Filtering driving cycles for assessment of electrified vehicles

Nikolce Murgovski, Xiaosong Hu, Lars Johannesson and Bo Egardt

***Abstract*—We present a method for pre-filtering driving cycles that are to be used for assessment of electrified vehicles. The method ensures that the vehicle may exactly follow the filtered velocity demanded by the driving cycle. Employing convex optimization, the method also allows optimal velocity shaping that minimizes the amount of wasted energy. We illustrate the method by an example of performance assessment of a hybrid electric bus in a series powertrain topology.**

***Index Terms*—driving cycle filtering, hybrid electric vehicle, power management, convex optimization, optimal control**

I. Introduction

Electrified vehicles are being of major interest in academia and industry due to their potential for improved powertrain efficiency and zero or low level of emissions, compared to conventional vehicles. The improved powertrain efficiency is mainly a result of an additional energy source, e.g. electric battery or supercapacitor, and an electric machine (EM) that may propel the vehicle alongside the internal combustion engine (ICE) (or completely replace the ICE, as in electric vehicles). The cost-effectiveness of the vehicle then strongly depends on the choice and size of powertrain components (electric buffer, ICE, EM), and the control strategy that decides the magnitude of power and energy delivered by these components.

Due to many competing powertrain solutions, the cost-effectiveness of the electrified vehicle is typically investigated before the manufacturing phase. Then, the potential of the vehicle is determined by simulating a vehicle model on certified driving cycles, or a set of driving cycles that mimic the typical daily usage of the vehicle (described by e.g. speed and road gradient as a function of time). The theoretically optimal performance is sought considering perfect knowledge of the driving cycle. This generally involves some type of optimization, which is typically not a trivial task. In terms of computational effort, especially challenging is the performance assessment of hybrid electric vehicles (HEVs), which possess both ICE and EM. To lower the computational burden, most of the fast vehicle dynamics are neglected and a backwards simulation model is used. In backwards simulation it is assumed that the vehicle exactly follows the demanded velocity, thus removing the velocity state from the problem. This leaves only one state in the problem, the state of energy (SOE) of the electric buffer, allowing the optimal solution to be pursued by dynamic programming, [6, 8, 14], Pontryagin's maximum principle, [3, 5], or convex optimization, [9–11, 15].

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The backwards simulation model, however, introduces also some difficulties. One limitation of this model is that it prohibits the usage of some driving cycles (with e.g. high acceleration demands), as these cycles may render the optimization problem infeasible. An example are artificial cycles constructed by speed limits changing in a staircase manner. To mitigate this problem, the driving cycles are pre-filtered before using them in optimization. This can be achieved by first measuring the actual speed of another vehicle following a reference velocity, and then optimizing the studied electrified vehicle over the filtered velocity profile. The other vehicle is typically an existing conventional vehicle, or a model of it, for which it is straight forward to obtain the optimal control strategy, by e.g. static optimization. This, however, does not guarantee feasibility, as the electrified vehicle may include downsized powertrain components, thus not being able to deliver the same performance as the conventional vehicle. Moreover, even if the problem is feasible, it may not be optimal to drive the electrified vehicle in the same way as the conventional vehicle. For example, an electrified vehicle may recuperate more braking energy with lower deceleration, and it is therefore beneficial to start braking sooner before reaching the stop. Obviously, the optimal solution can be obtained only when both the velocity shaping and control strategy are optimized simultaneously.

The contribution of this paper is a method for potential assessment of electrified vehicles using convex optimization [1]. The method is based on a forward simulation model involving two states, the vehicle velocity and SOE of the electric buffer. We present the method through an example of optimal control of a hybrid electric bus in a series powertrain topology, [5], but the final goal is to extend the method to other types of electrified vehicles and powertrain topologies.

The paper is organized as follows: the vehicle model and problem formulation are described in Section II and III; in Section IV the problem is rewritten from sampling in time to sampling in distance; convex remodeling is described in Section V; an example of optimally controlling a city bus is given in Section VI; and the paper is ended with discussions and conclusions in Section VII.

II. Vehicle model

The studied vehicle is an HEV in a series topology [5], as illustrated in Fig. 1. This powertrain does not have a direct mechanical link between the ICE and the wheels, but instead, the wheels are driven by an EM that receives energy from a battery or an engine-generator unit (EGU).

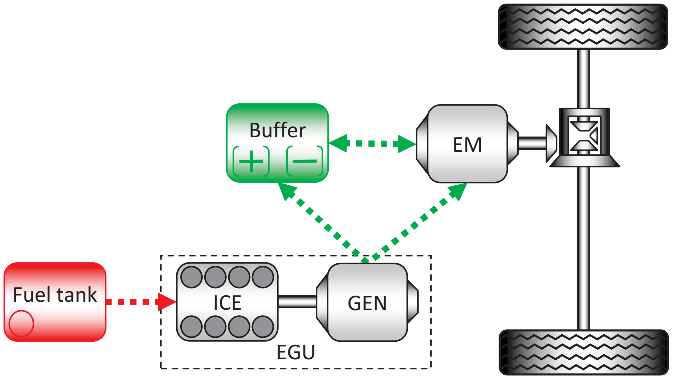

Fig. 1. Series PHEV powertrain model.

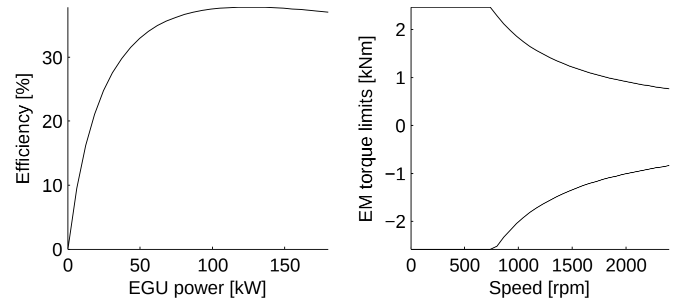

Fig. 2. EGU efficiency, left plot, and EM torque limits, right plot.

While following the driving mission, the vehicle meets two dissipative forces, aerodynamic drag and rolling resistance

$$F_a(t) = \frac{\rho_a c_d A_f}{2} v^2(t), \quad F_r(t) = mgc_r r \cos\alpha(t) \tag{1}$$

where $v(t)$ is longitudinal vehicle velocity, $\alpha(t)$ is road gradient, m is total vehicle mass, ρ_a ia air density, g is gravitational acceleration, c_d and c_r are aerodynamic and rolling coefficients and A_f is frontal area. Including the vehicle dynamics and road altitude, the traction force the vehicle delivers is

$$F_t(t) = \left(m + \frac{I}{r^2}\right)\frac{dv(t)}{dt} + mg\sin\alpha(t) + F_a(t) + F_r(t) \tag{2}$$

where r is wheels radius and I_v is rotational inertia (as seen at the wheels) of the wheels, differential, EM and all axels connecting them. Because the equation above covers the main contributors to vehicle inertia, we have neglected the rotational inertia of other components, as for example the EGU.

The traction force (2) is delivered entirely by the EM. The EM torque is bounded between speed-dependent limits, as depicted in Fig. 2. Mathematically, this constraint is described by

$$T_M(t) \in \left[\max\left\{b_1, \frac{b_2}{\omega(t)}\right\}, \min\left\{b_3, \frac{b_4}{\omega(t)}\right\}\right] \tag{3}$$

and includes a constant torque limit, at low speeds, and a constant power limit, at higher speeds where EM's field weakening is active. Here $\omega(t)$ is the EM speed which is directly related to the vehicle speed,

$$\omega(t) = \frac{\gamma}{r} v(t) \tag{4}$$

through the wheels radius and the differential gear ratio γ. We assume constant EM efficiencies, η_c for charging, and η_d for discharging. The efficiency of the inverter and differential gear are considered within the EM efficiencies.

The EGU fuel power is modeled as a quadratic function of generated power

$$P_f = a_0 + a_1 P_G(t) + a_2 P_G^2(t) \tag{5}$$

leading to the efficiency model depicted in Fig. 2. For the validity of this model, interested readers are referred to [11, 13]. In the studied application, the EGU is turned off only when the vehicle is not moving.

The battery is modeled by a constant open circuit voltage V_{oc} in a series connection to a resistor R. Then, the dissipative battery power is quadratic in battery current, which in terms of the internal battery power $P_B(t)$ can be described by

$$P_{Bd}(t) = R\frac{P_B^2(t)}{V_{oc}^2}. \tag{6}$$

The battery energy is given as

$$E_B(t) = -\int_0^t P_B(\tau)d\tau. \tag{7}$$

III. Problem Formulation

The optimization objective is to find the optimal control signals $P_G^*(t)$, $P_B^*(t)$ and $T_M^*(t)$ that minimize the amount of fuel,

$$Q_f \int_0^{t_f} P_f(\tau)e(\tau)d\tau \tag{8}$$

the vehicle will need in order to finish the driving mission. Here, Q_f is energy density of the fuel, and $e(t)$ is a binary signal that determines the engine on/off state and is given as

$$e(t) = \begin{cases} 1, & v(t) > 0 \\ 0, & v(t) = 0. \end{cases} \tag{9}$$

During the entire driving mission, both longitudinal velocity and battery energy have to be kept within given limits

$$v(t) \in [v_{min}(t), v_{max}(t)] \geq 0 \tag{10}$$
$$E_B(t) \in [E_{Bmin}, E_{Bmax}] \geq 0 \tag{11}$$

and energy has to be conserved, i.e.

$$v(0) = v(t_f) = 0 \tag{12}$$
$$E_B(0) = E_B(t_f). \tag{13}$$

The total driving time must not exceed a given threshold

$$t_f \leq T_{max}. \tag{14}$$

The model shall also satisfy the mechanical torque balance and electrical power balance equations

$$T_M(t) = \frac{r}{\gamma} F_t(t) + T_{brk}(t) \tag{15}$$

$$P_G(t)e(t) + P_B(t) = \omega(t)\max\left\{T_M(t)\eta_c, \frac{T_M(t)}{\eta_d}\right\} + P_{Bd}(t) \tag{16}$$

TABLE I
OPTIMIZATION PROBLEM.

minimize

$Q_f \frac{\gamma}{r} \int_0^{s_f} \frac{a_0 + a_1 \boldsymbol{P_G}(s) + a_2 \boldsymbol{P_G^2}(s)}{\boldsymbol{\omega}(s)} e(s) ds$

subject to

$\boldsymbol{T_M}(s) = A_1 \boldsymbol{\omega^2}(s) + 2A_2 \frac{d\boldsymbol{\omega}(s)}{ds} \boldsymbol{\omega}(s) + A_3(s) + \boldsymbol{T_{brk}}(s)$

$\boldsymbol{P_G}(s) e(s) + \boldsymbol{P_B}(s) = \boldsymbol{\omega}(s) \max \left\{ \boldsymbol{T_M}(s)\eta_c, \frac{\boldsymbol{T_M}(s)}{\eta_d} \right\} + R \frac{\boldsymbol{P_B^2}(s)}{V_{oc}^2}$

$\frac{d\boldsymbol{E_B}(s)}{ds} = -\frac{\gamma}{r} \frac{\boldsymbol{P_B}(s)}{\boldsymbol{\omega}(s)}$

$\frac{\gamma}{r} \int_0^{s_f} \frac{ds}{\boldsymbol{\omega}(s)} \leq T_{max}$

$\boldsymbol{\omega}(s) \in \frac{\gamma}{r} [v_{min}(s), v_{max}(s)]$

$\boldsymbol{\omega}(0) = \boldsymbol{\omega}(s_f) = 0$

$\boldsymbol{E_B}(s) \in [E_{Bmin}, E_{Bmax}]$

$\boldsymbol{E_B}(0) = \boldsymbol{E_B}(s_f)$

$\boldsymbol{T_M}(s) \in \left[\max \left\{ b_1, \frac{b_2}{\boldsymbol{\omega}(s)} \right\}, \min \left\{ b_3, \frac{b_4}{\boldsymbol{\omega}(s)} \right\} \right]$

$\boldsymbol{P_G}(s) \in [0, P_{Gmax}]$

$\boldsymbol{P_B}(s) \in [P_{Bmin}, P_{Bmax}]$

$\boldsymbol{T_{brk}}(s) \geq 0$

$\forall s \in [0, s_f]$

In the optimization problem the states are considered as optimization variables bound with equality constraints. Optimization variables, marked in bold for visibility, are: $\boldsymbol{T_{brk}}(s), \boldsymbol{T_M}(s), \boldsymbol{P_G}(s), \boldsymbol{P_B}(s), \boldsymbol{E_b}(s), \boldsymbol{\omega}(s)$.

TABLE II
CONVEX OPTIMIZATION PROBLEM.

minimize

$Q_f \frac{\gamma}{r} \int_0^{s_f} \left(\frac{a_0}{\sqrt{\boldsymbol{E}(s)}} + a_1 \boldsymbol{T_G}(s) + \frac{a_2 \gamma v_r(s)}{r} \boldsymbol{T_G^2}(s) \right) e(s) ds$

subject to

$\boldsymbol{T_M}(s) = A_1 \boldsymbol{E}(s) + A_2 \frac{d\boldsymbol{E}(s)}{ds} + A_3(s) + \boldsymbol{T_{brk}}(s)$

$\boldsymbol{T_G}(s) e(s) + \boldsymbol{T_B}(s) \geq \max \left\{ \boldsymbol{T_M}(s)\eta_c, \frac{\boldsymbol{T_M}(s)}{\eta_d} \right\} + \frac{R\gamma v_r(s)}{V_{oc}^2 r} \boldsymbol{T_B^2}(s)$

$\frac{d\boldsymbol{E_B}(s)}{ds} = -\frac{\gamma}{r} \boldsymbol{T_B}(s)$

$\frac{\gamma}{r} \int_0^{s_f} \frac{ds}{\sqrt{\boldsymbol{E}(s)}} \leq T_{max}$

$\boldsymbol{E}(s) \in \frac{\gamma^2}{r^2} [v_{min}^2(s), v_{max}^2(s)]$

$\boldsymbol{E}(0) = \boldsymbol{E}(s_f) = 0$

$\boldsymbol{E_B}(s) \in [E_{Bmin}, E_{Bmax}]$

$\boldsymbol{E_B}(0) = \boldsymbol{E_B}(s_f)$

$\boldsymbol{T_M}(s) \in [b_1, b_3]$

$\boldsymbol{T_M}(s) \frac{2\gamma}{r} v_r^3(s) \in [b_2, b_4] \left(3v_r^2(s) - \frac{r^2}{\gamma^2} \boldsymbol{E}(s) \right)$

$\boldsymbol{T_G}(s) \frac{2\gamma}{r} v_r^3(s) \in [0, P_{Gmax}] \left(3v_r^2(s) - \frac{r^2}{\gamma^2} \boldsymbol{E}(s) \right)$

$\boldsymbol{T_B}(s) \frac{2\gamma}{r} v_r^3(s) \in [P_{Bmin}, P_{Bmax}] \left(3v_r^2(s) - \frac{r^2}{\gamma^2} \boldsymbol{E}(s) \right)$

$\boldsymbol{T_{brk}}(s) \geq 0$

$\forall s \in [0, s_f]$

Optimization variables, marked in bold for visibility, are: $\boldsymbol{T_{brk}}(s), \boldsymbol{T_M}(s), \boldsymbol{T_G}(s), \boldsymbol{T_B}(s), \boldsymbol{E_b}(s), \boldsymbol{E}(s)$.

where we have neglected the power used by auxiliary devices. The torque $T_{brk}(t) \geq 0$ is delivered by the friction brakes.

IV. SAMPLING IN DISTANCE

The entire optimization problem is summarized here, but instead of sampling in time, the problem is rewritten with sampling in space. Note that this introduces a product or division with velocity where time derivative or integration is performed. For example, when sampling in space, the vehicle acceleration can be written as

$$\frac{dv(t)}{dt} = \frac{dv(s)}{ds} v(s). \tag{17}$$

Furthermore, instead of using the longitudinal vehicle velocity $v(s)$, the problem is rewritten in terms of $\omega(s)$. Finally, by introducing the EGU power limit P_{Gmax} and battery power limits P_{Bmin}, P_{Bmax}, the optimization problem can be summarized as in Table I. In order to represent the problem in a more compact form, we have grouped coefficients as follows

$$A_1 = \frac{\rho_a c_d A_f r^3}{2\gamma^3}, \quad A_2 = \frac{I + mr^2}{2\gamma^3} r \tag{18}$$

$$A_3(s) = \frac{mgr}{\gamma} (c_r \cos\alpha(s) + \sin\alpha(s)). \tag{19}$$

Due to the divisions $P_G(s)/\omega(s)$, $P_B(s)/\omega(s)$, and the products $d\omega(s)/ds\, \omega(s)$, $T_M(s)\omega(s)$, the problem is not convex in its present form, [1]. This problem is possible to solve with dynamic programming, but in order of obtaining a time efficient solution, we will show in the following section several steps to convexify the problem.

V. CONVEX MODELING

To eliminate the obviously non-convex elements, several variable changes are proposed

$$T_G(s) = \frac{P_G(s)}{\omega(s)}, \quad T_B(s) = \frac{P_B(s)}{\omega(s)}, \quad E(s) = \omega^2(s). \tag{20}$$

These changes, however, will introduce non-convexity at the power limits of some components. Take for example the EGU constraint

$$T_G(s) \leq \frac{P_{Gmax}}{\sqrt{E(s)}}. \tag{21}$$

The function at the right side of the inequality is convex in $E(s)$, while for the optimization problem to be convex, this function has to be concave. To remedy the problem, we will linearize the function about a reference velocity trajectory $v_r(s)$. Hence,

$$\frac{P_{Gmax}}{\sqrt{E(s)}} \approx P_{Gmax} \frac{r}{2\gamma v_r(s)} \left(3 - \frac{r^2}{\gamma^2 v_r^2(s)} E(s) \right). \tag{22}$$

Note that this approximation will always underestimate the function, which will make the limit of the approximated model lower than the limit of the original model. Hence, after the problem is solved and the actual power limits are applied, feasibility is guaranteed even when the optimal velocity is far from the reference. (However, the problem has to be solved in the first place, and the approximation may prevent finding a solution. Further discussed in Section VII.)

The importance of choosing a reference velocity that is closer to the optimal, is more relevant for the following approximation. The term involving the dissipative battery power

$$\frac{P_{Bd}(s)}{\omega(s)} = R \frac{T_B^2(s)}{V_{oc}^2} \sqrt{E(s)} \tag{23}$$

TABLE III
VEHICLE PARAMETERS.

Parameter	Value	Parameter	Value
m	15 t	$\eta_c = \eta_d$	85 %
A_f	7.54 m^2	P_{Bmin}	-66 kW
c_d	0.7	P_{Bmax}	79.2 kW
c_r	0.007	E_{Bmin}	0.2 kWh
r	0.509 m	E_{Bmax}	1.2 kWh
I	135.22 kgm^2	R	0.5 Ω
γ	4.7	V_{oc}	330 V

is neither convex nor concave in $T_B(s)$ and $E(s)$. We will approximate the losses by

$$\frac{P_{Bd}(s)}{\omega(s)} \approx R\frac{T_B^2(s)}{V_{oc}^2}\frac{v_r(s)\gamma}{r}. \tag{24}$$

Then, the idea is to iteratively repeat the optimization by using the recently obtained optimal velocity as a reference in the succeeding iteration. The procedure can be summarized as follows:

1) Choose a reference velocity $v_r(s)$.
2) Solve the convex problem in Table II and obtain the optimal velocity $v^*(s)$.
3) Assign a new reference velocity $v_r(s) = v^*(s)$.
4) Repeat steps 2) and 3) until the deference between optimal and reference velocity is under a given threshold, or maximum number of iterations has been reached.

In the convex problem in Table II, the electric power balance constraint (16) has been relaxed with inequality. It can be reasoned that this constraint will hold with equality at the optimum, as there is no need for the EGU and battery to deliver more power than what is needed by the EM. Further discussion on this topic can be found in [11]. The EM torque limit (3) has been broken into several constraints, two with constant torque limits and two with approximated power limits. Finally, the problem is written in discrete form using first order Euler discretization. We applied CVX [2, 4] to translate the problem into a form required by the solver, SeDuMi [7].

VI. OPTIMIZATION EXAMPLE

In this example we seek the optimal control of a city bus in a series HEV topology. The bus is equipped with a 180 kW diesel EGU and a 200 kW EM as depicted in Fig. 2. The remaining vehicle parameters are given in Table III. The bus operates on a bus line with 28 stops, as illustrated in Fig. 3. A time schedule is enforced that limits the driving time between each two consecutive stops. Hence, constraints similar to (14) have been applied for the 27 driving intervals between consecutive stops. The minimum allowed vehicle speed is zero, while for the maximum allowed speed we have taken the absolute maximum recorded speed of 20 measurements of a conventional diesel bus that has been operated on this line. As an initial reference velocity, we have used the average speed of the 20 measurements.

The convex problem (Table II) has been solved according to the procedure described in Section V. The algorithm converged

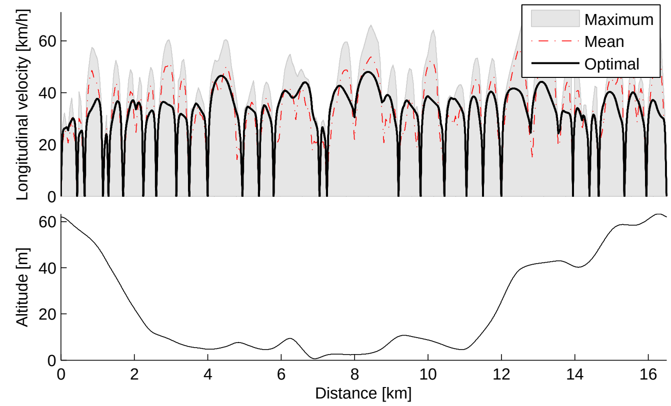

Fig. 3. Velocity profiles. The shaded region shows the maximum allowed velocity, the dash-dotted line is average velocity of performed measurements, and the solid line is the optimal vehicle velocity.

TABLE IV
OPTIMIZATION RESULTS.

	Initial reference value	Optimal value
Braking energy [kWh]	1.2	0.2
Aerodynamic drag [Wh]	28.6	27.7
Maximum speed [km/h]	66	48
Mean speed [km/h]	33	32

in four iterations when the speed difference between the optimal and reference velocity

$$\frac{\sum_s |v_r(s) - v^*(s)|}{s_f} \leq \epsilon. \tag{25}$$

has dropped below $\epsilon = 0.001$. The optimal velocity trajectory is given in Fig. 3, where it can be observed that the velocity is smoothen out such that wasted energy is minimized. Indeed, it is shown in Table IV that the maximum value of the optimal speed is much lower compared to the maximum value of the initial reference velocity. A significant difference is visible in total braking energy that has been greatly reduces in the optimal solution (computed as the integral of the term including A_1 in Table I, for negative accelerations).

The optimal fuel consumption is 32.5 l/100km, while the total driving time is 42.8 min. If we further include 8 s standstill interval per bus stop, the total time needed to finish the route would be 46.6 min.

VII. DISCUSSIONS AND CONCLUSIONS

We presented a strategy for optimally controlling electrified vehicles, where both longitudinal velocity and battery energy are considered states in the system. The method can be also used to pre-filter an initial reference velocity which may not be possible to drive using a backwards-simulation model.

We assumed in the studied example that the EGU is turned off only when the vehicle is standing still. The reason for this choice is not accidental, but intentionally made to avoid problems that convex optimization cannot bear. The optimal engine on/off control, which is a binary variable, cannot be decided by convex optimization. Instead, this strategy can be used only when all integer decisions are decided outside the convex optimization. Since in our example the distances where the vehicle is standing still are determined by a known signal (e.g. the maximum allowed velocity is zero at these distances), the on/off signal is safely found before starting the optimization. Improved, or near optimal, ICE on/off control in the frame of convex optimization is still an ongoing research, although some articles have been recently published on this topic [12].

Further studies are needed to carefully investigate the approximations performed to convexify the problem. For example, a badly chosen reference velocity may prevent the algorithm find a feasible solution, even though such solution might exist. Recall that the approximation (22) underestimates the actual power limit, so if the reference velocity is too far from the maximum feasible velocity, the approximation may render the problem infeasible. Future studies are also needed to extend this method to HEVs in parallel topology, either with fixed-geared, or continuously variable transmission.

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