19
25	563	143
24	881	349
23	1,282	639
22	1,767	1,012
21	2,334	1,468
20	2,985	2,008

Conclusion

In this article, we have developed a discrete-time, continuous-space model of a livestock–crop producer and used it to examine the long-run effects of alternative phosphorus runoff controls on optimal livestock production and manure application practices. In an empirical illustration, the model was used to assess the long-run effects of pollution controls on a representative midwestern finishing hog producer.

Placing either whole-farm or point-wise runoff controls on livestock–crop producers would reduce livestock supply and producer net income. For a given level of runoff reduction, whole-farm controls have a less adverse impact than point-wise controls. Whole-farm controls would preserve the fundamental relationships among marginal costs of application, allowing higher rates of manure application near the farm, where it is cheapest, and lower rates at greater distances, just as in the absence of controls. Whole-farm runoff controls are equivalent to a spatially uniform tax on phosphorus application; point-wise controls, on the other hand, are equivalent to a tax on phosphorus application that rises with the proximity to the livestock facility.

Prohibiting the manure application rate from exceeding the commercial fertilizer rate probably would have only a small effect on phosphorus runoff. In order to achieve significant runoff reductions from livestock–crop producing farms, restrictions would have to be placed on both manure and commercial fertilizer application. Constraining phosphorus application on livestock–crop farms but not on crop-only farms would be difficult to justify. Consideration should therefore be given to limiting phosphorus applications on both livestock–crop and crop-only farms. Phosphorus application limits, however, can be expected to have a substantially greater economic impact on livestock–crop producers than on crop-only producers.

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Notes

[1] Hog manure contains approximately .25% phosphorus by weight, implying that approximately 400 pounds of manure are required for one pound of phosphorus; a typical 0-45-0 fertilizer contains approximately 20% phosphorus by weight, implying that approximately five pounds of commercial fertilizer are required for one pound of phosphorus. Research strongly supports the assumption implicitly made here that manure phosphorus and commercial fertilizer phosphorus are perfect substitutes in crop production (Gilbertson et al.; Midwest Plan Service).

[2] The term $p^c\phi(x^*, a)$ will equal the current plus discounted future value marginal product of phosphorus input in steady state; as such, the function ϕ performs the role a long-run marginal product function would be expected to play. The function ϕ, however, depends on the discount factor δ; it cannot be constructed from the underlying technical production and carryover relations alone.

[3] The condition merely requires that the value marginal product of manure at the zero input level and the unit cost of commercial fertilizer both exceed the net unit cost of applying manure at a point adjacent to the livestock facility. Otherwise, it would be unprofitable to produce livestock, a degenerate case of no particular interest to us here.

[4] The negative of the shadow price, p^l/η, is the amount the farmer would be willing to pay to dispose of manure at its source; if a commercial manure disposal service was available at a lower cost than this, the farmer would profit from contracting the service. Because off-farm manure disposal is relatively rare in the U.S., we restrict our attention to the more common case of complete on-farm disposal of manure.

[5] Generally, the rate of phosphorus runoff is the mathematical product of the soil phosphorus level and the rate of erosion. The policies examined here, like most of the policies currently being considered for legislation, attempt to reduce soil phosphorus levels. Alternatively, policies could be designed to reduce phosphorus runoff by reducing soil erosion rates, say, by influencing producer tillage and cropping practices. Such runoff reduction policies are beyond the scope of the current research, but merit further consideration.

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Appendix

Consider first equation (18), which implicitly defines the steady-state level of phosphorus carryover z associated with a sustained level of phosphorus application x at a point a. Let $G(x, z, a) = z - g(x, z, a)$ and recall that $G(x, \bar{z}, a) \leq 0$ for all $x \geq 0$, where $\bar{z} > 0$ is the minimum carryover level. By assumption, $G(\cdot)$ is twice continuously differentiable, with $G_2 > 0$ and $G_{22} \geq 0$, so that, for every $x \geq 0$, there is a unique $z = z(x, a) \geq \bar{z}$ such that $G(x, z, a) = 0$, that is, such that (18) holds. By the Implicit Function Theorem, the solution function $z(\cdot)$ is continuously differentiable with $z_1(x, a) = g_1(x, z(x), a)/(1 - g_2(x, z(x, a))) \geq 0$. Thus at every point, for any sustained level of phosphorus application, there is a unique steady-state level of phosphorus carryover; moreover, the steady-state carryover level is a nondecreasing function of the application level.

Inserting $z(\cdot)$ in the right-hand side of (17), we see that the long-run marginal product function $\phi(\cdot)$ is well-defined and continuously differentiable for $x \geq 0$. Omitting arguments and differentiating, we obtain

$$\phi_1(x, a) = f_{11} + f_{12}z_1 + f_2 g_1(g_{12} + g_{22}z_1)/(\delta^{-1} - g_2)^2 + (f_2 g_{11} + f_2 g_{12} z_1 + g_1 f_{12} + g_1 f_{22} z_1)/(\delta^{-1} - g_2).$$

Since $g_2 < 1 < \delta^{-1}$, all first partials are nonnegative, all second partials are nonpositive, and f_{11} is strictly negative, it follows that $\phi(\cdot)$ is continuous in both its arguments and strictly decreasing in its first argument throughout its domain.