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A Linear Approximate Acreage Allocation Model

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It is shown that the first-order differential acreage allocation model developed by Bettendorf and Bloome and by Barten and Vanloot, and based on certainty equivalent profit maximization, may be extended to a levels version. The levels model, referred to as a linear approximate acreage allocation model, is potentially useful when panel or cross-sectional data are employed. An empirical application with U.S. state-level corn flex acreage data for the period 1991–95 indicates the feasibility of the approach. Estimated price and scale elasticities are generally larger than previous estimates, and are perhaps indicative of acreage response under the provisions of the 1996 Farm Act.

Key words: acreage allocation model, certainty equivalent profit maximization, normal flex acres, scale effects

Introduction

Over the years there has been considerable interest in estimating agricultural acreage supply equations (see, e.g., Burt and Worthington; Gallagher; Lee and Helmberger; Holt and Johnson; Shonkwiler and Maddala). Given the long history of government intervention in agriculture in the U.S. and elsewhere, a primary goal of these studies typically has been to develop a set of acreage response elasticity estimates, presumably for use in policy analysis and forecasting. In recent years, the basic acreage supply response framework has been extended to include, among other things, risk effects due to price and production uncertainty (see, e.g., Just; Chavas and Holt 1990, 1996; Krause, Lee, and Koo; Krause and Koo; Lin; Pope 1982; Pope and Just; Traill). Of interest is that most acreage supply models reported in the literature, and especially those that incorporate risk effects, have not been estimated within a systems framework. That is, total acreage constraints ordinarily have not been incorporated into model specifications in a manner analogous to that for other agricultural supply models (e.g., Chambers and Lee). The implication is that estimates of acreage scale elasticities defined as the response of a particular crop to an increase in total agricultural land typically have not been reported.

The few studies that have examined acreage supply decisions in a systems framework include Bewley, Young, and Coleman; Binkley and McKenzie; Coyle; Moore and

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Negri; Bettendorf and Blomme; Wu and Segerson; and Barten and Vanloot. Only Bewley, Young, and Coleman; Bettendorf and Blomme; and Barten and Vanloot report estimates of acreage scale elasticities. Further, only the empirical models developed by Coyle; Bettendorf and Blomme; and Barten and Vanloot are based on formal economic models, and therefore are the only ones for which homogeneity and reciprocity (symmetry) conditions may be logically imposed and tested.[1]

The Bettendorf and Blomme–Barten and Vanloot (BB-BV) model is of particular interest in situations where output price or revenue risks are potentially important in acreage planting decisions. As these authors show, it is possible to derive a system of (linear) acreage allocation models by using the basic mean-variance utility framework. The system they estimate is similar in its specification to Theil's Rotterdam demand system, and hence represents a first-order differential acreage allocation model. Among other things, these authors show that estimates of price *and* scale elasticities may be readily obtained from their model. There are times, however, when it is neither practical nor feasible to estimate a first-order differential acreage allocation system of the type developed by Bettendorf and Blomme and by Barten and Vanloot. Such instances occur, for example, when either cross-sectional or panel data (with a relatively small number of time-series observations) are employed.

The purpose of this study is to illustrate that the basic BB-BV framework may be modified to accommodate panel or cross-sectional data. Specifically, we show that a *levels version* of the basic BB-BV acreage allocation model, termed the linear approximate acreage allocation model, is readily attainable. This model, too, may be practicably employed in empirical analyses. The usefulness of our approach is demonstrated by estimating acreage supply response on corn normal flex acres (NFA) for eight Corn Belt states in the U.S. for the 1991–95 period.[2] Estimates of acreage supply response on NFA, particularly in the Corn Belt region, are of interest to policy makers in the U.S. because they are indicative of what future agricultural supply response might be in a free market environment.

In the next section we develop the basic modeling framework. The data used in the analysis are then discussed, followed by the presentation of the provisional model and elasticity estimates. Conclusions are offered in the final section.

An Acreage Allocation Model

Here we follow the basic set-up of Bettendorf and Blomme and of Barten and Vanloot, with several modifications. To begin, we assume a representative producer makes decisions about which crops to grow in a manner similar to that of an investor determining the composition of an investment portfolio. That is, we assume a representative

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[1] While Moore and Negri, and Wu and Segerson derived their model specifications by assuming expected profit maximization, they do not discuss or impose the implied symmetry conditions in their estimated acreage allocation models. Likewise, Binkley and McKenzie discuss symmetry conditions, but apparently do not use or otherwise test for reciprocity in their empirical analysis.

[2] In essence, normal flex acres represent the share of a farmer's base program acreage for a particular program crop (ordinarily 15%) that, during the period examined, could be planted to *any* crop desired without restriction. More importantly, there were no associated guarantees of direct price or income support from the federal government associated with flex acres.

farmer maximizes certainty equivalent (CE) profit, π, subject to a total land constraint. Of course, in the case of agriculture, important sources of risk include output price and crop yield uncertainty. Formally stated, the farmer's allocation problem[3] is to

$$(1) \qquad \max_{\mathbf{a}} CE(\pi) = \left\{ \mathbf{a}^T\mathbf{r}^e - \tfrac{1}{2}\lambda\mathbf{a}^T\Sigma\mathbf{a} \,|\, a_{tot} - \mathbf{i}^T\mathbf{a} \right\},$$

where $\mathbf{a}$ is an n-vector of acreage levels allocated among n crops, and $\mathbf{r}^e = (r_1^e, \ldots, r_n^e)^T$ is an n-vector of expected net returns with typical element given by[4]

$$(2) \qquad r_i^e = E(p_i y_i) = p_i^e y_i^e + \mathrm{cov}(p_i, y_i) - c_i,$$

where E is an expectation operator; p_i^e is the expected per bushel price of the ith crop; y_i^e denotes the expected yield per acre of the ith crop; $\mathrm{cov}(p_i, y_i)$ denotes the covariance between price and yield; and c_i is the per acre cost of production.[5] In (1), the $\{n \times n\}$ matrix Σ is a symmetric, positive definite second-moment matrix of expected returns per acre. That is,

$$(3) \qquad \Sigma = E\left\{ [\mathbf{r} - E(\mathbf{r})][\mathbf{r} - E(\mathbf{r})]^T \right\}$$

$$= \begin{bmatrix} \mathrm{var}(r_1) & \mathrm{cov}(r_1, r_2) & \cdots & \mathrm{cov}(r_1, r_n) \\ \mathrm{cov}(r_1, r_2) & \mathrm{var}(r_2) & \cdots & \mathrm{cov}(r_2, r_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(r_1, r_n) & \mathrm{cov}(r_2, r_n) & \cdots & \mathrm{var}(r_n) \end{bmatrix},$$

where

$$\mathrm{var}(r_i) = E\left[r_i - r_i^e\right]^2 \qquad \text{(variance of returns for } r_i\text{)},$$

and

$$\mathrm{cov}(r_i, r_j) = E\left[r_i - r_i^e\right]\left[r_j - r_j^e\right] \qquad \text{(covariance of returns between } r_i \text{ and } r_j\text{)}.$$

Also in (1), $\lambda \in \Re_{++}$ is a scalar coefficient of absolute risk aversion, and $\mathbf{i}^T\mathbf{a} = \sum_{i=1}^n a_i = a_{tot}$ denotes the land constraint, where $\mathbf{i} = (1, \ldots, 1)^T$ is an $\{n \times 1\}$ summation (unit) vector and a_{tot} represents total acres available. In this study, total acres available (a_{tot}) is treated as being exogenously determined.

The Lagrangian function associated with the optimization problem in (1) is

$$(4) \qquad \max_{\{\mathbf{a},\mu\}} L(\mathbf{a}, \mu) = \mathbf{a}^T\mathbf{r}^e - \tfrac{1}{2}\lambda\mathbf{a}^T\Sigma\mathbf{a} - \mu\left[a_{tot} - \mathbf{i}^T\mathbf{a}\right],$$

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[3] Of course, formal econometric models based on specifications similar to (1) have been used previously in agricultural supply response analysis. For example, Pope (1978) and Love and Buccola report estimates of systems of agricultural production equations derived from a framework similar to that specified in (1). Additional commentary on specification of econometric supply models based on CE profit maximization may be found in Pope (1982).

[4] Here a superscripted T denotes vector (matrix) transposition.

[5] Recall, for any pair of jointly distributed random variables x and y, that $E(xy) = E(x)E(y) + \mathrm{cov}(x, y)$. See Bohrnstedt and Goldberger for details.

where $\mu \in \Re_+$ is a scalar Lagrange multiplier associated with the total acreage constraint. By using summation notation, the Lagrangian function (4) may be alternatively stated as

$$\text{(5)} \quad \max_{\{a_1,\ldots,a_n,\mu\}} L(a_1, \ldots, a_n, \mu) = \sum_{i=1}^{n} a_i r_i^e - ½\lambda\left(\sum_{i=1}^{n} a_i^2 \operatorname{var}(r_i) + \sum_{i=1}^{n}\sum_{j=1}^{n}(1-\delta_{ij})a_i a_j \operatorname{cov}(r_i, r_j)\right) + \mu\left(a_{tot} - \sum_{i=1}^{n} a_i\right),$$

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ otherwise. The necessary first-order conditions derived from (5) are as follows:

$$\text{(6a)} \quad \frac{\partial L}{\partial a_i} = r_i^e - \lambda\left[a_i \operatorname{var}(r_i) + \sum_{j=1}^{n}(1-\delta_{ij})a_j \operatorname{cov}(r_i, r_j)\right] - \mu = 0, \quad i = 1, \ldots, n,$$

and

$$\text{(6b)} \quad \frac{\partial L}{\partial \mu} = a_{tot} - \sum_{j=1}^{n} a_j = 0.$$

Upon inspection, it may be readily verified that (6a) and (6b) are linear in the $n + 1$ unknowns, $\{a_1, \ldots, a_n\}$ and μ. By rewriting (6a) and (6b) so that the endogenous variables (i.e., $\{a_1, \ldots, a_n\}$ and μ) are isolated on the left-hand side and the predetermined variables (i.e., $\{r_1^e, \ldots, r_n^e\}$ and a_{tot}) are on the right-hand side, the first-order conditions in (6) may be expressed alternatively as

$$\text{(7a)} \quad a_1\lambda \operatorname{var}(r_1) + a_2\lambda \operatorname{cov}(r_1, r_2) + \ldots + a_n\lambda \operatorname{cov}(r_1, r_n) + \mu = r_1^e,$$

$$\text{(7b)} \quad a_1\lambda \operatorname{cov}(r_1, r_2) + a_2\lambda \operatorname{var}(r_2) + \ldots + a_n\lambda \operatorname{cov}(r_2, r_n) + \mu = r_2^e,$$

$$\vdots$$

$$\text{(7c)} \quad a_1\lambda \operatorname{cov}(r_1, r_n) + a_2\lambda \operatorname{cov}(r_2, r_n) + \ldots + a_n\lambda \operatorname{var}(r_n) + \mu = r_n^e,$$

$$\text{(7d)} \quad a_1 + a_2 + \ldots + a_n = a_{tot}.$$

In matrix form, the linear system in (7) may be written as

$$\text{(8)} \quad \begin{bmatrix} \lambda\operatorname{var}(r_1) & \lambda\operatorname{cov}(r_1,r_2) & \cdots & \lambda\operatorname{cov}(r_1,r_n) & 1 \\ \lambda\operatorname{cov}(r_1,r_2) & \lambda\operatorname{var}(r_2) & \cdots & \lambda\operatorname{cov}(r_2,r_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda\operatorname{cov}(r_1,r_n) & \lambda\operatorname{cov}(r_2,r_n) & \cdots & \lambda\operatorname{var}(r_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ \mu \end{bmatrix} = \begin{bmatrix} r_1^e \\ r_2^e \\ \vdots \\ r_n^e \\ a_{tot} \end{bmatrix},$$

or, more compactly, as

$$\mathbf{Ax} = \mathbf{b}, \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} \lambda\text{var}(r_1) & \lambda\text{cov}(r_1,r_2) & \dots & \lambda\text{cov}(r_1,r_n) & 1 \\ \lambda\text{cov}(r_1,r_2) & \lambda\text{var}(r_2) & \dots & \lambda\text{cov}(r_2,r_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda\text{cov}(r_1,r_n) & \lambda\text{cov}(r_2,r_n) & \dots & \lambda\text{var}(r_n) & 1 \\ 1 & 1 & \dots & 1 & 0 \end{bmatrix} \quad \text{(an } \{n+1\} \times \{n+1\} \text{ matrix);}$$

$$\mathbf{x} = \left[a_1, a_2, \dots, a_n, \mu\right]^T \quad \text{(an } \{n+1\} \times 1 \text{ vector);}$$

and

$$\mathbf{b} = \left[r_1^e, r_2^e, \dots, r_n^e, a_{tot}\right]^T \quad \text{(an } \{n+1\} \times 1 \text{ vector).}$$

By using the $\{n \times 1\}$ summation vector **i**, and by using the definition of the second-moment matrix Σ in (3), we may represent the matrix **A** in linear equation system (9) as

$$\mathbf{A} = \begin{bmatrix} \lambda\Sigma & \mathbf{i} \\ \mathbf{i}^T & 0 \end{bmatrix}. \tag{10}$$

Recall that Σ is, by definition, a symmetric, positive definite matrix, and that under constant absolute risk aversion (CARA) λ is a positive scalar constant. It therefore follows that the $\{n \times n\}$ matrix **M**, defined as $\mathbf{M} = \lambda\Sigma$ with typical elements $M_{ii} = \lambda\text{var}(r_i)$ and $M_{ij} = \lambda\text{cov}(r_i, r_j)$, $i \neq j$, also will be a symmetric, positive definite matrix, further implying that $\mathbf{M}^{-1}$ exists. By noting that $\mathbf{x} = (\mathbf{a}, \mu)^T$ and $\mathbf{b} = (\mathbf{r}, a_{tot})^T$, it follows that the linear system in (8) may be expressed as

$$\begin{bmatrix} \mathbf{M} & \mathbf{i} \\ \mathbf{i}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mu \end{bmatrix} = \begin{bmatrix} \mathbf{r}^e \\ a_{tot} \end{bmatrix}. \tag{11}$$

To solve (linear) equation system (11), we need only apply the partitioned inverse rule. Recall, for a linear system $\mathbf{Ax} = \mathbf{b}$, where **A** is an $\{n + 1\} \times \{n + 1\}$ matrix, that if **A** may be written as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},$$

and if $\mathbf{A}_{11}^{-1}$ exists, then by the partitioned inverse rule, the inverse of **A** is given by

$$(12)\qquad \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}_{11}^{-1}(\mathbf{I}_n + \mathbf{A}_{12}\mathbf{F}_2\mathbf{A}_{21}\mathbf{A}_{11}^{-1}) & -\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\mathbf{F}_2 \\ -\mathbf{F}_2\mathbf{A}_{21}\mathbf{A}_{11}^{-1} & \mathbf{F}_2 \end{bmatrix},$$

where

$$(13)\qquad \mathbf{F}_2 = \left(\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\right)^{-1}.$$

In (12), $\mathbf{I}_n$ is an $\{n \times n\}$ identity matrix (see Schott, pp. 247–48, for additional details). Terms in matrix **A** in (11) corresponding, respectively, to $\mathbf{A}_{11}, \mathbf{A}_{12}, \mathbf{A}_{21}$, and $\mathbf{A}_{22}$ in (12) are as follows:

$$(14a)\qquad \mathbf{A}_{11} = \mathbf{M} = \lambda\Sigma, \qquad \mathbf{A}_{12} = \mathbf{i},$$

$$(14b)\qquad \mathbf{A}_{21} = \mathbf{i}^T, \qquad \mathbf{A}_{22} = \mathbf{0}.$$

Substitution of (14) into (12) and (13) yields

$$(15)\qquad \mathbf{F}_2 = \left(0 - \mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}\right)^{-1} = -\left(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}\right)^{-1},$$

and therefore

$$(16)\qquad \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{M} & \mathbf{i} \\ \mathbf{i}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{M}^{-1}(\mathbf{I}_n - \mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1}) & \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1} \\ (\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1} & -(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1} \end{bmatrix}.$$

Note that the term $(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}$ in (15) and (16) is a (strictly positive) scalar. Now, the solution to (11) is obtained by premultiplying both sides by $\mathbf{A}^{-1}$, yielding

$$(17)\qquad \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{-1}(\mathbf{I}_n - \mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1}) & \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1} \\ (\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1} & -(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{r}^e \\ a_{tot} \end{bmatrix}.$$

The solution for the n-vector **a**, the vector of optimal acreage allocations, may be obtained directly from (17):

$$(18)\qquad \mathbf{a} = \mathbf{M}^{-1}\mathbf{i}\left(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}\right)^{-1}a_{tot} + \left(\mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{i}\left(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}\right)^{-1}\mathbf{i}^T\mathbf{M}^{-1}\right)\mathbf{r}^e.$$

Alternatively, the n-vector of optimal acreage decisions in (18) may be expressed compactly as

$$(19a)\qquad \mathbf{a} = \mathbf{b}a_{tot} + \mathbf{S}^*\mathbf{r}^e,$$

where

(19b) $$\mathbf{b} = \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1},$$

and

(19c) $$\begin{aligned}\mathbf{S}^* &= \mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1}\\ &= \mathbf{M}^{-1} - (\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{M}^{-1}\mathbf{i}\mathbf{i}^T\mathbf{M}^{-1}.\end{aligned}$$

To explore the restrictions inherent in (19), note first that matrix $\mathbf{S}^*$ as defined in (19c) is symmetric ($\mathbf{S}^* = \mathbf{S}^{*T}$). This follows because $\mathbf{i}\mathbf{i}^T$ is symmetric and because $\mathbf{M} = \lambda\Sigma$ is, by definition, symmetric, so that $\mathbf{M}^{-1}$ is also symmetric. Further, post-multiplying $\mathbf{S}^*$ by $\mathbf{i}$ yields

$$\begin{aligned}\mathbf{S}^*\mathbf{i} &= (\mathbf{M}^{-1} - \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1})\underline{\mathbf{i}}\\ &= \mathbf{M}^{-1}\mathbf{i} - \mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1}\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}\\ &= \mathbf{M}^{-1}\mathbf{i} - \mathbf{M}^{-1}\mathbf{i} = \mathbf{0}.\end{aligned}$$

That is, $\mathbf{S}^*\mathbf{i} = \mathbf{0}$ must hold. By similar arguments, it follows that $\mathbf{i}^T\mathbf{S}^* = \mathbf{0}^T$ must also hold. In other words, $\mathbf{S}^*$ will be at most of rank $n-1$. Now, premultiplying $\mathbf{b}$ in (19b) by $\mathbf{i}^T$ gives

$$\mathbf{i}^T\mathbf{b} = \mathbf{i}^T\mathbf{M}^{-1}\mathbf{i}(\mathbf{i}^T\mathbf{M}^{-1}\mathbf{i})^{-1} = 1.$$

Consequently, the linear restriction $\mathbf{i}^T\mathbf{b} = 1$ must hold as well.

To obtain a system of n (linear) acreage allocation share equations, simply divide both sides of (19a) by a_{tot}, the total acreage variable. Performing this operation and arranging terms yields the following system of n acreage allocation equations:[6]

(20a) $$\upsilon = \mathbf{b} + \mathbf{S}\mathbf{r}^e$$

or, alternatively,

(20b) $$\upsilon_i = b_i + \sum_j s_{ij}r_j^e, \quad i = 1, \ldots, n,$$

where $\mathbf{S}$ is defined such that $\mathbf{S} = \mathbf{S}^*/a_{tot}$ and $\upsilon = \mathbf{a}/a_{tot}$, an n-vector of acreage allocations (shares).

The system in (20) is an acreage allocation system. By making suitable stochastic assumptions, the system's parameters may be estimated econometrically.[7] Of interest is that the theoretically appealing properties of symmetry, homogeneity, and adding up may be readily maintained in estimation. Also, in equation system (20), the matrix $\mathbf{S}$ is

[6] Coyle derives a set of acreage equations nearly identical in specification to those in, respectively, equation systems (19) and (20) by assuming (expected) profit maximization and by applying duality theory.

[7] Alternatively, it is always possible to specify (20) so that the parameters in the matrix $\mathbf{M}^* = a_{tot}\mathbf{M}$, as defined in (19), are estimated directly. Given that such an approach results in an otherwise linear model being transformed into one that is nonlinear, there appears to be no obvious advantage to doing so.

a positive semi-definite matrix such that $\mathbf{x}^T\mathbf{S}\mathbf{x} > 0$ for all $\mathbf{x}$ *not* proportional to $\mathbf{i}$. If necessary, this "positivity" condition may be maintained in estimation by using the Cholesky factorization $\mathbf{S} = \mathbf{C}^T\mathbf{C}$ and estimating the Cholesky terms in $\mathbf{C}$ directly as described by Barten and Geyskens. The dependent variables in (20) are *acreage shares* as opposed to expenditure shares, as might be derived from a cost function or an indirect production function. Indeed, the system in (20) may be viewed as a *levels version* of Bettendorf and Blomme, and Barten and Vanloot's differential acreage allocation system. Accordingly, the levels model derived here may have more appeal than the BB-BV specification in situations where cross-sectional or panel data are employed.

Recall that (20) is motivated by the notion that a representative farmer is risk averse, and therefore seeks to optimally manage the total acreage portfolio. The risk-aversion parameter and the second moments of returns, however, cannot be separately identified in estimation. This said, the parameters in the model are easily interpretable in economic terms. The b_i parameters represent *acreage scale effects*, and therefore show how much more (less) acreage will be planted to the ith crop if total land availability increases. In addition, each s_{ii} will be positive, indicating that an increase in expected returns for the ith crop will increase acreage planted to that crop. A negative (positive) value for s_{ij} indicates that an increase in the jth crop's expected returns would decrease (increase) the share of the ith crop in total plantings. These coefficients can be transformed into elasticity formulae as follows:

(21) $$\varepsilon_{ij} = \frac{\partial a_i}{\partial p_j^e}\frac{p_j^e}{a_i} = \frac{s_{ij}}{\upsilon_i}\,p_j^e y_j^e, \quad \forall\, i, j \qquad \text{(price elasticities)}$$

and

(22) $$\eta_i = \frac{\partial a_i}{\partial a_{tot}}\frac{a_{tot}}{a_i} = \frac{b_i}{\upsilon_i}, \quad i = 1, \ldots, n \qquad \text{(scale elasticities).}$$

The η_i coefficients indicate the percentage increase (decrease) in acres planted to the ith crop due to a 1% increase in total acres available (i.e., due to a relaxation of the acreage constraint). The ε_{ij} have the usual interpretation as own- and cross-price acreage response elasticities.

An Empirical Application

The linear approximate acreage allocation model described in the previous section is applied to panel data for corn normal flex acres planted in the eight Corn Belt states during the 1991–95 period.[8] All data were obtained from various sources at the Economic Research Service (ERS) of the U.S. Department of Agriculture (USDA).[9] Specifically, a linear approximate acreage allocation model was estimated for corn ($i = 1$), soybeans ($i = 2$), and a category defined as "other" ($i = 3$). On average, the share of corn planted to total corn NFA over this period was 54%, while the share of soybeans

[8] The states included are Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, and Wisconsin.

[9] We are indebted to Bill Lin and Dick Heifner of the Field Crops Branch, Market and Trade Economics Division, ERS, for supplying much of the basic data used in the analysis.

in total corn NFA was 33%. It is not entirely clear which crops were planted on the remaining 13% defined as "other." Several possibilities include oats and winter wheat. Oats is a relatively minor crop in the Corn Belt region that is also planted in the spring. Alternatively, winter wheat is planted in the fall and harvested the following summer, but its role in Corn Belt field crop production is more important than that of oats. As a practical matter, state-level price and yield data and regional cost of production data are available for both commodities. For this reason, we originally experimented by using expected returns for both crops as a proxy for the (expected) returns associated with "other." Based on this preliminary analysis, we concluded that expected wheat returns are more representative of expected returns to "other," and so expected winter wheat returns will be used as a measure of expected returns to the "other" category throughout the remainder of the analysis.

To implement the model, it is necessary to compute expected returns per acre for each crop. That is, as indicated in equation (2), it is necessary to calculate expected price, expected yield, and the covariance of price and yield for each crop in each state over the sample period. Following Chavas, Pope, and Kao, and Choi and Helmberger, we use futures prices to represent expected prices. Specifically, average prices in March for harvest-time futures contracts for corn and soybeans [December Chicago Board of Trade (CBOT) contract for corn and November CBOT contract for soybeans] are used as a measure of expected prices for these commodities. Average futures prices during the *previous* September for the July CBOT contract are used for winter wheat. These futures prices are adjusted to an equivalent state-level measure by subtracting the average expected harvest-time state-level basis. Each expected state-level basis (b_{ijt}^e) is, in turn, determined as a rolling weighted average of the observed basis differential in the month preceding maturity for each contract during the most recent three years. Specifically, b_{ijt}^e is computed as[10]

$$b_{ijt}^e = \sum_{k=1}^{3} \omega_k \left(\tilde{p}_{ijt-k} - \tilde{f}_{it-k} \right), \tag{23}$$

where j indexes states and t indexes time, and ω_k are weights such that $\omega_1 = 1/2$, $\omega_2 = 1/3$, and $\omega_3 = 1/6$;[11] $\tilde{p}_{ijt}$ denotes the state-level monthly average harvest-time price received by farmers, where months represented are November for corn, October for soybeans, and June for winter wheat; $\tilde{f}_{it}$ is the corresponding monthly average nearby futures price (December for corn, November for soybeans, and July for wheat). Expected state-level harvest-time prices (p_{ijt}^e) are then determined by

$$p_{ijt}^e = f_{it} - b_{ijt}^e, \tag{24}$$

where f_{it} denotes the planting-time futures price, as described previously, and b_{ijt}^e is the expected basis, as defined in (23).

To obtain an estimate of expected per acre yields, the following formula is used:

[10] A similar scheme for computing expected prices—i.e., one that relies upon three-year rolling averages and, moreover, uses identical values for the ω_k weights—was employed by Chavas and Holt (1990, 1996).

[11] In what follows, we us the notation j, l = 1 (Illinois), 2 (Indiana), 3 (Iowa), 4 (Michigan), 5 (Minnesota), 6 (Missouri), 7 (Ohio), and 8 (Wisconsin).

(25) $$y^e_{ijt} = \delta_{ij} + \tfrac{1}{4}\left[\sum_{k=1}^{6} y_{ijt-k} - \max(y_{ijt-1}, \dots, y_{ijt-6}) - \min(y_{ijt-1}, \dots, y_{ijt-6})\right],$$

where δ_{ij} is an adjustment parameter which assures that deviations between observed and expected yields sum to zero over the sample period. Implicit in (25) is the assumption that farmers view historically very high (low) yields as being unrepresentative when forming expectations. To estimate covariance between prices and yields, let $u_{ijt} = \tilde{p}_{ijt} - p^e_{ijt}$ and $v_{ijt} = y_{ijt} - y^e_{ijt}$; i.e., let u_{ijt} denote the price expectation error and v_{ijt} denote the yield expectation error. The price and yield covariance, $\text{cov}(p_{ijt}, y_{ijt})$, is then determined as a weighted rolling average of the product of historical price and yield expectation errors. That is,

(26) $$\text{cov}(p_{ijt}, y_{ijt}) = \sum_{k=1}^{3} \omega_k u_{ijt-k} v_{ijt-k},$$

where the weights (the ω_k's) are as defined in (23). Finally, to compute expected per acre net returns, it is desirable to have state-level cost of production data for each crop. Unfortunately, such data are not available; historical data on costs of production for corn, soybeans, and winter wheat are, however, available for the entire Corn Belt or North Central region from ERS.

With this information, and by using expressions (23)–(26) to construct expected prices, yields, and price-yield covariances, expected state-level returns (i.e., the r^e_{ijt} values) were computed in accordance with equation (2). These data, along with information on corn-based NFA acres planted to corn, soybeans, and other crops in the Corn Belt, constitute the basic information used to estimate the linear approximate acreage allocation model.

The system to be estimated is specified as follows:

(27) $$\upsilon_{ijt} = b_i + \sum_{k=1}^{n} s_{ik} r^e_{ikt} + \sum_{l=1}^{7} c_{il} D_l + \nu_{ijt},$$

where c_{il} are parameters that denote (fixed) state-level effects (Wisconsin is omitted), D_l are corresponding state-level dummy variables, and ν_{ijt} is a mean-zero random error term. The parameter restrictions associated with (27) include: $\Sigma_i b_i = 1$, $\Sigma_i s_{ik} = 0$, and $\Sigma_i c_{il} = 0$ (adding up); $\Sigma_k s_{ik} = 0$ (homogeneity); and $s_{ik} = s_{ki}$ (symmetry). Because the covariance matrix associated with the error terms in (27) will be singular, an equation must be deleted in estimation (Barten 1969). Accordingly, the "other" crop category is omitted. Iterated seemingly unrelated regression estimates of (27), with symmetry and homogeneity restrictions imposed, were obtained by using TSP version 4.3.

Parameter estimates, multiplied by 100, are reported in table 1. To conserve space, values for omitted c_{il} parameters (i.e., fixed effects parameters) are not reported; however, they may be obtained by using adding-up conditions. Associated asymptotic standard errors and asymptotic p-values reported in table 1 were obtained by using White's heteroskedasticity-consistent covariance estimator.[12]

[12] Estimates of asymptotic standard errors, t-ratios, and p-values for omitted parameters and for price and scale elasticities were obtained by using the "Analyze" feature of TSP. That is, estimates of asymptotic standard errors were obtained by using the delta method, where the variance of an omitted parameter (elasticity) is calculated as a quadratic form of the White heteroskedasticity-consistent covariance matrix of the parameters. In each case, the vector used in the quadratic form is the analytical gradient of the omitted parameter (elasticity) with respect to estimated parameters.

Table 1. Estimated Acreage Allocation Model Parameters

Parameter	Estimate	Std. Error	t-Ratio	p-Value
b_1	49.536	2.565	19.316	< 0.001
b_2	18.844	1.668	11.295	< 0.001
b_3	31.620	2.027	15.601	< 0.001
s_{11}	0.199	0.028	7.129	< 0.001
s_{12}	-0.186	0.017	-10.780	< 0.001
s_{13}	-0.013	0.018	-0.690	0.490
s_{22}	0.227	0.027	8.308	< 0.001
s_{23}	-0.041	0.023	-1.810	0.070
s_{33}	0.054	0.023	2.340	0.019
c_{11}	10.997	2.208	4.980	< 0.001
c_{12}	6.366	2.463	2.584	0.010
c_{13}	12.556	2.508	5.007	< 0.001
c_{14}	-1.203	2.509	-0.479	0.632
c_{15}	11.146	3.261	3.418	0.001
c_{16}	-8.552	3.401	-2.515	0.012
c_{17}	-0.973	2.504	-0.388	0.698
c_{21}	11.745	1.034	11.355	< 0.001
c_{22}	15.745	1.462	10.767	< 0.001
c_{23}	8.120	2.172	3.738	< 0.001
c_{24}	7.326	1.066	6.870	< 0.001
c_{25}	7.309	2.082	3.510	< 0.001
c_{26}	20.987	1.079	19.456	< 0.001
c_{27}	18.111	1.162	15.580	< 0.001

Log Likelihood:	183.732
Corn:	$R^2 = 0.810$
Soybeans:	$R^2 = 0.854$
Other:	$R^2 = 0.920$

Notes: Values in the column headed "Std. Error" are asymptotic standard errors, computed by using White's heteroskedasticity-consistent estimator. R^2 denotes the square of the simple correlation between observed and fitted allocations. There are a total of 80 observations. For b_i, s_{ij}, and c_{il}, $i = 1$ (corn), 2 (soybeans), and 3 (other crop); $l = 1$ (Illinois), 2 (Indiana), 3 (Iowa), 4 (Michigan), 5 (Minnesota), 6 (Missouri), 7 (Ohio), and 8 (Wisconsin).

Several conclusions emerge from results reported in table 1. To begin, consider that 19 of 23 estimated parameters are "significant" at the $\alpha = 0.05$ level. Of interest is that estimated scale parameters are, in each case, highly significant, as indicated by their associated p-values. There are also substantial differences in scale response for these crops across states relative to Wisconsin, as suggested by the magnitude and significance of nearly all c_{il} parameters. In addition, each estimated s_{ii} parameter is statistically significant at the $\alpha = 0.05$ level and is positive, indicating that acreage planted to each crop will increase as expected returns to that crop increase. Results show that expected corn and soybean returns have a significant influence on share of corn NFA acres planted to corn and soybeans. Likewise, the parameter associated with expected returns to winter wheat is significant in the soybean allocation equation at the $\alpha = 0.10$ level, and vice versa. There apparently is no statistically significant relationship between corn and other, as approximated by expected returns to wheat.

Of additional interest is that matrix **S**, constructed from the s_{ij} coefficients in table 1, is positive semi-definite, as required. Furthermore, this positivity condition is satisfied automatically. Each equation also provides a reasonable fit to the data, as indicated by simple R^2 coefficients reported in table 1. Fitted shares for each equation at each sample point are positive, indicating that monotonicity is satisfied. Finally, results of a likelihood-ratio test revealed that, taken together, homogeneity and symmetry restrictions are *not* rejected at any usual significance levels. Specifically, the test statistic is 1.770, which, from the asymptotic $\chi^2_{(3)}$ distribution, is associated with a p-value of 0.622.[13] It therefore seems that the linear approximate acreage allocation model is a statistically valid *and* theoretically consistent representation of farmers' planting decisions on corn-based NFA in the Corn Belt region during the 1991–95 period. What, then, does this model indicate about acreage response elasticities?

To obtain further insights into the model's implied structure, estimates for the b_i and s_{ij} coefficients are converted into elasticities by using equations (21)–(22). Results, obtained at sample means, are reported in table 2. Note first that own-price elasticities for corn, soybeans, and other crops are relatively large (1.04, 1.54, and 0.61, respectively) and are, moreover, significantly different from zero at the $\alpha = 0.05$ level. Cross-price elasticities between corn and soybeans also are fairly large in magnitude, and are large relative to their asymptotic standard errors. The corn acreage elasticity with respect to the expected soybean price is −0.78, while the soybean acreage elasticity with respect to the expected corn price is −1.58. The elasticity of other acreage with respect to soybeans is −0.73, which is large in absolute terms and, moreover, is significant at the $\alpha = 0.10$ level.

On balance, these results indicate substantial interactions among expected returns for corn, soybeans, and winter wheat in farmers' acreage allocation decisions in the Corn Belt region. Scale elasticity estimates, reported in table 2, are also plausible. Results show that a 10% increase in total corn-based NFA during the sample period would have resulted in a 9.2% increase in acres planted to corn, a 5.7% increase in acres planted to soybeans, and a 25% increase in acres planted to "other." This latter

[13] Alternatively, in a linear acreage allocation model for western Canada, Coyle resoundingly rejects the symmetry/reciprocity restrictions.

Table 2. Estimated Own-Price, Cross-Price, and Scale Elasticities

Elasticity	Estimate	Std. Error	*t*-Ratio	*p*-Value
ε_{11}	1.038	0.146	7.129	< 0.001
ε_{12}	-0.776	0.072	-10.780	< 0.001
ε_{13}	-0.034	0.049	-0.690	0.490
ε_{21}	-1.580	0.147	-10.780	< 0.001
ε_{22}	1.539	0.185	8.308	< 0.001
ε_{23}	-0.176	0.097	-1.810	0.070
ε_{31}	-0.285	0.413	-0.690	0.490
ε_{32}	-0.731	0.404	-1.810	0.070
ε_{33}	0.608	0.260	2.340	0.019
η_1	0.916	0.047	19.316	< 0.001
η_2	0.566	0.050	11.295	< 0.001
η_3	2.504	0.161	15.601	< 0.001

Notes: All elasticities are evaluated at the sample means. Here, $i = 1$ denotes corn, $i = 2$ denotes soybeans, and $i = 3$ denotes winter wheat (as representative of the "other" crop category).

estimate is high in part because "other's" share is relatively small (13%). Overall, our scale elasticity estimates correspond favorably with those reported by Bettendorf and Blomme and by Barten and Vanloot.

These elasticity estimates, at least for corn and soybeans, are generally higher than previous estimates reported in the literature. For example, Chavas and Holt (1996) report own-price elasticities for corn and soybean acreage of 0.25 and 0.10, respectively. These authors also report a cross-price elasticity of -0.22 for corn with respect to soybeans and of -0.12 for soybeans with respect to corn. Chavas and Holt (1990) report generally similar magnitudes.[14] Of course, part of this discrepancy is attributable to our use of state-level panel data for a relatively short time period; previous estimates were obtained by using aggregate time-series data over a relatively long period. But this is not the entire story. Data used here are indicative of a situation where farmers were free to make planting decisions without the overriding influence of government programs. It therefore seems very likely that acres planted to various crops, at least in the Corn Belt region, will be more responsive to market incentives under provisions set forth in the 1996 Farm Act.

[14] Own-price elasticities reported by Chavas and Holt (1990) are 0.166 and 0.450 for corn and soybean acreage, respectively. Acreage elasticity estimates reported by these authors are similar in magnitude to those reported by Gallagher and by Lee and Helmberger. Of interest is that Lee and Helmberger also performed their analysis at the state level for the Corn Belt region. Their sample period, however, corresponded with a time in which government supply control and price support programs for field crops were omnipresent.

Conclusions

In a set of recent papers, Bettendorf and Blomme, and Barten and Vanloot developed an acreage allocation model that is similar in its specification to a Rotterdam demand system. While their model is potentially useful for estimating acreage response with time-series data, it has limited appeal when panel or cross-sectional data are employed. In this analysis, we illustrate that the BB-BV framework, which is consistent with certainty equivalent profit maximization and constant absolute risk aversion, may be extended to a levels version. The resulting linear approximate acreage allocation model is useful for maintaining the theoretically appealing properties of homogeneity, symmetry, and adding up. The modeling approach was applied to a panel of state-level corn NFA acreage data for the U.S. Corn Belt region during 1991–95. The estimated model fits the data well and, moreover, appears to be consistent with all of the requirements of theory, at least as dictated by certainty equivalent profit maximization. The results obtained are entirely plausible. They suggest, for example, strong and statistically significant interactions in acreage planting decisions for corn, soybeans, and "other" (as represented by expected wheat returns). Implied acreage elasticities are also generally larger than previous estimates. The framework presented here may be useful in the future for estimating farmers' planting decisions.

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